Research Article

Group Scheduling Problems with Time-Dependent and Position-Dependent DeJong's Learning Effect

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In this paper, we introduce a group scheduling model with time-dependent and position-dependent DeJong's learning effect. The objectives of scheduling problems are to minimize makespan, the total completion time, and the total weighted completion time, respectively. We show that the problems remain solvable in polynomial time under the proposed model.

1. Introduction

In classical scheduling problems, the scheduling models routinely assume that job processing times are known and fixed throughout the period of production process. However, this assumption may be unrealistic in many situations that the processing time of jobs may be shortened due to learning effect over time. Production scheduling problems with learning effect have been paid much attention in recent years. To the best of our knowledge, there is little research that considers position-dependent and time-dependent processing time group scheduling model. Besides, the existing learning effect scheduling model suffers the drawback that when a job's position or the starting processing time is sufficiently large in a schedule, its actual processing time is close to zero (infinity). This paper is to introduce a new scheduling model with position-dependent and time-dependent processing time, which overcomes the above shortcomings and is more general and realistic than the models existing in the literature.

The remaining part of this paper is organized as follows. In Section 2, we present a brief review of the existing scheduling model with learning effect. In Section 3, a precise formulation of the problem is given. Section 4 considers several single-machine scheduling problems with position-dependent and time-dependent DeJong's learning effect to minimize makespan, the total completion time, and the total

weighted completion time, respectively. The last section contains some conclusions of our model.

2. Review of Existing Models

Wright [1]; Biskup [2]; and Cheng and Wang [3] are among the pioneers that brought the learning effect into the field of scheduling research. Since then, learning effect has been widely employed in management science. To overcome the shortcoming associated with Wright's [1] definition of learning effect that the improvement of learning effect is infinite, DeJong [4] introduced a new learning scheduling model, which is more realistic. Wang et al. [5] studied time-dependent DeJong's learning effect model $p_{jr}^A = p_j (\alpha a \sum_{i=1}^{r-1} p_{[i]} + \beta)$, where $\alpha \geq 0$, $\beta \geq 0$, and $0 < a \leq 1$ are the parameters obtained empirically and $\alpha + \beta = 1$. Other studies have validated DeJong's learning model, e.g., Oko-lowski and Gawiejnowicz [6] and Ji et al. [7, 8]. More recent papers which have considered scheduling problems with learning effect include studies of Wang and Wang [9, 10]; Wang et al. [11]; Xu et al. [12]; Chen et al. [13]; Toksari and Arik [14]; Bai et al. [15]; Mustu and Eren [16]; Pei et al. [17]; Zhang et al. [18]; Wang and Wang [19, 20]; Wang et al. [21], and Przybylski [22].

In many production processes, the efficiency can be improved by grouping various parts and products with

similar designs. This phenomenon is known as group technology in the literature. Many advantages have been claimed through the wide applications of group technology, for instance, Ji et al. [23] and Wang and Liu [24]. For the latest results on group scheduling problems, we refer the reader to the studies of Zhang et al. [18, 25]; Wang and Wang [26]; and Lu et al. [27] among others.

3. Problem Formulation

We first define the notations which are used throughout this paper, followed by the descriptions of the problems:

- m : the number of groups ($m \geq 2$)
- n_i : the number of jobs in group G_i , $i = 1, 2, \dots, m$
- n : the total number of jobs, i.e., $n_1 + n_2 + \dots + n_m = n$
- G_i : group G_i , $i = 1, 2, \dots, m$
- $G_{[i]}$: group scheduled in the i th position in a sequence, $i = 1, 2, \dots, m$
- $J_{i[j]}$: the job scheduled in the j th position in group G_i
- p_{ij} : the normal processing time of job J_{ij} , $i = 1, 2, \dots, m$, $j = 1, 2, 3, \dots, n_i$
- $p_{i[j]}$: the normal processing time of job $J_{i[j]}$ scheduled in the j th position of group G_i in a sequence, $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n_i$
- s_i : the sequence-independent setup time of group G_i , $i = 1, 2, \dots, m$
- $p_{[i][j]}^A$: the actual processing time of job $J_{[i][j]}$ scheduled in the j th position of the i th group $G_{[i]}$ in a sequence, $i = 1, 2, \dots, m$ and $j = 1, 2, 3, \dots, n_i$

There are n independent jobs. All jobs are classified into m groups $G_1, G_2, \dots, G_m$ by the similarities and to be processed on a single machine. All the jobs are available at time zero, and job preemption is not allowed. A group setup time is required if the machine switches to process from one group to another. Jobs in the same group are processed consecutively and need no setup time. The machine can handle one job at a time. Each job J_{ij} has a normal processing time p_{ij} . The actual processing time of job will be shorter than its normal processing time due to the learning effect.

In this paper, we consider the following new time-dependent and position-dependent model:

$$p_{[i][r]}^A = p_{[i][r]} \left(M + (1 - M) a^{t_{[i][r]}} r^b \right), \quad (1)$$

$$i = 1, 2, \dots, m, r = 1, 2, 3, \dots, n_i,$$

where $0 < a < 1$, $b < 0$ is the learning index, $t_{[i][r]}$ is the total normal processing times of all jobs scheduled before $J_{[i][r]}$ in group $G_{[i]}$, i.e., $t_{[i][r]} = \sum_{j=1}^{r-1} p_{[i][j]}$. M represents “the factor of incompressibility” ($0 \leq M \leq 1$). If $M = 0$, the model simplifies to the classical learning model $p_{jr} = p_j r^a$. If $M = 1$, the processing time of job is constant. The objectives are to find the optimal job sequence in each group and the optimal group sequence such that makespan, the total completion time, and the total weighted completion time are

minimized, respectively. Using the three-field notation for scheduling problem, we denote our problems as

$$\begin{aligned} &1|GT, s_i, p_{[i][r]}^A = p_{[i][r]} \left(M + (1 - M) a^{t_{[i][r]}} r^b \right) | C_{\max}, \\ &1|GT, s_i, p_{[i][r]}^A = p_{[i][r]} \left(M + (1 - M) a^{t_{[i][r]}} r^b \right) | \sum C_{ij}, \\ &1|GT, s_i, p_{[i][r]}^A = p_{[i][r]} \left(M + (1 - M) a^{t_{[i][r]}} r^b \right) | \sum w_{ij} C_{ij}. \end{aligned} \quad (2)$$

Before proving the problems, some lemmas are introduced as follows.

Lemma 1. Let $F(x, r) = (1 - a^x (r + 1/r)^b)/x$, where $r \geq 1$, $x > 0$, $0 < a < 1$, and $b < 0$. Then, $F(x, r)$ is a decreasing function for all $x > 0$.

Proof. Taking the first derivative of $F(x, r)$ with respect to x , the following is obtained:

$$\frac{\partial F(x, r)}{\partial x} = \frac{a^x (r + 1/r)^b (1 - x \ln a) - 1}{x^2}. \quad (3)$$

Let

$$f(x, r) = a^x \left(\frac{r + 1}{r} \right)^b (1 - x \ln a) - 1. \quad (4)$$

Taking the first derivative of $f(x, r)$ with respect to x , the following is obtained:

$$\frac{\partial f(x, r)}{\partial x} = -a^x \left(\frac{r + 1}{r} \right)^b \ln^2 a. \quad (5)$$

Since $r \geq 1$, $x > 0$, $0 < a < 1$, and $b < 0$, we have $-a^x (r + 1/r)^b \ln^2 a < 0$. Hence, $f(x, r)$ is a decreasing function for all $x > 0$. Therefore, $f(x, r) < f(0, r)$. From $f(0, r) = (r + 1/r)^b - 1 < 0$, we obtain $f(x, r) < 0$. Hence, $\partial F(x, r)/\partial x < 0$ and $F(x, r)$ is a decreasing function for all $x > 0$.

Lemma 2. If the function $h(x)$ is a differentiable convex function for $a < b < c$, then

$$\begin{aligned} (1) & h(b) - h(a) = h'(\xi_1)(b - a), \quad a < \xi_1 < b, \\ (1) & h(c) - h(a) = h'(\xi_2)(c - a), \quad a < \xi_2 < c, \\ (1) & h'(\xi_1) \leq h'(\xi_2). \end{aligned} \quad (6)$$

Proof. From the Lagrange mean value theorem, the lemma can be easily obtained.

4. Result of Optimization

In this section, we address the application of our model to solve the single-machine group scheduling problems involving makespan minimization, the total completion time minimization, and the total weighted completion time minimization.

4.1. Makespan Minimization

Theorem 1. For the $1|GT, s_i, p_{[i][r]}^A = p_{[i][r]} \left(M + (1 - M) a^{t_{[i][r]}} r^b \right) | C_{\max}$ problem, the optimal schedule is obtained: (i)

the job sequence in each group is in the smallest normal processing time order (SPT) and (ii) the groups can be sequenced in any order.

Proof. Suppose a sequence is $\{G_{[1]}, G_{[2]}, \dots, G_{[m]}\}$. Therefore, the makespan of the sequence is obtained as follows:

$$C_{\max} = \sum_{i=1}^m s_{[i]} + \sum_{i=1}^m \sum_{j=1}^{n_i} p_{[i][j]} \left(M + (1-M)a \sum_{h=1}^{j-1} p_{[i][h]} r^b \right). \quad (7)$$

The first term of equation (7) is a constant. The value of the second term of equation (7) is determined by the job sequence in each group. Therefore, the makespan minimization of the studied model is independent of the sequence of group. Hence, the optimal group sequence can be scheduled in any order, so (ii) follows.

Next, we determine the optimal job sequence in each group to minimize the makespan by implementing the adjacent job exchange argument. Let π_1 and π'_1 be two job schedules of a group G_i where the difference between π_1 and π'_1 is a pairwise interchange of two adjacent jobs J_{ik} and J_{il} . That is, $\pi_1 = [S_1, J_{ik}, J_{il}, S_2]$ and $\pi'_1 = [S_1, J_{il}, J_{ik}, S_2]$, where

S_1 and S_2 are the partial sequences. The positions of J_{ik} and J_{il} in sequence π_1 are r and $r+1$, respectively, which are reverse in sequence π'_1 . Furthermore, we assume that t denotes the completion time of the last job in S_1 and $p_{ik} \leq p_{il}$. To prove π_1 dominates π'_1 , it suffices to show that $C_{il}(\pi_1) \leq C_{ik}(\pi'_1)$ and $C_{iu}(\pi_1) \leq C_{iu}(\pi'_1)$ for any job J_{iu} in S_2 . By definition, the completion time periods of jobs J_{ik} and J_{il} in sequence π_1 and π'_1 are given by

$$\begin{aligned} C_{ik}(\pi_1) &= t + p_{ik} \left(M + (1-M)a \sum_{h=1}^{r-1} p_{[i][h]} r^b \right), \\ C_{il}(\pi_1) &= t + p_{il} \left(M + (1-M)a \sum_{h=1}^{r-1} p_{[i][h]} r^b \right) \\ &\quad + p_{il} \left(M + (1-M)a \sum_{h=1}^{r-1} p_{[i][h]} + p_{ik} (r+1)^b \right), \\ C_{il}(\pi'_1) &= t + p_{il} \left(M + (1-M)a \sum_{h=1}^{r-1} p_{[i][h]} r^b \right), \\ C_{ik}(\pi'_1) &= t + p_{il} \left(M + (1-M)a \sum_{h=1}^{r-1} p_{[i][h]} r^b \right) \\ &\quad + p_{ik} \left(M + (1-M)a \sum_{h=1}^{r-1} p_{[i][h]} + p_{il} (r+1)^b \right). \end{aligned} \quad (8)$$

Thus, we can obtain

$$C_{ik}(\pi'_1) - C_{il}(\pi_1) = (1-M)a \sum_{h=1}^{r-1} p_{[i][h]} r^b p_{ik} p_{il} \left(\frac{1 - a^{p_{ik}} (r+1/r)^b}{p_{ik}} - \frac{1 - a^{p_{il}} (r+1/r)^b}{p_{il}} \right). \quad (9)$$

From $p_{ik} \leq p_{il}$ and Lemma 1, it is easy to obtain that $C_{ik}(\pi'_1) - C_{il}(\pi_1) \geq 0$. Hence, we have $C_{il}(\pi_1) \leq C_{ik}(\pi'_1)$.

Suppose J_{ij} is the first job in S_2 , we have

$$\begin{aligned} C_{ij}(\pi_1) &= C_{il}(\pi_1) + p_{ij} \left(M + (1-M)a \sum_{h=1}^{r-1} p_{[i][h]} + p_{il} + p_{ik} (r+2)^b \right), \\ C_{ij}(\pi'_1) &= C_{ik}(\pi'_1) + p_{ij} \left(M + (1-M)a \sum_{h=1}^{r-1} p_{[i][h]} + p_{il} + p_{ik} (r+2)^b \right). \end{aligned} \quad (10)$$

From $C_{il}(\pi_1) \leq C_{ik}(\pi'_1)$, we have $C_{ij}(\pi_1) \leq C_{ij}(\pi'_1)$ and $C_{iu}(\pi_1) \leq C_{iu}(\pi'_1)$ for any job J_{iu} in S_2 .

Repeating the job interchange argument for all jobs in each group not sequenced in the SPT order yields part (i) of Theorem 1 and we complete the proof of Theorem 1.

From Theorem 1, the problem $1|GT, s_i, p_{[i][r]}^A = p_{[i][r]} (M + (1-M)a^{t_{[i][r]}} r^b)|C_{\max}$ can be solved by using Algorithm 1.

Algorithm 1

Step 1. Jobs in each group are scheduled in nondecreasing order of the normal processing time (the SPT order)

Step 2. Groups are scheduled in any order

We address application of Algorithm 1 in the following example.

Example 1. There are five jobs classified into two groups G_1 and G_2 . $M = 0.5$, $a = 1/2$, $b = -1$, $G_1 = \{J_{11}, J_{12}\}$, $s_1 = 8$,

$p_{11} = 2$, $p_{12} = 6$, $G_2 = \{J_{21}, J_{22}, J_{23}\}$, $s_2 = 4$, $p_{21} = 10$, $p_{22} = 5$, and $p_{23} = 2$.

For the problem $1|GT, s_i, p_{[i][r]}^A = p_{[i][r]} (M + (1-M)a^{t_{[i][r]}} r^b)|C_{\max}$, by Algorithm 1, we solve this example as follows:

Step 1. In group G_1 , the optimal job sequence is $J_{11} \rightarrow J_{12}$. In group G_2 , the optimal job sequence is $J_{23} \rightarrow J_{22} \rightarrow J_{21}$.

Step 2. Groups are scheduled in any order. Therefore, the optimal schedule is $J_{11} \rightarrow J_{12} \rightarrow J_{23} \rightarrow J_{22} \rightarrow J_{21}$ or $J_{23} \rightarrow J_{22} \rightarrow J_{21} \rightarrow J_{11} \rightarrow J_{12}$.

Hence, the minimum makespan is as follows:

$$C_{\max} = s_1 + p_{[1][2]}^A + p_{[1][2]}^A + s_2 + p_{[2][1]}^A + p_{[2][2]}^A + p_{[2][3]}^A = 27.20. \quad (11)$$

4.2. Total Completion Time Minimization

Theorem 2. For the $1|GT, s_i, p_{[i][r]}^A = p_{[i][r]} (M + (1-M)a^{t_{[i][r]}} r^b)|\sum C_{ij}$ problem, the optimal schedule is obtained: (i) the job sequence in each group is in the smallest normal processing time order (SPT) and (ii) the groups can be sequenced in nondecreasing order of θ_i , where

$$\theta_i = \frac{s_i + \sum_{l=1}^{n_i} p_{i[l]} \left(M + (1-M)a \sum_{h=1}^{l-1} p_{i[h]} r^b \right)}{n_i}. \quad (12)$$

Proof. We still use the notation of Theorem 1 to prove the optimal job sequence in each group. From the proof of Theorem 1, we have $C_{ik}(\pi_1) \leq C_{il}(\pi'_1)$ and $C_{il}(\pi_1) \leq C_{ik}(\pi'_1)$. Hence, it is obtained that $C_{ik}(\pi_1) + C_{il}(\pi_1) \leq C_{il}(\pi'_1) + C_{ik}(\pi'_1)$ and π_1 dominates π'_1 . Therefore, we complete the proof of part (i).

Next, we determine the optimal group sequence. Let π_2 and π'_2 be two schedules where the difference between π_2 and π'_2 is a pairwise interchange of two adjacent groups G_i and G_j and job sequence in each group of π_2 and π'_2 is in SPT order. That is, $\pi_2 = [\delta_1, G_i, G_j, \delta_2]$ and $\pi'_2 = [\delta_1, G_j, G_i, \delta_2]$, where δ_1 and δ_2 are the partial sequences. Furthermore, denote t as the completion time of the last job in δ_1 . To prove π_2 dominates π'_2 , it suffices to address $\sum_{l=1}^{n_i} C_{i[l]}(\pi_2) + \sum_{l=1}^{n_j} C_{j[l]}(\pi_2) \leq \sum_{l=1}^{n_j} C_{j[l]}(\pi'_2) + \sum_{l=1}^{n_i} C_{i[l]}(\pi'_2)$. By definition, the completion times of jobs of groups G_i and G_j in π_2 are given by

$$\begin{aligned} C_{i[1]}(\pi_2) &= t + s_i + p_{i[1]}, \\ C_{i[2]}(\pi_2) &= t + s_i + p_{i[1]} + p_{i[2]}(M + (1 - M)a^{p_{i[1]}}2^b), \\ C_{i[n_i]}(\pi_2) &= t + s_i + \sum_{l=1}^{n_i} p_{i[l]}(M + (1 - M)a^{\sum_{h=1}^{l-1} p_{i[h]}} l^b), \\ C_{j[1]}(\pi_2) &= C_{i[n_i]}(\pi_2) + s_j + p_{j[1]}, \\ C_{j[n_j]}(\pi_2) &= C_{i[n_i]}(\pi_2) + s_j + \sum_{l=1}^{n_j} p_{j[l]}(M + (1 - M)a^{\sum_{h=1}^{l-1} p_{j[h]}} l^b). \end{aligned} \quad (13)$$

The completion times of jobs of groups G_j and G_i in π'_2 are given by

$$\begin{aligned} C_{j[1]}(\pi'_2) &= t + s_j + p_{j[1]}, \\ C_{j[2]}(\pi'_2) &= t + s_j + p_{j[1]} + p_{j[2]}(M + (1 - M)a^{p_{j[1]}}2^b), \\ C_{j[n_j]}(\pi'_2) &= t + s_j + \sum_{l=1}^{n_j} p_{j[l]}(M + (1 - M)a^{\sum_{h=1}^{l-1} p_{j[h]}} l^b), \\ C_{i[1]}(\pi'_2) &= C_{j[n_j]}(\pi'_2) + s_i + p_{i[1]}, \\ C_{i[n_i]}(\pi'_2) &= C_{j[n_j]}(\pi'_2) + s_i + \sum_{l=1}^{n_i} p_{i[l]}(M + (1 - M)a^{\sum_{h=1}^{l-1} p_{j[h]}} l^b). \end{aligned} \quad (14)$$

Thus, we have

$$\begin{aligned} &\sum_{l=1}^{n_j} C_{j[l]}(\pi'_2) + \sum_{l=1}^{n_i} C_{i[l]}(\pi'_2) - \sum_{l=1}^{n_i} C_{i[l]}(\pi_2) - \sum_{l=1}^{n_j} C_{j[l]}(\pi_2) \\ &= n_i \left(s_j + \sum_{l=1}^{n_j} p_{j[l]}(M + (1 - M)a^{\sum_{h=1}^{l-1} p_{j[h]}} l^b) \right) \\ &\quad - n_j \left(s_i + \sum_{l=1}^{n_i} p_{i[l]}(M + (1 - M)a^{\sum_{h=1}^{l-1} p_{i[h]}} l^b) \right). \end{aligned} \quad (15)$$

We have $\sum_{l=1}^{n_i} C_{i[l]}(\pi_2) + \sum_{l=1}^{n_j} C_{j[l]}(\pi_2) \leq \sum_{l=1}^{n_j} C_{j[l]}(\pi'_2) + \sum_{l=1}^{n_i} C_{i[l]}(\pi'_2)$, if and only if

$$\frac{s_i + \sum_{l=1}^{n_i} p_{i[l]}(M + (1 - M)a^{\sum_{h=1}^{l-1} p_{i[h]}} l^b)}{n_i} \leq \frac{s_j + \sum_{l=1}^{n_j} p_{j[l]}(M + (1 - M)a^{\sum_{h=1}^{l-1} p_{j[h]}} l^b)}{n_j}. \quad (16)$$

This completes the proof.

From Theorem 2, the problem $1|GT, s_i, p_{[i][r]}^A = p_{[i][r]}(M + (1 - M)a^{t_{[i][r]}} r^b) | \sum C_{ij}$ can be solved by the following algorithm:

Algorithm 2

Step 1. Jobs in each group are scheduled in a nondecreasing order of the normal processing time (the SPT order).

Step 2. To each group, calculate

$$\theta_i = \frac{s_i + \sum_{l=1}^{n_i} p_{i[l]}(M + (1 - M)a^{\sum_{h=1}^{l-1} p_{i[h]}} l^b)}{n_i}, \quad (17)$$

$$i = 1, 2, \dots, m.$$

Step 3. Groups are scheduled in the nondecreasing order of θ_i .

Obviously, it is easy to show that the total time for Algorithm 2 is $O(n \log n)$.

We show application of Algorithm 2 in the following example.

Example 2. As seen in Example 1, we change the objective to the total completion time minimization. By Algorithm 2, we solve the problem as follows:

Step 1: In group G_1 , the optimal job sequence is $J_{11} \rightarrow J_{12}$. In group G_2 , the optimal job sequence is $J_{23} \rightarrow J_{22} \rightarrow J_{21}$.

Step 2: To groups G_1 and G_2 , calculate

$$\theta_1 = \frac{8 + 2 * (0.5 + 0.5 * (1/2)^0 * 1^{-1}) + 6 * (0.5 + 0.5 * (1/2)^2 * 2^{-1})}{2} = 6.69,$$

$$\theta_2 = \frac{4 + 2 * (0.5 + 0.5 * (1/2)^0 * 1^{-1}) + 5 * (0.5 + 0.5 * (1/2)^2 * 2^{-1}) + 10 * (0.5 + 0.5 * (1/2)^7 * 3^{-1})}{3} = 4.61.$$

Therefore, the optimal group sequence is $G_2 \rightarrow G_1$ and the optimal schedule is $J_{23} \rightarrow J_{22} \rightarrow J_{21} \rightarrow J_{11} \rightarrow J_{12}$.

Hence, the minimum total completion time is $\sum C_{ij} = 79.65$.

4.3. Total Weighted Completion Time Minimization. In our model, we consider the group scheduling to minimize the total weighted completion time if jobs in each group satisfy an agreeable condition.

Theorem 3. For the $1|GT, s_i, p_{[i][r]}^A = p_{[i][r]}(M + (1 - M)a^{t_{[i][r]}r^b})| \sum w_{ij}C_{ij}$ problem, if jobs in each group have agreeable weighted normal processing time, i.e., $p_{ij} \leq p_{il}$ implies $w_{ij} \geq w_{il}$, for all jobs J_{ij} and J_{il} in G_i , $i = 1, 2, \dots, m$.

The optimal schedule satisfies the following: (i) the job in each group is in nondecreasing order of p_{ij}/w_{ij} (the WSPT order) and (ii) the groups can be sequenced in the nondecreasing order of φ_i , where

$$\varphi_i = \frac{s_i + \sum_{l=1}^{n_i} p_{i[l]} \left(M + (1 - M)a^{\sum_{h=1}^{l-1} p_{i[h]} l^b} \right)}{\sum_{h=1}^{n_i} w_{il}}. \quad (19)$$

Proof. First, we show the optimal job sequence in each group. Without loss of generality, we still use the notation of Theorem 1. From Theorem 1 and $p_{ik} \leq p_{il}$, we have $w_{ik} \geq w_{il}$. Therefore,

$$\begin{aligned} & w_{ik}C_{ik}(\pi'_1) + w_{il}C_{il}(\pi'_1) - w_{ik}C_{ik}(\pi_1) - w_{il}C_{il}(\pi_1) \\ &= (w_{ik} + w_{il})(p_{il} - p_{ik}) \left(M + (1 - M)a^{\sum_{h=1}^{r-1} p_{i[h]} r^b} \right) + w_{ik}p_{ik} \left(M + (1 - M)a^{\sum_{h=1}^{r-1} p_{i[h]} + p_{il}} (r + 1)^b \right) \\ &\quad - w_{il}p_{il} \left(M + (1 - M)a^{\sum_{h=1}^{r-1} p_{i[h]} + p_{ik}} (r + 1)^b \right) \\ &= (w_{ik} + w_{il})(p_{il} - p_{ik}) \left(M + (1 - M)a^{\sum_{h=1}^{r-1} p_{i[h]} r^b} \right) + w_{ik}p_{ik} \left(M + (1 - M)a^{\sum_{h=1}^{r-1} p_{i[h]} + p_{il}} (r + 1)^b \right) \\ &\quad - w_{ik}p_{ik} \left(M + (1 - M)a^{\sum_{h=1}^{r-1} p_{i[h]} (r + 1)^b} \right) \\ &\quad - w_{il}p_{il} \left(M + (1 - M)a^{\sum_{h=1}^{r-1} p_{i[h]} + p_{ik}} (r + 1)^b \right) + w_{il}p_{il} \left(M + (1 - M)a^{\sum_{h=1}^{r-1} p_{i[h]} (r + 1)^b} \right) \\ &\quad + (w_{ik}p_{ik} - w_{il}p_{il}) \left(M + (1 - M)a^{\sum_{h=1}^{r-1} p_{i[h]} (r + 1)^b} \right) \\ &\geq (w_{ik}p_{il} - w_{il}p_{ik}) \left(M + (1 - M)a^{\sum_{h=1}^{r-1} p_{i[h]} (r + 1)^b} \right) + w_{ik}p_{ik} \left(M + (1 - M)a^{\sum_{h=1}^{r-1} p_{i[h]} + p_{il}} (r + 1)^b \right) \\ &\quad - w_{ik}p_{ik} \left(M + (1 - M)a^{\sum_{h=1}^{r-1} p_{i[h]} (r + 1)^b} \right) \\ &\quad - w_{il}p_{il} \left(M + (1 - M)a^{\sum_{h=1}^{r-1} p_{i[h]} + p_{ik}} (r + 1)^b \right) + w_{il}p_{il} \left(M + (1 - M)a^{\sum_{h=1}^{r-1} p_{i[h]} (r + 1)^b} \right) \\ &= (w_{ik}p_{il} - w_{il}p_{ik}) \left(M + (1 - M)a^{\sum_{h=1}^{r-1} p_{i[h]} (r + 1)^b} \right) + (w_{ik}h'(\xi_1) - w_{il}h'(\xi_2))p_{ik}p_{il}. \end{aligned} \quad (20)$$

From Lemma 2, it is easy to show $h'(\xi_1) \geq h'(\xi_2)$. Since $p_{ik} \leq p_{il}$ and $w_{ik} \geq w_{il}$, it can be obtained $w_{ik}C_{ik}(\pi_1) + w_{il}C_{il}(\pi_1) \leq w_{ik}C_{ik}(\pi'_1) + w_{il}C_{il}(\pi'_1)$. From Theorem 1, we have $C_{iu}(\pi_1) \leq C_{iu}(\pi'_1)$ for any job J_{iu} in S_2 . Hence, the job

completion time of S_2 in π_1 is earlier than the same job completion time of S_2 in π'_1 . Thus, the job in each group is in nondecreasing order of p_{ij}/w_{ij} (the WSPT order) in the optimal schedule. This completes the proof of part (i).

Next, determine the optimal group sequence. We still use the notation of Theorem 2. To prove π_2 dominates π'_2 , it suffices to show that

$$\begin{aligned} & \sum_{l=1}^{n_i} w_{i[l]} C_{i[l]}(\pi_2) + \sum_{l=1}^{n_j} w_{j[l]} C_{j[l]}(\pi_2) \leq \sum_{l=1}^{n_j} w_{j[l]} C_{j[l]}(\pi'_2) + \sum_{l=1}^{n_i} w_{i[l]} C_{i[l]}(\pi'_2), \\ \text{For } & \sum_{l=1}^{n_j} w_{j[l]} C_{j[l]}(\pi'_2) + \sum_{l=1}^{n_i} w_{i[l]} C_{i[l]}(\pi'_2) - \sum_{l=1}^{n_i} w_{i[l]} C_{i[l]}(\pi_2) - \sum_{l=1}^{n_j} w_{j[l]} C_{j[l]}(\pi_2) \\ & = \left(\sum_{l=1}^{n_i} w_{i[l]} \right) \left(s_j + \sum_{l=1}^{n_j} p_{j[l]} \left(M + (1-M)a^{\sum_{h=1}^{l-1} p_{j[h]} l^b} \right) \right) - \left(\sum_{l=1}^{n_j} w_{j[l]} \right) \left(s_i + \sum_{l=1}^{n_i} p_{i[l]} M + (1-M)a^{\sum_{h=1}^{l-1} p_{i[h]} l^b} \right). \end{aligned} \quad (21)$$

We have $\sum_{l=1}^{n_i} w_{i[l]} C_{i[l]}(\pi_2) + \sum_{l=1}^{n_j} w_{j[l]} C_{j[l]}(\pi_2) \leq \sum_{l=1}^{n_j} w_{j[l]} C_{j[l]}(\pi'_2) + \sum_{l=1}^{n_i} w_{i[l]} C_{i[l]}(\pi'_2)$, if and only if

$$\frac{s_i + \sum_{l=1}^{n_i} p_{i[l]} \left(M + (1-M)a^{\sum_{h=1}^{l-1} p_{i[h]} l^b} \right)}{\sum_{l=1}^{n_i} w_{i[l]}} \leq \frac{s_j + \sum_{l=1}^{n_j} p_{j[l]} \left(M + (1-M)a^{\sum_{h=1}^{l-1} p_{j[h]} l^b} \right)}{\sum_{l=1}^{n_j} w_{j[l]}}. \quad (22)$$

Hence, the optimal groups can be sequenced in the nondecreasing order of φ_i , and this completes the proof of the theorem.

From Theorem 3, if w_{ij} and p_{ij} for jobs in each group have agreeable weighted normal processing time, the problem $1|GT, s_i, p_{[i][r]}^A = p_{[i][r]}(M + (1-M)a^{\sum_{h=1}^{l-1} p_{i[h]} l^b})| \sum w_{ij} C_{ij}$ can be solved by the following algorithm:

Algorithm 3

Step 1. Jobs in each group are scheduled in nondecreasing order of p_{ij}/w_{ij} (the WSPT order).

Step 2. Calculate

$$\varphi_i = \frac{s_i + \sum_{l=1}^{n_i} p_{i[l]} \left(M + (1-M)a^{\sum_{h=1}^{l-1} p_{i[h]} l^b} \right)}{\sum_{l=1}^{n_i} w_{i[l]}}, \quad (23)$$

$i = 1, 2, \dots, m.$

Step 3. Groups are scheduled in nondecreasing order of φ_i .

Obviously, it is easy to show that the total time for Algorithm 3 is $O(n \log n)$.

We present application of Algorithm 3 in the following example.

Example 3. Also turn to the Example 1, change the objective to the total weighted completion time minimization ($w_{11} = 2, w_{12} = 1, w_{21} = 1, w_{22} = 2$, and $w_{23} = 4$). Obviously, jobs in each group of this instance have the agreeable weighted normal processing times in Theorem 3. By Algorithm 3, we solve the problem as follows:

Step 1: In group G_1 , the optimal job sequence is $J_{11} \rightarrow J_{12}$. In group G_2 , the optimal job sequence is $J_{23} \rightarrow J_{22} \rightarrow J_{21}$.

Step 2: To groups G_1 and G_2 , calculate

$$\begin{aligned} \varphi_1 &= \frac{8 + 2 * (0.5 + 0.5 * (1/2)^0 * 1^{-1}) + 6 * (0.5 + 0.5 * (1/2)^2 * 2^{-1})}{3} = 4.46, \\ \varphi_2 &= \frac{4 + 2 * (0.5 + 0.5 * (1/2)^0 * 1^{-1}) + 5 * (0.5 + 0.5 * (1/2)^2 * 2^{-1}) + 10 * (0.5 + 0.5 * (1/2)^7 * 3^{-1})}{7} = 1.98. \end{aligned} \quad (24)$$

Therefore, the optimal group sequence is $G_2 \rightarrow G_1$ and the optimal schedule is $J_{23} \rightarrow J_{22} \rightarrow J_{21} \rightarrow J_{11} \rightarrow J_{12}$.

Hence, the minimum total weighted completion time is $\sum w_{ij} C_{ij} = 122.28$.

5. Conclusions

We study new group scheduling models with position-dependent and time-dependent learning effect. The actual processing time of a job is a function of the total normal processing time and the total number of jobs scheduled ahead of it, which is motivated by DeJong's learning schedule model. We show that the models can be solved polynomially when the objectives are the makespan minimization and the total completion time minimization. The total weighted completion time minimization is proved to be solved in polynomial time under an agreeable condition. It is suggested for future research to extend to other more practical learning effect scheduling models in the other machine environment, including multimachine and job-shop settings.

Data Availability

All data generated or analysed during this study are included in this article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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