

Research Article

On Periodic Dividends for the Classical Risk Model with Debit Interest

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A periodic dividend problem is studied in this paper. We assume that dividend payments are made at a sequence of Poisson arrival times, and ruin is continuously monitored. First of all, three integro-differential equations for the expected discounted dividends are obtained. Then, we investigate the explicit expressions for the expected discounted dividends, and the optimal dividend barrier is given for exponential claims. A similar study on a generalized Gerber–Shiu function involving the absolute time is also performed. To demonstrate the existing results, we give some numerical examples.

1. Introduction

Suppose the dynamics of the surplus process of an insurance company at time t is defined as the solution to

$$dR_t = \begin{cases} cdt - dS_t, & R_t \geq 0, \\ (c + \alpha R_t)dt - dS_t, & -\frac{c}{\alpha} \leq R_t < 0, \end{cases} \quad (1)$$

where $c > 0$ is the premium charged in the unit time and $\alpha > 0$ is the debit interest. $S_t = \sum_{i=1}^{N(t)} X_i$ is a compound Poisson process with intensity $\gamma > 0$ representing the total claim amounts until time t , and X_i is the i -th claim size. Suppose that the claim sizes are independent of each other and have common density function ζ . We also assume that X_i and $N(t)$ are mutually independent.

Model (1) means that the insurer can make loans at the rate $\alpha > 0$ once the process R_t goes below zero. When $R(t) < -c/\alpha$, the insurer can be prohibited to run its business, i.e., the insurer is absolutely ruined. The absolute ruin risk models have been investigated extensively, see Gerber and Yang [1], Cai [2], Yuen et al. [3], Wang and Yin [4], and Cai and Yang [5], among others.

In recent years, the research problems in risk theory are more and more closely related to real life. The risk model

with debit interest is a good example. The insurer cannot monitor the surplus continuously, and the dividend can be paid at some certain times. Because of its importance in real life, the topic of periodic dividends has become very popular in risk theory during the last twenty years. Considering dividends can only be made at some discrete times in practice, Albrecher et al. [6] put forward the periodic barrier dividends in this type of risk model. They assumed that both barrier dividends and ruin can only be observed at some randomized times. Considering the insurer monitors bankruptcy more closely than dividends, Avanzi et al. [7] investigated periodic dividend barrier strategy in the dual model where the solvency is monitored continuously. For those risk models with periodic dividends and bankruptcy, the reader can refer to Zhang and Cheung [8, 9], Avanzi et al. [10], Dong et al. [11], and Peng et al. [12], among others. Different to the papers mentioned above, the periodic dividend strategy in Peng et al. [12] is threshold strategy.

Now, we study surplus process (1) under periodic dividend strategy. We assume the ruin is continuously monitored as usual. Let $\{Z_i\}$ be the sequence of dividend observation times with $Z_0 = 0$ and $W_i = Z_i - Z_{i-1}$ ($i = 1, 2, \dots$) be an exponentially random variable with $\mathbb{E}W_i = 1/\beta$. In addition, we assume that

$W_k (k = 1, 2, \dots)$ are independent of $N(t)$ and X_i . Furthermore, we assume that no dividends are made at time 0. The modified surplus process is given by R_t^b with $b > 0$ being the constant level of the dividend barrier. Denote by $T = \inf\{t > 0: R_t^b < -(c/\alpha)\}$ the time of absolute ruin for R_t^b with the convention that $\inf \phi = \infty$.

Let

$$\nu(x, b) = \mathbb{E}_x \left[\sum_{Z_k \leq T} e^{-\sigma Z_k} (R_{Z_k}^b - b)_+ \right], \quad (2)$$

be the expected discounted dividend before time T , and $\sigma \geq 0$. The generalized Gerber–Shiu function is also our concern:

$$\Phi(x) = \mathbb{E}_x \left[e^{-\sigma T} s^{N(T)} \omega \left(R_{T-}^b, |R_T^b| \right) 1_{\{T < \infty\}} \right], \quad 0 < s \leq 1, \quad (3)$$

where 1_A denotes the indicator function of event A . $\omega(x, y) \geq 0$ is a bounded measurable function of $x \geq -(c/\alpha)$, $y > (c/\alpha)$, R_{T-}^b is the surplus prior to time T , and $|R_T^b|$ is the deficit at time T . When $s = 1$, $\Phi(x)$ is the Gerber–Shiu function proposed by Gerber and Shiu [13]. When $\omega(\cdot, \cdot) = 1$, $\Phi(x)$ describes the joint distribution of the absolute time T and $N(T)$:

$$\phi(x) = \mathbb{E}_x \left[s^{N(T)} e^{-\sigma T} 1_{\{T < \infty\}} \right]. \quad (4)$$

When $\omega(\cdot, \cdot) = 1$, $\sigma = 0$, $\Phi(x)$ is the probability generating function of $N(T)$. The topic on the number of claims has been studied by many scholars in various forms. Some scholars focus on some generalized Gerber–Shiu functions, see Li and Lu [14] and Wang et al. [15]; some only considered the Laplace transform, see Egidio dos Reis [16]; and some studied the density function, see Dickson [17] and Czarna et al. [18].

We arrange the paper as follows. In Section 2, we first derive a system of integro–differential equations for $\nu(x, b)$. For exponential claims, explicit results for $\nu(x, b)$ are given. The generalized Gerber–Shiu function is discussed in Section 3. In Section 4, some numerical results are shown to illustrate the given results.

2. Results for $\nu(x, b)$

2.1. Integro-differential Equations for $\nu(x, b)$. The integro-differential equation is a conventional method; we start this section with the expression for $\nu(x, b)$.

Theorem 1. For $-(c/\alpha) \leq x < 0$, we have

$$(\alpha x + c)\nu'(x, b) - (\sigma + \gamma)\nu(x, b) + \gamma \int_0^{x+c/\alpha} \nu(x-z, b)\zeta(z)dz = 0, \quad (5)$$

for $0 \leq x < b$, we have

$$c\nu'(x, b) - (\sigma + \gamma)\nu(x, b) + \gamma \int_0^{x+c/\alpha} \nu(x-z, b)\zeta(z)dz = 0, \quad (6)$$

and for $x > b$, we have

$$c\nu'(x, b) - (\sigma + \beta + \gamma)\nu(x, b) + \beta(x-b) + \beta\nu(b, b) + \gamma \int_0^{x+c/\alpha} \nu(x-z, b)\zeta(z)dz = 0. \quad (7)$$

In addition, we have the following relations:

$$\nu(b-, b) = \nu(b+, b), \quad (8)$$

$$\nu'(b-, b) = \nu'(b+, b), \quad (9)$$

$$\nu(0-, b) = \nu(0+, b), \quad (10)$$

$$\nu'(0-, b) = \nu'(0+, b), \quad (11)$$

$$\nu\left(-\frac{c}{\alpha}, b\right) = 0. \quad (12)$$

Proof. A standard method is used here, see also Yuen et al. [3] and Albrecher et al. [6]. For a small interval $(0, s)$, all the possible events are taken into account. Then, we have

$$\begin{aligned} \nu(x, b) &= e^{-(\beta+\sigma+\gamma)s} \nu(x+cs, b) + \beta s e^{-(\sigma+\gamma)s} \\ &\quad \cdot [x+cs-b+\nu(b, b)] 1_{\{x+cs>b\}} \\ &\quad + \beta s e^{-(\sigma+\gamma)s} \nu(x+cs, b) 1_{\{0<x+cs<b\}} \\ &\quad + \gamma s e^{-(\sigma+\gamma)s} \int_0^{x+cs+c/\alpha} \nu(x+cs-y, b)\zeta(y)dy + o(s), \end{aligned} \quad (13)$$

for $x \geq 0$, and for $-(c/\alpha) \leq x < 0$,

$$\begin{aligned} \nu(x, b) &= e^{-(\beta+\sigma+\gamma)s} \nu(t_\alpha(s, x), b) + \beta s e^{-(\sigma+\gamma)s} \nu(t_\alpha(s, x)) \\ &\quad + \gamma s e^{-(\sigma+\beta)s} \int_0^{t_\alpha(s, x)+c/\alpha} \nu(t_\alpha(s, x)-y, b)\zeta(y)dy + o(s), \end{aligned} \quad (14)$$

where $t_\alpha(s, x) = xe^{\alpha s} + c(e^{\alpha s} - 1)/\alpha$.

Taking derivative on s in (13) and letting $s \rightarrow 0$, we arrive at (6) and (7). By a similar method, (5) can be obtained from (14).

Continuity condition (8) can be obtained from (13), and (10) can be obtained by comparing (13) with (14). (9) can be obtained by using (6) and (7), while (11) is derived by (5) and (6). Let $x = -c/\alpha$ in (5), and we have boundary condition (12). \square

Remark 1. Obviously, (6) and (7) are the same as (2.1) and (2.2) in Yuen et al. [3], respectively.

2.2. *Explicit Expressions of $v(x, b)$.* In this section, the density function of claim sizes is supposed to be $f(x) = \mu e^{-\mu x}$, $x > 0$. Applying the operator $(d/dx + \mu)$ to (5), then we have

$$(\alpha x + c)v''(x, b) + (\alpha \mu x + c\mu + \alpha - \gamma - \sigma)v'(x, b) - \sigma \mu v(x, b) = 0, \tag{15}$$

for $(-c/\alpha) \leq x < 0$.

By the transforms $v(x, b) = k(z)$ and $z = -\mu(x + c/\alpha)$, (15) is reduced to a confluent hypergeometric equation:

$$zk''(z) + \left(1 - \frac{\sigma + \gamma}{\alpha} - z\right)k'(z) + \frac{\sigma}{\alpha}k(z) = 0, \quad z < 0. \tag{16}$$

It follows from Abramowitz and Stegun [19] that $k(x)$ admits the following expression:

$$k(z) = C_1 e^z U\left(1 - \frac{\gamma}{\alpha}, 1 - \frac{\sigma + \gamma}{\alpha}, -z\right) + C_2 z^{(\sigma + \gamma/\alpha)} M\left(\frac{\gamma}{\alpha}, 1 + \frac{\sigma + \gamma}{\alpha}, z\right), \tag{17}$$

where C_1 and C_2 are two constants, $M(a, b, z)$ is the first kind of confluent hypergeometric function, and $U(a, b, z)$ is the second kind of confluent hypergeometric function. Hence, the solution of (15) can be expressed as

$$v(x, b) = C_1 e^{-\mu(x + c/\alpha)} U\left(1 - \frac{\gamma}{\alpha}, 1 - \frac{\sigma + \gamma}{\alpha}, \mu\left(x + \frac{c}{\alpha}\right)\right) + C_2 \left(-\mu\left(x + \frac{c}{\alpha}\right)\right)^{(\sigma + \gamma/\alpha)} M\left(\frac{\gamma}{\alpha}, 1 + \frac{\sigma + \gamma}{\alpha}, -\mu\left(x + \frac{c}{\alpha}\right)\right). \tag{18}$$

Due to boundary condition (13), one immediately deduces $C_1 = 0$.

Using a similar procedure to (15), we deduce

$$cv''(x, b) - (\sigma + \gamma - c\mu)v'(x, b) - \sigma \mu v(x, b) = 0 \tag{19}$$

for $0 < x < b$ and

$$cv''(x, b) - (\gamma + \beta + \sigma - c\mu)v'(x, b) - (\sigma + \beta)\mu v(x, b) + \beta \mu [\gamma(b, b) + x - b] + \beta = 0, \tag{20}$$

for $x \geq b$. Solving (19) and (20) by the knowledge of the ordinary differential equation, one deduces

$$v(x, b) = C_3 e^{\rho_0 x} + C_4 e^{R_0 x}, \quad 0 \leq x < b, \tag{21}$$

$$v(x, b) = D_1 e^{\rho_\beta x} + D_2 e^{R_\beta x} + D_3 x + D_4, \quad x \geq b, \tag{22}$$

where $\rho_\beta > 0, R_\beta < 0$ are roots of the equation

$$r^2 + \left(\mu - \frac{\sigma + \gamma + \beta}{c}\right)r - \frac{(\sigma + \beta)\mu}{c} = 0. \tag{23}$$

Clearly, $v(x, b)$ is bounded. Hence, $D_1 = 0$. By (18), (21), and (22) and (8)–(11), we get the following equations:

$$C_3 + C_4 = C_2 \Delta_1, \tag{24}$$

$$C_3 \rho_0 + C_4 R_0 = C_2 \Delta_2, \tag{25}$$

$$D_2 e^{R_\beta b} + D_3 b + D_4 = C_3 e^{\rho_0 b} + C_4 e^{R_0 b}, \tag{26}$$

$$D_2 R_\beta e^{R_\beta b} + D_3 = C_3 \rho_0 e^{\rho_0 b} + C_4 R_0 e^{R_0 b}, \tag{27}$$

where

$$\Delta_1 = \left(\frac{-c\mu}{\alpha}\right)^{(\sigma + \gamma/\alpha)} M\left(\frac{\gamma}{\alpha}, 1 + \frac{\sigma + \gamma}{\alpha}, -\frac{c\mu}{\alpha}\right),$$

$$\Delta_2 = \frac{\sigma + \gamma}{c} \Delta_1 - \frac{\gamma \mu}{\sigma + \alpha + \gamma} \left(\frac{-c\mu}{\alpha}\right)^{(\sigma + \gamma/\alpha)} M\left(1 + \frac{\gamma}{\alpha}, 2 + \frac{\sigma + \gamma}{\alpha}, -\frac{c\mu}{\alpha}\right). \tag{28}$$

Substituting (22) into (7) and equating the coefficients of x lead to

$$D_3 = \frac{\beta}{\beta + \sigma}, \tag{29}$$

and then equating the constant term yields

$$\beta D_2 e^{R_\beta b} - \sigma D_4 = \left(\frac{\gamma}{\mu - c - \beta b}\right) D_3 + \beta b. \tag{30}$$

Solving (24)–(30), we get

$$v(x, b) = \frac{(R_0 - \rho_0)h(\beta)(-\mu(x + (c/\alpha)))^{\sigma + \gamma/\alpha} M((\gamma/\alpha), 1 + (\sigma + \gamma/\alpha), -\mu(x + (c/\alpha)))}{H(\beta, b)}, \quad -\frac{c}{\alpha} < x < 0, \tag{31}$$

$$v(x, b) = \frac{(R_0 \Delta_1 - \Delta_2)h(\beta)}{H(\beta, b)} e^{\rho_0 x} + \frac{(\Delta_2 - \rho_0 \Delta_1)h(\beta)}{H(\beta, b)} e^{R_0 x}, \quad 0 \leq x \leq b, \tag{32}$$

$$v(x, b) = D_2 e^{R_\beta x} + D_3 x + D_4, \quad x > b,$$

where

$$D_2 = \frac{[\rho_0(R_0\Delta_1 - \Delta_2)e^{(\rho_0 - R_\beta)b} + R_0(\Delta_2 - \rho_0\Delta_1)e^{(R_0 - R_\beta)b}]h(\beta)}{R_\beta H(\beta, b)} - \frac{\beta}{R_\beta(\beta + \sigma)}e^{-R_\beta b},$$

$$D_3 = \frac{\beta}{\beta + \sigma},$$

$$D_4 = \frac{[(1 - (\rho_0/R_\beta))(R_0\Delta_1 - \Delta_2)e^{\rho_0 b} + (1 - (R_0/R_\beta))(\Delta_2 - \rho_0\Delta_1)e^{R_0 b}]h(\beta)}{H(\beta, b)} - \frac{\beta}{\beta + \sigma}\left(b - \frac{1}{R_\beta}\right), \tag{33}$$

$$H(\beta, b) = [(\sigma + \beta)\rho_0 - \sigma R_\beta](\Delta_2 - R_0\Delta_1)e^{\rho_0 b} + [(\sigma + \beta)R_0 - \sigma R_\beta](\rho_0\Delta_1 - \Delta_2)e^{R_0 b},$$

$$h(\beta) = \frac{\beta[\gamma R_\beta + \mu(\beta + \sigma - cR_\beta)]}{\mu(\beta + \sigma)}.$$

Remark 2. For $\beta \rightarrow \infty$, it is easy to check that $R_\beta \rightarrow -\mu$. Letting $\beta \rightarrow \infty$ in (31) and (32) and noticing $\rho_0 + R_0 = -\mu + (\sigma + \gamma)/c$, we have

$$V_c(x, b) = \frac{(\rho_0 - R_0)(1 + (\alpha x/c))^{(\sigma+\gamma)/\alpha} M(\gamma/\alpha, 1 + (\sigma + \gamma/\alpha), -\mu(x + c/\alpha))}{(\rho_0 + \mu)\Delta_1 - (\gamma\mu/\gamma + \alpha + \sigma)M(1 + (\gamma/\alpha), 2 + (\sigma + \gamma/\alpha), -c\mu/\alpha)}, \quad -\frac{c}{\alpha} < x < 0,$$

$$V_c(x, b) = \frac{(\Delta_2 - R_0\Delta_1)e^{\rho_0 x} + (\rho_0\Delta_1 - R_0)e^{R_0 x}}{(\rho_0 + \mu)\Delta_1 - (\gamma\mu/\gamma + \alpha + \sigma)M(1 + (\gamma/\alpha), 2 + (\sigma + \gamma/\alpha), -c\mu/\alpha)}, \quad 0 \leq x \leq b,$$

which are in accordance to (3.4) in Yuen et al. [3].

When the initial surplus $-c/\alpha < x \leq b$, we can investigate the optimal dividend barrier b^* which can maximize $v(x, b)$ before absolute ruin. By (31) and (32), we identify that their numerators have nothing to do with the barrier b , and their denominators are $H(\beta, b)$. We also find that their numerators are bigger than 0. So, the optimal barrier b^* solves $H'(\beta, b^*) = 0$, i.e.,

$$b^* = \frac{1}{\rho_0 - R_0} \ln \frac{R_0 [(\sigma + \beta)R_0 - \sigma R_\beta] (\rho_0\Delta_1 - \Delta_2)}{\rho_0 [(\sigma + \beta)\rho_0 - \sigma R_\beta] (R_0\Delta_1 - \Delta_2)}. \tag{35}$$

When $\beta \rightarrow \infty, b^* \rightarrow (1/\rho_0 - R_0) \ln (R_0^2(\rho_0\Delta_1 - \Delta_2)/\rho_0^2(R_0\Delta_1 - \Delta_2))$, which is the same as (3.5) in Yuen et al. [3].

Remark 3. When the claim sizes are general distributions, e.g., Erlang(n) or mixture of exponential distribution, the explicit expression for $v(x, b)$ cannot be provided for $(-c/\alpha) < x < 0$. Then, some numerical methods will be helpful, e.g., Yu et al. [20] and Zhang et al. [21].

3. Results for $\Phi(x)$

Using a similar method for Theorem 1, we can obtain the following results for the generalized Gerber–Shiu function $\Phi(x)$.

Theorem 2. *The generalized Gerber–Shiu function $\Phi(x)$ admits the following expressions:*

$$c\Phi'(x) - (\gamma + \beta + \sigma)\Phi(x) + \beta\Phi(b) + \gamma s \int_0^{x+c/\alpha} \Phi(x-z)\zeta(z)dz + \gamma s \int_{x+c/\alpha}^\infty \omega(x, z-x)\zeta(z)dz = 0, \quad x \geq b, \tag{36}$$

$$c\Phi'(x) - (\sigma + \gamma)\Phi(x) + \gamma s \int_0^{x+c/\alpha} \Phi(x-z)\zeta(z)dz + \gamma s \int_{x+c/\alpha}^\infty \omega(x, z-x)\zeta(z)dz = 0, \quad 0 \leq x < b, \tag{37}$$

$$(\alpha x + c)\Phi'(x) - (\sigma + \gamma)\Phi(x) + \gamma s \int_0^{x+(c/\alpha)} \Phi(x-z)\zeta(z)dz + \gamma s \int_{x+(c/x)}^\infty \omega(x, z-x)\zeta(z)dz = 0, \quad -\frac{c}{\alpha} \leq x < 0, \quad (38)$$

with the following conditions:

$$\Phi(0+) = \Phi(0-), \quad (39)$$

$$\Phi(b+) = \Phi(b-), \quad (40)$$

$$\Phi'(b+) = \Phi'(b-), \quad (41)$$

$$\Phi'(0+) = \Phi'(0-), \quad (42)$$

$$\Phi\left(-\frac{c}{\alpha}\right) = \frac{\gamma s}{\gamma + \sigma} \int_0^\infty \omega\left(-\frac{c}{\alpha}, -\frac{c}{\alpha} - y\right)\zeta(y)dy. \quad (43)$$

3.1. *Explicit Expressions of $\phi(x)$.* We also assume that the claim sizes have the density function $f(x) = \mu e^{-\mu x}$ ($x > 0$), similar to Section 2.2, and we have

$$\begin{aligned} (\alpha x + c)\phi''(x) + (\alpha\mu x + c\mu + \alpha - \gamma - \sigma)\phi'(x) + [\gamma\mu s - (\sigma + \gamma)\mu]\phi(x) &= 0, \quad -\frac{c}{\alpha} < x < 0, \\ c\phi''(x) - (\sigma + \gamma - c\mu)\phi'(x) - [(\sigma + \gamma)\mu - \gamma\mu s]\phi(x) &= 0, \quad 0 \leq x < b, \end{aligned} \quad (44)$$

$$c\phi''(x) + (c\mu - \gamma - \beta - \sigma)\phi'(x) - [(\gamma + \beta + \sigma)\mu - \gamma\mu s]\phi(x) + \beta\mu\phi(b) = 0, \quad x \geq b.$$

Then, solving the above system of equations leads to

$$\phi(x) = c_1 M_1(x, s) + c_2 M_2(x, s), \quad -\frac{c}{\alpha} \leq x < 0,$$

$$\phi(x) = c_3 e^{\rho_{0s}x} + c_4 e^{R_{0s}x}, \quad 0 \leq x < b, \quad (45)$$

$$\phi(x) = d_1 e^{\rho_{\beta s}x} + d_2 e^{R_{\beta s}x} + d_4, \quad x \geq b,$$

where

$$\begin{aligned} M_1(x, s) &= e^{-\mu(x+(c/\alpha))} U\left(1 - \frac{\gamma s}{\alpha}, 1 - \frac{\sigma + \gamma}{\alpha}, \mu\left(x + \left(\frac{c}{\alpha}\right)\right)\right), \\ M_2(x, s) &= \left(-\mu\left(x + \left(\frac{c}{\alpha}\right)\right)\right)^{(\sigma+\gamma)/\alpha} M\left(\frac{\gamma s}{\alpha}, 1 + \frac{\sigma + \gamma}{\alpha}, -\mu\right. \\ &\quad \cdot \left.\left(x + \left(\frac{c}{\alpha}\right)\right)\right), \end{aligned} \quad (46)$$

and $\rho_{\beta s} > 0, R_{\beta s} < 0$ are solutions of the equation

$$cr^2 + (\gamma + \beta + \sigma - cu)r - (\gamma + \beta + \sigma - \gamma s)\mu = 0. \quad (47)$$

Since $\phi(x)$ is a bounded function, then $d_1 = 0$. By conditions (40)–(43), we have

$$\frac{\Gamma(\sigma + \gamma/\alpha)}{\Gamma(\gamma + \alpha + \sigma - \gamma s/\alpha)} c_1 = \frac{\gamma s}{\sigma + \gamma}, \quad (48)$$

$$c_3 + c_4 = c_1 M_1(0, s) + c_2 M_2(0, s), \quad (49)$$

$$c_3 \rho_{0s} + c_4 R_{0s} = c_1 M_1'(0, s) + c_2 M_2'(0, s), \quad (50)$$

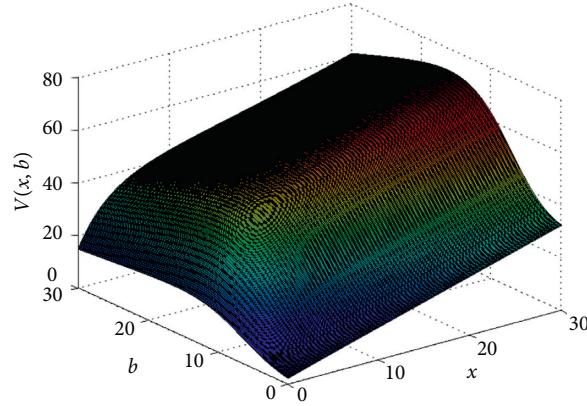
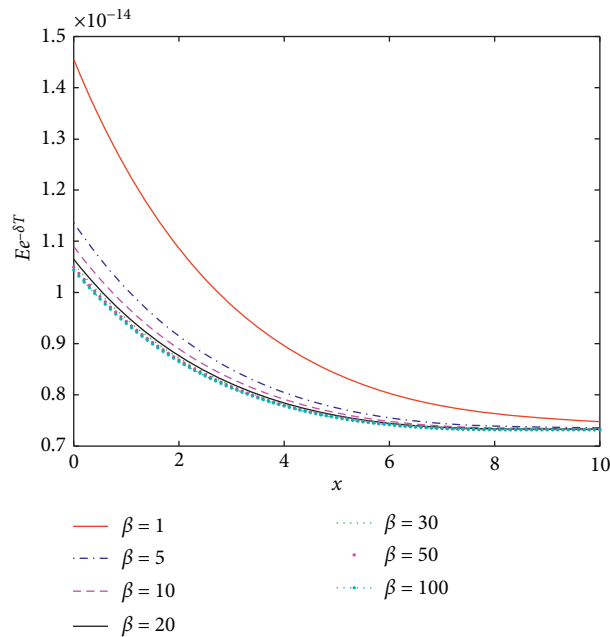
$$c_3 e^{\rho_{0s}b} + c_4 e^{R_{0s}b} = d_2 e^{R_{\beta s}b} + d_4, \quad (51)$$

$$c_3 \rho_{0s} e^{\rho_{0s}b} + c_4 R_{0s} e^{R_{0s}b} = d_2 R_{\beta s} e^{R_{\beta s}b}. \quad (52)$$

Inserting (45) into (36) and comparing the coefficients of constants yield

$$(\gamma s - \gamma - \sigma)d_4 + \beta d_2 e^{R_{\beta s}b} = 0. \quad (53)$$

Solving (48)–(53), one obtains

FIGURE 1: Influence of x and b on the expected discounted dividend $v(x, b)$.FIGURE 2: Influence of β on $\mathbb{E}_x[\exp - \sigma T]$.

$$\begin{aligned}
 c_1 &= \frac{\gamma s \Gamma(\gamma + \alpha + \sigma - \gamma s / \alpha)}{(\sigma + \gamma) \Gamma(\sigma + \gamma / \alpha)}, \\
 c_2 &= \frac{\gamma s \Gamma(\gamma + \alpha + \sigma - \gamma s / \alpha) [(\ell_1 + \ell_2) M_1'(0, s) + (\ell_1 \rho_{0s} + \ell_2 R_{0s}) M_1(0, s)]}{(\sigma + \gamma) \Gamma(\sigma + \gamma / \alpha) [(\ell_1 \rho_{0s} + \ell_2 R_{0s}) M_2(0, s) - (\ell_1 + \ell_2) M_2'(0, s)]}, \\
 c_3 &= \frac{\gamma s \Gamma(\gamma + \alpha + \sigma - \gamma s / \alpha) \ell_1 [M_1'(0, s) M_2(0, s) - M_1(0, s) M_2'(0, s)]}{(\sigma + \gamma) \Gamma(\sigma + \gamma / \alpha) [(\ell_1 \rho_{0s} + \ell_2 R_{0s}) M_2(0, s) - (\ell_1 + \ell_2) M_2'(0, s)]}, \\
 c_4 &= \frac{\gamma s \Gamma(\gamma + \alpha + \sigma - \gamma s / \alpha) \ell_2 [M_1'(0, s) M_2(0, s) - M_1(0, s) M_2'(0, s)]}{(\sigma + \gamma) \Gamma(\sigma + \gamma / \alpha) [(\ell_1 \rho_{0s} + \ell_2 R_{0s}) M_2(0, s) - (\ell_1 + \ell_2) M_2'(0, s)]}, \\
 d_2 &= \frac{\gamma s \Gamma(\gamma + \alpha + \sigma - \gamma s / \alpha) (\sigma + \gamma - \gamma s) [M_1'(0, s) M_2(0, s) - M_1(0, s) M_2'(0, s)]}{(\sigma + \gamma) \Gamma(\sigma + \gamma / \alpha) [(\ell_1 \rho_{0s} + \ell_2 R_{0s}) M_2(0, s) - (\ell_1 + \ell_2) M_2'(0, s)]} e^{-R_{\beta s} b}, \\
 d_4 &= \frac{\beta \gamma s \Gamma(\gamma + \alpha + \sigma - \gamma s / \alpha) [M_1'(0, s) M_2(0, s) - M_1(0, s) M_2'(0, s)]}{(\sigma + \gamma) \Gamma(\sigma + \gamma / \alpha) [(\ell_1 \rho_{0s} + \ell_2 R_{0s}) M_2(0, s) - (\ell_1 + \ell_2) M_2'(0, s)]}
 \end{aligned} \tag{54}$$

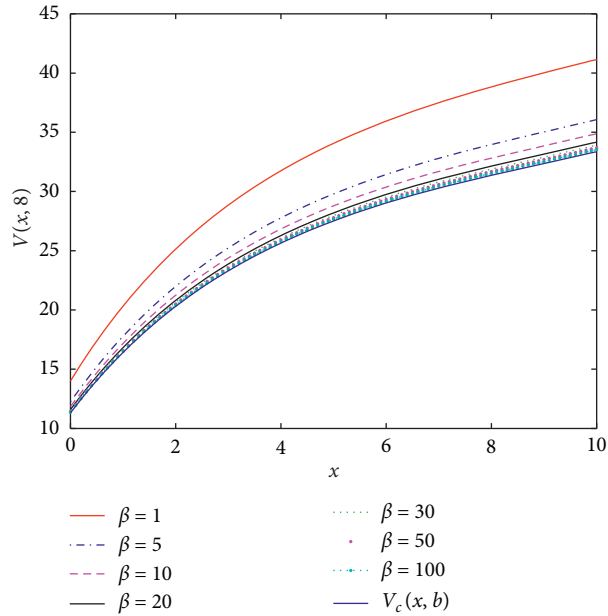


FIGURE 3: Influence of β on $v(x, b)$.

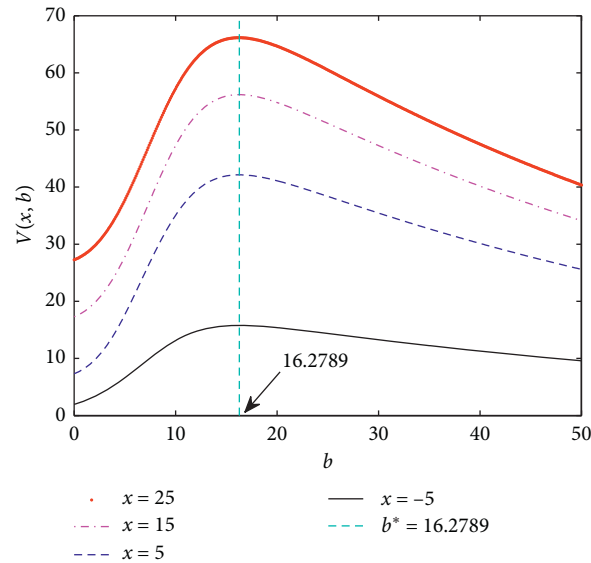


FIGURE 4: $v(x, b)$ as a function of b .

where

$$\begin{aligned}
 \ell_1 &= \frac{(R_{0s} - R_{\beta s})(\sigma + \gamma - \gamma s) + \beta R_{0s} e^{-\rho_{0s} b}}{R_{0s} - \rho_{0s}}, \\
 \ell_2 &= \frac{(\rho_{0s} - R_{\beta s})(\sigma + \gamma - \gamma s) + \beta \rho_{0s} e^{-R_{0s} b}}{\rho_{0s} - R_{0s}}.
 \end{aligned}
 \tag{55}$$

4. Numerical Examples

In this last section, we give some numerical examples to illustrate the results. In Figures 1–4, we assume that the claim size follows exponential distribution with mean 2, the premium rate $c = 4$, the Poisson intensity $\gamma = 0.8$, the

discounted factor $\sigma = 0.04$, and the debit interest $\alpha = 0.06$. Figure 1 depicts the profiles of $v(x, b)$ for $(x, b) \in (0, 30) \times (0, 30)$. We find that $v(x, b)$ is not a monotone function of b . When $b = 8$, the curves of $\mathbb{E}_x[e^{-\sigma T} 1_{\{T < \infty\}}]$ and $v(x, b)$ are given in Figures 2 and 3 for several different β , respectively. It is shown that $v(x, b)$ is a decreasing function of β , while $\mathbb{E}_x[e^{-\sigma T} 1_{\{T < \infty\}}]$ is an increasing function of β . The bigger β means the more frequent observation, which can lead to the earlier ruin. By Figure 3, we also find that $v(x, b)$ is more and more close to $V_c(x, b)$ as β becomes more and more large. The profiles of $v(x, b)$ are given in Figure 4 for some different initial surplus x . By Figure 4, we know that the optimal barrier is $b^* = 16.2789$ which has nothing to do with the initial surplus.

TABLE 1: Impact of β and α on b^* when $c = 4$, $\gamma = 0.8$, $\mu = 0.5$, and $\sigma = 0.04$.

α	$\beta = 1$	$\beta = 5$	$\beta = 10$	$\beta = 20$	$\beta = 30$	$\beta = 50$	$\beta = 100$	$\beta \rightarrow \infty$
0.04	14.4939	15.9856	16.2774	16.4446	16.5045	16.5541	16.5925	16.6320
0.06	14.4954	15.9870	16.2789	16.4460	16.5059	16.5556	16.5940	16.6335
0.07	14.4960	15.9877	16.2795	16.4467	16.5066	16.5563	16.5947	16.6341
0.084	14.5125	16.0042	16.2960	16.4632	16.5231	16.5727	16.6111	16.6506

The numerical results for b^* are given in Table 1 for various observation intensity β and debit interest α . The optimal dividend barrier b^* is an increasing function of β , which is also an increasing function of α . More frequent observation will give rise to more dividend payments in the initial stage and earlier absolute ruin. In order to avoid the earlier absolute ruin, a bigger dividend barrier will be needed to decrease the dividend payments at the beginning.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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