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Research Article

Design of Robust Adaptive Fuzzy Controller for a Class of Single-Input Single-Output (SISO) Uncertain Nonlinear Systems

Jieqiong Lin,[1] Jiakang Zhou,[1] Mingming Lu,[1] Hao Wang,[1] and Allen Yi[2]

[1]*Key Laboratory of Micro/Nano and Ultra-precision Manufacturing (Jilin Province), School of Mechatraonic Engineering, Changchun University of Technology, Changchun 130012, China*
[2]*Department of Industrial, Welding and Systems Engineering, Ohio State University, 210 Baker Systems Building, 1971 Neil Avenue, Columbus, OH 43210, USA*

Correspondence should be addressed to Mingming Lu; lumm@ccut.edu.cn

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In order to solve the precision and stability control problems of nonlinear uncertain systems applied in machining systems, in this paper, a robust adaptive fuzzy control technique based on Dynamic Surface Control (DSC) method is proposed for the generalized single-input single-output (SISO) uncertain nonlinear system. A first-order low-pass filter is introduced in each step of the traditional robust control method to overcome the "calculation expansion" problem, and Takagi–Sugeno (T-S) fuzzy logic system is applied to approximate an uncertain nonlinear function of unknown structure in the system. The designed robust adaptive fuzzy controller is applied to the 3D elliptical vibration cutting (3D EVC) device system model, and the effectiveness of the controller design is verified by analysis of position tracking, speed tracking, and tracking error. The results of studies show that the robust adaptive fuzzy controller can effectively suppress the jitter problem of the three-dimensional elliptical vibration cutting device so that the control object can be stabilized quickly even if it has a little jitter at the beginning. It can be smoothed to move along the ideal displacement and velocity signals. It is verified that the designed controller has strong robust adaptability.

1. Introduction

Adaptive control technology provides a very effective mean to solve the uncertainty in the system [1–3]. Since the 1970s, adaptive control has attracted great attention in the field of control theory [2, 4, 5]. In actual engineering control, the model is always nonlinear and contains some uncertainty. Uncertain nonlinear systems are susceptible to two types of uncertainties: known structural uncertainties, i.e., parameter uncertainties [6] and uncertainties in structural unknowns, i.e., modeling errors and external disturbances [7]. Considering nonlinear systems with known uncertainties in structure, in the past research, significant results have been achieved in adaptive techniques for feedback linearization of nonlinear systems [8–10]. For nonlinear systems with structurally unknown uncertainties, robust control strategies can be applied. Since some controlled systems cannot know the bounds of the unknown uncertainty of their structure, adaptive control and robust control methods cannot be used for controller design. However, for such problems, neural networks and fuzzy control can approximate uncertain continuous functions of the unknown structure in the system [11, 12]. Ge et al. researched a robust adaptive neural network control method for a perturbed strictly feedback nonlinear system, which can guarantee the final boundedness in the case of unknown structural uncertainty [13]; Cao et al. proposed a new system control strategy using the global approximation property of the fuzzy system to approximate the unknown function of the designed system and ensure the stability of the whole system [14].

However, in the system described above, it is satisfied that the unknown nonlinearity and the control input appear in the same state space model equation. In order to overcome the limitations of such systems, relevant research scholars have proposed adaptive post-push technology [4, 6, 15, 16], furthermore successfully solved the constraint of the matching conditions imposed on the nonlinearity of the system. However, adaptive post-pushing technology has some disadvantages such as overparameterization. In order to solve this shortcoming, Yang et al. designed the controller with adaptive post-pushing technology and robust control technology, which made the closed-loop system globally consistent and bounded under the influence of unknown nonlinearity and parameter uncertainty factors [17]. In addition, robust adaptive neural networks and fuzzy controllers are studied for systems with matching conditions imposed by the abovementioned nonlinearities [18, 19]. Especially for high-order systems and multivariable systems, when neural network basis functions or fuzzy rule numbers are used to approximate uncertain nonlinear functions, many parameters need to be adjusted, resulting in a "dimension disaster" problem. As a result, the computational burden becomes too large and the "calculation expansion" problem arises at the same time, which is not conducive to engineering applications [19, 20]. Yang et al. investigated an adaptive control technique based on the T-S fuzzy system approximator, which adjusted the adaptive parameters of each system to two and solved the problem of "dimensionality disaster" [21]. Hedrick et al. proposed a "Dynamic Surface Control" (DSC) method that overcame the "computational expansion" problem by introducing a first-order low-pass filter at each step of the pushback technique [22]. Sui et al. proposed a new stochastic finite-time stability theorem combined with the backstepping technique and an adaptive fuzzy stochastic finite-time control method. Simulation results proved the effectiveness of the method [23]. Mu et al. proposed finite-time switching mode manifolds and corresponding nonsingular controllers, and the reduced system is obtained from terminal sliding mode control (TSMC), explained that TSMC is a finite-time control by the homogeneous theory [24].

Backstepping technology is a systematic controller synthesis method for uncertain systems. It is a regression design method that combines the selection of the Lyapunov function with the design of the controller. The reason why backstepping technology is widely used in many industrial control fields is mainly because the method eliminates the constraint that the system uncertainty and meets the matching conditions, thereby solving the control problem of relatively complex nonlinear systems. With the continuous development of intelligent control technologies such as neural networks and fuzzy systems, backstepping technology has gained great potential for development. In particular, the introduction of neural networks and adaptive technologies has greatly promoted the application of the backstepping method. Sui et al. proposed a finite-time adaptive fuzzy decentralized control method by combining backstepping recursive design and Lyapunov function theory. The experimental result has proved that the developed control strategy can make the output of the system have good tracking performance in a limited time [25]. Soon after, Sui et al. presented a finite-time switching control method based on the backstepping recursive technique and the common Lyapunov function method and applied it to a mass spring-damper system to prove the effectiveness [26].

In this paper, a robust adaptive fuzzy control technique based on "Dynamic Surface Control" (DSC) method is proposed for generalized single-input single-output (SISO) uncertain nonlinear systems. The T-S fuzzy logic system is used to approximate the uncertain nonlinear function with an unknown structure in the system. The main contributions of the paper include the following:

(1) In the case of unknown structural uncertainty, the resulting closed-loop system is ultimately bounded.
(2) Only one function needs to have a T-S fuzzy logic system approximation and each subsystem needs only one parameter to be adjusted, which overcomes the problem of "dimensionality disaster." In turn, the calculation amount of the control algorithm is greatly reduced, and the problem of "calculation expansion" is solved.

The chapters of this paper are distributed as follows. Section 1 mainly introduces the expressions and assumptions of the questions and provides some preliminary knowledge. A design of robust adaptive fuzzy controller based on "Dynamic Surface Control" (DSC) method and Lyapunov stability analysis are described in Section 2. Section 3 uses MATLAB to carry out simulation research to prove the effectiveness of the proposed method. Finally, the conclusion is reflected in Section 4.

2. Problem Formulation and Preparation

2.1. Problem Preparation

2.1.1. Fuzzy Control Theory. Fuzzy Logic Control, referred to as Fuzzy Control, is a computer digital control technology based on fuzzy set theory, fuzzy linguistic variables, and fuzzy logic reasoning. Professor L. A. Zadeh of the University of California, United States, published the famous "Fuzzy Sets" paper [27] and proposed the concepts of fuzzy sets and fuzzy algorithms.

The fuzzy control system is mainly composed of four parts: fuzzification, rule base, fuzzy reasoning machine, and defuzzification:

(1) Fuzzification: the main function is to select the input amount of the fuzzy controller and convert it into a fuzzy amount that can be recognized by the system. It consists of three steps. First, the input volume is processed to meet the needs of fuzzy control. Second, the input amount is scaled. Third, the fuzzy language value of each input and the corresponding membership function are determined.
(2) Rule base: it consists of a fuzzy "If-Then" rule set. The fuzzy rule base contains many control rules and is a key step in the transition from actual control experience to fuzzy controllers.
(3) Fuzzy reasoning machine: based on combined reasoning or independent reasoning for fuzzy rule bases, a variety of fuzzy reasoning machines are proposed, which mainly implement knowledge-based reasoning decisions.

(4) Defuzzification: the main function is to convert the control amount obtained by reasoning into the control output.

There are two types of fuzzy control systems: Mamdani and Takagi–Sugeno (T-S).

Consider a fuzzy system approximating a continuous multidimensional function $y = f(x)$, where $x = (x_1, x_2, \ldots, x_n)^T \in U$ is the input vector and $y \in V \subset R$ is the output vector. The fuzzy logic system constitutes a mapping from subspace U to subspace V, as shown in Figure 1. The system usually consists of the following P $(P > 1)$ fuzzy rules:

If $x_1 = A^i_{h_1}$, $x_2 = A^i_{h_2}, \ldots, x_n = A^i_{h_n}$, then $y_i = B^i_{h_1,h_2,\ldots,h_n}$, $i = 1, 2, \ldots, P$, where $A^i_{h_1,h_2,\ldots,h_n}$ and $B^i_{h_1,h_2,\ldots,h_n}$, respectively, represent the input fuzzy set and the output fuzzy set.

If $B^i_{h_1,h_2,\ldots,h_n}$ is a single fuzzy set, then it is a Mamdani-type fuzzy system. If $B^i_{h_1,h_2,\ldots,h_n}$ is a function, it is a Takagi–Sugeno (T-S)-type fuzzy system [28]. Because the T-S-type fuzzy system is used in this paper, the Mamdani-type fuzzy system will not be introduced too much. After deblurring the abovementioned T-S fuzzy system with product fuzzy reasoning, the output of the T-S fuzzy system becomes

$$F(x) = \sum_{i=1}^{K} y_i \xi_i(x), \tag{1}$$

where $\xi_i(x)$ represents a fuzzy basis function, $y_i = a^i_1 x_1 + a^i_2 x_2 + \cdots + a^i_n x_n$; then, formula (1) can be changed to

$$F(x) = \xi(x) A_x x, \tag{2}$$

where

$$\xi(x) = [\xi_1(x), \xi_2(x), \ldots, \xi_K(x)],$$
$$A_x = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{K1} & a_{K2} & \cdots & a_{Kn} \end{bmatrix}. \tag{3}$$

The proposed T-S-type fuzzy system provides convenient conditions for the analytical design of the fuzzy control system. Because complex nonlinear systems will be decomposed into a series of linear subsystems, the originally difficulty to parse and describe the system will not be complicated by the followers of each rule in the T-S fuzzy system.

Assuming that the output universe is a compact set within $\mathbf{R}^r$, then for any given real continuous functions U and $f(x)$ on $\forall \varepsilon > 0$, there exists a fuzzy system $F(x)$ of form (2), such that

$$\sup_{x \in U} \| f(x) - F(x) \| \le \varepsilon. \tag{4}$$

For the T-S-type fuzzy system, $F(x) = \hat{f}(x, \mathbf{A}_x)$, and the detailed proof process is given in [29, 30].

2.1.2. Dynamic Surface Control. Dynamic surface control is developed on the basis of Backstepping Control. Backstepping method has obvious advantages in implementing robust control or adaptive control of uncertain nonlinear systems. However, the backstepping method itself does not have a good solution to the term expansion caused by the derivation of virtual control and the problem caused by the term expansion. This disadvantage is particularly prominent in higher-order systems. It is to overcome the computational complexity of the traditional inverse design. The main advantages of the dynamic surface control method are as follows: (1) it can eliminate the expansion of differential terms and make the controller and parameter design simple; (2) it can reduce the number of neural network and fuzzy system input variables used for modeling; and (3) there is no need to approximate the bounded error in the stability analysis, thereby avoiding the cyclical argument.

The following briefly introduces the design of dynamic surface control methods.

Take a second-order nonlinear system as an example:

$$\begin{cases} \dot{x}_1 = x_2 + a f_1(x_1), \\ \dot{x}_2 = u + b f_2(\overline{x}_2), \\ y = x_1, \end{cases} \tag{5}$$

where $\overline{x}_2 = [x_1, x_2]^T$ represents the system state variable and u represents the system input.

Define the first error surface $S_1 = x_1 - y_r$, where y_r represents the system reference input signal, and derive the error surface S_1 to obtain the following formula:

$$\dot{S}_1 = x_2 + x_{2d} - x_{2d} + a f_1(x_1) - \dot{y}_r, \tag{6}$$

where x_{2d} represents the intermediate virtual control rate.

In order to overcome the expansion of the differential term, a new low-pass filter is introduced to obtain a new variable z_2:

$$\tau_2 \dot{z}_2 + z_2 = x_{2d}, \tag{7}$$

where τ_2 represents the time constant of the first-order low-pass filter.

By introducing a first-order filter in a nonlinear system, the problem of term expansion caused by the derivation of virtual control in high-order systems is avoided. The design and stability analysis of the dynamic surface control method proves that the literature [31] gives specific steps.

2.1.3. Statement of Problem. Consider a class of nonlinearly indeterminate single-input single-output (SISO) systems that are subject to external disturbances:

$$\begin{cases} \dot{x}_i = g_i(\overline{x}_i) x_{i+1} + f_i(\overline{x}_i) + d_i(x,t), \\ \dot{x}_n = g_n(x) u + f_n(x) + d_n(x,t), \\ y = x_1, \end{cases} \tag{8}$$

where $\overline{x}_i = [x_1, \ldots, x_i]^T$, $x = (x_1, \ldots, x_n)^T$ is the system state vector; $u, y \in R$ is the input and output of the control system; $f_i(\cdot)$ is the unknown smooth nonlinear function; $g_i(\cdot)$ is the unknown smooth virtual control nonlinear gain function; and $d_i(x,t)$ $(1 \le i \le n)$ is the uncertain external disturbance, and bounded.

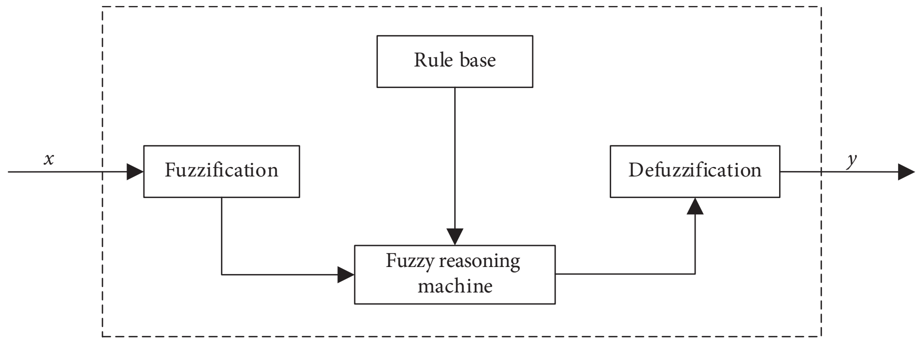


Figure 1: Structure diagram of the fuzzy logic system.

Define the tracking error z, track a given reference signal $y_r(t)$, and assume that the given reference signal $y_r(t)$ is bounded:

$$z = y(t) - y_r(t). \tag{9}$$

Make assumptions about the abovementioned system.

Assumption 1. The unknown smooth virtual control nonlinear gain function $g_i(\cdot)$ satisfies $0 \le a_{\min} \le g_i(\overline{x}_i, t) \le b_{\max}$, where $a_{\min}$ and $b_{\max}$ represent the upper and lower bounds of the parameter and are some unknown constant.

Assumption 2. It is uncertain that the external disturbance $d_i(x,t)(1 \le i \le n)$ is bounded, and $|d_i(x,t)| \le d_i$, d_i is an unknown normal number.

Assumption 3. The weight of the fuzzy system θ_i^* and the approximation error ε_i are bounded.

2.2. Design of Robust Adaptive Fuzzy Controller

2.2.1. Controller Design. Consider the first subsystem of the system shown in equation (8), defining the first tracking error $z_1 = x_1 - y_r$; then,

$$\dot{z}_1 = g_1(\overline{x}_1, t)x_2 + f_1(x_1, t) + d_1 - \dot{y}_r. \tag{10}$$

Using a T-S-type fuzzy system to approximate $f_1(x_1, \omega)$, then

$$\begin{aligned} f_1(x_1, \omega) &= \boldsymbol{\xi}_1(x_1)\mathbf{A}_1 x_1 + \varepsilon_1 = \boldsymbol{\xi}_1(x_1)\mathbf{A}_1 z_1 + \boldsymbol{\xi}_1(x_1)\mathbf{A}_1 y_r + \varepsilon_1 \\ &= q_{\theta 1}\boldsymbol{\xi}_1\boldsymbol{\omega}_1 + \boldsymbol{\xi}_1(x_1)\mathbf{A}_1 y_r + \varepsilon_1, \end{aligned} \tag{11}$$

where ε_1 represents the approximation error, $q_{\theta 1}$ is an unknown constant, and $q_{\theta 1} = \|\mathbf{A}_1\|$, $\mathbf{A}_1 = q_{\theta 1}\mathbf{A}_1^m$, and $\boldsymbol{\omega}_1 = \mathbf{A}_1 z_1$; then,

$$\dot{z}_1 = g_1(\overline{x}_1, t)x_2 + q_{\theta 1}\boldsymbol{\xi}_1(x)\boldsymbol{\omega}_1 + l_1 - \dot{y}_r, \tag{12}$$

where $l_1 = \boldsymbol{\xi}_1(x_1)\mathbf{A}_1 y_r + \varepsilon_1 + d_1$, $\|l_1\| \le \|\boldsymbol{\xi}_1(x_1)\mathbf{A}_1 y_r + \varepsilon_1 + d_1\| \le a_{\min}\theta_1\varphi_1(x_1)$, $\theta_1 = a_{\min}^{-1}\max(\|\mathbf{A}_1 y_r\|, \|\varepsilon_1 + d_1\|)$, and $\varphi_1(x_1) = 1 + \|\xi_1\|$.

Therefore, the intermediate stabilization function ψ_2 and the parameter adaptation rate are

$$\psi_2 = -k_1 z_1 + \dot{y}_r - \widehat{\lambda}_1\rho_1(x_1)z_1, \tag{13}$$

$$\dot{\widehat{\lambda}}_1 = \mu_1\left[\rho_1(x_1)z_1^2 - \sigma_1\left(\widehat{\lambda}_1 - \lambda_1^0\right)\right], \tag{14}$$

$$\rho_1(x_1) = \frac{1}{4\iota_1^2}\boldsymbol{\xi}_1\boldsymbol{\xi}_1^T + \frac{1}{4\kappa_1^2}\varphi_1^2, \tag{15}$$

where k_1, μ_1, ι_1, and κ_1 are normal numbers; $\widehat{\lambda}_1$ is an estimated value of λ_1; $\lambda_1 = a_{\min}^{-1}\max(q_{\theta 1}^2, \theta_1^2)$; and λ_1^0 is the initial value.

At this point, the estimated value s_2 of ψ_2 is obtained:

$$s_2 = \psi_2 - \tau_2\dot{s}_2, \tag{16}$$

where τ is the time constant.

Considering the $i-\text{th}(2 \le i \le n-1)$ subsystems of the system shown in equation (8) and defining the $i-\text{th}$ tracking error: $z_i = x_i - s_i$; then,

$$\dot{z}_i = g_i(\overline{x}_i, t)x_{i+1} + f_i(\overline{x}_i, t) + d_i - \dot{s}_i. \tag{17}$$

Using the T-S fuzzy system to approximate the uncertain function $f_i(\overline{x}_i, t)$, then

$$\begin{aligned} f_i(\overline{x}_i, t) &= \boldsymbol{\xi}_i(\overline{x}_i)\mathbf{A}_i\overline{x}_i^T + \varepsilon_i = \boldsymbol{\xi}_i(\overline{x}_i)\mathbf{A}_i\begin{bmatrix} z_1 + y_r \\ z_2 + s_2 \\ \cdots \\ z_i + s_i \end{bmatrix} + \varepsilon_i \\ &= q_{\theta i}\boldsymbol{\xi}_i\boldsymbol{\omega}_i + d_i', \end{aligned} \tag{18}$$

where $d_i' = \boldsymbol{\xi}_i\mathbf{A}_i^1 y_r + \boldsymbol{\xi}_i\sum_{j=2}^{i}\mathbf{A}_i^j s_j + \varepsilon_i$, $q_{\theta i} = \|\mathbf{A}_i^1\|$, $\mathbf{A}_i^1 = q_{\theta i}\mathbf{A}_i^m$, and $\boldsymbol{\omega}_i = \mathbf{A}_i^m\overline{z}_i$; then,

$$\dot{z}_i = g_i(\overline{x}_i, t)x_{i+1} + q_{\theta i}\boldsymbol{\xi}_i(x)\boldsymbol{\omega}_i + l_i - \dot{s}_i, \tag{19}$$

where $l_i = d_i + d_i'$, $\|l_i\| \le \|d_i + \boldsymbol{\xi}_i\mathbf{A}_i^1 y_r + \boldsymbol{\xi}_i\sum_{j=2}^{i}\mathbf{A}_i^j + \varepsilon_i\| \le a_{\min}\theta_i\varphi_i(x_i)$, $\theta_i = a_{\min}^{-1}\max(\|\mathbf{A}_i^1 y_r\|, \|\sum_{j=2}^{i}\mathbf{A}_i^j\|, \|\varepsilon_i + d_i\|)$, and $\varphi_i(x_i) = 1 + \|\xi_i\|$.

Therefore, the intermediate stabilization function ψ_{i+1} and the parameter adaptation rate are

$$\psi_{i+1} = -k_i z_i + \dot{s}_i - \hat{\lambda}_i \rho_i(\overline{x}_i) z_i, \tag{20}$$

$$\dot{\hat{\lambda}}_i = \mu_i\left[\rho_i(\overline{x}_i) z_i^2 - \sigma_i\left(\hat{\lambda}_i - \lambda_i^0\right)\right], \tag{21}$$

$$\rho_i(\overline{x}_i) = \frac{1}{4\iota_i^2}\xi_i\xi_i^T + \frac{1}{4\kappa_i^2}\varphi_i^2, \tag{22}$$

where k_i, μ_i, ι_i, and κ_i are normal numbers; $\hat{\lambda}_i$ is an estimated value of λ_i; $\lambda_i = a_{\min}^{-1}\max(q_{\theta i}^2, \theta_i^2)$; and λ_i^0 is the initial value.

At this point, the estimated value s_{i+1} of ψ_{i+1} is obtained:

$$s_{i+1} = \psi_{i+1} - \tau_{i+1}\dot{s}_{i+1}. \tag{23}$$

Define the $n-$th tracking error: $z_n = x_n - s_n$; similarly, there is

$$\begin{aligned}\dot{z}_n &= g_n(x,t)u + f_n(\overline{x}_n,t) + d_n - \dot{s}_n,\\ f_i(\overline{x}_n,t) &= \boldsymbol{\xi}_n(\overline{x}_n)\mathbf{A}_n\overline{x}_n^T + \varepsilon_n = \boldsymbol{\xi}_n(\overline{x}_n)\mathbf{A}_n\begin{bmatrix} z_1 + y_r \\ z_2 + s_2 \\ \cdots \\ z_n + s_n\end{bmatrix} + \varepsilon_n\\ &= q_{\theta n}\boldsymbol{\xi}_n\boldsymbol{\omega}_n + d_n',\end{aligned} \tag{24}$$

where $d_n' = \boldsymbol{\xi}_n\mathbf{A}_n^1 y_r + \boldsymbol{\xi}_n\sum_{j=2}^n \mathbf{A}_n^j s_j + \varepsilon_n$, $q_{\theta n} = \|\mathbf{A}_n^1\|$, $\mathbf{A}_n^1 = q_{\theta n}\mathbf{A}_n^m$, and $\boldsymbol{\omega}_n = \mathbf{A}_n^m\overline{z}_n$; then,

$$\dot{z}_n = g_n(x,t)u + q_{\theta n}\boldsymbol{\xi}_n(x)\boldsymbol{\omega}_n + l_n - \dot{s}_n, \tag{25}$$

where $l_n = d_n + d_n'$, $\|l_n\| \le \|d_n + \boldsymbol{\xi}_n\mathbf{A}_n^1 y_r + \boldsymbol{\xi}_n\sum_{j=2}^n \mathbf{A}_n^j + \varepsilon_n\| \le a_{\min}\theta_n\varphi_n(x)$, $\theta_n = a_{\min}^{-1}\max(\|\mathbf{A}_n^1 y_r\|, \|\sum_{j=2}^n \mathbf{A}_i^n\|, \|\varepsilon_n + d_n\|)$, and $\varphi_n(x) = 1 + \|\xi_n\|$.

Therefore, select the control rate and parameter adaptation rate:

$$u = -k_n z_n + \dot{s}_n - \hat{\lambda}_n\rho_n(x) z_n, \tag{26}$$

$$\dot{\hat{\lambda}}_n = \mu_n\left[\rho_n(x) z_n^2 - \sigma_n\left(\hat{\lambda}_n - \lambda_n^0\right)\right], \tag{27}$$

$$\rho_n(x) = \frac{1}{4\iota_n^2}\xi_n\xi_n^T + \frac{1}{4t_n^2}\varphi_n^2, \tag{28}$$

where k_n, μ_n, ι_n, and κ_n are normal numbers; $\hat{\lambda}_n$ is an estimated value of λ_n; $\lambda_n = a_{\min}^{-1}\max(q_{\theta n}^2, \theta_n^2)$; and λ_n^0 is the initial value.

2.2.2. Lyapunov Stability Analysis. Define a new tracking error:

$$y_{i+1} = s_{i+1} - \psi_{i+1}, \quad i = 1, 2, \ldots, n-1. \tag{29}$$

From equation (23), $\dot{s}_i = ((-(s_i + \psi_i))/\tau_i) = ((-y_i)/\tau_i)$; then,

$$\dot{y}_2 = \dot{s}_2 - \dot{\psi}_2 = -\frac{y_2}{\tau_2} + \left(-\frac{\partial\psi_2}{\partial z_1}\dot{z}_1 - \frac{\partial\psi_2}{\partial x_1}\dot{x}_1 - \frac{\partial\psi_2}{\partial\hat{\theta}_1}\dot{\hat{\theta}}_n - \frac{\partial\psi_2}{\partial\hat{\lambda}_1}\dot{\hat{\lambda}}_1 + \ddot{y}_r\right) = -\frac{y_2}{\tau_2} + \mathrm{O}_2\left(z_1, z_2, y_2, \hat{\theta}_1, \hat{\lambda}_1, y_r, \dot{y}_r, \ddot{y}_r\right), \tag{30}$$

where $\mathrm{O}_2(\cdot)$ is the remainder. Similarly, there is

$$\dot{y}_{i+1} = \dot{s}_{i+1} - \dot{\psi}_{i+1} = -\frac{y_{i+1}}{\tau_{i+1}} + \mathrm{O}_{i+1}(z_1, \ldots, z_{i+1}, \cdot\, y_2, \ldots y_i, \hat{\theta}_1, \ldots \hat{\theta}_i, \hat{\lambda}_1, \ldots, \hat{\lambda}_i, y_r, \dot{y}_r, \ddot{y}_r). \tag{31}$$

According to the $i-$th tracking error equation and equation (29), $x_{i+1} = z_{i+1} + s_{i+1}$ and $s_{i+1} = y_{i+1} + \psi_{i+1}$, and the tracking error can be expressed as follows:

$$\begin{cases}\dot{z}_1 = g_1(\overline{x}_1,t)z_1 + g_1(\overline{x}_1,t)y_1 + g_1(\overline{x}_1,t)\psi_1 + q_{\theta 1}\boldsymbol{\xi}_1(x)\boldsymbol{\omega}_1 + l_1 - \dot{y}_r,\\ \dot{z}_i = g_i(\overline{x}_i,t)z_{i+1} + g_i(\overline{x}_i,t)y_{i+1} + g_i(\overline{x}_i,t)\psi_{i+1} + q_{\theta i}\boldsymbol{\xi}_i(x)\boldsymbol{\omega}_i + l_i - \dot{s}_i,\\ \dot{z}_n = g_n(x,t)u + q_{\theta n}\boldsymbol{\xi}_n(x)\boldsymbol{\omega}_n + l_n - \dot{s}_n.\end{cases} \tag{32}$$

Then, bring equations (13)~(15), (19)~(21), and (26)~(28) into the abovementioned equation and obtain the following expression:

$$\begin{cases} \dot{z}_1 = g_1 z_1 + g_1 y_1 - g_1 k_1 z_1 + g_1 \dot{y}_r - g_1 \widehat{\lambda}_1 \left(\dfrac{1}{4\iota_1^2} \boldsymbol{\xi}_1 \boldsymbol{\xi}_1^T + \dfrac{1}{4\kappa_1^2} \varphi_1^2 \right) z_1 + q_{\theta 1} \boldsymbol{\xi}_1 (x) \boldsymbol{\omega}_1 + l_1 - \dot{y}_r, \\ \dot{z}_i = g_i z_{i+1} + g_i y_{i+1} - g_i k_i z_i + g_i \dot{s}_i - g_i \widehat{\lambda}_i \left(\dfrac{1}{4\iota_i^2} \boldsymbol{\xi}_i \boldsymbol{\xi}_i^T + \dfrac{1}{4\kappa_i^2} \varphi_i^2 \right) z_i + q_{\theta i} \boldsymbol{\xi}_i (x) \boldsymbol{\omega}_i + l_i - \dot{s}_i, \\ \dot{z}_n = -g_n k_n z_n + g_n \dot{s}_n - g_n \widehat{\lambda}_n \left(\dfrac{1}{4\iota_n^2} \boldsymbol{\xi}_n \boldsymbol{\xi}_n^T + \dfrac{1}{4t_n^2} \varphi_n^2 \right) z_n + q_{\theta n} \boldsymbol{\xi}_n (x) \boldsymbol{\omega}_n + l_n - \dot{s}_n. \end{cases} \tag{33}$$

Define the closed-loop system Lyapunov function as

$$V = \frac{1}{2} \sum_{i=1}^{n} \left(z_i^2 + \widehat{\lambda}_i^{\,T} a_{min} \mu_i^{-1} \widetilde{\lambda}_i \right) + \frac{1}{2} \sum_{i=1}^{n-1} y_{i+1}^2, \tag{34}$$

where $\widehat{\lambda}_i = \lambda_i - \widehat{\lambda}_i$.

In the following, it will be proved that the given arbitrary $\zeta > 0$, the existence of l_i, and ι_i, κ_i, σ_i, and μ_i make the solution of the closed-loop system consistent and bounded.

The derivative of the closed-loop system Lyapunov function with respect to time is

$$\dot{V} = \sum_{i=1}^{n} \left(z_i \dot{z}_i - \widehat{\lambda}_i^{\,T} a_{\min} \mu_i^{-1} \dot{\widehat{\lambda}}_i \right) + \sum_{i=1}^{n-1} y_{i+1} \dot{y}_{i+1}. \tag{35}$$

Deduced analysis can be drawn

$$\begin{aligned} \dot{V} \le & \sum_{i=1}^{n-1} \Bigg(g_i z_{i+1} z_i + g_i y_{i+1} z_i - a_{\min} k_i z_i^2 - a_{\min} \widehat{\lambda}_i \frac{1}{4\iota_i^2} \boldsymbol{\xi}_i \boldsymbol{\xi}_i^T z_i^2 \\ & - g_i \widehat{\lambda}_i \frac{1}{4\kappa_i^2} \varphi_i^2 z_i^2 \\ & + q_{\theta i} \boldsymbol{\xi}_i (\overline{x_i}) \boldsymbol{\omega}_i z_i + l_i z_i - \widehat{\lambda}_i^{\,T} a_{\min} \mu_i^{-1} \dot{\widehat{\lambda}}_i \Bigg) + g_1 \dot{y}_r z_1 - \dot{y}_r z_1 \\ & + \sum_{i=2}^{n} \left(g_i \dot{s}_i z_i - \dot{s}_i z_i \right) \\ & - a_{\min} k_n z_n^2 - g_n \widehat{\lambda}_n \frac{1}{4t_n^2} \boldsymbol{\xi}_n \boldsymbol{\xi}_n^T z_n^2 - a_{\min} \widehat{\lambda}_n \frac{1}{4\kappa_n^2} \varphi_n^2 z_n^2 + q_{\theta n} \boldsymbol{\xi}_n (x) \boldsymbol{\omega}_n z_n \\ & + l_n z_n - \widehat{\lambda}_n^{\,T} a_{\min} \mu_n^{-1} \dot{\widehat{\lambda}}_n + \sum_{i=1}^{n-1} \left(-\frac{y_{i+1}^2}{\tau_{i+1}} + \left| y_{i+1} \mathrm{O}_{i+1} \right| \right). \end{aligned} \tag{36}$$

However,

$$\begin{aligned} q_{\theta i} \boldsymbol{\xi}_i \boldsymbol{\omega}_i z_i &= q_{\theta i} \boldsymbol{\xi}_i \boldsymbol{\omega}_i z_i - \iota_i^2 \boldsymbol{\omega}_i^T \boldsymbol{\omega}_i + \iota_i^2 \boldsymbol{\omega}_i^T \boldsymbol{\omega}_i = -\iota_i^2 \left(\boldsymbol{\omega}_i - \frac{q_{\theta i}^2}{2\iota_i^2} \boldsymbol{\xi}_i z_i \right)^2 \\ &\quad + \frac{q_{\theta i}^2}{4\iota_i^2} \boldsymbol{\xi}_i \boldsymbol{\xi}_i^T z_i^2 + \iota_i^2 \boldsymbol{\omega}_i^T \boldsymbol{\omega}_i \\ &\le \frac{q_{\theta i}^2}{4\iota_i^2} \boldsymbol{\xi}_i \boldsymbol{\xi}_i^T z_i^2 + \iota_i^2 \boldsymbol{\omega}_i^T \boldsymbol{\omega}_i \le a_{\min} \widehat{\lambda}_i \frac{1}{4\iota_i^2} \boldsymbol{\xi}_i \boldsymbol{\xi}_i^T z_i^2 \\ &\quad + a_{\min} \widetilde{\lambda}_i \frac{1}{4\iota_i^2} \boldsymbol{\xi}_i \boldsymbol{\xi}_i^T z_i^2 + \iota_i^2 \boldsymbol{\omega}_i^T \boldsymbol{\omega}_i, \\ l_i z_i &\le a_{\min} \widehat{\theta}_i \varphi_i (\overline{x}_i) \| z_i \| + a_{\min} \widetilde{\theta}_i \varphi_i (\overline{x}_i) \| z_i \| \\ &\le g_i \widehat{\theta}_i \varphi_i (\overline{x}_i) \| z_i \| + a_{\min} \widetilde{\theta}_i \varphi_i (\overline{x}_i) \| z_i \|. \end{aligned} \tag{37}$$

At the same time, it can be known from $\dot{s}_i = ((-(s_i + \psi_i))/\tau_i) = ((-y)_i/\tau_i)$:

$$\begin{aligned} g_1 \dot{y}_r z_1 - \dot{y}_r z_1 &\le \frac{1 + b_{\max}}{\tau_i} z_i^2 + (1 + b_{\max}) \dot{y}_r^2 \le \frac{1 + b_{\max}}{4} z_i^2 \\ &\quad + (1 + b_{\max}) \mathrm{O}_0^2, \\ g_i \dot{s}_i z_i - \dot{s}_i z_i &\le g_i \left| \dot{s}_i z_i \right| + \left| \dot{s}_i z_i \right| \le (g_i + 1) \left| \frac{y_i}{\tau_i} \right| \left| z_i \right| \le \frac{1 + b_{\max}}{\tau_i} z_i^2 \\ &\quad + \frac{1 + b_{\max}}{4\tau_i} y_i^2. \end{aligned} \tag{38}$$

Then, equation (36) can be changed to

$$\begin{aligned} \dot{V} \le & \sum_{i=1}^{n-1} \left(g_i z_{i+1} z_i + g_i y_{i+1} z_i \right) + \sum_{i=1}^{n} \Bigg(a_{\min} \widetilde{\lambda}_i \frac{1}{4\iota_i^2} \boldsymbol{\xi}_i \boldsymbol{\xi}_i^T z_i^2 + \iota_i^2 \boldsymbol{\omega}_i^T \boldsymbol{\omega}_i \Bigg) \\ & + g_i \widehat{\theta}_i \varphi_i (\overline{x}_i) \| z_i \| + a_{\min} \widetilde{\theta}_i \varphi_i (\overline{x}_i) \| z_i \| - g_i \widehat{\lambda}_i \frac{1}{4\kappa_i^2} \varphi_i^2 z_i^2 g_n \\ & - g_n \widehat{\lambda}_n \frac{1}{4t_n^2} \boldsymbol{\xi}_n \boldsymbol{\xi}_n^T z_n^2 + \left(\frac{1 + b_{\max}}{4} - a_{\min} k_1 \right) z_1^2 \\ & + (1 + b_{\max}) \mathrm{O}_0^2 + \sum_{i=2}^{n} \left(- \left(- \frac{1 + b_{\max}}{\tau_i} + a_{\min} k_i \right) z_i^2 \right) \\ & + \sum_{i=2}^{n} \left(\frac{1 + b_{\max}}{4\tau_i} \right) y_i^2 + \sum_{i=1}^{n-1} \left(- \frac{y_{i+1}^2}{\tau_{i+1}} + \left| y_{i+1} \mathrm{O}_{i+1} \right| \right). \end{aligned} \tag{39}$$

Since, $g_i \widehat{\theta}_i \varphi_i (\overline{x}_i) \| z_i \| + a_{\min} \widetilde{\theta}_i \varphi_i (\overline{x}_i) \| z_i \| - g_i \widehat{\lambda}_i (1/4\kappa_i^2) \varphi_i^2 z_i^2 \le \theta_i \varphi_i (\overline{x_i}) \| z_i \| - g_i \widehat{\lambda}_i (1/4\kappa_i^2) \varphi_i^2 z_i^2 \le (\theta_i^2/4t_n^2) \varphi_i^2 (\overline{x}) z_i^2 + \kappa_i^2 - g_i \widehat{\lambda}_i (1/4\kappa_i^2) \varphi_i^2 z_i^2 \le g_i \kappa_i \le b_{\max} \kappa_i$, $g_i z_{i+1} z_i \le z_i^2 + (b_{\max}/4) z_{i+1}^2$ and $g_i y_{i+1} z_i \le z_i^2 + (b_{\max}/4) y_{i+1}^2$. Bring equations (21) and (22) into the abovementioned equation; then,

$$\begin{aligned} \dot{V} \le & \left(\frac{1 + b_{\max}}{4} - a_{\min} k_1 - 2 \right) z_1^2 + \sum_{i=2}^{n} \left(- \left(- \frac{1 + b_{\max}}{\tau_i} + a_{\min} k_i - 2 \right) z_i^2 \right) \\ & + \left(\frac{1 + b_{\max}}{4} - a_{\min} k_n \right) z_n^2 - \sum_{i=1}^{n} \left(\frac{\sigma_i}{2\lambda_{\max} a_{\min} \mu_i^{-1}} \widetilde{\lambda}_i^T \mu_i^{-1} \widetilde{\lambda}_i \right) \\ & + \sum_{i=1}^{n-1} \left(\frac{b_{\max}}{4} y_{i+1}^2 - \frac{3 - b_{\max}}{4\tau_{i+1}} + \left| y_{i+1} \mathrm{O}_{i+1} \right| \right) + \sum_{i=1}^{n} (\delta_i') + \iota^2 \| \boldsymbol{\omega}^2 \|, \end{aligned} \tag{40}$$

where $\delta_i' = (1 + b_{\max})\mathrm{O}_0^2 + b_{\max}\kappa_i + (\sigma_i/2)|\lambda_i^* - \lambda_i^0|^2$, $\iota = (\iota_1^2 + \iota_2^2 + \cdots + \iota_n^2)^{1/2}$, and $\boldsymbol{\omega} = [\boldsymbol{\omega}_1, \boldsymbol{\omega}_2, \ldots, \boldsymbol{\omega}_n]^T$.

Since $\boldsymbol{\omega}_i = \mathbf{A}_i^m \overline{z}_i^T$, $\|\mathbf{A}_\mathbf{i}^m\| \leq 1$; however,

$$\boldsymbol{\omega} = \begin{bmatrix} \boldsymbol{\omega}_1 \\ \boldsymbol{\omega}_2 \\ \cdots \\ \boldsymbol{\omega}_n \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1^m & 0 & \cdots & 0 \\ \mathbf{A}_2^{m1} & \mathbf{A}_2^{m2} & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ \mathbf{A}_n^{m1} & \mathbf{A}_n^{m2} & \cdots & \mathbf{A}_n^{mn} \end{bmatrix} \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \\ \cdots \\ \mathbf{z}_n \end{bmatrix} = \mathbf{Az}. \quad (41)$$

This can be obtained as

$$\|\boldsymbol{\omega}\| \leq \|\mathbf{A}\|\|\mathbf{z}\| \leq \|\mathbf{z}\|. \quad (42)$$

First inspection the properties of equation (8):

Let $|\mathrm{O}_{i+1}|$ be the maximum value of M_{i+1} on the set Ω, and let $(1/\tau_{i+1}) = ((3 - b_{\max})/4\tau_{i+1})^{-1}((b_{\max}/4) + (\mathrm{M}_{i+1}^2/2\psi) + \psi_0)$, and since $|y_{i+1}\mathrm{O}_{i+1}| \leq (y_{i+1}^2\mathrm{M}_{i+1}^2/2\psi) + (\psi/2)$, then

$$\begin{aligned} &\frac{b_{\max}}{4}y_{i+1}^2 - \frac{3 - b_{\max}}{4\tau_{i+1}} + \left|y_{i+1}\mathrm{O}_{i+1}\right| \\ &\leq -\left(\frac{b_{\max}}{4} + \frac{\mathrm{M}_{i+1}^2}{2\psi} + \psi_0\right)y_{i+1}^2 + \frac{b_{\max}}{4}y_{i+1}^2 + \frac{y_{i+1}^2\mathrm{M}_{i+1}^2\mathrm{O}_{i+1}^2}{2\psi\mathrm{M}_{i+1}^2} + \frac{\psi}{2} \\ &= -\psi_0 y_{i+1}^0 - \left(1 - \frac{\mathrm{O}_{i+1}^2}{\mathrm{M}_{i+1}^2}\right)\frac{y_{i+1}^2\mathrm{M}_{i+1}^2}{2\psi} + \frac{\psi}{2}. \end{aligned} \quad (43)$$

Let be $(\sigma_i/2\lambda_{\max}a_{\min}\mu_i^{-1}) = \psi_0$, $k_1 = b_{\max}^{-1}(2 + ((1 + b_{\max})/4) + \psi_0)$, $k_i = b_{\max}^{-1}(2 + ((1 + b_{\max})/\tau_i) + (b_{\max}/4) + \psi_0)$, $k_n = b_{\max}^{-1}(((1 + b_{\max})/\tau_i) + \psi_0)$, and ψ_0 is a normal number. Then, equation (36) becomes

$$\begin{aligned} \dot{V} \leq &-\psi_0\sum_{i=1}^n z_i^2 - \psi_0\sum_{i=1}^n\left(\widetilde{\lambda}_i^T a_{\min}\mu_i^{-1}\widetilde{\lambda}_i\right) - \psi_0\sum_{i=1}^{n-1}y_{i+1}^2 \\ &+ \sum_{i=1}^n\left(\iota_i^2\boldsymbol{\omega}_i^T\boldsymbol{\omega}_i\right) + \chi \leq -2\psi_0 V + \chi, \end{aligned} \quad (44)$$

where $z = [z_1, z_2, \ldots, z_n]^T$, $\chi = \sum_{i=1}^n(\delta_i') + \sum_{i=1}^{n-1}(\psi/2)$, then

$$V(t) \leq \frac{\chi}{2\psi_0} + V(t_0)e^{-2\psi_0(t-t_0)}, \quad \forall t \geq t_0 \geq 0. \quad (45)$$

Obviously, for any $\zeta_1 > 0$, exist $\mathrm{T} > 0$ so that for all $t \geq t_0 + \mathrm{T}$, $\|z_1\| \leq \zeta_1$ is established. Then, all solutions of the closed-loop system are consistent and bounded, that is, by adjusting the controller parameters so that the tracking error $z_1 = y(t) - y_r(t)$ is as small as possible to achieve the desired tracking accuracy.

3. Simulation Studies of Three-Dimensional Elliptical Vibration Cutting System

3.1. System Transformation. Make the following assumptions for system (8).

Assumption 4. System (8) has a strong relative r.

Under the condition of hypothesis 3.1, there is a differential homeomorphic map $(\mathbf{u}, \mathbf{v}) = T(\mathbf{x})$ so that equation (8) becomes

$$\begin{cases} y^{(r)} = a_1(t) + [b_1(t) + g_1(\mathbf{u}, \mathbf{v})]u + f_1(\mathbf{u}, \mathbf{v}) + d(\mathbf{u}, \mathbf{v}), \\ \dot{v} = q(\mathbf{u}, \mathbf{v}), \end{cases} \quad (46)$$

where $L_g^r h(\mathbf{x}) = f_1(\mathbf{u}, \mathbf{v})$, $L_g L_f^{r-1}h(\mathbf{x}) = g_1(\mathbf{u}, \mathbf{v})$, $\dot{v} = q(\mathbf{u}, \mathbf{v})$ denotes zero dynamics of the system, $a_1(t)$ and $b_1(t)$ are known functions that are continuously bounded, and $a_1(t) = b_1(t) = 0$ is assumed.

Assumption 5. System (1) is of the minimum phase and pairs and the variable $\mathbf{u}$ satisfies the Lipschitz condition:

$$|q(\mathbf{u}, \mathbf{v}) - q(0, \mathbf{v})| \leq L|\mathbf{u}|, \quad (47)$$

where L is a constant.

Under the assumption of 5, if $\mathbf{u}$ is bounded, then $\mathbf{v}$ must be bounded.

Suppose the system has no zero dynamics, i.e., $\mathbf{v} = 0$.

Let $f(\mathbf{x}) = f_1(\mathbf{u}, 0)$, $g(\mathbf{x}) = g_1(\mathbf{u}, 0)$, and $\mathbf{u} = T^{-1}(\mathbf{x})$; then, equation (8) can be changed to

$$y^{(r)} = g(\mathbf{x})u + f(\mathbf{x}) + d(\mathbf{x}, \mathbf{t}). \quad (48)$$

For the three-dimensional elliptical vibration assisted cutting system of the previous research [21], considering this type of single-input single-output (SISO) nonlinear Wiener system, it can be expressed as follows:

$$y^{(3)} = g(x,t)u(t) + f(x,t) + d(t), \quad (49)$$

where $g(x,t)$ and $f(x,t)$ are nonlinear functions, $g(x,t) > 0$, and $d(t)$ is external interference.

Based on the LabVIEW simulation analysis, five fuzzy sets are defined for each variable in the system model to be fuzzy: {NL, NM, ZE, PM, PL}. These fuzzy sets are described by the following membership functions:

$$\begin{cases} \Gamma_{\mathrm{NL}} = \exp\left[-(x+1)^2\right], \\ \Gamma_{\mathrm{NM}} = \exp\left[-(x+0.5)^2\right], \\ \Gamma_{\mathrm{ZE}} = \exp\left[-x^2\right], \\ \Gamma_{\mathrm{PM}} = \exp\left[-(x-0.5)^2\right], \\ \Gamma_{\mathrm{PL}} = \exp\left[-(x-1)^2\right]. \end{cases} \quad (50)$$

According to the control algorithm proposed in Section 2, the control rate, intermediate stabilization function, and parameter adaptation rate are selected as follows:

$$\begin{aligned} &\begin{cases} \psi_2 = -5z_1 + y_d, \\ \psi_3 = -0.05z_2 + s_2 - -\widehat{\lambda}_2\rho_2 z_2, \\ u = -0.02z_3 + s_3 - -\widehat{\lambda}_3\rho_3 z_3, \end{cases} \\ &\begin{cases} \dot{\widehat{\lambda}}_2 = 0.2\left[\rho_2 z_2^2 - 0.5\widehat{\lambda}_2\right], \\ \dot{\widehat{\lambda}}_3 = 0.1\left[\rho_3 z_3^2 - 1.5\widehat{\lambda}_3\right], \end{cases} \\ &\rho_i(\overline{x}_i) = \frac{1}{4\iota_i^2}\xi_i\xi_i^T + \frac{1}{4\kappa_i^2}\varphi_i^2, \quad i = 2, 3. \end{aligned} \quad (51)$$

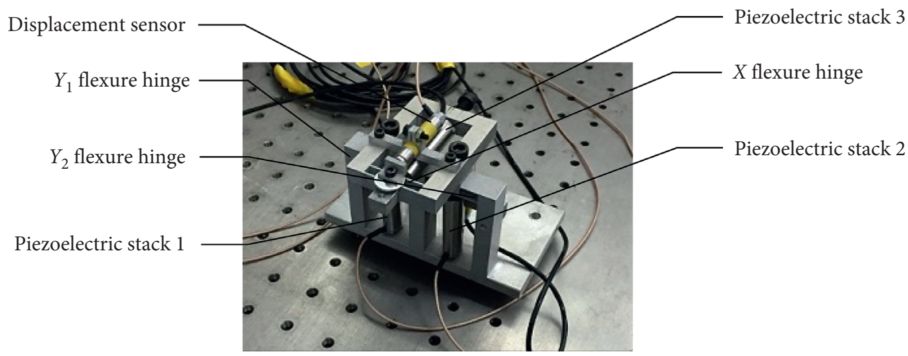


FIGURE 2: Structure of the 3D EVC system.

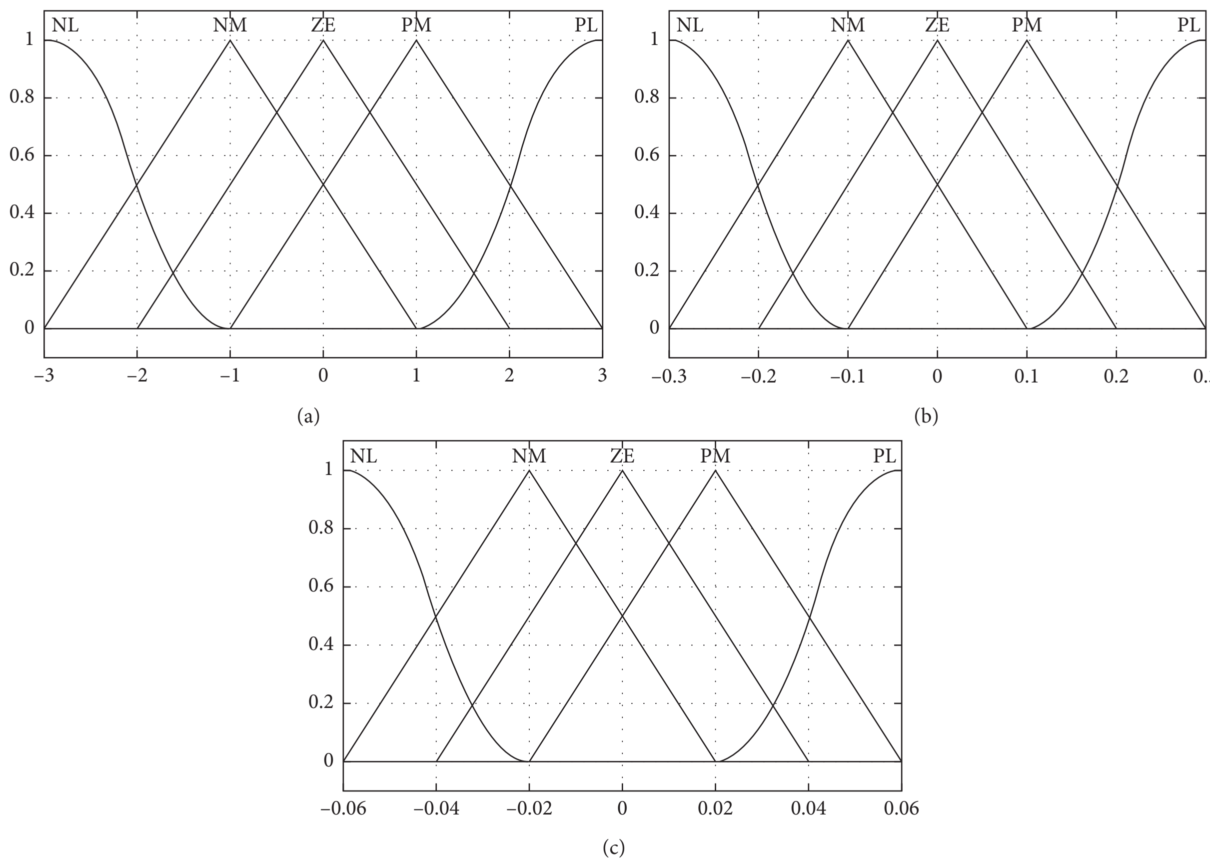
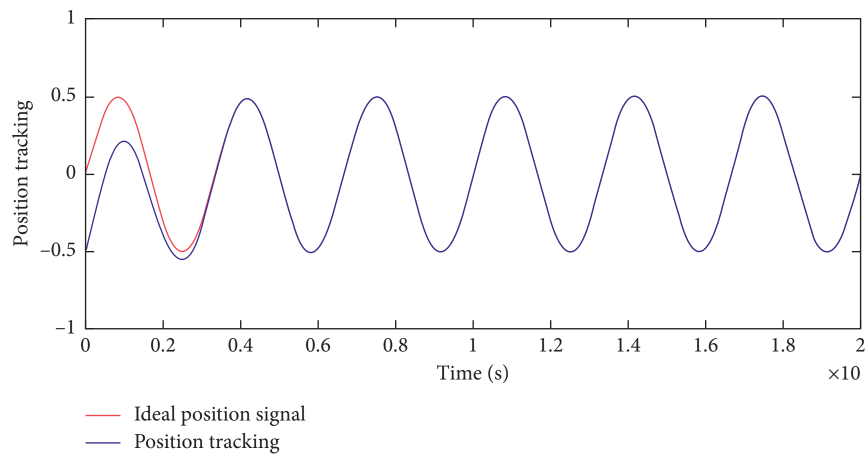


FIGURE 3: Membership function graph.

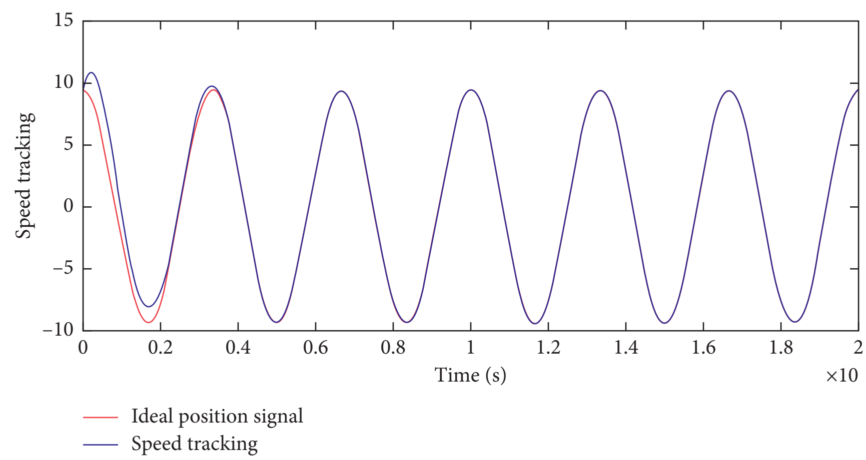
3.2. Description of Three-Dimensional Elliptical Vibration Cutting System. The three-dimensional elliptical vibration assisted cutting device (shown in Figure 2) is driven mainly by two parallel, vertically placed piezoelectric stacks in a nonresonant manner. The three piezoelectric stacks are, respectively, distributed on the upper flexible hinge and the lower flexible hinge, and each of the piezoelectric stacks is placed in parallel with a displacement sensor. The 3D EVC system can be viewed as three single-input single-output (SISO) systems using a nonlinear Wiener model to describe the 3D EVC system. The system identification results are detailed in [32].

3.3. Simulation Studies. In order to further illustrate the effectiveness of the proposed method, the three-dimensional elliptical vibration cutting device developed by our group was used to design the nonlinear Wiener control system as an example. The robust adaptive fuzzy control design and simulation research were carried out. A type of single-input and single-output nonlinear control system that can express its system equations is as follows:

$$\begin{cases} y^{(3)} = f_1(x,t) + g_1(x,t)u(t) + d_1(t), \\ x = y, \end{cases} \tag{52}$$



(a)



(b)

Figure 4: Simulation results: (a) the duration curve of the position tracking signal; (b) the duration curve of the speed tracking signal.

where $f_1(x,t)$ and $g_1(x,t)$ represent nonlinear functions, $g_1(x,t) > 0$; and $d_1(t)$ are unknown external disturbances.

The robust adaptive fuzzy control simulation is carried out by taking the system identification result of the $X+$ direction subsystem of the designed three-dimensional elliptical vibration cutting device as an example. In the simulation, the same membership function as in equation (50) is taken, and use MATLAB software to generate the membership function graph shown in Figure 3. The external interference is taken as $d_1(t) = \sin(t)$; the initial values of the system variables x_1, x_2, and x_3 are set to $[0.2, 2\pi, 0]$; and the reference tracking signal is $y_d = \sin(\pi t)$. The simulation results of position tracking and speed tracking are shown in Figure 4.

The tracking error curve of robust adaptive fuzzy control for a three-dimensional elliptical vibratory cutting system is shown in Figure 4. It can be seen from Figures 4 and 5 that the nonlinear model identified by the three-dimensional elliptical vibration cutting device can effectively suppress the jitter problem by using the robust adaptive fuzzy controller. The control object has a little jitter at the beginning, but it can be quickly stabilized and smoothed to move along the ideal displacement and velocity signals. Figures 6(a) and 6(b) are simulation results of the estimated parameters $\widehat{\lambda_2}$ and $\widehat{\lambda_3}$,

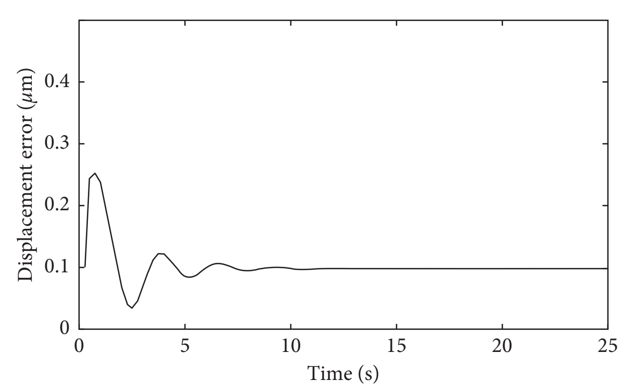


Figure 5: The tracking error curve of robust adaptive fuzzy control for a three-dimensional elliptical vibratory cutting system.

respectively. The parameter estimation simulation curve shows that the proposed controller has strong robust adaptability and can freely adjust its own parameters according to the needs of the controlled system.

The robust adaptive fuzzy control of the $X+$ directional subsystem of the abovementioned three-dimensional elliptical vibration cutting system can make the system tend to be

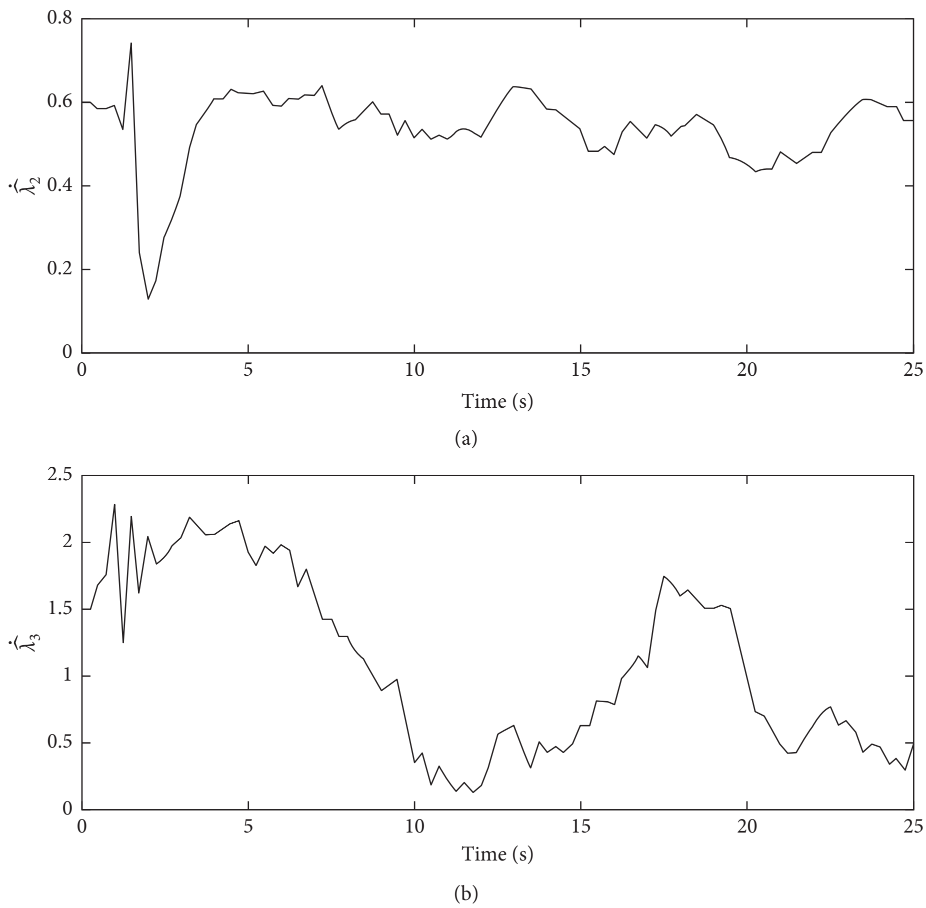


Figure 6: Simulation results: (a) parameter estimation $\hat{\dot{\lambda}}_2$; (b) parameter estimation $\hat{\dot{\lambda}}_3$.

stable in a short time, is not affected by external disturbances and has strong robust adaptability.

4. Conclusions

In this paper, the design of a robust adaptive fuzzy controller for a class of single-input single-output (SISO) uncertain nonlinear systems is proposed, and the corresponding design steps are given. The conclusions reached are as follows:

(1) The most notable features of the proposed control algorithm are as follows: in the case of unknown structural uncertainty, the resulting closed-loop system is ultimately bounded. Only one function needs to have a T-S fuzzy logic system approximation, and each subsystem needs only one parameter to be adjusted, which overcomes the problem of "dimensionality disaster." Thereby, the calculation amount of the control algorithm is greatly reduced and the problem of "calculation expansion" is solved.

(2) The simulation research on the nonlinear control system of the three-dimensional elliptical vibration cutting device developed by our group is carried out. It is verified that the robust adaptive fuzzy control can make the system tend to be stable in a short time, is not affected by external disturbances, and has strong robust adaptability.

Future research will focus on extending the parameter optimization problem of the analytical control system to prove that the proposed solution has more effective engineering application value.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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