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Research Article

On Highly Dimensional Elastic and Nonelastic Interaction between Internal Waves in Straight and Varying Cross-Section Channels

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This manuscript studies the computational solutions of the highly dimensional elastic and nonelastic interaction between internal waves through the fractional nonlinear $(4+1)$-dimensional Fokas equation. This equation is considered as the extension model of the two-dimensional Davey–Stewartson (DS) and Kadomtsev–Petviashvili (KP) equations to a four spatial dimensions equation with time domain. The modified Khater method is employed along the Atangana–Baleanu (AB) derivative operator to construct many novel explicit wave solutions. These solutions explain more physical and dynamical behavior of that kind of the interaction. Moreover, 2D, 3D, contour, and stream plots are demonstrated to explain the detailed dynamical characteristics of these solutions. The novelty of our paper is shown by comparing our results with those obtained in previous published research papers.

1. Introduction

Internal waves are waves that spread inside a stream, with gradients of intensity [1–3]. The surface gravity waves pass along the broad pressure boundary between air and water, while internal waves migrate inside the ocean over gradients of intensity [4–7]. Perturbations of these gradients of intensity are preserved by momentum, which creates a propagating motion [8–11].

Globally, internal waves play a significant role in the ocean, providing nutrients to surface waters that facilitate the growth of phytoplankton, the foundation of the ocean food chain [12–15]. Created primarily by the tide's interaction with ocean floor and water topography, internal waves may bring the energy from these forces through the entire ocean basins [16, 17]. As internal waves pass through the continental shelf, they interact with the topography, and as the gravity of the surface steepens and splits on the sea, internal waves steep their energy in the shelf and dissipate it [18, 19]. When the internal waves rise, they turn into nonlinear waves of fluid that may assume several forms (e.g., solitons, bores, and boluses), all of which have the potential to bring deep water that has different properties (probably colder, higher in nutrients, lower in oxygen, lower in pH) across the shelf and into shallower waters [20–22].

Depending on the potential of the nonlinear partial differential equation to describe several complicated processes in diverse fields such as physiology, plasma physics, hydrodynamics, fluid mechanics, and optics, numerous precise and computational schemes such as in [23–26] have been developed. Using inspired schemes, computational and technical advances are seen as the basic usefulness of solving these phenomena [27–31]. Such schemes have recently been regarded as simple methods for discovering the different formulas of moving wave solutions to these dynamic phenomena [32–34]. However, in the nonlinear partial differential equation (NLPDE), with an integer instruction, several researchers have struggled to extract and formulate certain complex phenomena [35, 36]. The fractional equation is then deemed an appropriate solution to this issue because it includes a nonlocal property that is not NLPDE-based with an integer [37–40].

In this research, we study the nonlinear fractional $(4+1)$-dimensional Fokas model that is mathematically given by [41–43]

$$4D_t^{\alpha}U_x - U_{xxxy} + U_{xyyy} + 12U_yU_x + 12UU_{xy} - 6U_{wz} = 0, \quad (0<\alpha<1), \tag{1}$$

where U is the function of the elastic and nonelastic interaction between internal waves in straight and varying cross-section channels. Implementation of the following AB-derivative definitions on equation (1) with the following wave transformation $U(x,y,z,w,t) = \psi(\zeta), \zeta = (((\alpha-1)\lambda t^{-\alpha n})/B(\alpha)\sum_{n=0}^{\infty}(-(\alpha/1-\alpha))^n\Gamma(1-\alpha n)) + k_4 w + k_1 x + k_2 y + k_3 z$, where k_i and λ $(i=1,2,3,4)$ are arbitrary constants [37–40], while $B(\alpha)$ is a normalized function, converts the fractional PDE into the next integer order ODE:

$$\alpha_1\psi'' + \alpha_2\psi'''' + \alpha_3\psi'^2 + \alpha_4\psi\psi'' = 0, \tag{2}$$

where $\alpha_1 = (4k_1\lambda - 6k_3k_4), \alpha_2 = (k_1k_2^3 - k_1^3k_2), \alpha_3 = 12k_1k_2$, and $\alpha_4 = -6k_3k_4$.

The remaining parts of our research paper are organized as follows: Section 2 employs the modified Khater system [44–49] to provide the nonlinear fractional Fokas model with novel solitary solutions. Section 3 describes the outcomes and provides the physical description of the sketches seen. This work is concluded in Section 4.

2. Applications

Usage of the modified Khater technique via the concepts of homogeneous equilibrium on equation (2) provides general solutions:

$$\psi(\zeta) = \sum_{i=1}^{m} a_i K^{i\varphi(\zeta)} + \sum_{i=1}^{m} b_i K^{-i\varphi(\zeta)} + a_0 = a_1K^{\varphi(\zeta)} + a_2K^{2\varphi(\zeta)} + a_0 + b_2K^{-2\varphi(\zeta)} + b_1K^{-\varphi(\zeta)}, \tag{3}$$

where $a_i, b_j\,(i,j=0,1,2,\ldots,), a_m \neq 0$, or $b_m \neq 0$. Additionally, $\varphi(\zeta)$ is the solution function of $\varphi'(\zeta) = (1/\ln(K))[\delta + \rho K^{f(\zeta)} + \varkappa K^{-f(\zeta)}]$ where δ, ρ, and $\varkappa$ are arbitrary constants. Using equation (3) through its auxiliary equation in the modified Khater technique's framework gives the following families for the above-mentioned arbitrary constants.

Family I:

$$\begin{aligned} &a_1 \longrightarrow 0,\\ &a_2 \longrightarrow 0,\\ &b_1 \longrightarrow \frac{b_2\delta}{\varkappa},\\ &\alpha_1 \longrightarrow -\frac{\alpha_4\left(12a_0\varkappa^2 - b_2\delta^2 - 8b_2\rho\varkappa\right)}{12\varkappa^2},\\ &\alpha_2 \longrightarrow -\frac{\alpha_4 b_2}{12\varkappa^2},\\ &\alpha_3 \longrightarrow \alpha_4. \end{aligned} \tag{4}$$

Family II:

$$\begin{aligned} &a_1 \longrightarrow \frac{a_2\delta}{\rho},\\ &b_1 \longrightarrow 0,\\ &b_2 \longrightarrow 0,\\ &\alpha_1 \longrightarrow -\frac{\alpha_4\left(-a_2\delta^2 + 12a_0\rho^2 - 8a_2\rho\varkappa\right)}{12\rho^2},\\ &\alpha_2 \longrightarrow -\frac{a_2\alpha_4}{12\rho^2},\\ &\alpha_3 \longrightarrow \alpha_4. \end{aligned} \tag{5}$$

Consequently, the explicit solutions of equation (1) are given in the following forms.

In case of $\delta^2 - 4\rho\varkappa < 0, \rho \neq 0$, we obtain

$$\begin{aligned} U(\xi)_{I,1} &= a_0 + \frac{2b_2\rho\left(\delta\sqrt{4\rho\varkappa-\delta^2}\tan\left((1/2)\xi\sqrt{4\rho\varkappa-\delta^2}\right) - \delta^2 + 2\rho\varkappa\right)}{\varkappa\left(\delta - \sqrt{4\rho\varkappa-\delta^2}\tan\left((1/2)\xi\sqrt{4\rho\varkappa-\delta^2}\right)\right)^2},\\ U(\xi)_{I,2} &= a_0 + \frac{2b_2\rho\left(\delta\sqrt{4\rho\varkappa-\delta^2}\cot\left((1/2)\xi\sqrt{4\rho\varkappa-\delta^2}\right) - \delta^2 + 2\rho\varkappa\right)}{\varkappa\left(\delta - \sqrt{4\rho\varkappa-\delta^2}\cot\left((1/2)\xi\sqrt{4\rho\varkappa-\delta^2}\right)\right)^2},\\ U(\xi)_{II,1} &= \frac{a_2\left(-\left(\delta^2-4\rho\varkappa\right)\sec^2\left((1/2)\xi\sqrt{4\rho\varkappa-\delta^2}\right) - 4\rho\varkappa\right)}{4\rho^2} + a_0,\\ U(\xi)_{II,2} &= \frac{a_2\left(-\left(\delta^2-4\rho\varkappa\right)\csc^2\left((1/2)\xi\sqrt{4\rho\varkappa-\delta^2}\right) - 4\rho\varkappa\right)}{4\rho^2} + a_0. \end{aligned} \tag{6}$$

In case of $\delta^2 - 4\rho\varkappa > 0, \rho \neq 0$, we obtain

$$U(\xi)_{I,3} = a_0 - \frac{2b_2\rho\left(\delta\sqrt{\delta^2-4\rho\varkappa}\tanh\left((1/2)\xi\sqrt{\delta^2-4\rho\varkappa}\right)+\delta^2-2\rho\varkappa\right)}{\varkappa\left(\sqrt{\delta^2-4\rho\varkappa}\tanh\left((1/2)\xi\sqrt{\delta^2-4\rho\varkappa}\right)+\delta\right)^2}, \tag{7}$$

$$U(\xi)_{I,4} = a_0 - \frac{2b_2\rho\left(\delta\sqrt{\delta^2-4\rho\varkappa}\coth\left((1/2)\xi\sqrt{\delta^2-4\rho\varkappa}\right)+\delta^2-2\rho\varkappa\right)}{\varkappa\left(\sqrt{\delta^2-4\rho\varkappa}\coth\left((1/2)\xi\sqrt{\delta^2-4\rho\varkappa}\right)+\delta\right)^2}, \tag{8}$$

$$U(\xi)_{II,3} = \frac{a_2\left(-\left(\delta^2-4\rho\varkappa\right)\operatorname{sech}^2\left((1/2)\xi\sqrt{\delta^2-4\rho\varkappa}\right)-4\rho\varkappa\right)}{4\rho^2}+a_0, \tag{9}$$

$$U(\xi)_{II,4} = \frac{a_2\left(\left(\delta^2-4\rho\varkappa\right)\operatorname{csch}^2\left((1/2)\xi\sqrt{\delta^2-4\rho\varkappa}\right)-4\rho\varkappa\right)}{4\rho^2}+a_0. \tag{10}$$

In case of $\rho\varkappa > 0, \varkappa \neq 0, \rho \neq 0$, and $\delta = 0$, we obtain

$$\begin{aligned} U(\xi)_{I,5} &= a_0 + \frac{b_2\rho\cot^2(\xi\sqrt{\rho\varkappa})}{\varkappa}, \\ U(\xi)_{I,6} &= a_0 + \frac{b_2\rho\tan^2(\xi\sqrt{\rho\varkappa})}{\varkappa}, \\ U(\xi)_{II,5} &= \frac{a_2\varkappa\tan^2(\xi\sqrt{\rho\varkappa})}{\rho}+a_0, \\ U(\xi)_{II,6} &= \frac{a_2\varkappa\cot^2(\xi\sqrt{\rho\varkappa})}{\rho}+a_0. \end{aligned} \tag{11}$$

In case of $\rho\varkappa < 0, \varkappa \neq 0, \rho \neq 0$, and $\delta = 0$, we obtain

$$\begin{aligned} U(\xi)_{I,7} &= a_0 + \frac{b_2\rho\cot^2(\xi\sqrt{\rho}\sqrt{\varkappa})}{\varkappa}, \\ U(\xi)_{I,8} &= a_0 + \frac{b_2\rho\tan^2(\xi\sqrt{\rho}\sqrt{\varkappa})}{\varkappa}, \\ U(\xi)_{II,7} &= \frac{a_2\varkappa\tan^2(\xi\sqrt{\rho}\sqrt{\varkappa})}{\rho}+a_0, \\ U(\xi)_{II,8} &= \frac{a_2\varkappa\cot^2(\xi\sqrt{\rho}\sqrt{\varkappa})}{\rho}+a_0. \end{aligned} \tag{12}$$

In case of $\delta = 0$ and $\varkappa = -\rho$, we obtain

$$U(\xi)_{I,9} = a_0 + b_2\tanh^2(\xi\varkappa), \tag{13}$$

$$U(\xi)_{II,9} = a_2\coth^2(\xi\varkappa) + a_0. \tag{14}$$

In case of $\delta = (\varkappa/2) = \kappa$ and $\rho = 0$, we obtain

$$U(\xi)_{I,10} = a_0 + \frac{b_2 e^{\kappa\xi}}{2\left(e^{\kappa\xi}-2\right)^2}. \tag{15}$$

In case of $\delta = \rho = \kappa$ and $\varkappa = 0$, we obtain

$$U(\xi)_{II,10} = \frac{1}{4}a_2\operatorname{csch}^2\left(\frac{\kappa\xi}{2}\right)+a_0. \tag{16}$$

In case of $\varkappa = 0, \delta \neq 0$, and $\rho \neq 0$, we obtain

$$U(\xi)_{II,11} = \frac{2a_2\delta^2e^{\delta\xi}}{\rho\left(\rho e^{\delta\xi}-2\right)^2}+a_0. \tag{17}$$

In case of $\delta = \rho = 0$ and $\varkappa \neq 0$, we obtain

$$U(\xi)_{I,11} = a_0 + \frac{b_2}{\xi^2\varkappa^2}. \tag{18}$$

In case of $\delta = \varkappa = 0$ and $\rho \neq 0$, we obtain

$$U(\xi)_{II,12} = \frac{a_2}{\xi^2\rho^2}+a_0. \tag{19}$$

In case of $\delta = 0$ and $\varkappa = \rho$, we obtain

$$\begin{aligned} U(\xi)_{I,12} &= a_0 + b_2\cot^2(C+\xi\varkappa), \\ U(\xi)_{II,13} &= a_2\tan^2(C+\xi\varkappa)+a_0. \end{aligned} \tag{20}$$

In case of $\rho = 0, \delta \neq 0$, and $\varkappa \neq 0$, we obtain

$$U(\xi)_{I,13} = \frac{a_0\varkappa\left(\varkappa-\delta e^{\delta\xi}\right)^2+b_2\delta^3e^{\delta\xi}}{\varkappa\left(\varkappa-\delta e^{\delta\xi}\right)^2}. \tag{21}$$

In case of $\delta^2 - 4\rho\varkappa = 0$, we obtain

$$\begin{aligned} U(\xi)_{I,14} &= a_0 - \frac{b_2\delta^3\xi(\delta\xi+4)}{4\varkappa^2(\delta\xi+2)^2}, \\ U(\xi)_{II,14} &= \frac{2a_2\varkappa(\delta\xi+2)\left(2\rho\varkappa(\delta\xi+2)-\delta^3\xi\right)}{\delta^4\xi^2\rho}+a_0. \end{aligned} \tag{22}$$

3. Results and Discussion

This section shows our obtained solutions and their novelty. Also, we compare our obtained solutions with those of previously published articles to show the similarity and

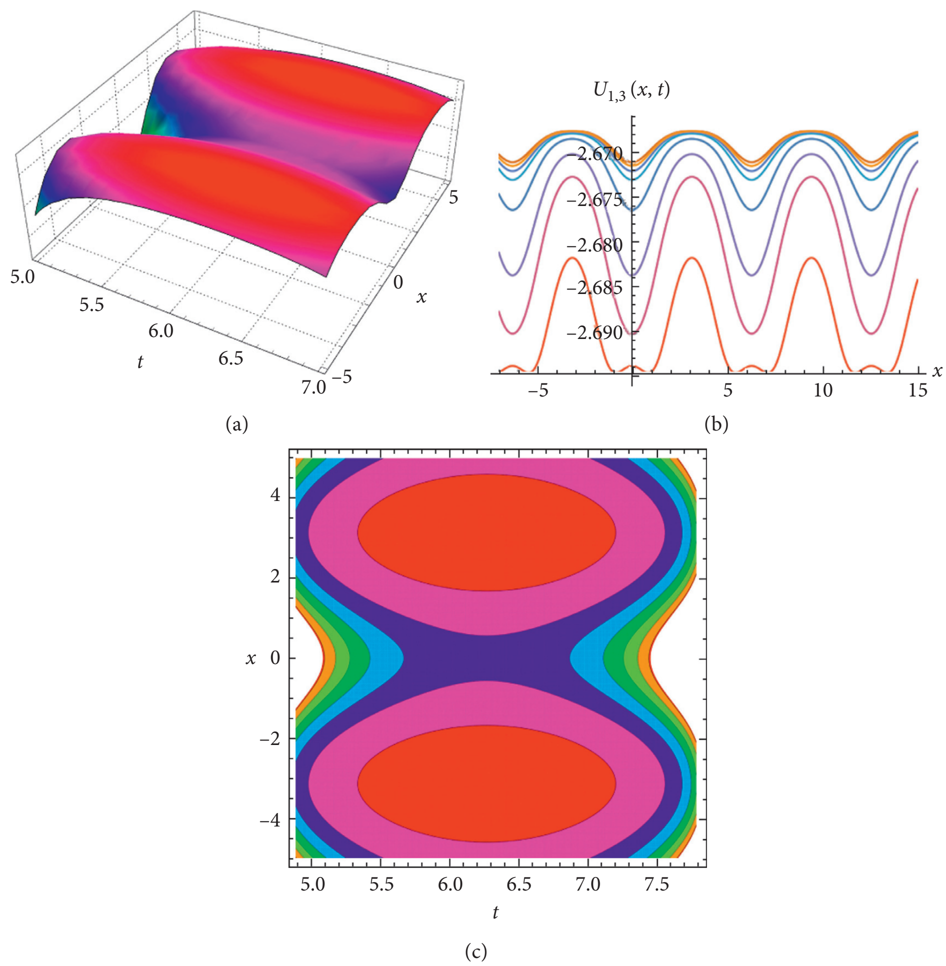


FIGURE 1: Solitary wave solutions equation (7) in three, two, and contour plots.

difference between our and their solutions. Our discussion is divided into three main parts, which are the used analytical method, obtained solutions, and figure interpretation:

(1) The used computational scheme:

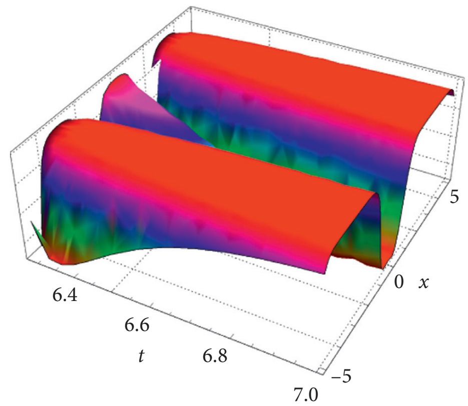
The modified Khater method have been used for the first time for applying to the fractional nonlinear $(4+1)$-dimensional Fokas equation. This modified method is considered as one of the most general analytical schemes in this field; especially, it covers more than twelve recent analytical schemes [50].

(2) The obtained solutions:

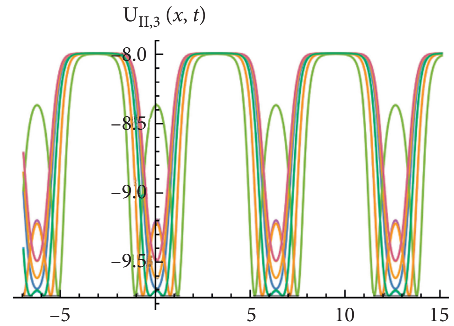
This part gives a comparison between our obtained solutions and those obtained in previously accepted papers. In [41–43] by Wan-Jun Zhang and Tie-Cheng Xia, Ruoxia Yao, Yali Shen, and Zhibin Li, and Wei Li and Yinping Liu, respectively, who applied the Hirota bilinear method, the bilinear form, and Hirota method, receptively to a fractional nonlinear $(4+1)$-dimensional Fokas equation, many distinct types of solutions for these fractional nonlinear models were obtained. All our obtained solutions of the investigated model are new and different from those obtained in [41–43].

(3) The figures interpretation:

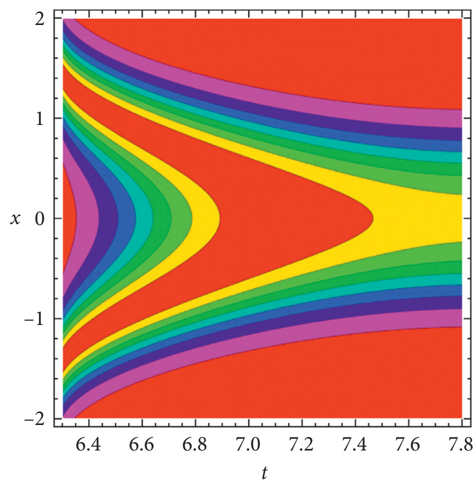
We have represented some of our obtained solutions in three distinct types of figures (3D, 2D, and contour plots) to explain kink, antikink, periodic, and singular shapes to illustrate the perspective view of the solution, the wave propagation pattern of the wave along x-axis, and the overhead view of the solution for the following values of the parameters:



(a)

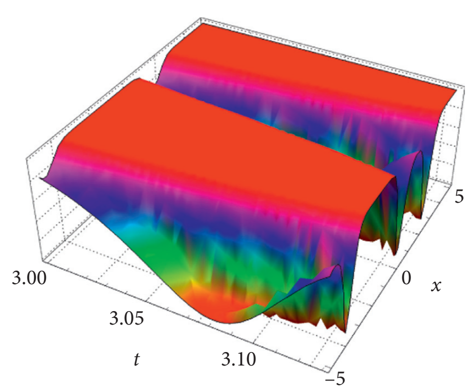


(b)

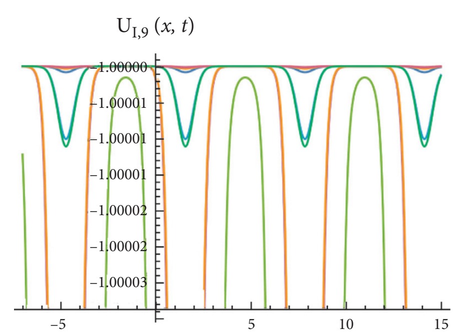


(c)

FIGURE 2: Solitary wave solutions equation (9) in three, two, and contour plots.

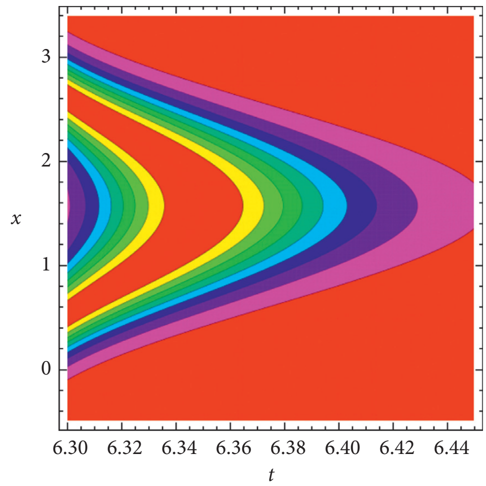


(a)



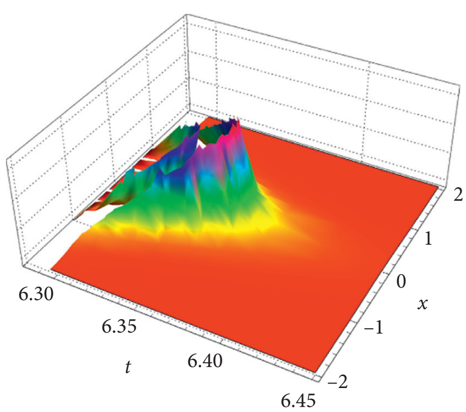
(b)

FIGURE 3: Continued.

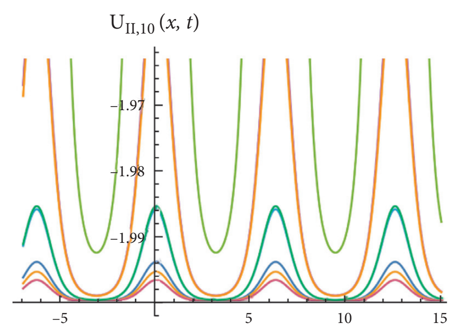


(c)

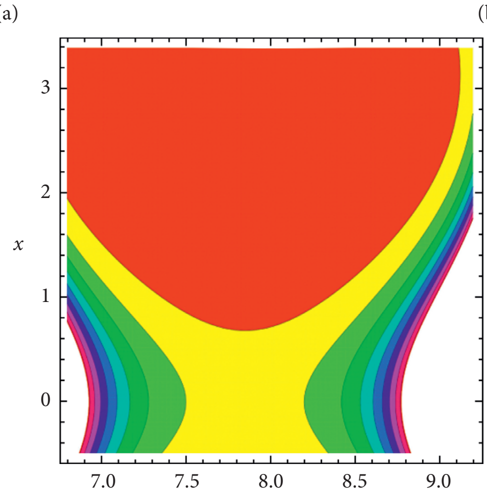
FIGURE 3: Solitary wave solutions equation (13) in three, two, and contour plots.



(a)



(b)



(c)

FIGURE 4: Solitary wave solutions equation (16) in three, two, and contour plots.

$$\left[a_0 = -2, b_2 = 1, \delta = 5, k_1 = -1, k_2 = 3, k_3 = 4, k_4 = \frac{33}{2}, \lambda = -6, \rho = 2, w = -\frac{2}{3}, y = 1, z = 2, \varkappa = 3a_2 = 7, \right.$$
$$\& a_0 = 6, \delta = 3, k_1 = 5, k_2 = 3, k_3 = 4, k_4 = \frac{33}{2}, \lambda = 9, \rho = 1, w = -\frac{2}{3}, y = 1, z = 2, \varkappa = 2 \,\& a_0 = -2, b_2 = 1,$$
$$\delta = 0, k_1 = -1, k_2 = 3, k_3 = 4, k_4 = \frac{33}{2}, \lambda = -6, \rho = -3, w = -\frac{2}{3}, y = 1, z = 2, \varkappa = 3 \,\& a_2 = 1,\ a_0 = -2, \delta = 0,$$
$$\left. \kappa = 2, k_1 = -1, k_2 = 3, k_3 = 4, k_4 = \frac{33}{2}, \lambda = -6, w = -\frac{2}{3}, y = 1, z = 2\right]. \quad (23)$$

4. Conclusion

This research paper has successfully investigated the nonlinear fractional nonlinear $(4+1)$-dimensional Fokas model via the modified Khater method that has used the Atangana–Baleanu derivative operator to convert the fractional form of the studied model to a nonlinear ordinary differential equation with an integer order. Many distinct exact traveling and solitary wave solutions have been obtained. These solutions have been illustrated via various sketches (Figures 1–4) that explain more novel properties of the considered fractional models. The accuracy and novelty of our obtained solutions have been explained. The powerfulness and effectiveness of the used techniques are also explained and verified.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflicts of interest.

Authors' Contributions

Mostafa M. A. Khater and Qiang Zheng are responsible for the analytical simulation. Haiyong Qin and Raghda A. M. Attia are responsible for the final editing and revision of the whole paper.

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