Research Article

Maximum Obtainable Energy Harvesting Power from Galloping-Based Piezoelectrics

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In this paper, the maximum obtainable energy from a galloping cantilever beam is found. The system consists of a bluff body in front of wind which was mounted on a cantilever beam and supported by piezoelectric sheets. Wind energy caused the transverse vibration of the beam and the mechanical energy of vibration is transferred to electrical charge by use of piezoelectric transducer. The nonlinear motion of the Euler–Bernoulli beam and conservation of electrical energy is modeled by lumped ordinary differential equations. The wind forces on the bluff body are modeled by quasisteady aeroelasticity approximation where the fluid and solid corresponding dynamics are disconnected in time scales. The linearized motion of beam is limited by its yield stress which causes to find a limit on energy harvesting of the system. The theory founded is used to check the validity of previous results of researchers for the effect of wind speed, tip cross-section geometry, and electrical load resistance on onset speed to galloping, tip displacement, and harvested power. Finally, maximum obtainable average power in a standard RC circuit as a function of deflection limit and synchronized charge extraction is obtained.

1. Introduction

Piezoelectric energy harvester uses the ambient energy and transfers it into electric charge [1–7]. The parametric study and the design of piezoelectric energy harvesting from galloping motion is studied by Barrero-Gil et al. [1]. First, experimental results are obtained by Sirohi and Mahadik [2, 3] using a cantilever beam exposed to air with constant velocity in a wind tunnel. Simulation of galloping cantilever coupled with a piezoelectric transducer in an electric circuit is performed by Abdelkefi et al. [4]. Analytical solution of that system of equations is presented in the work of Tan and Yan [5]. As the values for the harvested power of Abdelkefi et al. [4] and Tan and Yan [5] in some figures (watts) are beyond the order of the magnitude of experimental data of Sirohi and Mahadik [2, 3]

and Jamalabadi et al. [6, 7] (milliwatts), this paper addressed the problem to the linear assumption of force-deflection relation for the Euler–Bernoulli beam. This research proposes to consider the limitation of the yield stress of piezoelectric material as the maximum point of mechanical stability as well as energy harvesting.

2. Mathematical Model

The schematic of the system is shown in Figure 1. A bluff body exposed to the free stream is mounted on an Euler–Bernoulli cantilever beam. The two piezoelectric wafers are attached on free surfaces of beam which are in an electric circuit with electric impedance. The y -direction galloping of the bluff body in the first mode of the structure is modeled by Abdelkefi et al. [4] by

FIGURE 1: Schematic of the galloping piezoelectric energy harvester for the case of wind direction normal to the cantilever beam.

$$\ddot{y}(t) + (2\xi\omega - A)\dot{y}(t) - B\phi(L)^2\dot{y}(t)^3 + \omega^2 y(t) + \phi(L)\theta_p V(t) = 0, \quad (1)$$

$$\begin{aligned} y(t) &= \phi(L)y_{\max} \sin(\Omega t), \\ V(t) &= \sqrt{RC}\Omega\phi(L)y_{\max} \sin(\Omega t + \alpha). \end{aligned} \quad (3)$$

$$\dot{V}(t) = -\frac{V(t)}{RC_p} + \frac{\theta_p}{\phi(L)C_p} \dot{y}(t). \quad (2)$$

The integration of equations (1) and (2) term for half period of motion, the onset of galloping, the maximum deflection of the beam, and power harvesting which are

By assuming the following functions for motion and voltage,

$$U_{\min} = \frac{2(2\xi\omega + C)}{\rho_{\text{air}} b_{\text{tip}} a_1 \left(\phi^2(L)L_{\text{tip}} + \phi(L)\phi'(L)L_{\text{tip}}^2 + 1/3\phi'^2(L)L_{\text{tip}}^3 \right)}, \quad (4)$$

$$A_{\max} = \frac{\phi(L)}{\Omega} \sqrt{\frac{(C + 2\xi\omega - A)}{0.75B}}, \quad (5)$$

$$P_{\max} = \frac{C(2\xi\omega + C - A)}{0.75B}. \quad (6)$$

Before going further to derive the optimal values of the system parameters, same as of Tan and Yan [5], it should be

noticed that the maximum bending moment at the base of the beam is calculated by

$$M_{\text{base}}(\dot{y}) = \frac{(2/3)w_{\text{PZT}} \left[((t_b/2) + t_{\text{PZT}})^3 - t_{\text{PZT}}^3 \right] + (1/12)w_b t_b^3}{((t_b/2) + t_{\text{PZT}})} \sigma_Y, \quad (7)$$

where the yielding stress of piezoelectric material is about 31.2 MPa in the experiment.

3. Results

The numerical (solving equations (1) and (2)) and analytical (equations (5) and (6)) solutions are calculated based on the data provided [4]. The change in the amplitudes of the displacement of the tip of the cantilever beam with the free stream velocities at different electrical impedances from the numerical and analytical solutions are shown in Figure 2. As shown in

Figure 2, the analytical and numerical solutions are in a good agreement. In addition, the limitation considered in equation (7) affected the results, where soon after the onset of galloping, the system experienced the tear in the piezoelectric sheet. To better assess the consequences of the constraint of equation (7), the vertical axis of Figure 2 is plotted in a log scale. As shown, the maximum displacement that the beam can bear is less than the order of centimeter, and for such stiffness, the order of the deflection before failure is millimeter.

The variation in the harvested power with the electrical impedances at different free stream velocities from the

FIGURE 2: Analytical solutions (dash lines), numerical solutions (symbols), and corrected solutions (lines) of the amplitudes of the tip displacement versus the electrical impedance and free stream velocity.

analytical and numerical solutions is revealed in Figure 3. As shown in Figure 3, the results of numerical and analytical methods are in a good agreement. Additionally, the limitation considered in equation (7) affected the results, where soon after the onset of galloping, the system experienced the tear in the piezoelectric wafers and the harvesting of the wind energy is stopped. To see the significances of the constraint of equation (7) on the system clearer, the vertical axis in Figure 3 is schemed in a log

scale. As exposed, the maximum harvested power in the electric circuit is less than the order of 10^{-1} watts, and for other cases, the order of the harvested power before failure is milliwatts. The results are in a good agreement with the experimental results [2, 3, 6, 7].

By differentiating equation (6) with respect to the parameter C , the optimal design of the electric circuit for the galloping system is obtained as $(\partial P_{\max}/\partial C = 0 \implies C_o = (A - 2\xi\omega/2))$

$$\begin{aligned}
 R_o &= \frac{\theta_p^2 + C_p (A - 2\xi\omega/2)^2 \pm \sqrt{(\theta_p^2 + C_p (A - 2\xi\omega/2)^2)^2 - 4C_p (A - 2\xi\omega/2)^2 (C_p \omega^2 + \theta_p^2)}}{C_p (A - 2\xi\omega) (C_p \omega^2 + \theta_p^2)}, \\
 U_o^{\text{onset}} &= \frac{4\xi\omega}{\rho_{\text{air}} b_{\text{tip}} a_1} \frac{1}{\phi^2(L) L_{\text{tip}} + \phi(L) \phi'(L) L_{\text{tip}}^2 + 1/3 \phi'^2(L) L_{\text{tip}}^3}, \\
 A_{\max,o} &= \phi(L) \sqrt{\frac{2\xi\omega - A}{1.5B\Omega^2}}, \\
 P_{\max,o} &= -\frac{(2\xi\omega - A)^2}{3B}, \\
 \Omega &= \omega \sqrt{\frac{1}{1 - 0.5R_{co} C_p (A - 2\xi\omega)}}.
 \end{aligned} \tag{8}$$

Analytical solutions, numerical solutions, and corrected solutions of the amplitudes of the harvested power versus the parameter C and free stream velocity are plotted in Figure 4. As shown, again the limited range of parameter C is allowed

and the maximum obtainable power should be searched through those values.

When the value of expression under the square root in equation (8) for R_o is negative (the velocities higher than

FIGURE 3: Analytical solutions (dash lines), numerical solutions (symbols), and corrected solutions (lines) of the amplitudes of the harvested power versus the electrical impedance and free stream velocity.

FIGURE 4: Analytical solutions (dash lines), numerical solutions (symbol), and corrected solutions (lines) of the amplitudes of the harvested power versus the parameter C and free stream velocity.

TABLE 1: Regular circuits.

Name	Output	Schematic
Standard RC	$V_{\max} = (\theta / (1/R\omega) \sin \varphi + C_p \cos \varphi) u_{\max}$ $u_{\max} = (U/\omega) \sqrt{(-4\rho b L U a_1 + 8(C + R\theta^2 \sin^2 \varphi)/3\rho b L U a_3)}$ $P_{\text{ave}} = (V_{\max}^2 / 2R)$	
Synchronized charge extraction	$V_{\max} = (2\theta/C_p) u_{\max}$ $u_{\max} = (U/\omega) \sqrt{((-4\rho b L U a_1 + 8C + 32\theta^2/C_p)/3\rho b L U a_3)}$ $P_{\text{ave}} = (C_p \omega V_{\max}^2 / 2\pi)$	

TABLE 2: New circuit interfaces.

Name	Output	Schematic
Parallel-synchronized	$V_c = (2\Theta/C_p(1-\gamma) + \pi/R\omega)u_m$ $u_m = \sqrt{\frac{(1/2)C_b\omega\pi + (4\pi\Theta^2((1/C_pR\omega) + (1-\gamma^2/2\pi)))/C_p[(\pi/C_pR\omega) + (1-\gamma)]^2 - (1/4)\rho hLUa_1\omega\pi/(3/16)\rho hLUa_3\omega\pi((\omega^2/U^2) + \beta^2)}{(1/2)C_b\omega\pi + (4\pi\Theta^2((1/C_pR\omega) + (1-\gamma^2/2\pi)))/C_p[(\pi/C_pR\omega) + (1-\gamma)]^2 - (1/4)\rho hLUa_1\omega\pi/(3/16)\rho hLUa_3\omega\pi((\omega^2/U^2) + \beta^2)}}$ $P_{ave} = (V_{max}^2/R)$	
Series-synchronized	$V_c = (2\Theta(1+\gamma)R\omega/\pi(1-\gamma) + 2(1+\gamma)C_pR\omega)u_m$ $u_m = \sqrt{\frac{((1/2)C_b\omega\pi + (2\pi\Theta^2(1+\gamma)/C_p[\pi(1-\gamma) + 2C_pR\omega(1+\gamma)]) - (1/4)\rho hLUa_1\omega\pi/(3/16)\rho hLUa_3\omega\pi((\omega^2/U^2) + \beta^2))}{((1/2)C_b\omega\pi + (2\pi\Theta^2(1+\gamma)/C_p[\pi(1-\gamma) + 2C_pR\omega(1+\gamma)]) - (1/4)\rho hLUa_1\omega\pi/(3/16)\rho hLUa_3\omega\pi((\omega^2/U^2) + \beta^2))}}$ $P_{ave} = (4\Theta^2(1+\gamma)^2R\omega^2/[\pi(1-\gamma) + 2(1+\gamma)C_pR\omega]^2)u_m^2$

FIGURE 5: Schematic of the galloping piezoelectric energy harvester for the case of wind direction parallel to the cantilever beam.

FIGURE 6: Maximum obtainable various galloping piezoelectric energy harvester for the case of wind direction parallel to the cantilever beam.

16.1 m/s), the optimal values of the C ($(\partial C/\partial R) = 0 \implies C_o = (\theta_p^2/2C_p\Omega)$) is found by

$$R_o = (C_p\Omega)^{-1},$$

$$U_o^{\text{onset}} = \frac{4\xi\omega + \theta_p^2}{C_p\Omega\rho_{\text{air}}b_{\text{tip}}a_1} \frac{1}{\phi^2(L)L_{\text{tip}} + \phi(L)\phi'(L)L_{\text{tip}}^2 + 1/3\phi'^2(L)L_{\text{tip}}^3},$$

$$A_{\text{max},o} = \sqrt{\frac{(\theta_p^2/(2C_p\Omega)) + 2\xi\omega - A}{0.75B\Omega^2}} \phi(L),$$

$$P_{\text{max},o} = \frac{(\theta_p^2/(2C_p\Omega)) + 2\xi\omega - A}{1.5BC_p\Omega},$$

$$\Omega = \sqrt{\omega^2 + \frac{\theta_p^2}{2C_p}}.$$
(9)

TABLE 3: Nomenclature.

Symbol	Description
A	$(\rho_{\text{air}} U b_{\text{tip}} a_1 / 2) (\phi^2(L) L_{\text{tip}} + \phi(L) \phi'(L) L_{\text{tip}}^2 + \frac{1}{3} \phi'^2(L) L_{\text{tip}}^3)$
B	$(\rho_{\text{air}} b_{\text{tip}} a_3 / 2U) (\phi(L) \int_0^{L_{\text{tip}}} \phi(L) + s(\phi(L))^3 ds + \phi(L) \int_0^{L_{\text{tip}}} s(\phi(L)) \int_0^{L_{\text{tip}}} s\phi'(L)^3 + ds)$
C	$(R\theta_p^2 / (1 + C_p^2 \Omega^2 R^2))$
C_b	$2\zeta \omega_n / \phi^2(L_t)$
M	$1/\phi^2(L_t)$
K	$\omega_n^2 / \phi^2(L_t)$
Θ	$\chi' \phi(L_t)$
ρ_{air}	Air density
L	Length of the beam
F_{tip}	$(\rho_{\text{air}} U^2 b_{\text{tip}} / 2) \int_0^{L_{\text{tip}}} a_1 (\dot{y}_L + s\dot{y}'_L / U) + a_3 (\dot{y}_L + s\dot{y}'_L / U)^3 ds$
M_{tip}	$(\rho_{\text{air}} U^2 b_{\text{tip}} / 2) \int_0^{L_{\text{tip}}} a_1 (\dot{y}_L + s\dot{y}'_L / U) + a_3 (\dot{y}_L + s\dot{y}'_L / U)^3 ds$
M_{base}	$F_{\text{tip}} L + M_{\text{tip}}$
b_{tip}	Width of the tip body
L_{tip}	Length of the tip body
a_1, a_3	Aerodynamic force coefficients
R	Load resistance
Q	Quality factor
ξ	Damping ratio of the structure
ϕ	Mode shape of the structure
θ_p	Electromechanical coefficient of piezoelectric material
σ_Y	Yield strength of piezoelectric material
C_p	Capacity of piezoelectric layer
U	Wind velocity
V	Piezoelectric voltage
β	$(\phi'(L_t) / \phi(L_t))$
Ω	Angular velocity of the motion
χ	First natural angular velocity of the cantilever beam
ω	$e^{-\pi/2Q}$
γ	Cantilever-beam displacement

As for the current parameters, this value of the C parameter is less than the allowed values of C parameter; for harvested power, the discussion is not necessary.

Analytical solutions for galloping-based piezoelectric energy harvesters with various interfacing circuits are summarized in Table 1. By assuming $V = V_m \cos(\omega t + \varphi)$ and $u = u_m \cos(\omega t)$ where $\tan \varphi = (1/RC_p \omega)$, the maximum obtainable average power in a standard RC circuit as a function of deflection limit is

$$P_{\text{max}} = \frac{R}{2} (\omega \theta \sin \varphi \delta_{\text{max}})^2, \quad (10)$$

and for the synchronized charge extraction,

$$P_{\text{max}} = \frac{2\omega}{\pi C_p} (\theta \delta_{\text{max}})^2. \quad (11)$$

For the case of wind direction parallel to the cantilever beam (see Figure 5), the governing equations are

$$M \ddot{u}(t) + C_b \dot{u}(t) + K u(t) + \Theta V(t) = \frac{1}{2} \rho h L U^2 \left[a_1 \left(\frac{\dot{u}(t)}{U} + \beta u(t) \right) + a_3 \left(\frac{\dot{u}(t)}{U} + \beta u(t) \right)^3 \right], \quad (12)$$

$$I + C_p \dot{V} - \Theta \dot{u} = 0.$$

Various circuit interfaces are shown in Table 2. Maximum obtainable various galloping piezoelectric energy harvester for the case of wind direction parallel to the cantilever beam is plotted in Figure 6. The beam data is obtained from the energy harvester 2 in Table 3 of Zhao and Yang [8].

4. Conclusion

In this study, the nonlinear model of the galloping cantilever beam used for piezoelectric energy harvesting is simulated numerically with respect to the failure criteria as a limit of the maximum obtainable power. The ideal case of such

system is compared with the case of maximum stress limited due to the yielding stress of piezoelectric material. The results show that the mechanical limits of the system do not allow us to obtain the anticipated values in theory, and the feasible values are 2-3 orders of magnitude lower than prediction values. Hence, the fracture limitation should be considered in the process of the design of galloping-based energy harvesters with piezoelectric materials. Furthermore, the current research proposes for engineering applications, and designing the control system for the amplitude of galloping is necessary as well. Finally, maximum obtainable average power in a standard RC circuit as a function of deflection limit and synchronized charge extraction is obtained. In addition, four electrical interfaces in galloping-based energy harvesters are assessed. The results are for a feeble coupling SCE circuit, which is reasonable at higher wind while SSHI suits low wind speed. The standard circuit is suggested for strong electromechanical pairing, and the SCE has the best strength against the wind and can produce the highest value of power.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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