Research Article

Finite-Time Terminal Sliding Mode Tracking Control of a VTOL UAV via the Generalized NDOB

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In this paper, we propose a finite-time sliding mode trajectory tracking control methodology for the vertical takeoff and landing unmanned aerial vehicle (VTOL UAV). Firstly, a system error model of trajectory tracking task is established based on Rodrigues parameters by considering both external and internal uncertainties. According to the cascade property, the system model is divided into translational and rotational subsystems, and a hierarchical control structure is hence proposed. Then, a finite-time generalized nonlinear disturbance observer (NDOB) is proposed, based on which the finite-time convergence result of equivalent disturbance estimation can be acquired. Finally, by introducing a tan-type compensator into the traditional terminal sliding mode control (SMC), the finite-time convergence result of the closed-loop control system is acquired based on Lyapunov stability analysis. Simulation results show the effectiveness of the proposed methodology.

1. Introduction

In recent years, vertical takeoff and landing unmanned aerial vehicles (VTOL UAVs) have been widely investigated in the aspects of battlefield rescue, community logistics, scientific exploration, and disaster detection. However, the VTOL UAV system is regarded as an underactuated system with a second-order nonholonomic constraint. It inevitably suffers from parameter perturbation and complicated aerodynamic disturbance. Thus, the trajectory tracking task is a challenge work due to its nonlinearity, strong coupling and under-actuated properties, internal and external uncertainties, etc.

Euler angles are widely used to establish the system model in a lot of previous works [1–6]. However, a simplified kinematics is usually expressed as

$$\dot{\Psi} = \omega, \quad (1)$$

where Ψ and ω are the Euler angle and angular velocity of the rigid body, respectively. It is reported in [7] that equation (1) is the simplified form of the original kinematics under the following two ideal assumptions: (1) the system only rotates

in one degree-of-freedom (DOF) at a time, and (2) the roll/pitch DOF changes only if the pitch/roll DOF is horizontal. Meanwhile, dynamics of Euler representation suffers from gimbal lock issue, which results in singularities during large-angle flight maneuvers. The above problems can be solved using quaternion-based representation, but unfortunately, the quaternion-based representation is not unique due to the double-fold covering between mapping of S^3 (trispheres) and $SO(3)$ [8]. In this paper, Rodrigues parameters (RPs) are introduced as the attitude representation, which is a three-dimensional vector without restrictions.

The trajectory tracking accuracy is usually affected by system uncertainties. To solve this problem, numerous approaches have been reported, such as sliding mode control (SMC) [9–14], adaptive control [15, 16], model predictive control [17–20], fuzzy control [21, 22], and neural networks [23–25]. However, the aforementioned works have some disadvantages, which make them difficult to be implemented in practice [8]. To overcome the system uncertainties in motion control, the concept of disturbance estimation methodology has been widely investigated in recent years [26]. For this approach, the disturbance observer (DOB) is

usually employed to estimate the total disturbance online based on the control input and system output. Among the existing methods, nonlinear DOB (NDOB) has been widely used recently [27–29]. Most existing results only use the first-order structure to estimate the disturbance. Although this structure can be widely used according to its intuitional structure and simple parameter tuning, the estimation performance of high-order time-varying disturbance is largely limited due to its simple structure. In [30], a generalized NDOB is proposed to deal with the time-varying disturbance. It is shown that the estimation performance for time-varying disturbances of the generalized NDOB is much better than that of the traditional NDOB; however, it cannot acquire the finite-time convergence result theoretically. In this paper, we propose a modified generalized NDOB to estimate the system uncertainties, based on which finite-time convergence result can be acquired theoretically.

The modified generalized NDOB can only estimate the system uncertainties online for cancelation; thus, it should be used along with the outer-loop controller for the trajectory tracking task. The backstepping technique has been widely used due to the cascade property of the VTOL UAV [31–34]. Although this method can provide strictly stability result, the real-time property will be reduced due to calculation explosion of the backstepping technique. To overcome this problem, the terminal SMC method was introduced in [35], and a tan-type feedback term was applied to acquire the finite-time convergence result of the outer-loop controller.

From the descriptions above, a finite-time SMC based on the modified NDOB is proposed for the VTOL UAV in this paper. By selecting RPs to represent the attitude, system error model is established for the trajectory tracking object. Considering the cascade property, hierarchical technique is introduced as the control structure for trajectory tracking. Then, finite-time controllers, which contain modified generalized NDOB and terminal SMC, are proposed for both translational and rotational subsystems. Finally, the global finite-time stability result is achieved based on the bounded property of the coupling term between each subsystem. This work is motivated in the following two aspects. Firstly, since the VTOL UAV is an underactuated system with the nonholonomic constraint, the hierarchical control structure is adopted for control system implementation. Then, the trajectory tracking problem can be separated into the controller design problem for translational and rotational subsystems, respectively. Secondly, to acquire faster convergence rate and higher control accuracy, finite-time convergence theory and DOB-based concept are introduced for the controller design. The finite-time generalized NDOB and terminal SMC methodologies are proposed to formulate the controller for each subsystem. The main contributions of this research are summarized as follows:

- (1) A modified generalized NDOB is proposed by introducing a nonlinear feedback into the traditional structure to acquire finite-time convergence of disturbance estimation

- (2) The finite-time terminal SMC method is proposed by using a tan-type function as a feedback compensator
- (3) The Rodrigues theorem is employed to analyze the property of coupling term between each subsystem, based on which finite-time stability of the closed-loop system is obtained

This paper is organized as follows. The trajectory tracking task is considered to establish the system error model based on RPs in Section 2. In Section 3, according to the cascade property based on hierarchical technique, the system error model is divided into translational and rotational subsystems, and finite-time generalized NDOB-based terminal SMC is proposed for each subsystem. In Section 4, global finite-time Lyapunov stability is obtained. In Section 5, effectiveness of the proposed methodology is demonstrated by numerical simulations. Conclusions are summarized in Section 6.

2. Problem Formulation

In this section, the finite-time convergence theorem is firstly introduced. Then, the system error model of the VTOL UAV for trajectory tracking task is established based on RPs, and the control object is also presented.

2.1. Preliminaries

Definition 1. The operator $\text{sign}(\cdot)$ for the vector $\mathbf{x} = [x_1, \dots, x_n]^T \in \mathcal{R}^n$ is defined as

$$\text{sign}^q(\mathbf{x}) = \begin{bmatrix} x_1 \cdot |x_1|^{q-1} \\ \dots \\ x_n \cdot |x_n|^{q-1} \end{bmatrix}. \quad (2)$$

Lemma 1 (see [36]). Consider the system in the following form:

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}), \\ \mathbf{f}(0) &= 0, \end{aligned} \quad (3)$$

where $\mathbf{x} = [x_1, \dots, x_n]^T \in \mathcal{R}^n$ is a state vector and $\mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), \dots, f_n(\mathbf{x})]^T \in \mathcal{R}^n$ is a known continuous vector field. Assume that there exists a continuous positive definite Lyapunov function $V(\mathbf{x})$ which satisfies the following inequality:

$$\dot{V}(\mathbf{x}(t)) + \kappa_1 V^\kappa(\mathbf{x}(t)) \leq 0, \quad \forall t > t_0, \quad (4)$$

where $\kappa_1 > 0$ and $0 < \kappa < 1$. Then, the origin of the system is globally stable in finite time t_s :

$$t_s \leq t_0 + \left(\frac{1}{\kappa_1(1-\kappa)} \right) V^{1-\kappa}(\mathbf{x}_{t_0}). \quad (5)$$

Lemma 2 (see [37]). Considering the system in equation (3), assume that there exists a continuous positive definite Lyapunov function $V(\mathbf{x})$ which satisfies the following inequality:

$$\dot{V}(\mathbf{x}(t)) + \kappa_2 V(\mathbf{x}(t)) + \kappa_3 V^\kappa(\mathbf{x}(t)) \leq 0, \quad \forall t > t_0, \quad (6)$$

where $\kappa_2 > 0$, $\kappa_3 > 0$, and $0 < \kappa < 1$. Then, the origin of the system is globally stable in finite time t_s :

$$t_s \leq t_0 + \left(\frac{1}{\kappa_2(1-\kappa)} \right) \ln \left(\frac{\kappa_2 + \kappa_3 V^{1-\kappa}(\mathbf{x}(t_0))}{\kappa_3} \right). \quad (7)$$

2.2. System Description. In this work, we use RPs to establish the system model. Defining $\mathbf{r}$ and α as the unit vector of the rotation axis and the rotation angle, the RPs are given as $\rho = \mathbf{r} \tan(\alpha/2)$. The VTOL UAV model is described as follows [31]:

$$\begin{cases} \dot{\xi} = \mathbf{v}, \\ m\dot{\mathbf{v}} = \mathbf{e}_3 mg - R\mathbf{e}_3 T, \\ \dot{\rho} = H(\rho)\boldsymbol{\omega}, \\ J\dot{\boldsymbol{\omega}} = -\boldsymbol{\omega} \times J\boldsymbol{\omega} + \boldsymbol{\tau}, \end{cases} \quad (8)$$

where $\xi, \mathbf{v} \in \mathcal{R}^3$ denote the position and velocity of the rigid body, $\mathbf{e}_3 = [0 \ 0 \ 1]^T$ is the unit vector of the z -axis, m and J are the masses of the inertial matrix of the rigid body, T and $\boldsymbol{\tau} \in \mathcal{R}^{3 \times 3}$ are the control thrust and torque, and $R \in \text{SO}(3)$ denotes the attitude transition matrix. R in terms of RPs is expressed as

$$R = \frac{(1 - \rho^T \rho)I_3 + 2\rho\rho^T - 2[\rho \times]}{1 + \rho^T \rho}, \quad (9)$$

where I_3 is a third-order identity matrix and the operator $[\times]$ denotes the skew symmetric matrix.

In this research, a trajectory tracking task is considered, and the object is used to design the control thrust T_d and torque τ_d to make the VTOL UAV to track the desired trajectory quickly and accurately. The vector $[\xi_d \ \dot{\xi}_d \ \ddot{\xi}_d]^T$ is the desired trajectory, which contains the desired position, velocity, and acceleration.

The following system error states are introduced as

$$\begin{cases} \tilde{\xi} = \xi - \xi_d, \\ \tilde{\mathbf{v}} = \mathbf{v} - \dot{\xi}_d, \\ \tilde{\rho} = \rho \oplus \rho_d^{-1}, \\ \tilde{\boldsymbol{\omega}} = \boldsymbol{\omega} - \tilde{R}\boldsymbol{\omega}_d, \end{cases} \quad (10)$$

where ρ_d and $\boldsymbol{\omega}_d$ can be acquired according to the desired rotation matrix, ρ_d^{-1} is known as inverse of ρ_d , which is extracted as $\rho_d^{-1} = -\rho_d$, and operator $\oplus$ represents the production of RPs, which is expressed as follows with two RPs:

$$\rho_1 \oplus \rho_2 = \frac{\rho_1 + \rho_2 + \rho_1 \times \rho_2}{1 - \rho_1 \cdot \rho_2}, \quad (11)$$

and $\tilde{R} = RR_d^T$ is regarded as the attitude error matrix.

The system error model is given as

$$\begin{cases} \dot{\tilde{\xi}} = \tilde{\mathbf{v}}, \\ \dot{\tilde{\mathbf{v}}} = g\mathbf{e}_3 - \left(\frac{1}{m}\right)R\mathbf{e}_3 T_d - \ddot{\xi}_d + \mathbf{d}_1, \\ \dot{\tilde{\rho}} = H(\tilde{\rho})\tilde{\boldsymbol{\omega}}, \\ \dot{\tilde{\boldsymbol{\omega}}} = J^{-1}[-\boldsymbol{\omega} \times J\boldsymbol{\omega} + \boldsymbol{\tau}_d + \mathbf{d}_2] - (\tilde{R}\dot{\boldsymbol{\omega}}_d - [\tilde{\boldsymbol{\omega}} \times] \tilde{R}\boldsymbol{\omega}_d). \end{cases} \quad (12)$$

Both system uncertainties and external disturbances are considered in this work. It is assumed that the mass and inertia error are defined as

$$\begin{aligned} \Delta m &= m - m_0, \\ \Delta J &= J - J_0. \end{aligned} \quad (13)$$

The Euler-Lagrange equations can be written as

$$\begin{cases} \ddot{\tilde{\xi}} = -\left(\frac{R\mathbf{e}_3 T_d}{m_0}\right) + g\mathbf{e}_3 - \ddot{\xi}_d + \mathbf{f}_1, \\ \ddot{\tilde{\rho}} = -H(\tilde{\rho})(\tilde{R}\dot{\boldsymbol{\omega}}_d - [\tilde{\boldsymbol{\omega}} \times] \tilde{R}\boldsymbol{\omega}_d) + H(\tilde{\rho})J_0^{-1}\boldsymbol{\tau}_d + \mathbf{f}_2, \\ \overline{H}(\tilde{\rho}, \dot{\tilde{\rho}})\tilde{\boldsymbol{\omega}} - H(\tilde{\rho})J_0^{-1}L(\boldsymbol{\omega})J_0^*, \end{cases} \quad (14)$$

where $\overline{H}(\tilde{\rho}, \dot{\tilde{\rho}})$ is the time derivative of $H(\tilde{\rho})$. The equivalent disturbances $\mathbf{f}_1$ and $\mathbf{f}_2$ on system dynamics are expressed as

$$\begin{cases} \mathbf{f}_1 = \left(\frac{m}{m_0}\right)\mathbf{d}_1 + \left(\frac{\Delta m}{m_0}\right)(g\mathbf{e}_3 - \ddot{\xi}_d - \ddot{\tilde{\xi}}), \\ \mathbf{f}_2 = H(\tilde{\rho})J_0^{-1}[\mathbf{d}_2 - \Delta J\dot{\tilde{\boldsymbol{\omega}}} - L(\boldsymbol{\omega})\Delta J^* - \Delta J(\tilde{R}\dot{\boldsymbol{\omega}}_d - [\tilde{\boldsymbol{\omega}} \times] \tilde{R}\boldsymbol{\omega}_d)], \end{cases} \quad (15)$$

where J^* denotes the vector form of the diagonal elements of J and $L(\cdot) = \text{skew}(\cdot)\text{diag}(\cdot)$, where $\text{skew}(\cdot)$ and $\text{diag}(\cdot)$ denote the skew symmetric matrix and diagonal matrix of a vector.

It is assumed that the first-order time derivative of equivalent disturbances $\mathbf{f}_1$ and $\mathbf{f}_2$ is bounded as

$$\begin{aligned} \|\dot{\mathbf{f}}_1\| &\leq \bar{d}_1, \\ \|\dot{\mathbf{f}}_2\| &\leq \bar{d}_2. \end{aligned} \quad (16)$$

From the descriptions above, the following control system design procedure is presented based on the Euler-Lagrange equations in equation (14). From equation (10), we can obtain that $\xi = \xi_d$ and $\mathbf{v} = \dot{\xi}_d$ when $\tilde{\xi} = 0$ and $\tilde{\mathbf{v}} = 0$. Since desired attitudes ρ_d , $\boldsymbol{\omega}_d$, and $\dot{\boldsymbol{\omega}}_d$ are obtained by the rotation matrix R_d , the VTOL UAV can obtain the desired acceleration by stabilizing the attitude subsystem. Thus, trajectory tracking control object can be equivalently transformed into the stabilization of the error system in

equation (14) with the equivalent disturbances described in equation (15). Consequently, the control object turns to design the thrust T_d and torque τ_d for the stabilization of equilibrium points $\xi = 0$, $\bar{v} = 0$, $\bar{p} = 0$, and $\bar{\omega} = 0$, with total disturbance in equation (15).

3. Control System Design

According to the cascade property of the system model and the control object, the hierarchical technique is first adopted to implement the control structure. Then, a finite-time terminal SMC methodology based on the modified NDOB is proposed for both translational and rotational subsystems, respectively.

3.1. Hierarchical Control Structure. From equation (12), we find that the VTOL UAV is a cascade system. Thus, the overall closed-loop system could be divided according to the convergent speed of each part according to the singular perturbation theory. Since the translational subsystem will converge after the convergence of the rotational subsystem, it is regarded that the translational subsystem is a slow subsystem, while the rotational subsystem is a fast subsystem. Thus, hierarchical technique can be introduced to implement the control system. The controllers for translational and rotational subsystems can be designed separately.

Figure 1 shows the hierarchical control structure proposed in this paper. The controller for the translational subsystem is first designed to acquire the desired thrust T_d and desired transition matrix R_d , which make the UAV to track a desired trajectory. Then, the desired attitude of rotational subsystem ρ_d can be acquired according to rotation matrix R_d . At last, the rotational controller can calculate the desired torque vector τ_d to stabilize the rotational subsystem.

In order to separate these two subsystems, the translational error dynamics can be rewritten as

$$\ddot{\xi} = -\left(\frac{R_d \mathbf{e}_3 T_d}{m_0}\right) + g \mathbf{e}_3 - \ddot{\xi}_d + \mathbf{f}_1 + \delta, \quad (17)$$

where

$$\delta = (I_3 - \tilde{R})(R_d \mathbf{e}_3 T_d) \quad (18)$$

is regarded as the coupling term between the two subsystems. During the design of the translational controller, it is assumed that the rotational controller has already converged with $\delta = 0$. Although it is assumed that $\tilde{R} = I_3$ during the design of the translational controller, the strict Lyapunov stability is still analyzed based on the original dynamics of equation (14).

To analyze the stability of the closed-loop system, it is necessary to analyze the property of the coupling term δ . According to the Rodrigues theorem, the Euclidean norm of δ is given as

$$\begin{aligned} \|\delta\| &= T_d \left[(R_d \mathbf{e}_3)^T (2I_3 - \tilde{R}^T - \tilde{R}) (R_d \mathbf{e}_3) \right]^{(1/2)}, \\ &= T_d \left| \sin\left(\frac{\alpha}{2}\right) \right| \left[-(R_d \mathbf{e}_3)^T ([\mathbf{r} \times]^2) (R_d \mathbf{e}_3) \right]^{(1/2)}. \end{aligned} \quad (19)$$

Since $(R_d \mathbf{e}_3)^T ([\mathbf{r} \times]^2) (R_d \mathbf{e}_3)$ is a quadratic form, we have

$$\lambda_{\min}([\mathbf{r} \times]^2) \leq (R_d \mathbf{e}_3)^T ([\mathbf{r} \times]^2) (R_d \mathbf{e}_3) \leq \lambda_{\max}([\mathbf{r} \times]^2), \quad (20)$$

where operators $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalue of a matrix. Notice that the characteristic polynomial of $[\mathbf{r} \times]^2$ is $\lambda^3 + 2\lambda^2 + \lambda = 0$ with the roots 0 and -1; thus,

$$0 \leq \|\delta\| \leq 2T_d \left| \sin\left(\frac{\alpha}{2}\right) \right| = 2T_d \left| \cos\left(\frac{\alpha}{2}\right) \right| \|\bar{p}\|, \quad (21)$$

which means $\|\delta\|$ equals 0 after the convergence of the rotational subsystem, and the upper bound of $\|\delta\|$ is $2T_d$.

3.2. Modified NDOB. The controller for each subsystem consists of two parts: inner-loop DOB and outer-loop controller. In this section, a modified generalized NDOB is proposed to enable the estimated disturbance to converge in finite time. For the translational dynamics, the traditional generalized NDOB can be designed as follows [30]:

$$\begin{cases} \dot{\mathbf{z}} = -R \mathbf{e}_3 T_d + m_0 g \mathbf{e}_3 - m_0 \ddot{\xi}_d + \hat{\mathbf{f}}_1, \\ \hat{\mathbf{f}}_1 = \lambda_1 (m_0 \dot{\xi} - \mathbf{z}) + \lambda_2 \int_0^t (m_0 \dot{\xi} - \mathbf{z}) d\tau. \end{cases} \quad (22)$$

By substituting equation (22) into translational dynamics, we have

$$\ddot{\mathbf{f}}_1 + \lambda_1 \dot{\mathbf{f}}_1 + \lambda_2 \mathbf{f}_1 = \ddot{\mathbf{f}}_1, \quad (23)$$

which indicates that the estimation error is bounded with positive definite λ_1 and λ_2 . To acquire finite-time stabilization of the estimation error, the generalized NDOB of equation (22) is modified into the following form:

$$\begin{cases} \dot{\mathbf{z}}_1 = -R \mathbf{e}_3 T_d + m_0 g \mathbf{e}_3 - m_0 \ddot{\xi}_d + \hat{\mathbf{f}}_1, \\ \hat{\mathbf{f}}_{1,i} = \lambda_1 \text{sign}^{q_1} (m_0 \dot{\xi}_i - \mathbf{z}_{1,i}) + \lambda_2 \int_0^t \text{sign}^{(2q_1-1)} (m_0 \dot{\xi}_i - \mathbf{z}_{1,i}) d\tau, \end{cases} \quad (24)$$

where $i = 1 \sim 3$ denotes the three axes and λ_1 and λ_2 are positive matrices.

Similarly, the modified generalized NDOB for the rotational subsystem is designed as follows:

$$\begin{cases} \dot{\mathbf{z}}_2 = -L(\omega) J_0^* + \tau_d - J_0 (\tilde{R} \dot{\omega}_d - [\tilde{\omega} \times] \tilde{R} \omega_d) + \hat{\mathbf{f}}_2, \\ \hat{\mathbf{f}}_{2,i} = \lambda_3 \text{sign}^{q_2} (J_0 \tilde{\omega}_i - \mathbf{z}_{2,i}) + \lambda_4 \int_0^t \text{sign}^{(2q_2-1)} (J_0 \tilde{\omega}_i - \mathbf{z}_{2,i}) d\tau, \end{cases} \quad (25)$$

where $i = 1 \sim 3$ denotes the three axes and λ_3 and λ_4 are positive matrices.

Theorem 1. The modified generalized NDOB in equation (24) for the translational subsystem and equation (25) for the rotational subsystem will enable the disturbance estimation

FIGURE 1: Hierarchical control structure of the VTOL UAV.

error converge to a sufficiently small region in finite time under assumptions in equation (16).

Proof. For the translational subsystem, by defining the following error variables,

$$\begin{aligned} \varphi_1 &= m_0 \dot{\xi} - z_1, \\ \varphi_2 &= [\varphi_{2,1} \quad \varphi_{2,2} \quad \varphi_{2,3}]^T, \\ \varphi_{2,i} &= f_{1,i} - \hat{f}_{1,i} + \lambda_1 |\varphi_{1,i}|^{q_1} \text{sign}(\varphi_{1,i}), \\ &\quad i = 1 \sim 3, \end{aligned} \quad (26)$$

it follows that

$$\begin{cases} \dot{\varphi}_{1,i} = \varphi_{2,i} - \lambda_1 |\varphi_{1,i}|^{q_1} \text{sign}(\varphi_{1,i}), \\ \dot{\varphi}_{2,i} = -\lambda_2 |\varphi_{1,i}|^{(2q_1-1)} \text{sign}(\varphi_{1,i}) + \dot{f}_{1,i}. \end{cases} \quad (27)$$

For the proposed translational NDOB, Lyapunov function is selected as

$$\begin{aligned} V_{\text{DOB}} &= \sum_{i=1}^3 V_i, \\ V_i &= \vartheta_i^T P \vartheta_i, \quad \vartheta_i = \begin{bmatrix} |\varphi_{1,i}|^{q_1} \text{sign}(\varphi_{1,i}) \\ \varphi_{2,i} \end{bmatrix}. \end{aligned} \quad (28)$$

P is defined as

$$P = \begin{bmatrix} \frac{\lambda_1}{q_1} & -\lambda_2 \\ -\lambda_2 & 2 \end{bmatrix}. \quad (29)$$

The dynamic equation of ϑ_i is given as

$$\dot{\vartheta}_i = A_i \vartheta_i + B f_{1,i}, \quad (30)$$

with

$$\begin{aligned} A_i &= \begin{bmatrix} -q_1 \mu_i \lambda_1 & q_1 \mu_i \\ -\lambda_2 \mu_i & 0 \end{bmatrix}, \\ B &= \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \end{aligned} \quad (31)$$

where $\mu_i = |\varphi_{1,i}|^{(q_1-1)} > 0$. For proper defined λ_1 and λ_2 , matrix A_i is a Hurwitz matrix. Thus, there exists a positive definite matrix Q_i such that

$$A_i^T P + P A_i = -Q_i. \quad (32)$$

Thus, the first-order time derivative of V_i is

$$\dot{V}_i \leq -(\lambda_{\min}(Q_i) \|\vartheta_i\| - 2\bar{d}_1 \|B^T P\|) \|\vartheta_i\|. \quad (33)$$

Matrix A_i can also be expressed as

$$\begin{aligned} A_i &= A_{1,i} A_{2,i}, \\ A_{1,i} &= \begin{bmatrix} q_1 \mu_i & 0 \\ 0 & \mu_i \end{bmatrix}, \\ A_{2,i} &= \begin{bmatrix} \lambda_1 & -1 \\ \lambda_2 & 0 \end{bmatrix}, \end{aligned} \quad (34)$$

and it follows that

$$\lambda_{\min}(-A_i) = \lambda_{\min}(A_{1,i} A_{2,i}) \geq \lambda_{\min}(A_{1,i}) \cdot \lambda_{\min}(A_{2,i}). \quad (35)$$

Since $A_{1,i}$ is a diagonal matrix and $\varrho_1 \mu_i < \mu_i$, there exists

$$\lambda_{\min}(A_{1,i}) = \begin{cases} 1, |\varphi_{1,i}| < \left(\frac{1}{\varrho_1}\right)^{(1/\varrho_1-1)}, \\ \varrho_1 \mu_i, |\varphi_{1,i}| \geq \left(\frac{1}{\varrho_1}\right)^{(1/\varrho_1-1)}. \end{cases} \quad (36)$$

If $|\varphi_{1,i}| \geq (1/\varrho_1)^{(1/\varrho_1-1)}$, then $\|\vartheta_i\| \geq (1/\varrho_1)^{(\varrho_1/\varrho_1-1)}$, and we have

$$\lambda_{\min}(Q_i) \geq 2\varrho_1 \mu_i \lambda_{\min}(A_{2,i}) \lambda_{\min}(P). \quad (37)$$

Substituting equation (37) into equation (33), we have

$$\begin{aligned} \dot{V}_i &\leq -(2\varrho_1 \mu_i \lambda_{\min}(A_{2,i}) \lambda_{\min}(P) |\varphi_{1,i}|^{\varrho_1} - 2\bar{d}_1 \|B^T P\|) \|\vartheta_i\|, \\ &\leq -(2\varrho_1 |\varphi_{1,i}|^{(2\varrho_1-1/\varrho_1-1)} \lambda_{\min}(A_{2,i}) \lambda_{\min}(P) - 2\bar{d}_1 \|B^T P\|) \|\vartheta_i\|. \end{aligned} \quad (38)$$

With $\varrho_1 \in (0.5, 1)$, there exists a constant κ such that

$$\varrho_1 \left(\frac{1}{\varrho_1}\right)^{(2\varrho_1-1/\varrho_1-1)} \geq \kappa. \quad (39)$$

Notice that λ_1 and λ_2 are adjusted to make sure $\kappa \lambda_{\min}(A_{2,i}) \lambda_{\min}(P) > \bar{d}_1 \|B^T P\|$, letting $\eta_{1,i} = 2\kappa \lambda_{\min}(A_{2,i}) \lambda_{\min}(P) - \bar{d}_1 \|B^T P\|$, and we can obtain that

$$\begin{aligned} \dot{V}_i &\leq -\Lambda_{1,i} V_i^{(1/2)}, \\ \Lambda_{1,i} &= \left(\frac{\eta_{1,i}}{\sqrt{(\lambda_{\max}(P))}} \right). \end{aligned} \quad (40)$$

Thus, $\|\vartheta_i\|$ converges to the region $\|\vartheta_i\| \leq (1/\varrho_1)^{(1/\varrho_1-1)}$ in finite time $t_{1,i}$, defined as

$$t_{1,i} \leq \left(\frac{2}{\Lambda_{1,i}} \right) V_i^{(1/2)}(\vartheta_i(t)) \leq \left(\frac{2}{\Lambda_{1,i}} \right) V_i^{(1/2)}(\vartheta_i(t_0)). \quad (41)$$

If $\|\vartheta_i\| < (1/\varrho_1)^{(1/\varrho_1-1)}$, then $|\varphi_i| < (1/\varrho_1)^{(1/\varrho_1-1)}$, and we have

$$\lambda_{\min}(Q_i) \geq 2\lambda_{\min}(A_{2,i}) \lambda_{\min}(P). \quad (42)$$

Substituting equation (42) into equation (33), we have

$$\dot{V}_i \leq -(2\lambda_{\min}(A_{2,i}) \lambda_{\min}(P) |\varphi_{1,i}|^{\varrho_1} - 2\bar{d}_1 \|B^T P\|) \|\vartheta_i\|. \quad (43)$$

If

$$\left(\frac{1}{\varrho_1} \right)^{(1/\varrho_1-1)} > \|\vartheta_i\| > \frac{\bar{d}_1 \|B^T P\|}{\lambda_{\min}(A_{2,i}) \lambda_{\min}(P)} \quad (44)$$

is satisfied, there exists $\eta_{2,i} > 0$ such that

$$\begin{aligned} \dot{V}_i &\leq -\Lambda_{2,i} V_i^{(1/2)}, \\ \Lambda_{2,i} &= \left(\frac{\eta_{2,i}}{\sqrt{(\lambda_{\max}(P))}} \right). \end{aligned} \quad (45)$$

Then, we can obtain

$$t_{2,i} \leq \left(\frac{2}{\Lambda_{2,i}} \right) V_i^{(1/2)}(\vartheta_i(t)) \leq \left(\frac{2}{\Lambda_{2,i}} \right) V_i^{(1/2)}(\vartheta_i(t_{1,i})). \quad (46)$$

Therefore, $\|\vartheta_i\|$ can converge to the region

$$\|\vartheta_i\| \leq \frac{\bar{d}_1 \|B^T P\|}{\lambda_{\min}(A_{2,i}) \lambda_{\min}(P)} \quad (47)$$

in finite time $T_i = t_{1,i} + t_{2,i}$.

Notice that if the suitable parameters are selected to make $\lambda_{\min}(A_{2,i}) \lambda_{\min}(P)$ sufficiently large, the defined error variable ϑ can converge to a sufficiently small region of 0 in finite time, which indicates that the disturbance estimation error will converge to a sufficiently small region of 0 in finite time.

Similarly, for the rotational subsystem, by defining the following error states,

$$\begin{aligned} \varphi_3 &= J_0 \tilde{\omega} - \mathbf{z}_2, \\ \varphi_4 &= \mathbf{f}_2 - \hat{\mathbf{f}}_2 + \lambda_3 |\varphi_3|^{\varrho_2} \text{sign}(\varphi_3), \end{aligned} \quad (48)$$

it follows that

$$\begin{cases} \dot{\varphi}_3 = \varphi_4 - \lambda_3 |\varphi_3|^{\varrho_2} \text{sign}(\varphi_3), \\ \dot{\varphi}_4 = -\lambda_4 |\varphi_3|^{(2\varrho_2-1)} \text{sign}(\varphi_3) + \dot{\mathbf{f}}_2. \end{cases} \quad (49)$$

The estimation error dynamics of the rotational subsystem in equation (49) has the same form as equation (27) of the translational subsystem; thus, the finite-time convergence results of the rotational subsystem can be obtained similarly. This completes the proof.

3.3. Finite-Time Terminal SMC. With the disturbance estimation of the DOB, the estimated system uncertainties can be compensated in the outer-loop controller. In this section, a finite-time terminal SMC is proposed to stabilize each subsystem along with the proposed NDOB [35]. The sliding surface for the translational subsystem is defined as follows:

$$s_1 = \dot{\tilde{\xi}} + \alpha_1 \tilde{\xi} + \alpha_2 \tilde{\xi}^{p_1/q_1}, \quad (50)$$

where α_1 and α_2 are positive constants and p_1 and q_1 are odd positive integers satisfying $p_1 < q_1$. With the above form of the sliding surface, if $s_1 = 0$, the convergence time can be calculated as follows [35]:

$$t \leq t_0 + \left(\frac{q_1}{2\alpha_2(q_1 - p_1)} \right) \ln \left(\frac{\alpha_1 [s_{1,i}(t_0)^{(2(q_1-p_1)/q_1)}] + 2\alpha_2}{2\alpha_2} \right), \quad (51)$$

which implies that if the sliding surface equals 0, the system states will converge to the equilibrium point in finite time.

The translational controller is designed as follows:

$$R_d \mathbf{e}_3 T_d = m_0 \left(g \mathbf{e}_3 - \ddot{\hat{\mathbf{f}}}_d - \hat{\mathbf{f}}_1 \right) + m_0 \begin{bmatrix} F_{1,1} \\ F_{1,2} \\ F_{1,3} \end{bmatrix} + m_0 \left(\alpha_1 \dot{\hat{\xi}} + \left(\frac{\alpha_2 p_1}{q_1} \right) \tilde{\xi}^{(p_1 - q_1/q_1)} \dot{\hat{\xi}} + M_1 \text{sign}(s_1) \right), \quad (52)$$

where M_1 is a sufficiently small positive constant which reflects the convergence region of the estimation error of the proposed NDOB; $F_{1,i}$ is defined as

$$F_{1,i} = - \left(\frac{\beta_1 \sin(\pi s_{1,i}^2 / 2k_{t_{1,i}}^2) \cos(\pi s_{1,i}^2 / 2k_{t_{1,i}}^2)}{s_{1,i}} \right) - \left(\frac{\beta_2 \cos^2(\pi s_{1,i}^2 / 2k_{t_{1,i}}^2) \tan^{\eta_1}(\pi s_{1,i}^2 / 2k_{t_{1,i}}^2)}{s_{1,i}} \right), \quad i = 1 \sim 3, \quad (53)$$

where β_1 and β_2 are positive constants and η_1 is a positive constant satisfying $\eta_1 < 1$. $k_{t_{1,i}}$ is selected such that $s_{1,i} \in \Omega_{s_{1,i}} \triangleq \{\mathcal{R}, |s_{1,i}| < k_{t_{1,i}}\}$.

For the rotational subsystem, the sliding surface is defined as

$$s_2 = \dot{\tilde{\mathbf{p}}} + \alpha_3 \tilde{\mathbf{p}} + \alpha_4 \tilde{\mathbf{p}}^{(p_2/q_2)}, \quad (54)$$

where α_3 and α_4 are positive constants and p_2 and q_2 are odd positive integers satisfying $p_2 < q_2$.

The rotational controller is designed as follows:

$$\begin{aligned} \boldsymbol{\tau}_d = & -J_0 H^{-1}(\tilde{\mathbf{p}}) \hat{\mathbf{f}}_2 - J_0 H^{-1}(\tilde{\mathbf{p}}) \bar{H}(\tilde{\mathbf{p}}, \dot{\tilde{\mathbf{p}}}) \tilde{\boldsymbol{\omega}} + L(\boldsymbol{\omega}) J_0^* \\ & + J_0 (\bar{\mathbf{R}} \dot{\boldsymbol{\omega}}_d - [\tilde{\boldsymbol{\omega}} \times] \bar{\mathbf{R}} \boldsymbol{\omega}_d) + J_0 H^{-1}(\tilde{\mathbf{p}}) \begin{bmatrix} F_{2,1} \\ F_{2,2} \\ F_{2,3} \end{bmatrix} \\ & + J_0 H^{-1}(\tilde{\mathbf{p}}) \left(\alpha_3 \dot{\tilde{\mathbf{p}}} + \left(\frac{\alpha_4 p_2}{q_2} \right) \tilde{\mathbf{p}}^{(p_2 - q_2/q_2)} \dot{\tilde{\mathbf{p}}} + M_2 \text{sign}(\mathbf{s}_2) \right), \end{aligned} \quad (55)$$

where M_2 is a sufficiently small positive constant which reflects the convergence region of the estimation error of the proposed NDOB; $F_{2,i}$ is defined as

$$F_{2,i} = - \left(\frac{\beta_3 \sin(\pi s_{2,i}^2 / 2k_{t_{2,i}}^2) \cos(\pi s_{2,i}^2 / 2k_{t_{2,i}}^2)}{s_{2,i}} \right) - \left(\frac{\beta_4 \cos^2(\pi s_{2,i}^2 / 2k_{t_{2,i}}^2) \tan^{\eta_2}(\pi s_{2,i}^2 / 2k_{t_{2,i}}^2)}{s_{2,i}} \right), \quad i = 1 \sim 3, \quad (56)$$

where β_3 and β_4 are positive constants and η_2 is a positive constant which satisfies $\eta_2 < 1$. $k_{t_{2,i}}$ is selected such that $s_{2,i} \in \Omega_{s_{2,i}} \triangleq \{\mathcal{R}, |s_{2,i}| < k_{t_{2,i}}\}$.

4. Stability Analysis

In this section, we analyze the Lyapunov stability of the overall closed-loop system.

Theorem 2. For original system error model equation (14) satisfying the assumption in equation (16), with the NDOB proposed in equation (24), sliding surface of equation (50), controller equation (52) for the translational subsystem and the NDOB proposed in equation (25), sliding surface of equation (54), controller equation (55) for the rotational subsystem, the system tracking error will converge in finite time.

Proof. For rotational dynamics, the Lyapunov function is selected as follows:

$$V_{\text{rot}} = \sum_{i=1}^3 \frac{k_{t_{2,i}}^2}{\pi} \tan\left(\frac{\pi s_{2,i}^2}{2k_{t_{2,i}}^2}\right). \quad (57)$$

According to the L'Hospital rule, it is shown that

$$\lim_{k_{t_{2,i}} \rightarrow \infty} \left(\frac{k_{t_{2,i}}^2}{\pi} \right) \tan\left(\frac{\pi s_{2,i}^2}{2k_{t_{2,i}}^2}\right) = \left(\frac{1}{2}\right) s_{2,i}^2, \quad (58)$$

which means that the tan-type Lyapunov function reduces to standard quadratic ones if $k_{t_{2,i}} \rightarrow \infty$.

The first-order time derivative of V_1 is obtained as

$$\dot{V}_{\text{rot}} = \sum_{i=1}^3 \sec^2\left(\frac{\pi s_{2,i}^2}{2k_{t_{2,i}}^2}\right) s_{2,i} \dot{s}_{2,i} = \begin{bmatrix} \sec^2\left(\frac{\pi s_{2,1}^2}{2k_{t_{2,1}}^2}\right) s_{2,1} \\ \sec^2\left(\frac{\pi s_{2,2}^2}{2k_{t_{2,2}}^2}\right) s_{2,2} \\ \sec^2\left(\frac{\pi s_{2,3}^2}{2k_{t_{2,3}}^2}\right) s_{2,3} \end{bmatrix}^T \dot{\mathbf{s}}_2. \quad (59)$$

Differentiating sliding surface equation (54) with respect to time and then substituting rotational controller equation (55), one can obtain

$$\begin{aligned}\dot{s}_2 &= \ddot{\tilde{\mathbf{p}}} + \alpha_3 \dot{\tilde{\mathbf{p}}} + \left(\frac{\alpha_4 p_2}{q_2}\right) \tilde{\mathbf{p}}^{((p_2 - q_2)/q_2)} \dot{\tilde{\mathbf{p}}}, \\ &= \tilde{\mathbf{f}}_2 - M_2 \text{sign}(s_2) + \begin{bmatrix} F_{2,1} \\ F_{2,2} \\ F_{2,3} \end{bmatrix}.\end{aligned}\quad (60)$$

By substituting rotational controller equation (60) into equation (59), we can obtain

$$\begin{aligned}\dot{V}_{\text{rot}} &= - \sum_{i=1}^3 \left[\beta_{3,i} \tan\left(\frac{\pi s_{2,i}^2}{2k_{t_{2,i}}^2}\right) + \beta_{4,i} \tan^{\eta_1}\left(\frac{\pi s_{2,i}^2}{2k_{t_{2,i}}^2}\right) \right] \\ &\quad - \begin{bmatrix} \sec^2\left(\frac{\pi s_{2,1}^2}{2k_{t_{2,i}}^2}\right) s_{2,1} \\ \sec^2\left(\frac{\pi s_{2,2}^2}{2k_{t_{2,i}}^2}\right) s_{2,2} \\ \sec^2\left(\frac{\pi s_{2,3}^2}{2k_{t_{2,i}}^2}\right) s_{2,3} \end{bmatrix}^T (M_2 \text{sign}(s_2) - \tilde{\mathbf{f}}_2).\end{aligned}\quad (61)$$

According to Theorem 1, the estimation error of the NDOB will converge to a sufficiently small region in finite time. Assume that if $t > T_{\text{rot},1}$, the estimation error of the rotational NDOB satisfies $|\tilde{f}_{2,i}| \leq M_2$, $i = 1 \sim 3$; thus, we can conclude that the second part of the above inequality is negative definite. Thus, we have the following inequality which holds if $t > T_{\text{rot}}$:

$$\dot{V}_{\text{rot}} \leq -\beta_3 V_{\text{rot}} - \beta_4 V_{\text{rot}}^{\eta_1}. \quad (62)$$

According to Lemma 2, the sliding surface of rotation dynamics will converge in finite time $T_{\text{rot},2}$ as

$$T_{\text{rot},2} \leq T_{\text{rot}} + \left(\frac{1}{\alpha_3(1-\eta)}\right) \ln\left(\frac{\alpha_3 + \alpha_4 V^{1-\eta_1}(s(T_{\text{rot},1}))}{\alpha_4}\right). \quad (63)$$

According to the definition of s_2 , when $s_2 = 0$, the error states $\tilde{\mathbf{p}}$ and $\dot{\tilde{\mathbf{p}}}$ will converge to the equilibrium point in finite time according to equation (51):

$$\begin{aligned}T_{\text{rot}} &\leq T_{\text{rot},2} \\ &+ \max_{i=1 \sim 3} \left\{ \frac{q_2}{2\alpha_3(q_2 - p_2)} \ln\left(\frac{\alpha_3 [s_{2,i}(t_0)]^{(2(q_2 - p_2)/q_2)} + 2\alpha_4}{2\alpha_4}\right) \right\}.\end{aligned}\quad (64)$$

For the translational dynamics, the Lyapunov function is defined as

$$V_{\text{trans}} = \sum_{i=1}^3 \frac{k_{t_{1,i}}^2}{\pi} \tan\left(\frac{\pi s_{1,i}^2}{2k_{t_{1,i}}^2}\right). \quad (65)$$

Similarly, the first-order time derivative of V_{trans} is obtained as

$$\begin{aligned}\dot{V}_{\text{trans}} &= - \sum_{i=1}^3 \left[\beta_{1,i} \tan\left(\frac{\pi s_{1,i}^2}{2k_{t_{1,i}}^2}\right) + \beta_{2,i} \tan^{\eta_2}\left(\frac{\pi s_{1,i}^2}{2k_{t_{1,i}}^2}\right) \right] \\ &\quad - \begin{bmatrix} \sec^2\left(\frac{\pi s_{1,1}^2}{2k_{t_{1,i}}^2}\right) s_{1,1} \\ \sec^2\left(\frac{\pi s_{1,2}^2}{2k_{t_{1,i}}^2}\right) s_{1,2} \\ \sec^2\left(\frac{\pi s_{1,3}^2}{2k_{t_{1,i}}^2}\right) s_{1,3} \end{bmatrix}^T (M_1 \text{sign}(s_1) - \tilde{\mathbf{f}}_1)\end{aligned}\quad (66)$$

when $t \geq T_{\text{rot}}$.

Similarly, assume that if $t > T_{\text{trans},1}$, the estimation error of the translational NDOB satisfies $|\tilde{f}_{1,i}| \leq M_1$, $i = 1 \sim 3$. If $t > \max\{T_{\text{trans},1}, T_{\text{rot}}\}$, we have

$$\dot{V}_{\text{trans}} \leq -\beta_1 V_{\text{trans}} - \beta_2 V_{\text{trans}}^{\eta_2}. \quad (67)$$

According to Lemma 2, the sliding surface of translational dynamics will converge in finite time $T_{\text{trans},2}$ as

$$T_{\text{trans},2} = \max\{T_{\text{trans},1}, T_{\text{rot}}\} + \left(\frac{1}{\alpha_1(1-\eta_2)}\right) \ln\left(\frac{\alpha_1 + \alpha_2 V^{1-\eta_2}(s(\max\{T_{\text{trans},1}, T_{\text{rot}}\}))}{\alpha_2}\right). \quad (68)$$

According to the definition of s_1 , when $s_1 = 0$, the error states $\tilde{\xi}$ and $\dot{\tilde{\xi}}$ will converge to the equilibrium point in finite time as

$$T_{\text{trans}} \leq T_{\text{trans},2} + \max_{i=1 \sim 3} \left\{ \left(\frac{q_2}{2\alpha_1(q_2 - p_1)}\right) \ln\left(\frac{\alpha_1 [s_{1,i}(t_0)]^{(2(q_2 - p_1)/q_2)} + 2\alpha_2}{2\alpha_2}\right) \right\}. \quad (69)$$

This completes the proof.

5. Simulations

In this section, numerical simulations are carried out in Matlab/Simulink to show the effectiveness of our methodology. The VTOL UAV system is considered with the following parameters: $m = 2 \text{ kg}$, $J_x = J_y = 0.1 \text{ kg} \cdot \text{m}^2$, and $J_z = 0.24 \text{ kg} \cdot \text{m}^2$. The control parameters are selected as follows: $\lambda_1 = 8.0$, $\lambda_2 = 2.0$, $\lambda_3 = 10.0$, $\lambda_4 = 2.0$, $\rho_1 = \rho_2 = 0.7$, $p_1 = 5$, $p_2 = 11$, $q_1 = 7$, $q_2 = 13$, $\eta_1 = \eta_2 = 0.5$, $k_{t1} = 50.0$, $k_{t2} = 30.0$, $\alpha_1 = 1.2$, $\alpha_2 = 0.2$, $\alpha_3 = 20.0$, and $\alpha_4 = 1.0$. Simulation step is 0.02 sec.

5.1. Case 1. In this section, a spiral rising trajectory is chosen as the desired trajectory. The adaptive backstepping and PD approaches are considered for comparison. The analytical expression of the desired trajectory is given as

$$\xi_d = \begin{bmatrix} 15 \sin\left(\frac{2t}{10}\right) & 15 \cos\left(\frac{2t}{10}\right) & -0.5t - 5 \end{bmatrix}. \quad (70)$$

Both aerodynamics disturbances on translational and rotational subsystems are given as

$$\begin{aligned} \mathbf{d}_1 &= \begin{bmatrix} 0.1 \sin(\pi t) + \cos\left(\frac{\pi t}{10}\right) + 3 \\ 0.1 \sin(\pi t) + 1.5 \cos\left(\frac{\pi t}{10}\right) + 4 \\ 0.1 \sin(\pi t) + 2 \cos\left(\frac{\pi t}{10}\right) + 5 \end{bmatrix} \text{ N}, \\ \mathbf{d}_2 &= \begin{bmatrix} 0.1 \sin(\pi t) + 0.2 \cos\left(\frac{\pi t}{10}\right) + 0.1 \\ 0.1 \sin(\pi t) + 0.2 \cos\left(\frac{\pi t}{10}\right) + 0.2 \\ 0.1 \sin(\pi t) + 0.2 \cos\left(\frac{\pi t}{10}\right) + 0.3 \end{bmatrix} \text{ N} \cdot \text{m}. \end{aligned} \quad (71)$$

For the comparisons among the proposed method, adaptive backstepping and PD approaches of trajectory tracking performance are demonstrated in Figure 2. We can find that the trajectory tracking accuracy of the proposed methodology is higher than the accuracy of the adaptive backstepping and PD controller. For the adaptive backstepping method, the convergence rate of the adaptive algorithm is less than that of the proposed finite-time generalized NDOB. Notice that the PD controller is an error-based method, and the control output cannot reject the disturbance actively. The traditional PD controller cannot decrease the tracking error effectively under time-varying disturbances. Thus, the accuracy of the PD controller is less than the other two methods. The tracking error of position, velocity, RPs, and angular velocity of the proposed approach is shown in Figures 3–6. It is shown that the proposed methodology can make the tracking error converge quickly

FIGURE 2: Trajectory tracking results of the proposed method (comparisons with the adaptive backstepping and PD controller).

FIGURE 3: Position tracking error of the proposed method.

FIGURE 4: Velocity tracking error of the proposed method.

FIGURE 5: RP tracking error of the proposed method.

FIGURE 6: Angular velocity tracking error of the proposed method.

and accurately. The estimation errors of both translational and rotational subsystems are shown in Figures 7 and 8. The proposed finite-time generalized NDOB can estimate the system uncertainties accurately. The root mean square (RMS) error of the above methods is shown in Table 1. It is clear that the tracking error of our method is less than that of the other two methods.

5.2. Case 2. In this section, the estimation performance of the proposed generalized NDOB is analyzed in detail. The comparisons with the traditional generalized NDOB [30] are also carried out. The analytical expression of the desired trajectory is chosen as

$$\xi_d = [10 \cos(0.1t + 2) \quad 10 \sin(0.1t + 2) \quad -t - 3]. \quad (72)$$

The aerodynamics disturbances are the same as Case 1, and we assume that there is an extra external disturbance acting on the translational dynamics at times of 10 sec and 30 sec. In Figure 9, the trajectory tracking effect of the proposed method is compared with the traditional method, and the projection of the results on every 2D plane is also presented. The specified tracking error comparisons are expressed in Figures 10 and 11. Figure 12 shows the

FIGURE 7: Disturbance estimation error for the translational subsystem.

FIGURE 8: Disturbance estimation error for the rotational subsystem.

TABLE 1: Performance comparisons of the root mean square error.

	$\tilde{\xi}_1$ (m)	$\tilde{\xi}_2$ (m)	$\tilde{\xi}_3$ (m)
Proposed methodology	0.1418	0.1471	0.0346
Adaptive backstepping	0.1675	0.4125	0.2144
PD approach	0.4993	0.8987	0.9023

estimation error of the proposed and traditional NDOB. Simulation results in Figure 9 show that the sudden change of external disturbances brings the system with the tracking error. In the simulation, the same observer gain is chosen for both modified and traditional generalized NDOB. From Figure 12, we can find that, by using the proposed observer, the convergent time is less than 0.5 sec, while the convergent time of the traditional observer is more than 2.5 sec. Moreover, for the continuous time-varying disturbances, the estimation error of the traditional method is larger than that of the proposed method. This will in turn affect the tracking accuracy. The specified tracking error is depicted in Tables 2 and 3. It is shown that the max cross-tracking error of the proposed approach is

FIGURE 9: Trajectory tracking results of the proposed method (comparison with the traditional generalized NDOB).

FIGURE 10: Position tracking error comparisons.

less than the traditional method in both steady and transient states.

From the descriptions above, we can conclude that the proposed approach can successfully deal with the trajectory tracking task of a VTOL UAV. With the hierarchical control structure, the translational controller can obtain the desired thrust and attitude to stabilize the trajectory tracking error, while the rotational controller can enable the aircraft to track the desired attitude quickly and accurately. Comparing with the traditional control scheme, our proposed methodology can obtain higher controller

FIGURE 11: Velocity tracking error comparisons.

FIGURE 12: Comparisons of disturbance estimation for the translational subsystem.

accuracy, as well as faster convergence rate. Meanwhile, the proposed NDOB can decrease the estimation error, which will provide higher control accuracy. In a word, comparing with the traditional method, the proposed methodology can acquire less convergence time and higher control accuracy.

TABLE 2: Comparisons of the max cross-tracking error.

	In steady state (m)	In transient state (m)
Proposed methodology	0.1211	0.3738
Generalized NDOB	0.4968	0.9763

TABLE 3: Performance comparisons of the root mean square error.

	$\tilde{\xi}_1$ (m)	$\tilde{\xi}_2$ (m)	$\tilde{\xi}_3$ (m)
Proposed methodology	0.0919	0.1370	0.0776
Generalized NDOB	0.2701	0.3631	0.2193

6. Conclusions

In this research, we focus on the trajectory tracking task of a VTOL UAV, and a modified generalized NDOB-based finite-time terminal SMC methodology is proposed. Different from the most existing works, RPs are used in this paper to establish the system error model, based on which the system error model is divided into translational and rotational subsystems according to the cascade property. By introducing the nonlinear feedback, the finite-time generalized NDOB is proposed to acquire better disturbance estimation performance; hence, terminal SMC technique is applied for the desired trajectory tracking performance. Simulations show that the modified generalized NDOB can bring with higher convergent speed and estimation accuracy, and the proposed methodology has better control performance compared with the traditional methods.

The proposed methodology can only deal with the trajectory tracking task for a single VTOL UAV. However, this work does not contain the trajectory planning problem of the aircraft with environmental obstacles. In future works, the trajectory planning problem with complicated obstacles will be considered. Also, the complex tasks that require multiple UAVs will be investigated. Moreover, the experimental setup will be implemented to validate the effectiveness of the corresponding methods.

Data Availability

The simulation data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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