Research Article

Model of the Secondary Path between the Input Voltage and the Output Force of an Active Engine Mount on the Engine Side

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The active engine mount (AEM) provides an effective solution to improve the acoustic and vibration comfort of a car. The same AEM can be installed for different engines and different vehicle bodies and attenuates the engine vibration, which is one of the most pressing challenges. To study this problem, this paper develops a mathematical model of a secondary path between the input voltage and output force of the AEM on the engine side considering the frequency-dependent characteristic of the stiffness, which includes the structure parameters of the AEM as well as the dynamics of the actuator, the fluid in the inertia track, the foundation (vehicle body), and the attenuated vibrating object (AEM preload or engine). The proposed model is validated by three test cases without vibration excitation, which are performed with different AEM preloads and foundations. The AEM is considered as an active part and passive part, the mass of which is determined experimentally. Parameter effect on the dynamic characteristics of the secondary path of the AEM is studied based on three tests and a numerical simulation.

1. Introduction

The engine is the main excitation source of the vehicle. Isolating the vehicle body from engine vibration is one of the most pressing challenges. Compared with the passive engine mount or semiactive engine mount, the active engine mount (AEM) achieves a significant performance improvement in decreasing the wide frequency band vibration [1]. Several concepts for AEM have been proposed [2–5]. The common AEM [6–9] is similar to the conventional hydraulic engine mount (HEM) with an actuator, such as a solenoid or a voice (moving) coil actuator, as shown in Figure 1; the AEM is excited by a controller, and it generates active output forces to suppress the high-frequency engine vibrations (no less than 20 Hz). The secondary path between the input voltage of the AEM and the output force of the AEM on the engine side can be used to study the dynamic characteristics of the AEM and improve the performance of the model-based AEM control. The

existing research methods of the secondary path can be divided into two categories: identification and theoretical modelling.

Identification, which is also termed experimental modelling, is a mathematical model derived from measurements. Harmonic identification [10], LMS- or FBLMS-based finite impulse response (FIR) identification [11], and subspace identification [12] are applied in the secondary path research to control. Seba et al. [12] identified the magnitude and phase plot of the frequency response of the secondary path between the force point actuator inputs and the accelerometer outputs, which was applied to the feedforward control. Buttelmann et al. [13] identified the Bode plot of the secondary path between the active absorber and the disturbance sensor on the chassis side of the engine mount. Hausberg et al. [14] identified the secondary path between the active engine mount and the accelerometer located on the chassis side of the engine mount for the feedforward control. Fakhari et al. [15, 16]

FIGURE 1: Schematic diagram of the electromagnetic AEM.

identified the transfer function of the secondary path between the input current of the ARM and the transmitted force on the engine side by curve fitting, which is expressed by the poles and zeros of the transfer function. Identification can be applied to the control strategy, but it is difficult to study the parameter effect on the secondary path over a short time.

However, there are fewer studies on the analytical analysis of the model of the secondary path. The theoretical model of the secondary path can be obtained by applying equations such as those derived from physics. Lee and Lee [17] proposed the model of the transfer function between the control voltage and the position of the electromagnetic actuator. Mansour et al. [18] proposed the secondary path between the actuator displacement and the active force of the AEM on the engine side. Lee and Lee [19] considered that the actuator dynamics was included in the secondary path, and the transmitted forces to the engine and chassis were not equal. Then, the theoretical model of the transfer function between the control force of the electromagnetic actuator and the output force of the AEM on the engine side was proposed, which was employed by Fakhari and Ohadi [20] for robust control. Considering the frequency-dependent characteristic of the volumetric stiffness of the main liquid chamber and the complex stiffness of the main rubber spring, Hausberg et al. [21] proposed the model of the transfer function between the control voltage and the output force of the AEM on the engine side. The transfer function of the secondary path contains fluid dynamics in the inertia track and actuator dynamics, structural parameters of the cross-sectional area of the fluid channel, decouple plate, and main fluid chamber bulking.

This work aims to find a solution for the secondary path of the AEM control in a situation with different vibrating objects (AEM preload or engine) and different foundations (chassis). The primary focus of this article is to analytically derive expressions for the secondary path between the input voltage and the output force of the AEM on the engine side. Considering the frequency-dependent characteristic of stiffness, the derived model

includes the structure parameters of the AEM, dynamics of the actuator, dynamics of the fluid in the inertia track, dynamics of the attenuated vibrating object (AEM preload or engine), and dynamics of the foundation (chassis). To identify the mass of the active part and passive part of the AEM and verify the feasibility of the derived model of the secondary path, three experimental investigations are performed, and the effects of the experimental temperatures on the experimental data are analysed. The effect of key parameters related to the proposed model on the dynamic characteristics of the AEM of the secondary path is also discussed.

The article is organized as follows. Section 2 shows the analytic derivation of the model of the secondary path. Section 3 analyses the effects of the experimental temperatures on the experimental data, identifies the mass of the active part and passive part of the AEM, and verifies the model feasibility. Section 4 presents the analysis of the parameter effects, which is followed by the conclusions in Section 5.

2. Model

2.1. Structure of the Electromagnetic AEM. A typical electromagnetic AEM in Figure 1 is selected for this study [1, 21], which is mainly composed of a main rubber spring, two fluid chambers, an inertia track, a diaphragm, and a moving coil (voice coil) actuator. When the engine operates at low frequencies (engine starting, less than 20 Hz), the electromagnetic AEM works with the moving coil (voice coil) actuator turned off, which can be considered a passive traditional hydraulic engine mount (HEM). When the engine operates at high frequencies (no less than 20 Hz), the moving coil is energized. Then, the moving coil (voice coil) actuator produces a Lorentz force and drives the decouple plate and diaphragm up and down. The main rubber spring is deformed by the change in hydraulic pressure in the main liquid chamber, which isolates the vibration transmitted between the engine and the vehicle body. The displacement of the engine connection of the AEM is different from the displacement of the other parts of the AEM. Then, the AEM can be considered the active part of the AEM (engine side) and passive part of the AEM (vehicle body side). The engine connection of the AEM and the moving equivalent main rubber spring are considered the active part of the AEM. The active part of the AEM and the preload (engine or attenuated vibrating object) on the AEM are considered the active part of the AEM system. The remaining parts of the AEM are considered the passive part of the AEM. The passive part of the AEM and foundation (vehicle body) are considered the passive part of the AEM system.

2.2. Lumped Model of the Electromagnetic AEM. The lumped model of electromagnetic AEM is shown in Figure 2, which includes the electromagnetic and actuator model, fluid

model, elastomeric model, dynamic model of fluid in the inertia track, active part, and passive part.

2.2.1. Elastomeric Model. The stiffness and damping of the elastic rubber are frequency-dependent characteristics. Elastic rubber, such as the main rubber spring of the HEM, is modeled using a spring in parallel to a damper (Kelvin-Voigt Model) [22–24], which is consistent for the dynamic stiffness over frequency, but the damping characteristics are only an estimation. A four-parameter complex stiffness model [21, 25] that models a linearized form with two dampers and one spring in parallel to another spring, which neglects the amplitude dependency considering small amplitudes and the linear model, is consistent for the dynamic stiffness and damping characteristics over frequency. The four-parameter complex stiffness main rubber spring model can be equivalent to a frequency-dependent stiffness and damping two-parameter model, which is expressed by the following equation:

$$k_{eq,r} = k_1 + \frac{c_1 k_2 s + c_1 c_2 s^2}{k_2 + (c_1 + c_2)s} = k_r^*(\omega) + i\omega c_r^*(\omega), \quad (1)$$

where $k_r^*(\omega)$ and $c_r^*(\omega)$ are the frequency-dependent equivalent stiffness and damping of the main rubber spring of AEM, respectively. k_1 and k_2 are the springs of the four-parameter main rubber spring model, and c_1 and c_2 are the dampers of the four-parameter main rubber spring model. The complex stiffness of the moving voice coil actuator and the volumetric stiffness of the main liquid chamber are also expressed by (1).

2.2.2. Fluid Model. The change in hydraulic pressure in the main liquid chamber is caused by the up and down movement of the moving coil actuator. Some fluid in the main liquid chamber will flow to the compensation chamber through the inertia track, and some fluid in the main liquid chamber will contribute to the bulging volume of the main

liquid chamber; the fluid in the main liquid chamber forces the main rubber spring to move up and down and consequently attenuates the vibration from the engine. The fluid is considered incompressible. The continuity equations can be expressed as follows:

$$A_a(\dot{x}_a(t) - \dot{x}_p(t)) = A_r(\dot{x}_e(t) - \dot{x}_p(t)) + A_1\dot{x}_1(t) + A_i\dot{x}_i(t). \quad (2)$$

The volume change of the main liquid chamber causes a pressure change in the chamber. According to the volumetric compliance, the volumetric stiffness of the main liquid chamber can be expressed as follows:

$$k_{eq,1}x_1(t) = p_1(t)A_1. \quad (3)$$

According to Newton's law, the dynamics model of the fluid in the inertia track is given by

$$(p_1(t) - p_2(t))A_i = m_i \ddot{x}_i(t) + d_i R_i A_i^2 \dot{x}_i(t), \quad (4)$$

where A_r is the equivalent piston area of the main rubber spring. $x_e(t)$ is the displacement of the active part of the AEM. $p_1(t)$ is the pressure of the main fluid chamber. The volumetric stiffness of the compensation chamber is far smaller than that of the main liquid chamber [6, 10]. Then, the volumetric stiffness of the compensation chamber $p_2(t)$ can be neglected. $x_1(t)$ and A_1 are the displacement and equivalent area of the main fluid chamber bulking area, respectively. $k_{e,1}$ is the equivalent volumetric stiffness of the main liquid chamber at the harmonic voltage excitation.

$$d_i = R_i A_i^2, \quad (5)$$

where A_i is the cross-sectional area of the inertia track. m_i , R_i , and $x_i(t)$ are the mass, damping, and displacement of the fluid in the inertia track, respectively. m_i is expressed by the following equation:

$$m_i = \rho_i A_i l_i, \quad (6)$$

where ρ_i and l_i are the fluid density in the inertia track and length of the inertia track.

2.2.3. Electromagnetic and Actuator Model. The dynamic characteristics of the electromagnetic AEM secondary path are studied under harmonic voltage excitation. The AEM is in an equilibrium state under preload, and a harmonic voltage is applied to the circuit of the moving coil actuator. The harmonic voltage excitation can be expressed as follows:

$$u(t) = U \sin(\omega t), \quad (7)$$

where U is the amplitude of the harmonic voltage and ω is the circle frequency of the harmonic voltage, which is equal to the harmonic excitation applied on the AEM.

The moving (voice) coil actuator subsystem consists of a permanent magnet and a moving coil [26–30]. The electrical differential equation of the voltage applied on the moving coil actuator can be expressed as follows:

$$u(t) = K_M \dot{x}_a(t) + R_i(t) + L \frac{d_i(t)}{dt}, \quad (8)$$

where $u(t)$, R , L , and $x_a(t)$ are the input voltage applied to the moving coil actuator, electrical resistance of the moving coil, inductance of the moving coil, and position of the moving diaphragm, respectively.

The magnetic flux generated by the permanent magnet interacts with the current in the moving coil, and the moving coil actuator produces a Lorentz force, which can be expressed as follows:

$$f_a(t) = Bli(t) = K_M i(t), \quad (9)$$

where B , l , and $i(t)$ are the field density, wire length, and coil current, respectively. K_M is the force factor of the moving coil actuator.

The moving coil actuator moves up and down under the interaction of the Lorentz force and main liquid chamber pressure. The differential equation of the mechanical motion of the moving coil actuator is given by

$$\begin{aligned} f_a(t) - P_1(t)A_a &= m_a(\ddot{x}_a(t) - \ddot{x}_p(t)) + c_a^*(\dot{x}_a(t) - \dot{x}_p(t)) \\ &\quad + k_a^*(x_a(t) - x_p(t)), \end{aligned} \quad (10)$$

where m_a is the equivalent mass of the moving voice coil actuator (including the moving coil, coil carriage, decouple plate, and diaphragm). k_a^* and c_a^* are the stiffness and damping of the moving voice coil actuator, respectively. A_a and $x_a(t)$ are the equivalent area and vertical displacement of the decouple plate and diaphragm, respectively. $x_p(t)$ is the vertical displacement of the passive part of the AEM.

2.2.4. Dynamics Model of the Main Rubber Spring and Chassis. The active part and passive part of the AEM system are considered the mass-spring-damper system. The dynamics of the active part of the AEM (or system) can be written by Newton's law as follows:

$$\begin{aligned} m_e \ddot{x}_e(t) &= p_1(t)A_r - c_r^*(\dot{x}_e(t) - \dot{x}_p(t)) - k_r^*(x_e(t) \\ &\quad - x_p(t)) = f_e(t). \end{aligned} \quad (11)$$

The dynamics of the passive part of AEM (or system) can be expressed as follows:

$$\begin{aligned} m_p \ddot{x}_p(t) &= c_r^*(\dot{x}_e(t) - \dot{x}_p(t)) + k_r^*(x_e(t) - x_p(t)) \\ &\quad - p_1(t)(A_{di} - A_a) + c_a^*(\dot{x}_a(t) - \dot{x}_p(t)) \\ &\quad + k_a^*(x_a(t) - x_p(t)) - f_a(t) \\ &= c_r^*(\dot{x}_e(t) - \dot{x}_p(t)) + k_r^*(x_e(t) - x_p(t)) \\ &\quad - p_1(t)A_{di} - m_a \ddot{x}_a(t), \end{aligned} \quad (12)$$

where m_e is the equivalent mass of the active part of the AEM (or AEM system) at the engine connection and m_p is the equivalent mass of the passive part of the AEM (or AEM system) at the attachment part. c_r^* and k_r^* are the dynamic stiffness and damping of the main rubber spring, respectively. A_{di} is the equivalent area of the internal diameter of the AEM on the decouple plate and diaphragm.

2.3. Model of the Secondary Path. From equation (1) to equation (12), the transfer functions of the secondary path between the input voltage and the output force of the AEM on the engine side can be expressed in the Laplace domain as follows:

$$\frac{f_e(s)}{u(s)} = \frac{D_c EF(1 - k_{eq,r}) + k_{eq,r}(D_{kr,e} - 1)(D_{km,m,a}s^2 - D_c E(-\varepsilon_{d,r} + D_{kr,e} + \varepsilon_{a,r}D_{ma}s^2))}{D_c F}, \quad (13)$$

$$\left\{ \begin{array}{l} \varepsilon_{a,r} = \frac{A_a}{A_r}, \\ \varepsilon_{1,r} = \frac{A_1}{A_r}, \\ \varepsilon_{i,r} = \frac{A_i}{A_r}, \\ \varepsilon_{d,r} = \frac{A_{di}}{A_r}, \end{array} \right. \quad (14)$$

$$\left\{ \begin{array}{l} D_a = k_{eq,a} + m_a s^2 + \frac{k_m^2 s}{Ls + R}, \\ D_c = Ls + R, \\ D_{ka,a} = \frac{k_{eq,a}}{D_a}, \\ D_{km,a} = \frac{k_m}{D_a}, \\ D_{m,a} = \frac{m_a}{D_a}, \\ D_{km,m,a} = \frac{k_m m_a}{D_a}, \\ D_{ka,m,a} = \frac{k_{eq,a} m_a}{D_a}, \\ D_{ka,km,m,a} = \frac{k_{eq,a} k_m m_a}{D_a}, \end{array} \right. \quad (15)$$

$$\left\{ \begin{array}{l} D_e = k_{eq,r} + m_e s^2, \\ D_{kr,e} = \frac{k_{eq,r}}{k_{eq,r} + m_e s^2}, \\ D_{k1,i} = \frac{k_{eq,1}}{sd_i + m_i s^2}, \end{array} \right. \quad (16)$$

$$\left\{ \begin{array}{l} k_{eq,r} = k_1 + \frac{c_1 k_2 s + c_1 c_2 s^2}{k_2 + (c_1 + c_2)s} = k_r^*(\omega) + i\omega c_r^*(\omega), \\ k_{eq,a} = k_3 + \frac{c_3 k_4 s + c_3 c_4 s^2}{k_4 + (c_3 + c_4)s} = k_a^*(\omega) + i\omega c_a^*(\omega), \\ k_{eq,1} = k_5 + \frac{c_5 k_6 s + c_5 c_6 s^2}{k_6 + (c_5 + c_6)s} = k_1^*(\omega) + i\omega c_1^*(\omega), \end{array} \right. \quad (17)$$

$$E = \frac{(((D_{kr,e} + \varepsilon_{a,r} - 1)D_{km,m,a} - \varepsilon_{a,r}D_{ka,km,m,a})s^2)(k_{eq,r}(1 - D_{kr,e}) + (m_c + D_{ka,m,a})s^2) + \varepsilon_{a,r}D_{km,a}}{D_c((\varepsilon_{1,r}^2/k_{eq,1}) + (\varepsilon_{i,r}D_{k1,i}/A_r k_{eq,1}) + (\varepsilon_{a,r}^2/D_a) + (1/D_e) + (((D_{kr,e} - \varepsilon_{d,r} + \varepsilon_{a,r}D_{m,a}s^2)(1 - D_{ka,a})(\varepsilon_{a,r} - 1))/((k_{eq,r}(1 - D_{kr,e}) + (m_c + D_{ka,m,a})s^2))))}, \quad (18)$$

$$F = k_{eq,r}(1 - D_{kr,e}) + (m_c + D_{ka,m,a})s^2. \quad (19)$$

where (14) is the structure factor equation, (15) is the dynamics factor of the moving coil actuator, and (16) is the dynamics factor equation of fluid in the inertia track and the dynamics factor equation of the active part of the AEM (or AEM system). $k_{eq,r}$, $k_{eq,a}$, and $k_{eq,1}$ in (17) are the complex stiffness of the main rubber spring, complex stiffness of the moving coil actuator, and volumetric complex stiffness of the main liquid chamber. Equation (18) is the coupling dynamics factors of the active part and passive part of the AEM system. Equation (19) is the coupling factors of the AEM system.

3. Experimental and Simulation Validation

3.1. Experimental Setup. As shown in Figure 3, an experimental setup is performed to validate the proposed model of the secondary path between the input voltage and the output force of the AEM on the engine side. The experimental setup includes a computer, an AEM, a signal generator, a power amplifier, an oscilloscope, LMS SCADAS, and a UT320 gauge. The power amplifier can amplify the signal from the signal generator. The oscilloscope measured the voltage signal from the power amplifier, and the AEM is driven by

FIGURE 3: Experimental setup for parameter identification and simulation validation.

the voltage. The acceleration signals from the engine connection are acquired at a sampling frequency of 10 kHz by LMS SCADAS. Three test cases are performed considering different preloads on the AEM when there is only an input voltage, and the input voltage varies in steps of 2 Hz between 20 Hz and 50 Hz and steps of 5 Hz between 50 Hz and 250 Hz. Test A is a no-preload test for the AEM test case, which is performed to study the effect of the temperature on the experimental results, identify the mass of the active part of AEM (moving equivalent mass of the main rubber spring and engine connection), and validate the proposed model of the secondary path of AEM. Tests B and C validate the proposed model of the secondary path of AEM.

3.2. Effect of the Temperature and Input Voltage Duration on the Experimental Results. The dynamical behaviours of the rubber [31, 32] and electromagnetic force [33] are sensitive to temperature. A longer input voltage time corresponds to a higher temperature. Then, the effects of the temperature and input voltage duration on the Lorentz force of the moving coil, stiffness and damping of the main rubber spring, and diaphragm should be considered in the test. The three test results have been collected at an ambient temperature of 28°C,

and the surface temperature of the AEM measured by the UT320 gauge does not exceed 40°C. When the frequency of the input voltage exceeds 100 Hz, the maximal surface temperature of the AEM is 35°C. When the input voltage decreases, the surface temperature of the AEM slowly increases.

The relative error is applied to estimate the effect of the temperature and input voltage duration on the experimental results, which can be expressed as follows:

$$\delta_{f,n} = \frac{(A_{f,n} - \overline{A_f})}{\overline{A_f}} \times 100\%, \quad (20)$$

where $\delta_{f,n}$ and $A_{f,n}$ are the relative error and acceleration on the engine connection, respectively. $\overline{A_f}$ is the mean of all accelerations when the frequency is f , and n is the surface temperature of the AEM.

The effects of the temperature of 28–40°C on the accelerations on the engine connection at various frequencies (22 Hz, 30 Hz, 38 Hz, 46 Hz, 65 Hz, 80 Hz, and 95 Hz) are investigated when the input voltage increases. At the ambient temperature of 28°C, once the voltage of a certain frequency is applied on the AEM, the experimental data are collected, and experimental data with a duration of 30 s are collected at a certain temperature.

Figure 4 shows that the relative error of the acceleration first decreases and subsequently increases when the temperature increases from 28°C to 40°C, which indicates that the acceleration first decreases and subsequently increases with the temperature. The acceleration first decreases, which is consistent with the experimental results of the solenoid showing the magnetic flux density and Lorentz force decrease when the current input decreases from 0 to 1 s [34]. The acceleration increases with the temperature from 29°C to 40°C, which is consistent with the experimental results of Xue et al. [33]. The relative errors of the accelerations on the engine connection at various frequencies do not exceed 5% when the temperature increases from 28°C to 40°C. Then, the effects of the temperature or input voltage duration on experimental results can be neglected.

3.3. Parameter Identification and Simulation Validation.

The structural and performance parameter identification methods for the semi-AEM [35, 36], moving coil actuator [37], and AEM [16, 30, 38] mainly include the finite element analysis and experimental evaluation. The elastomeric model is frequency-dependent and preload-dependent [39, 40]. Equations (9) and (15) only show the frequency-dependent characteristic of the elastomeric model. To simulate the secondary path of the AEM with different preloads, the complex stiffness of the main rubber spring, complex stiffness of the moving coil actuator, and volumetric complex stiffness of the main liquid chamber are identified by solving a nonlinear least-square curve fitting according (15). The parameters of complex stiffness $k_{eq,r}$, $k_{eq,a}$, and $k_{eq,l}$ are shown in Table 1. The parameters of the moving coil actuator are obtained by part dismantling and finite element analysis. The area of the part of AEM is obtained by part dismantling, as shown in Table 2.

Figure 5 shows the amplitudes of the secondary path between the acceleration at the engine connection and the input voltage of AEM without preload, and the amplitudes at 2 V, 4 V, 6 V, and 8 V are equal to each other at a certain frequency. When there is no AEM preload, the output force $F(t)$ can be calculated by multiplying the mass of the active part of the system and the measured acceleration at the engine connection. The amplitudes of the secondary path between the output force at the engine connection and the input voltage of AEM are obtained for various excitation frequencies. In addition, there is a phase difference of the secondary path between the output force and the applied voltage of AEM at each excitation frequency. The equivalent mass of the active part and passive part of AEM can be identified by curve fitting according to (12), which is shown in Figure 6. The identified equivalent mass is shown in Table 2. Tests B and C in Figures 7 and 8 show the amplitude ratio and phase difference of the AEM with preloads of 60 N and 1400 N, respectively. Figures 7 and 8 validate the proposed model of the secondary path between the input voltage and the output force of the AEM on the engine side.

The curve shapes of test in this paper are consistent with those of the test for measurement of the amplitude and phase of the secondary path of AEM carried out by Hausberg et al.

FIGURE 4: Relative error of the temperature and input voltage duration on the experimental results.

[24]. Therefore, the difference between simulation and experimental results in this paper is mainly caused by the stiffness of the moving coil actuator in the proposed secondary path model, which is validated by the parameter effect analysis shown in Figure 9(a) in Section 4.2. The measured value of the stiffness of the moving coil actuator is obtained by parameter identification test, which can be validated in the papers [24, 38]. Therefore, there is difference between the measured value obtained by parameter identification test and the actual value $k_{eq,a}$ in the proposed secondary path model. The actual value $k_{eq,a}$ in the proposed secondary path model is influenced by the interaction between liquid in the main fluid chamber, electrical circuit of moving coil actuator, and diaphragm. On the other hand, the amplitude of the simulation change with the different values of $k_{eq,a}$, but the amplitude curve shapes of the simulation do not change with the different values of $k_{eq,a}$. Then the curve shapes of the simulation are consistent with those of the test with preload in this paper. And the proposed secondary path model of AEM is validated by three different tests with different preloads. Therefore, the difference between simulation and experimental results can be negligible.

4. Parameter Effect Analysis

The AEM is mainly used to attenuate the engine vibration. Then, according to equation (13) and the previous three experiments, the effect of the parameters on the amplitude and phase of the secondary path is analysed based on the mass of the active part system, which is 140.1 kg.

4.1. Structure Impact Factor. According to equation (11), a smaller structure impact factor ($\varepsilon_{a,r}$, $\varepsilon_{l,r}$, $\varepsilon_{i,r}$, or $\varepsilon_{d,r}$) corresponds to a larger equivalent piston area of the main rubber spring, a larger output force $f_e(t)$, and larger amplitude and phase of the secondary path. This result is

TABLE 1: Complex stiffness parameters of the AEM.

Parameter (unit, symbol)	Preload = 1400 N	Preload = 60 N	Preload = 0
Spring of main rubber spring 1 (N/mm, k_1)	417.9	358.2	331.6
Spring of main rubber spring 2 (N/mm, k_2)	48.16	39.57	36.81
Damper of main rubber spring 1 (N·s/m, c_1)	259.1	207.9	189.2
Damper of main rubber spring 2 (N·s/m, c_2)	11.16	11.62	11.71
Spring of moving voice coil actuator 1 (N/mm, k_3)	47.28	43.59	42.16
Spring of moving voice coil actuator 2 (N/mm, k_4)	12.6	10.86	10.21
Damper of moving voice coil actuator 1 (N·s/m, c_3)	57.64	55.27	54.37
Damper of moving voice coil actuator 2 (N·s/m, c_4)	1.06	1.084	1.09
Bulge spring of main rubber spring 1 (N/mm, k_5)	180.4	175.4	173.6
Bulge spring of main rubber spring 2 (N/mm, k_6)	272.9	234.7	222.9
Bulge damper of main rubber spring 1 (N·s/m, c_5)	1525.7	1169.4	1070.6
Bulge damper of main rubber spring 2 (N·s/m, c_6)	62.9	66.2	67.3
Passive part of the AEM system (kg)	200	40	11

TABLE 2: Parameters of the AEM.

Parameter	Value	Unit	Symbol
Equivalent mass of active part of AEM	0.1	kg	m_e
Equivalent mass of passive part of AEM	4	kg	m_p
Equivalent mass of moving voice coil actuator	0.1058	kg	m_a
Equivalent piston area of main rubber spring	3872.4	mm ²	A_r
Equivalent bulking area	2362.2	mm ²	A_1
Equivalent area of decouple plate and diaphragm	1683.9	mm ²	A_a
Cross-sectional area of the inertia track	92.5	mm ²	A_i
Equivalent area of internal diameter of the AEM	4473.4	mm ²	A_{di}
Length of the inertia track	328.8	mm	l_i
Fluid density	1000	kg/m ³	ρ_i
Fluid damping in the inertia track	2.8×10^8	N·s/m ⁵	R_i
Moving coil resistance	3.7	Ω	R
Moving coil inductance	0.99	mH	L
Voice coil constant	15.9	kgm/As ²	K_M

FIGURE 5: Amplitudes of the secondary path of AEM without preload.

consistent with the numerical simulation results, as shown in Figure 10. There is a small change in the phase of the secondary path.

4.2. Complex Stiffness. According to (10) and (11), a smaller complex stiffness of the moving coil actuator corresponds to a

larger amplitude of the output force $f_e(t)$ and larger amplitude and phase of the secondary path. This result is consistent with the numerical simulation results, as shown in Figure 9. Figure 9 shows that a smaller spring k_3 corresponds to a larger phase of the secondary path, while a smaller damper c_3 or c_4 corresponds to a smaller phase. The effect of spring k_4 on the phase below 240 Hz is different from that more than 240 Hz.

FIGURE 6: Test A for the mass identification of the active part of AEM and the proposed model validation of the secondary path of AEM.

FIGURE 7: Test B for experimental validation of the secondary path of AEM with a preload of 60 N.

Figure 11 shows that only in the low-frequency zone does the larger spring k_1 of the complex stiffness of the main rubber spring correspond to the larger amplitude and phase of the secondary path of the AEM between the output force and the input voltage. The changes in spring k_2 and damper c_1 and c_2 do not affect the amplitude.

According to equation (1), a larger k_5 corresponds to a larger volumetric complex stiffness of the main liquid chamber, a larger amplitude of the output force of the AEM on the engine side according to (3) and (11), a larger amplitude of the secondary path, and a smaller phase, as shown in Figure 12. The effect of k_6 and c_5 on the amplitude of the output force is similar to that of temperature k_5 . The effect of

c_6 on the phase of the secondary path is similar to that of temperature k_5 .

4.3. Mass. The model of the secondary path of the AEM includes the mass of the moving coil actuator, mass of the passive part of the AEM system, and mass of the active part of the AEM system according to (13). Figure 13(a) shows that the larger mass of the moving coil actuator corresponds to the larger amplitude of the secondary path of the AEM and smaller phase between the applied voltage and the output force of AEM at high frequencies. Figure 13(b) shows that the larger mass of the passive parts of the AEM system

FIGURE 8: Test C for experimental validation of the secondary path of AEM with a preload of 1400 N.

FIGURE 9: Amplitude and phase of the secondary path of AEM with different complex stiffness values of the moving coil actuator. (a) Different k_3 . (b) Different k_4 . (c) Different c_3 . (d) Different c_4 .

FIGURE 10: Amplitude and phase of the secondary path of the AEM with different equivalent piston areas of the main rubber spring.

FIGURE 11: Amplitude and phase of the secondary path of the AEM with different complex stiffness values of the main rubber spring. (a) Different k_1 . (b) Different k_2 , c_1 , and c_2 .

FIGURE 12: Amplitude and phase of the secondary path of the AEM with different volumetric complex stiffness values of the main liquid chamber. (a) Different k_5 . (b) Different k_6 . (c) Different c_5 . (d) Different c_6 .

FIGURE 13: Amplitude and phase of the secondary path of the AEM with different masses. (a) Different moving coil actuators. (b) Different passive parts of the system. (c) Different active parts and passive parts of the system.

corresponds to the smaller amplitude of the secondary path of the AEM below 60 Hz. The change in mass of the passive parts of the AEM system does not affect the amplitude of

output force more than 60 Hz or the phase from 20 Hz to 250 Hz. The elastomeric model is preload-dependent, so the complex stiffness of the main rubber spring, moving coil

FIGURE 14: Amplitude and phase of the secondary path of the AEM with different magnetic fields and coils. (a) Different electrical resistances of the moving coil. (b) Different inductances of the moving coil. (c) Different force factors.

actuator, and main liquid chamber varies with different masses of the active part of the system. Figure 13(c) shows the amplitude and phase of the secondary path of the AEM for different active parts of the system, passive parts of the system, and different complex stiffnesses. The larger masses of the active parts and passive parts of the AEM system correspond to the lower frequency of the maximum amplitude of the secondary path of the AEM.

4.4. Magnetic Field and Coil. The Lorentz force varies with the parameter of the magnetic field and coil, which affects the amplitude of the output force of the AEM on the engine side according to equations (7)–(11). Therefore, the larger electrical resistances of the moving coil correspond to the smaller coil current. Then, the Lorentz force and pressure of the main fluid chamber decrease, the output force f_e increases, and the amplitude of the secondary path increases. This result is consistent with the numerical simulation results, as shown in Figure 14(a). Figure 14(a) also shows that the larger electrical resistances of the moving coil correspond to the smaller phase. Figure 14(b) shows that the larger inductance of the moving coil corresponds to the smaller amplitude of the secondary path of the AEM and the phase at high frequency. According to equation (9), equation (10), and equation (11), a larger force factor corresponds to larger Lorentz force and pressure of the main fluid chamber. Then, the output force f_e increases, and the amplitude of the secondary path increases, which is consistent with the numerical simulation results, as shown in Figure 14(c).

5. Conclusion

A mathematical model of the secondary path between the input voltage and output force of the AEM on the engine side is proposed considering the frequency-dependent stiffness of the main rubber spring, moving coil actuator, and main liquid chamber. The proposed model includes the structure parameters of the AEM, dynamics of the actuator, dynamics of the foundation (vehicle body), and dynamics of the attenuated vibrating object (AEM preload or engine).

Three experiments are performed when the input voltage is applied on the AEM without an applied vibration. The experimental temperature hardly affects the experimental data. The masses of the passive part and active part of the AEM are identified by Test A. Three experiments validate the feasibility of the proposed model.

The effects of the parameters on the dynamics of the secondary path are analysed according to the numerical simulation and the three experiments. The masses of the active parts and passive parts of the system strongly affect the amplitude and phase of the secondary path. The equivalent piston area of the main rubber spring has a large effect on the amplitude of the secondary path but hardly affects the phase of the secondary path. The parameters of the complex stiffness, magnetic field, and coil affect the dynamics of the secondary path. The selected parameters can affect the dynamics of the secondary path between the input voltage and the output force of the AEM on the engine side.

Data Availability

Data used to support the findings of this work are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

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