

## Research Article

# Differential Equation Models for Education Charges of Universities in China and the Applications

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In recent years, with the expansion of university enrollment in China, the cost of running a school is getting higher and higher. Under the circumstance of insufficient state investment in higher education, the education charge paid by students has become an important source of the university income. It has become a hot topic of social concern to formulate a reasonable charging model to enable more students to enter higher education institutions. In this paper, we mainly put forward the basic differential equation models describing the problem of higher education charges in China. Through the qualitative analysis of these two basic models, we draw several conditions for universities to maintain or stabilize their education charges and give some suggestions on macrocontrol of university education charges in China.

## 1. Introduction to the Present Situation of Higher Education in China

Since the 1990s, under the guidance of the strategy of “rejuvenating the country through science and education and sustainable development (Deng Xiaoping, the chief designer of China, first put forward the important conclusion that science and technology are the first productive force. In 1997, the 15th Congress of the Communist Party of China clearly put forward the strategy of rejuvenating the country through science and education: to fully implement the idea that science and technology are the first productive force, to adhere to education as the basis, and to put science and technology and education in an important position in economic and social development),” the Chinese government has implemented “Project 211 (Project 211 is a construction project of about 100 universities and a number of key disciplines facing the 21<sup>st</sup> century),” “Project 985 (Project 985 is guided by the key areas and major needs urgently needed by the state or industry and focuses on building a number of platform bases around the national

development strategy and the frontier of disciplines),” and “2011 Collaborative Innovation Plan (2011 Collaborative Innovation Plan is called the innovation ability promotion program of colleges and universities)” successively. The “first-class university and first-class discipline construction of higher education” has also been deployed since 2016. China’s higher education has achieved a great leap both in quantity and quality due to this series of important measures. Since 1999, China has experienced a huge increase in the enrollment in higher education (see Table 1). The level of the popularization of higher education is also rising.

From the table, we can easily see that China’s gross enrollment rate has considerably increased. In 2002, the higher education gross enrollment rate in China reached 15.0%. After 16 years of development, the gross enrollment rate of higher education in China reached an astonishing 48.1% in 2018. Correspondingly, the number of students enrolled in 2018 reached 87.679 million. The Chinese government’s financial investment in higher education is also increasing year by year. In 1993, the Chinese government has formulated the Outline of China’s Educational

TABLE 1: Development of higher education in China from 1999 to 2018.

Year	Number of schools	Enrollment in higher education (thousand)	Gross enrollment rate (%)	Number of teaching staff expenses (RMB)	Per public finance budget education expenses	Per public finance public funds (RMB)
1999	1071	1689	10.5	1065	7201.24	2962.37
2000	1041	2335	12.5	1113	7309.58	2921.23
2001	1225	2848	13.3	1214	6818.23	2613.56
2002	1396	3408	15.0	1304	6177.96	2453.47
...	...	...	...	...	...	...
2014	2526	7835	37.5	2336	16102.72	7637.97
2015	2560	8024	40.0	2369	18143.57	8280.08
2016	2596	8153	42.7	2405	18747.65	8067.26
2017	2631	8421	45.7	2443	20298.63	8506.02
2018	2663	8768	48.1	2488	22007.77	9222.23

Remark. The data cited in this paper come from the *China Statistical Yearbook* (1999–2018) and the *China Education Funds Statistical Yearbook* (1999–2018).

Reform and Development in which, for the first time, the Chinese government put forward the target of government spending on education, taking for 4% of GDP. This goal has already been achieved in 2012 (see Table 2), and the proportion has remained above 4% for seven consecutive years (see [1]).

At the same time, we have also realized that although China's higher education has made great achievements in recent decades, there are still many problems or challenges. According to the statistical data, in terms of national financial investment in education, the world's average level is about 7%. Developed countries have reached about 9% and economically underdeveloped countries only reached 4.1% (see [2]). Therefore, there is still a long way to go for China to invest in education funds (see [3]). As far as the current development of education in China is concerned, two issues have aroused wide concern. The first issue to which most Chinese scholars pay attention is how to solve the problem of higher education charges and the problem of tuition payment for the poor students with the premise of a stable educational financial input. The second issue is how to coordinate the relationship among the development of higher education, talent training, and educational equity in order to avoid social contradictions. These two issues are two crucial social problems that China is facing today. Through the years, these problems have been studied from the perspective of educational economics, sociology, and law by various scholars in the field of educational theories (see [2, 4–12]). And, they put forward some corresponding measures on how to calculate the cost of colleges and universities. In order to survive and develop, colleges and universities must pay attention to the dual development of economic and social benefits, improve the efficiency of the use of funds, and further optimize the allocation of educational resources.

## 2. The Analysis of the Theory of Higher Education Charges in China

At present, the model of raising funds for running universities in China is still mainly based on the state investment and is supplemented by the investment of private and

other social forces. This is different from all state investment models implemented in the planned economy in the past. Before the 1990s, higher education institutions are regarded as nonprofit units, so all expenditures are provided by the state finance, while the education department and the finance department do not account for the cost of higher education.

Now, we know that higher education is a special activity process and resource consumption. Higher education cost is the value of educational resources consumed in the process of higher education activities. Because higher education can train technical talents and promote economic development, the government is the beneficiary of higher education. And, because higher education can bring great future benefits to the individuals, the individuals are also beneficiaries of higher education. At the same time, society and universities can also gain their own benefits through the development of higher education. Therefore, from the perspective of market economy theory, it is necessary to implement the cost compensation and allocation system in higher education at present.

According to Martin Throw's theory of the stages of higher education, education in China has changed from elitism to popularity. The process of popularization of higher education is not only the expansion of the educational scale, but also the change of cost sharing. In 1986, Johnston put forward the cost-sharing theory of higher education (see [7]). He believes that the cost of higher education should be reasonably shared by the beneficiaries. Beneficiaries should compensate the cost according to the level of income and the ability to bear it. Therefore, the main body of cost sharing in higher education should include the government, the individuals, the society, and the universities. In the way of sharing, the government shares the cost through financial allocation, the individual by paying tuition and miscellaneous expenses, the society by donating, and the colleges and universities by means of income generation from school-run industries and transformation of scientific research achievements (see [5]). In fact, the tuition charge of higher education is only related to the educational cost shared by the educator. According to the principle of complementarity and affordability of education fees, we believe that the main

TABLE 2: The proportion of national financial investment in education to GDP in 2008–2018.

Year	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
%	3.48	3.59	3.66	3.93	4.28	4.11	4.10	4.26	4.22	4.14	4.11

Remark. The data cited come from the *China Education Funds Statistical Yearbook* (2008–2018).

influencing factors of higher education fees are the educational costs shared by educators and the unpaid tuition fees of poor students.

In this paper, we aim to establish differential equation models of the higher education cost by analyzing the theory of total cost, input, and output, and then we study the higher educational charges of universities.

### 3. Differential Equation Models Based on Cost Sharing in Higher Education

3.1. *Establishment of Models.* According to the theory of sharing in the higher educational cost and the variable relation of difference equation (see [4, 6]), we suppose that the continuous variable  $N$  is the number of students in the university; then, we establish the following functional variables which affect the change of the higher education cost:

- $R(N)$ : educational charge of colleges and universities
- $\gamma(N)$ : individually shared educational costs
- $p(N)$ : unpaid tuition of per poor student
- $q(N)$ : allowance of per unpaid students

From the analysis on the sharing of educational costs in institutions of higher education, we know that the functions  $\gamma(N)$ ,  $\gamma(N)$ , and  $p(N)$  satisfy the following condition: as  $\gamma(N)$  increases,  $R(N)$  also increases; as  $\gamma(N)$  increases,  $R(N)$  decreases. So, we can suppose that the relationship between the change rate of  $R(N)$  and the function  $\gamma(N)$  is a positive linear correlation, and the relationship between the change rate of  $R(N)$  and the function  $p(N)$  is also linear and negative. At the same time, as the function  $p(N)$  increases,  $q(N)$  also increases. So, we have

$$q(N) = \alpha p(N), \tag{1}$$

where  $\alpha$  is the subsidy coefficient. We can obtain the following differential equation model (I) for the university educational charge:

$$R'(N) = \delta\gamma(N) - \sigma[p(N) + q(N)] = \delta\gamma(N) - \sigma(1 + \alpha)p(N). \tag{2}$$

Because the function  $p(N)$  of the unpaid charge can increase with the increase in the individually shared educational cost  $\gamma(N)$ , we have that

$$p(N) = g(\gamma(N)), \tag{3}$$

where  $g(\gamma)$  is the monotonically increasing functions. Therefore, we obtain the following differential equation model (II) for the university educational charge:

$$R'(N) = \delta\gamma(N) - \beta g(\gamma(N)), \tag{4}$$

where  $\beta = \sigma(1 + \alpha)$ .

Furthermore, we can obtain the function  $\gamma(N)$  by the function  $R(N)$ . So, we have

$$\gamma(N) = f(R(N)), \tag{5}$$

where  $f(R)$  is the monotonically increasing function.

Therefore, we obtain the following differential equation model (III) for the university educational charge:

$$R' = \delta f(R) - \sigma g(f(R)), \tag{6}$$

where  $\delta > 0$  and  $\sigma > 0$ .

3.2. *Qualitative Analysis of Educational Charge Model of Universities.* From model (I), we can obtain sufficient and necessary conditions for the constant charge of higher education:

$$\delta\gamma(N) = \sigma g(\gamma(N)). \tag{7}$$

This shows that the necessary and sufficient conditions for colleges and universities to keep their educational charge unchanged are as follows: maintaining a balance between the cost of per education shared by individuals and the cost of per unpaid expenses of poor students. In order to achieve this balance, there must exist a positive solution  $N_0$  for the above equation.

Therefore, we can obtain the number  $N_0$  of students enrolled in colleges and universities and individually shared per cost of education  $\gamma_0 = \gamma(N_0)$ .

At present, China has set an upper limit of 25% for the individual share of education costs. But in fact, the proportion of poor students in colleges and universities has exceeded 30%. This is unbalanced. The objective reason for the imbalance is that the cost of higher education is difficult to calculate. It is easy to make the charge standard exceed the upper limit of the proportion of the education cost shared by individuals. Therefore, balance can be achieved and problems be solved by increasing state finance investment in education, encouraging and attracting social funds to run schools, or by improving the payment ability of poor students by student loans, scholarships, work aids, etc.

Furthermore, by the discriminating method of function extreme, we have the following sufficient condition for keeping the educational charges of colleges and universities invariant:

$$(1) \text{ As } \gamma'(N_0) > 0, 0 < g'(\gamma(N_0)) < \delta/\sigma, \text{ or } \gamma'(N_0) < 0, g''(\gamma(N_0)) > \delta/\sigma, \text{ we have} \\ R'(N_0) = 0, R''(N_0) > 0. \tag{8}$$

Therefore, there exists the minimum value of educational charges in colleges and universities, i.e.,  $R(N_0)$ . This means that under the conditions at which the per cost  $\gamma(N)$  of education shared by individuals increases, the minimum charge  $R(N_0)$  of

colleges and universities would maintain unchanged as long as the growth rate  $g'(\gamma(N_0))$  of the per unpaid expenses for poor students is controlled within a certain limit. But, if the growth rate  $g'(\gamma(N_0))$  of the per unpaid expenses of poor students exceeded one degree  $\delta/\sigma$ , we can keep the minimum charges  $R(N_0)$  of colleges and universities unchanged by reducing the cost  $\gamma(N)$  of per education shared by individuals.

- (2) As  $\gamma'(N_0) < 0$ ,  $0 < g'(\gamma(N_0)) < \delta/\sigma$ , or  $\gamma'(N_0) > 0$ ,  $g'(\gamma(N_0)) > \delta/\sigma$ , we have

$$\begin{aligned} R'(N_0) &= 0, \\ R''(N_0) &< 0. \end{aligned} \quad (9)$$

Therefore, there exists the maximum value of educational charges in colleges and universities, i.e.,  $R(N_0)$ . This means that under the conditions at which the per cost  $\gamma(N)$  of education shared by individuals decreases, the maximum charge  $R(N_0)$  of colleges and universities would maintain unchanged as long as the growth rate  $g'(\gamma(N_0))$  of the per unpaid expenses of poor students is controlled within a certain limit. But, if the growth rate  $g'(\gamma(N_0))$  of the per unpaid expenses for poor students exceeded one degree  $\delta/\sigma$ , we can keep the maximum charge  $R(N_0)$  of colleges and universities unchanged by raising the cost  $\gamma(N)$  of per education shared by individuals. However, this situation is equivalent to high tuition charges, and some impoverished students will default more tuition charges or drop out of school because of their inability to pay. Therefore, we should try our best to avoid this problem. From model (II), we can know that the necessary and sufficient conditions for the constant charge of higher education are as follows:

$$\delta f(R) = \sigma g(f(R)). \quad (10)$$

It is also a balance. Here, we ask that there is a positive solution for the above equation, i.e.,  $\gamma_0 = f(R_0)$ . Because  $R_0$  is the equilibrium point of the differential equation model (II), we can obtain the following stability conclusion for educational charges  $R_0$  by the stability criterion of solutions (see [8]).

- (3) As  $f'(\gamma_0) > 0$ ,  $0 < g'(\gamma(N_0)) < \delta/\sigma$ , we know that  $R_0$  is the equilibrium point of the differential equation model (II). This means that under the conditions at which the per cost  $\gamma = f(R)$  of education shared by individuals increases, we can maintain the stability conclusion for educational charges  $R_0$  as long as the growth rate of the per unpaid expenses of poor students is controlled within a certain limit.
- (4) As  $f'(\gamma_0) < 0$ ,  $g'(\gamma(N_0)) > \delta/\sigma$ , we know that  $R_0$  is also the equilibrium point of the differential equation model (II). This means that if the growth rate of the per unpaid expenses of poor students exceeded one degree  $\delta/\sigma$ , we can keep the stability conclusion of  $R_0$

by reducing the cost of per education shared by individuals.

## 4. Conclusion

In the past 30 years, China has experienced a huge increase in the enrollment in higher education. So, we pay attention on how to solve the problem of higher education charges and the problem of tuition payment for the poor students with the premise of a stable educational financial input. In this paper, we mainly put forward the basic differential equation model describing the problem of higher education charges in China. Through the qualitative analysis of these two basic models, we draw several conditions for universities to maintain or stabilize their education charges and give some new conclusions and suggestions on macrocontrol of university education charges.

## Data Availability

The data used to support the findings of this study are available from the *China Statistical Yearbook* (1999–2018) and the *China Education Funds Statistical Yearbook* (1999–2018).

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

## Authors' Contributions

The study was carried out in collaboration among all authors. All authors read and approved the final manuscript.

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