Research Article

Decision-Making Based on q-Rung Orthopair Fuzzy Soft Rough Sets

Yinyu Wang ¹, Azmat Hussain ², Tahir Mahmood ², Muhammad Irfan Ali ³,
Hecheng Wu,¹ and Yun Jin ⁴

¹College of Economic and Management, Nanjing University of Aeronautics and Astronautics, Nanjing 211106, China

²Department of Mathematics and Statistics, Faculty of Basic and Applied Sciences, International Islamic University Islamabad, Islamabad, Pakistan

³Department of Mathematics, Islamabad Model College for Boys, G-11/1, Islamabad, Pakistan

⁴Wuxi Vocational College of Science and Technology, Wuxi 214028, China

Correspondence should be addressed to Tahir Mahmood; tahirbakhath@iiu.edu.pk

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Recently, Yager presented the new concept of q-rung orthopair fuzzy (q-ROF) set (q-ROFS) which emerged as the most significant generalization of Pythagorean fuzzy set (PFS). From the analysis of q-ROFS, it is clear that the rung q is the most significant feature of this notion. When the rung q increases, the orthopair adjusts in the boundary range which is needed. Thus, the input range of q-ROFS is more flexible, resilient, and suitable than the intuitionistic fuzzy set (IFS) and PFS. The aim of this manuscript is to investigate the hybrid concept of soft set (S, S) and rough set with the notion of q-ROFS to obtain the new notion of q-ROF soft rough (q-ROFS_R) set (q-ROFS_{RS}). In addition, some averaging aggregation operators such as q-ROFS_R weighted averaging (q-ROFS_{RWA}), q-ROFS_R ordered weighted averaging (q-ROFS_{ROWA}), and q-ROFS_R hybrid averaging (q-ROFS_{RHA}) operators are presented. Then, the basic desirable properties of these investigated averaging operators are discussed in detail. Moreover, we investigated the geometric aggregation operators, such as q-ROFS_R weighted geometric (q-ROFS_{RWG}), q-ROFS_R ordered weighted geometric (q-ROFS_{ROWG}), and q-ROFS_R hybrid geometric (q-ROFS_{RHG}) operators, and proposed the basic desirable characteristics of the investigated geometric operators. The technique for multicriteria decision-making (MCDM) and the stepwise algorithm for decision-making by utilizing the proposed approaches are demonstrated clearly. Finally, a numerical example for the developed approach is presented and a comparative study of the investigated models with some existing methods is brought to light in detail which shows that the initiated models are more effective and useful than the existing methodologies.

1. Introduction

Decision-making has always been a hot topic under consideration by the researchers. Multicriteria decision-making (MCDM) has a high prospective and discipline manner to improve and evaluate multiple conflicting criteria in all areas of decision-making. In this competitive environment, an enterprise needs the most accurate and rapid response to change the customer needs. So, MCDM has the ability to handle successfully the evaluation process of multiple contradictory criteria. For an intelligent decision, the experts

analyze each and every character of an alternative and then they take the decision. For an intelligent and successful decision, the experts require careful preparation and analysis of each and every character for an alternative and then they can take a good decision if they are armed with all the data and information that they need. To handle this complexity, Zadeh [1] originated the dominant and pioneer concept of the fuzzy set. For each domain in the fuzzy set, a value is assigned from the unit interval and is called membership grade (MemG). From the inception of the fuzzy set, it has been generalized in different directions in which one of the

most significant concepts is intuitionistic fuzzy (IF) set (IFS). Atanassov [2] initiated this dominant concept of IFS which is characterized by two mappings called MemG and non-membership grade (NMemG). IFS is defined on the basis of restriction that the sum of MemG and NMemG must not exceed the unit interval $[0, 1]$. The notion of IFS appears as a hot research area after its origination. In literature, researchers used different techniques to handle the ranking with score or accuracy functions but all these techniques had some drawbacks. So, Ali et al. [3] initiated a graphical technique for ranking the IF values. Xu [4] investigated the series of aggregation operators such as IF weighted averaging (IFWA), IF ordered weighted averaging (IFOWA), and IF hybrid averaging (IFHA) operators under IF environment. The series of geometric operators, namely, IF weighted geometric (IFWG), IF ordered weighted geometric (IFOWG), and IF hybrid geometric (IFHG) operators, were presented by Xu and Yager [5]. Zhao et al. [6] initiated the idea of generalized IFWA, generalized IFOWA, and generalized IFHA operators by utilizing the IF information. Wang and Liu [7, 8] originated the concepts of geometric and averaging aggregation operators by utilizing Einstein operations. Zeng et al. [9] investigated a novel score function of intuitionistic fuzzy and then presented its application in decision-making. In a different scenario, the professional experts are restricted to provide their choices in the range of IFS. To cover this shortcoming, Yager [10] investigated the powerful paradigm of Pythagorean fuzzy (PF) set (PFS) in which the square sum of MemG and NMemG must lie between the real numbers 0 and 1. PFS relaxes and widens the boundary range by providing additional space to the decision-maker. Yager [11] originated the geometric and averaging aggregation operation by using PF information. Peng and Yang [12] initiated the concept of subtraction and division operators and proved some of its basic properties. Peng and Yang [13] investigated the notion of PF Choquet integral average and PF Choquet integral geometric operators. Garg [14, 15] proposed some PF Einstein averaging and PF Einstein geometric operators and presented their basic characteristics. Garg [16] investigated confidence PF weighted and ordered weighted averaging operators with their basic properties. Wei and Lu [17] originated the notions of PF power averaging and geometric operators and presented their desirable characteristics of these investigated operators. Wei [18] presented some interaction averaging and geometric operators by using PF information. The concept of Hamacher operations for PF averaging and geometric operators was presented by Wu and Wei [19]. In the PF environment, the decision-makers are restricted to their boundary limitation and they cannot provide their preferred values freely. Due to these restrictions, some decisive information cannot be effectively handled by PFS.

Recently, Yager [20] presented the new concept of q -rung orthopair fuzzy (q -ROF) set (q -ROFS) from which the most significant generalization of PFS emerged. In q -ROFS, the sum of q th power of MemG and q th power of NMemG must be confined to the unit interval $[0, 1]$ and, furthermore, when the rung q increases, then the range of orthopair satisfies the boundary restriction which is needed.

Thus, the concept of q -ROFS is more useful and powerful than IFS and PFS because these are the special cases of q -ROFS. The basic properties of q -ROFS are proposed by Yager and Alajlan [21] and have been utilized in knowledge representation. Ali [22] proposed another view of q -ROFS by using the idea of orbits. Liu and Wang [23] proposed the concepts of q -ROF weighted averaging (q -ROFWA) and q -ROF weighted geometric (q -ROFWG). Liu and Liu [24] presented the combined study of Bonferroni mean (BM) operators with q -ROFS to investigate the q -ROF BM operators and also study q -ROF geometric BM operators with their desirable properties. Jana et al. [25] initiated the q -ROF Dombi averaging and geometric aggregation operators with their fundamental desirable characteristics. Wang et al. [26] investigated the combined concept of Muirhead means (MM) operators with q -ROFS to get the new aggregation operators that are q -ROF MM operators. Joshi and Gegov [27] initiated the concept of the confidence level of experts to the original information under q -ROF environment to propose some aggregation operators such as confidence q -ROFWA (C q -ROFWA) and confidence q -ROFWG (C q -ROFWG), C q -ROFOWA, and C q -ROFOWG operators. Yang et al. [28] presented the idea of q -RO normal fuzzy sets and defined the operational laws and score function for it. They also initiated some aggregation operators for the same concept that is q -RONFWA and q -RONFOWG. Furthermore, Hussain et al. [29] proposed hesitant q -ROFWA and hesitant q -ROFWG operators and discussed their desirable properties. Hussain et al. [30] proposed the generalized and group generalized averaging operation by using q -ROF information.

The dominant theory of rough set was first proposed by Pawlak [31] which generalized the classical set theory to cope with the imprecise, vague, and uncertain information. By the definition of Pawlak rough set, a universal set is characterized by two approximation sets known as upper and lower approximations. The lower approximation consists of those alternatives which contain a subset and the upper approximation consists of those alternatives having nonempty intersection with a subset. Further equivalence relation plays a key role in Pawlak rough set for approximations but this condition too restricts the practical and theoretical aspects of rough set. So, researchers used the generalized structure by using the nonequivalence structure; for details, see [32–38]. From the inception, researchers used the hybrid study of rough set theory with different concepts. The hybrid study of rough set and IFS was proposed by Chakrabarty et al. [39] to obtain the notion of IF rough set (IFRS) and IFRS became the hot and progressive research area for the researchers; for details, see [40–43]. Zhou and Wu [44] proposed the combined study of rough set and IFS by using crisp and fuzzy approximation. Zhou and Wu [45] initiated the constructive and axiomatic approach under the IF rough environment. Hussain et al. [46] investigated the idea of rough PF ideals by using the algebraic structure of semi-groups. Zeng [47] proposed a new MCDM technique based on probabilistic information by using the PF environment. Hussain et al. [48] proposed the notion of q -ROF rough set by utilizing fuzzy β -covering and fuzzy β -covering

neighborhood. Molodtsov [49] originated the prominent and pioneer concept of soft set (S_tS) which generalized the classical set theory and is free from inherent complexity which the contemporary theories faced. It is observed that S_tS has a very close relation with fuzzy set and rough set. The S_tS theory is an essential concept and powerful mathematical tool for coping the uncertain, ambiguous, and imprecise data. Maji et al. [50, 51] proposed the hybrid notion of S_tS with fuzzy set and IFS to obtain fuzzy S_tS and IFS_tS which play a key role among these theories. Ali et al. [52] improved some existing definitions and operations in S_tS theory. The concept of generalized IFS_tS was proposed by Agarwal et al. [53]. Arora and Garg [54] presented the concept of IFS_t weighted averaging (IFS_tWA) and IFS_t weighted geometric (IFS_tWG) operators. Garg and Arora [55] proposed the notion of some power averaging and geometric aggregation operators by utilizing generalized IFS_tS . Arora [56] initiated the notion of IFS_tWA and IFS_tWG by using the Einstein operations. Feng et al. [57] improved some existing literature related to generalized IF S_tS and proposed some new operations for the developed concept. The combined study S_tS , rough set, and PFS were presented by Hussain et al. [58] to achieve the new concept of soft rough PFS and PF soft rough set. Riaz and Hashmi [59] presented the hybrid study of S_tS , rough set, PFS, and m-polar fuzzy set to get the new notion of Pythagorean m-polar fuzzy soft rough set. Hussain et al. [60] originated the hybrid structure of S_tS with q-ROFS to get the prominent notion of q-ROF soft (q-ROFS $_t$) set (q-ROFS $_tS$) and proposed some aggregation operators such as q-ROFS $_t$ weighted averaging (q-ROFS $_tWA$), q-ROFS $_t$ ordered weighted averaging (q-ROFS $_tOWA$), and q-ROFS $_t$ hybrid averaging (q-ROFS $_tHA$).

q-Rung orthopair fuzzy soft rough sets, a hybrid intelligent structure of soft sets, rough sets, and q-rung orthopair fuzzy sets are a powerful mathematical tool to deal with indeterminate, inconsistent, and incomplete information, which has caught the attention of the researchers. From the analysis, it is observed that aggregation operators have great importance in decision-making to aggregate the collective evaluated information of different sources into a single value. According to the best of our knowledge up till now, no application of the aggregation operators with the hybridization of q-ROFS with soft set and rough set is reported in q-ROF environment. Therefore, this motivates the current work of q-ROFS $_t$ rough study, and, further, we will investigate aggregation operators based on soft rough information that are q-ROFS $_tRWA$, q-ROFS $_tROWA$, q-ROFS $_tRHA$, q-ROFS $_tRWG$, q-ROFS $_tROWG$, and q-ROFS $_tRHG$ operators.

The design of the remaining sections of the manuscript is summarized as follows: Section 2 consists of a brief study of the basic notions connecting the link with the coming sections. Section 3 is devoted to investigating the hybrid concept of S_tS and rough set with the notion of q-ROFS to obtain the new concept of q-ROFS $_tRS$. In Section 4, we presented the averaging aggregation operators such as q-ROFS $_tRWA$, q-ROFS $_tROWA$, and q-ROFS $_tRHA$. Furthermore, the basic desirable properties of investigated averaging operators that are Idempotency, Boundedness,

Monotonicity, shift invariance, and Homogeneity are investigated in detail. Section 5 is devoted to the geometric aggregation operators such as q-ROFS $_tRWG$, q-ROFS $_tROWG$, and q-ROFS $_tRHG$. Moreover, the basic desirable characteristics of these investigated geometric operators that are Idempotency, Boundedness, Monotonicity, shift invariance, and Homogeneity are investigated in detail. In Section 6, the technique for MCDM and the stepwise algorithm for decision-making are demonstrated by utilizing the proposed approach. In Section 7, a numerical example for the developed approach is presented and a comparative study of the investigated models with some existing methods is given in detail which shows that the investigated models are more effective and useful than existing approaches. The final Section 8 consists of a conclusion of the manuscript.

2. Preliminaries

This section consists of some basic notions including IFS, PFS, q-ROFS, and q-ROFS $_tS$ which will be helpful in on word sections.

Definition 1 (see [2]). Consider a universe Y and an IFS $\mathcal{T}$ on set Y denoted and defined as

$$\mathcal{T} = \{ \langle s, \beta_{\mathcal{T}}(s), \vartheta_{\mathcal{T}}(s) \rangle | s \in Y \}, \quad (1)$$

in which $\beta_{\mathcal{T}}, \vartheta_{\mathcal{T}}: Y \longrightarrow [0, 1]$ denotes the MemG and NMemG of an alternative $s \in Y$ to the set $\mathcal{T}$ having the condition that $0 \leq \beta_{\mathcal{T}}(s) + \vartheta_{\mathcal{T}}(s) \leq 1$. $\pi_{\mathcal{T}} = 1 - (\beta_{\mathcal{T}}(s) + \vartheta_{\mathcal{T}}(s))$ is called hesitancy degree of $s \in Y$.

Definition 2 (see [10]). Consider a universe Y and a PFS $\mathcal{T}$ on set Y defined and denoted as

$$\mathcal{T} = \{ \langle s, \beta_{\mathcal{T}}(s), \vartheta_{\mathcal{T}}(s) \rangle | s \in Y \}, \quad (2)$$

in which $\beta_{\mathcal{T}}, \vartheta_{\mathcal{T}}: Y \longrightarrow [0, 1]$ denotes the MemG and NMemG of an alternative $s \in Y$ to the set $\mathcal{T}$ having the condition that $0 \leq (\beta_{\mathcal{T}}(s))^2 + (\vartheta_{\mathcal{T}}(s))^2 \leq 1$. $\pi_{\mathcal{T}} = \sqrt{1 - (\beta_{\mathcal{T}}(s))^2 - (\vartheta_{\mathcal{T}}(s))^2}$ is called hesitancy degree of $s \in Y$.

Definition 3 (see [20]). Consider Y as a universe of discourse and a q-ROFS $\mathcal{T}$ on set Y is an object of the form

$$\mathcal{T} = \{ \langle s, \beta_{\mathcal{T}}(s), \vartheta_{\mathcal{T}}(s) \rangle_q | s \in Y \text{ and } q \geq 1 \}, \quad (3)$$

in which $\beta_{\mathcal{T}}, \vartheta_{\mathcal{T}}: Y \longrightarrow [0, 1]$ shows the MemG and NMemG of an alternative $s \in Y$ to the set $\mathcal{T}$ having the condition that $0 \leq (\beta_{\mathcal{T}}(s))^q + (\vartheta_{\mathcal{T}}(s))^q \leq 1$. The hesitancy degree is shown as $\pi_{\mathcal{T}} = \sqrt[q]{1 - (\beta_{\mathcal{T}}(s))^q - (\vartheta_{\mathcal{T}}(s))^q}$ for each $s \in Y$.

Molodtsov [49] originated the prominent and pioneer concept of soft set (S_tS) which generalized the classical set theory and is free from inherent complexity which the contemporary theories faced. It is observed that S_tS has very close relation with fuzzy set and rough set. The S_tS theory is an essential concept and powerful mathematical tool for

coping the uncertain, ambiguous and imprecise data which is defined as:

Definition 4 (see [49]). Let Y be a fixed set and $\mathcal{E}$ be set of parameters with $\mathcal{B} \subseteq \mathcal{E}$. Then the pair $(\mathcal{T}, \mathcal{B})$ is said to be S_tS , where $\mathcal{T}$ is function given as $\mathcal{T}: \mathcal{B} \longrightarrow P(Y)$. $P(Y)$ denotes the collection of all subsets of Y .

Definition 5 (see [50]). Let $(\mathcal{T}, \mathcal{B})$ a S_tS over Y with $\mathcal{B} \subseteq \mathcal{E}$. Then $(\mathcal{T}^*, \mathcal{B})$ is known as fuzzy S_tS over Y , where $\mathcal{T}^*$ is a function given as $\mathcal{T}^*: \mathcal{B} \longrightarrow F(Y)$. $F(Y)$ denotes the collection of all fuzzy subsets of Y and mathematically it is gives as

$$\mathcal{T}_{c_j}^* = \{ \langle s_i, \beta_j(s_i) \rangle | s_i \in Y \}. \quad (4)$$

Definition 6 (see [60]). Consider a universal set Y . Let $\mathcal{E}$ be set of parameters and $\mathcal{B} \subseteq \mathcal{E}$. Then a q-ROFS_tS is a pair $(\mathcal{T}, \mathcal{B})$ over set Y and $\mathcal{T}$ is a mapping given as $\mathcal{T}: \mathcal{B} \longrightarrow q\text{-ROFS}_t(Y)$ in which q-ROFS_tS(Y) contains the collection of all q-ROFSs. Then, the q-ROFS_tS is denoted and defined as

$$\mathcal{T}_{c_j}(s_i) = \{ \langle s_i, \beta_j(s_i), \vartheta_j(s_i) \rangle_q | s_i \in Y \text{ and } q \geq 1 \}, \quad (5)$$

in which $\beta_j(s_i), \vartheta_j(s_i)$ represents the MemG and NMemG of an alternative $s_i \in Y$ to the set $\mathcal{T}_{c_j}$ satisfying that $0 \leq (\beta_j(s_i))^q + (\vartheta_j(s_i))^q \leq 1$.

$\pi_{\mathcal{T}} = \sqrt[q]{1 - (\beta_j(s_i))^q - (\vartheta_j(s_i))^q}$ is known to be hesitancy degree of $s_i \in Y$. For simplicity $\mathcal{T}_{c_j}(s_i) = (s_i, \beta_j(s_i), \vartheta_j(s_i))_q$ is denoted as $\mathcal{T}_{c_j} = (\beta_{ij}, \vartheta_{ij})$ if there is no confusion and is called q-ROFS_t number (q-ROFS_tN).

Considering two q-ROFS_tNs $\mathcal{T}_{c_j} = (\beta_{1j}, \vartheta_{1j})$ for $(j = 1, 2)$, reference [20] defined the following operation are defined on them:

- (i) $\mathcal{T}_{c_1} \cup \mathcal{T}_{c_2} = (\max(\beta_{11}, \beta_{12}), \min(\vartheta_{11}, \vartheta_{12}))$
- (ii) $\mathcal{T}_{c_1} \cap \mathcal{T}_{c_2} = (\min(\beta_{11}, \beta_{12}), \max(\vartheta_{11}, \vartheta_{12}))$
- (iii) $\mathcal{T}_{c_1} \oplus \mathcal{T}_{c_2} = (\sqrt[q]{\beta_{11}^q + \beta_{12}^q - \beta_{11}^q \beta_{12}^q}, \vartheta_{11} \vartheta_{12})$
- (iv) $\mathcal{T}_{c_1} \ominus \mathcal{T}_{c_2} = (\beta_{11} \beta_{12}, \sqrt[q]{\vartheta_{11}^q + \vartheta_{12}^q - \vartheta_{11}^q \vartheta_{12}^q})$
- (v) $\mathcal{T}_{c_1} \leq \mathcal{T}_{c_2}$ if $(\beta_{11} \leq \beta_{12}, \vartheta_{11} \geq \vartheta_{12})$
- (vi) $\mathcal{T}_{c_1}^c = (\vartheta_{11}, \beta_{11})$ where $\mathcal{T}_{c_1}^c$ represents the complement of $\mathcal{T}_{c_1}$
- (vii) $\alpha \mathcal{T}_{c_1} = (\sqrt[q]{1 - (1 - \beta_{11}^q)^\alpha}, \vartheta_{11}^\alpha)$ for $\alpha \geq 1$
- (viii) $\mathcal{T}_{c_1}^\alpha = (\beta_{11}^\alpha, \sqrt[q]{1 - (1 - \vartheta_{11}^q)^\alpha})$ for $\alpha \geq 1$

3. q-ROF Soft Rough Set (q-ROFS_tRS)

The concept of S_tS theory, is the generalization of classical set theory and is free from inherent complexity which the contemporary theories faced. The S_tS theory and rough set theory are essential concepts and powerful mathematical tools for coping the uncertain, ambiguous, and imprecise data. Motivated from the combining study of soft rough set, this section is devoted to the hybridized study of q-ROFS with S_tS and rough set to obtain the new concept of

q-ROFS_tRS. Some basic operations, a new score function, and some desirable characteristics of the proposed concept are investigated in detail.

Definition 7. Let $(\mathcal{T}, \mathcal{E})$ be a q-ROFS_tS over Y . Any subset $\mathcal{L}$ of $Y \times \mathcal{E}$ is set to a q-ROFS_t relation from Y to $\mathcal{E}$ and is defined as

$$\mathcal{L} = \{ \langle (s_i, c_j), \beta(s_i, c_j), \vartheta(s_i, c_j) \rangle | (s_i, c_j) \in Y \times \mathcal{E} \}, \quad (6)$$

where $\beta: Y \times \mathcal{E} \longrightarrow [0, 1]$ and $\vartheta: Y \times \mathcal{E} \longrightarrow [0, 1]$ denote the MemG and NMemG with $0 \leq [\beta(s_i, c_j)]^q + [\vartheta(s_i, c_j)]^q \leq 1$ for all $(s_i, c_j) \in Y \times \mathcal{E}$.

If $Y = \{s_1, s_2, \dots, s_m\}$ and $\mathcal{E} = \{c_1, c_2, \dots, c_n\}$, then, q-ROFS_t relation $\mathcal{L}$ from Y to $\mathcal{E}$ can be presented in Table 1.

Definition 8. Consider a universal Y , $\mathcal{E}$ as the set of parameters and $(\mathcal{T}, \mathcal{E})$ as a q-ROFS_tS. Let $\mathcal{L}$ be an arbitrary q-ROFS_t relation from set Y to $\mathcal{E}$. The pair $(\mathcal{T}, \mathcal{E}, \mathcal{L})$ is said to be q-ROFS_t approximation space. For any optimum decision normal object $\mathcal{M} \in q\text{-ROFS}(\mathcal{E})$, then the lower and upper approximation of $\mathcal{M}$ with respect to approximation space $(\mathcal{T}, \mathcal{E}, \mathcal{L})$ are denoted and defined as

$$\underline{\mathcal{L}}(\mathcal{M}) = \{ \langle s_i, \underline{\beta}_j(s_i), \underline{\vartheta}_j(s_i) \rangle | s_i \in Y \}, \quad (7)$$

$$\overline{\mathcal{L}}(\mathcal{M}) = \{ \langle s_i, \overline{\beta}_j(s_i), \overline{\vartheta}_j(s_i) \rangle | s_i \in Y \}, \quad (8)$$

where $\underline{\beta}_j(s_i) = \bigwedge_{c_j \in \mathcal{E}} [\beta_{\mathcal{L}}(s_i, c_j) \wedge \beta_{\mathcal{M}}(c_j)]$, $\underline{\vartheta}_j(s_i) = \bigvee_{c_j \in \mathcal{E}} [\vartheta_{\mathcal{L}}(s_i, c_j) \vee \vartheta_{\mathcal{M}}(c_j)]$ and $\overline{\beta}_j(s_i) = \bigvee_{c_j \in \mathcal{E}} [\beta_{\mathcal{L}}(s_i, c_j) \vee \beta_{\mathcal{M}}(c_j)]$, $\overline{\vartheta}_j(s_i) = \bigwedge_{c_j \in \mathcal{E}} [\vartheta_{\mathcal{L}}(s_i, c_j) \wedge \vartheta_{\mathcal{M}}(c_j)]$ such that $0 \leq [\underline{\beta}_j(s_i)]^q + [\underline{\vartheta}_j(s_i)]^q \leq 1$ and $0 \leq [\overline{\beta}_j(s_i)]^q + [\overline{\vartheta}_j(s_i)]^q \leq 1$.

It is observed that $\underline{\mathcal{L}}(\mathcal{M})$ and $\overline{\mathcal{L}}(\mathcal{M})$ are two q-ROFSs in Y . Thus, the operators $\underline{\mathcal{L}}(\mathcal{M}), \overline{\mathcal{L}}(\mathcal{M}): q\text{-ROFS}_t(\mathcal{E}) \longrightarrow q\text{-ROFS}_t(Y)$ are, respectively, known as lower and upper q-ROFS_tR approximation operators. Therefore, q-ROFS_tRS is a pair $\mathcal{L}(\mathcal{M}) = (\underline{\mathcal{L}}(\mathcal{M}), \overline{\mathcal{L}}(\mathcal{M})) = (s_i, (\underline{\beta}_j(s_i), \underline{\vartheta}_j(s_i)), (\overline{\beta}_j(s_i), \overline{\vartheta}_j(s_i)))$.

For simplicity, we can write $\mathcal{L}(\mathcal{M}) = (\underline{\mathcal{L}}(\mathcal{M}), \overline{\mathcal{L}}(\mathcal{M})) = (s_i, (\underline{\beta}_j(s_i), \underline{\vartheta}_j(s_i)), (\overline{\beta}_j(s_i), \overline{\vartheta}_j(s_i)))$ as $\mathcal{L}_{c_j}(\mathcal{M}_i) = (\mathcal{L}_{c_j}(\mathcal{M}_i), \overline{\mathcal{L}_{c_j}(\mathcal{M}_i)}) = ((\underline{\beta}_{ij}, \underline{\vartheta}_{ij}), (\overline{\beta}_{ij}, \overline{\vartheta}_{ij}))$ and call q-ROFS_tR number (q-ROFS_tRN), if there is no confusion.

Remark 1

- (a) If $q = 1$ is fixed, then the developed q-ROFS_tR approximation operators reduce to IFS_tR approximation operators
- (b) If $q = 2$ is fixed, then the developed q-ROFS_tR approximation operators reduce to PFS_tR approximation operators

Consider the following example to better understand the concept of q-ROFS_tR approximation operators.

Example 1. Suppose a decision-maker Z buys a house, as given in set $Y = \{s_1, s_2, s_3, s_4, s_5\}$ under consideration. Let the

TABLE 1: q-ROFS_t relation $\mathcal{L}$ from Y to $\mathcal{E}$.

$\mathcal{L}$	c_1	c_2	$\dots$	c_n
s_1	$(\beta(s_1, c_1), \vartheta(s_1, c_1))$	$(\beta(s_1, c_2), \vartheta(s_1, c_2))$	$\dots$	$(\beta(s_1, c_n), \vartheta(s_1, c_n))$
s_2	$(\beta(s_2, c_1), \vartheta(s_2, c_1))$	$(\beta(s_2, c_2), \vartheta(s_2, c_2))$	$\dots$	$(\beta(s_2, c_n), \vartheta(s_2, c_n))$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
s_m	$(\beta(s_m, c_1), \vartheta(s_m, c_1))$	$(\beta(s_m, c_2), \vartheta(s_m, c_2))$	$\dots$	$(\beta(s_m, c_n), \vartheta(s_m, c_n))$

parameter set $\mathcal{E} = \{c_1, c_2, c_3, c_4\}$ where c_1 = beautiful, c_2 = size, c_3 = expensive, and c_4 = location. A decision-maker Z wants to purchase a house from the available houses which fulfill the utmost extent of given parameters. Consider the decision-maker Z presents the gorgeous of houses in form of q-ROFS_t relation $\mathcal{L}$ from set Y to $\mathcal{E}$ and is given in Table 2.

Consider a decision-maker Z presents the optimum normal decision object $\mathcal{M}$ which is a q-ROF subset over parameter set $\mathcal{E}$; that is,

$$\mathcal{M} = \{(c_1, 0.9, 0.2), (c_2, 0.4, 0.6), (c_3, 0.8, 0.4), (c_4, 0.5, 0.1)\}. \quad (9)$$

Now, by using equations (7) and (8), we have

$$\begin{aligned}
\underline{\beta}_1(s_1) &= 0.4, \\
\underline{\vartheta}_1(s_1) &= 0.6, \\
\underline{\beta}_2(s_2) &= 0.3, \\
\underline{\vartheta}_2(s_2) &= 0.7, \\
\underline{\beta}_3(s_3) &= 0.2, \\
\underline{\vartheta}_3(s_3) &= 0.9, \\
\underline{\beta}_4(s_4) &= 0.4, \\
\underline{\vartheta}_4(s_4) &= 0.6, \\
\underline{\beta}_5(s_5) &= 0.3, \\
\underline{\vartheta}_5(s_5) &= 0.7, \\
\overline{\beta}_1(s_1) &= 0.9, \\
\overline{\vartheta}_1(s_1) &= 0.1, \\
\overline{\beta}_2(s_2) &= 0.9, \\
\overline{\vartheta}_2(s_2) &= 0.1, \\
\overline{\beta}_3(s_3) &= 0.95, \\
\overline{\vartheta}_3(s_3) &= 0.1, \\
\overline{\beta}_4(s_4) &= 0.93, \\
\overline{\vartheta}_4(s_4) &= 0.1, \\
\overline{\beta}_5(s_5) &= 0.9, \\
\overline{\vartheta}_5(s_5) &= 0.1.
\end{aligned} \quad (10)$$

Now, to get the lower and upper q-ROFS_tR approximation operators,

TABLE 2: q-ROFS_t relation from set Y to $\mathcal{E}$ for $q \geq 3$.

$\mathcal{L}$	c_1	c_2	c_3	c_4
s_1	(0.9, 0.4)	(0.8, 0.2)	(0.7, 0.3)	(0.65, 0.2)
s_2	(0.8, 0.5)	(0.5, 0.1)	(0.85, 0.2)	(0.3, 0.7)
s_3	(0.6, 0.9)	(0.2, 0.6)	(0.6, 0.1)	(0.95, 0.3)
s_4	(0.7, 0.4)	(0.93, 0.4)	(0.4, 0.2)	(0.5, 0.1)
s_5	(0.3, 0.7)	(0.78, 0.25)	(0.8, 0.15)	(0.7, 0.4)

$$\begin{aligned}
\underline{\mathcal{L}}(\mathcal{M}) &= \{(s_1, 0.7, 0.6), (s_2, 0.3, 0.7), (s_3, 0.2, 0.9), \\
&\quad (s_4, 0.4, 0.6), (s_5, 0.3, 0.7)\}, \\
\overline{\mathcal{L}}(\mathcal{M}) &= \{(s_1, 0.9, 0.1), (s_2, 0.9, 0.1), (s_3, 0.95, 0.1), \\
&\quad (s_4, 0.93, 0.1), (s_5, 0.9, 0.1)\}.
\end{aligned} \quad (11)$$

Therefore,

$$\begin{aligned}
\mathcal{L}(\mathcal{M}) &= (\underline{\mathcal{L}}(\mathcal{M}), \overline{\mathcal{L}}(\mathcal{M})) = \{(s_1, (0.7, 0.6), (0.9, 0.1)), \\
&\quad (s_2, (0.3, 0.7), (0.9, 0.1)), (s_3, (0.2, 0.9), \\
&\quad (0.95, 0.1)), (s_4, (0.4, 0.6), (0.93, 0.1)), \\
&\quad (s_5, (0.3, 0.7), (0.9, 0.1))\}.
\end{aligned} \quad (12)$$

Definition 9. Consider $\mathcal{L}_{c_j}(\mathcal{M}_1) = (\underline{\mathcal{L}}_{c_j}(\mathcal{M}_1), \overline{\mathcal{L}}_{c_j}(\mathcal{M}_1))$ for $(j = 1, 2)$ are the two q-ROFS_tRNs. Then, the following operators are defined on them:

- (i) $\underline{\mathcal{L}}_{c_1}(\mathcal{M}_1) \cup \underline{\mathcal{L}}_{c_2}(\mathcal{M}_1) = \{(\underline{\mathcal{L}}_{c_1}(\mathcal{M}_1) \cup \underline{\mathcal{L}}_{c_2}(\mathcal{M}_1)), (\overline{\mathcal{L}}_{c_1}(\mathcal{M}_1) \cup \overline{\mathcal{L}}_{c_2}(\mathcal{M}_1))\}$
- (ii) $\underline{\mathcal{L}}_{c_1}(\mathcal{M}_1) \cap \underline{\mathcal{L}}_{c_2}(\mathcal{M}_1) = \{(\underline{\mathcal{L}}_{c_1}(\mathcal{M}_1) \cap \underline{\mathcal{L}}_{c_2}(\mathcal{M}_1)), (\overline{\mathcal{L}}_{c_1}(\mathcal{M}_1) \cap \overline{\mathcal{L}}_{c_2}(\mathcal{M}_1))\}$
- (iii) $\underline{\mathcal{L}}_{c_1}(\mathcal{M}_1) \oplus \underline{\mathcal{L}}_{c_2}(\mathcal{M}_1) = \{(\underline{\mathcal{L}}_{c_1}(\mathcal{M}_1) \oplus \underline{\mathcal{L}}_{c_2}(\mathcal{M}_1)), (\overline{\mathcal{L}}_{c_1}(\mathcal{M}_1) \oplus \overline{\mathcal{L}}_{c_2}(\mathcal{M}_1))\}$
- (iv) $\underline{\mathcal{L}}_{c_1}(\mathcal{M}_1) \otimes \underline{\mathcal{L}}_{c_2}(\mathcal{M}_1) = \{(\underline{\mathcal{L}}_{c_1}(\mathcal{M}_1) \otimes \underline{\mathcal{L}}_{c_2}(\mathcal{M}_1)), (\overline{\mathcal{L}}_{c_1}(\mathcal{M}_1) \otimes \overline{\mathcal{L}}_{c_2}(\mathcal{M}_1))\}$
- (v) $\underline{\mathcal{L}}_{c_1}(\mathcal{M}_1) \subseteq \underline{\mathcal{L}}_{c_2}(\mathcal{M}_1) = (\underline{\mathcal{L}}_{c_1}(\mathcal{M}_1) \subseteq \underline{\mathcal{L}}_{c_2}(\mathcal{M}_1))$ and $(\overline{\mathcal{L}}_{c_1}(\mathcal{M}_1) \subseteq \overline{\mathcal{L}}_{c_2}(\mathcal{M}_1))$
- (vi) $\alpha \underline{\mathcal{L}}_{c_1}(\mathcal{M}_1) = (\alpha \underline{\mathcal{L}}_{c_1}(\mathcal{M}_1), \alpha \overline{\mathcal{L}}_{c_1}(\mathcal{M}_1))$ for $\alpha \geq 1$
- (vii) $(\underline{\mathcal{L}}_{c_1}(\mathcal{M}_1))^\alpha = ((\underline{\mathcal{L}}_{c_1}(\mathcal{M}_1))^\alpha, (\overline{\mathcal{L}}_{c_1}(\mathcal{M}_1))^\alpha)$ for $\alpha \geq 1$
- (viii) $\underline{\mathcal{L}}_{c_1}(\mathcal{M}_1)^c = (\underline{\mathcal{L}}_{c_1}(\mathcal{M}_1)^c, \overline{\mathcal{L}}_{c_1}(\mathcal{M}_1)^c)$, where $\underline{\mathcal{L}}_{c_j}(\mathcal{M}_1)^c$ are $\overline{\mathcal{L}}_{c_j}(\mathcal{M}_1)^c$, the complements of q-ROFS_tR approximation operators $\underline{\mathcal{L}}_{c_j}(\mathcal{M}_1)$ and $\overline{\mathcal{L}}_{c_j}(\mathcal{M}_1)$, that is, $\underline{\mathcal{L}}_{c_j}(\mathcal{M}_1)^c = (\vartheta_{ij}, \beta_{ij})$

- (ix) $\mathcal{L}(\mathcal{M}_1) = \mathcal{L}(\mathcal{M}_2)$ iff $\underline{\mathcal{L}}(\mathcal{M}_1) = \underline{\mathcal{L}}(\mathcal{M}_2)$ and $\overline{\mathcal{L}}(\mathcal{M}_1) = \overline{\mathcal{L}}(\mathcal{M}_2)$

Definition 10. Let $\mathcal{L}_{c_1}(\mathcal{M}_1) = (\mathcal{L}_{c_1}(\mathcal{M}_1), \overline{\mathcal{L}_{c_1}(\mathcal{M}_1)}) = ((\underline{\beta}_{11}, \underline{\vartheta}_{11}), (\overline{\beta}_{11}, \overline{\vartheta}_{11}))$ be a q-ROFS_tRN. Then, the score function is given as

$$SC(\mathcal{L}_{c_1}(\mathcal{M}_1)) = \frac{1}{2} \left(\underline{\beta}_{11}^q + \overline{\beta}_{11}^{-q} - \underline{\vartheta}_{11}^q - \overline{\vartheta}_{11}^{-q} \right), \quad (13)$$

$$SC(\mathcal{L}_{c_1}(\mathcal{M}_1)) \in [-1, 1] \text{ and } q \geq 1.$$

The greater the score value, the greater the q-ROFS_tRN.

Proposition 1. Let $(\mathcal{T}, \mathcal{E}, \mathcal{L})$ be q-ROFS_t approximation space. For any two $\mathcal{L}(\mathcal{M}_1) = (\underline{\mathcal{L}}(\mathcal{M}_1), \overline{\mathcal{L}}(\mathcal{M}_1))$ and $\mathcal{L}(\mathcal{M}_2) = (\underline{\mathcal{L}}(\mathcal{M}_2), \overline{\mathcal{L}}(\mathcal{M}_2))$ q-ROFS_tRSs over a common universe set Y , then the following properties hold:

- (i) $\sim(\sim \mathcal{L}(\mathcal{M}_1)) = \mathcal{M}_1$, where $\sim \mathcal{L}(\mathcal{M}_1)$ is the complement of $\mathcal{L}(\mathcal{M}_1)$, $\mathcal{L}(\mathcal{M}_1) \cup \mathcal{L}(\mathcal{M}_2) = \mathcal{L}(\mathcal{M}_2) \cup \mathcal{L}(\mathcal{M}_1)$, $\mathcal{L}(\mathcal{M}_1) \cap \mathcal{L}(\mathcal{M}_2) = \mathcal{L}(\mathcal{M}_2) \cap \mathcal{L}(\mathcal{M}_1)$
- (ii) $\sim(\mathcal{L}(\mathcal{M}_1) \cup \mathcal{L}(\mathcal{M}_2)) = (\sim \mathcal{L}(\mathcal{M}_1)) \cap (\sim \mathcal{L}(\mathcal{M}_2))$
- (iii) $\sim(\mathcal{L}(\mathcal{M}_1) \cap \mathcal{L}(\mathcal{M}_2)) = (\sim \mathcal{L}(\mathcal{M}_1)) \cup (\sim \mathcal{L}(\mathcal{M}_2))$
- (iv) If $\mathcal{M}_1 \subseteq \mathcal{M}_2$, then $\mathcal{L}(\mathcal{M}_1) \subseteq \mathcal{L}(\mathcal{M}_2)$
- (v) $\mathcal{L}(\mathcal{M}_1 \cup \mathcal{M}_2) \supseteq \mathcal{L}(\mathcal{M}_1) \cup \mathcal{L}(\mathcal{M}_2)$
- (vi) $\mathcal{L}(\mathcal{M}_1 \cap \mathcal{M}_2) \subseteq \mathcal{L}(\mathcal{M}_1) \cap \mathcal{L}(\mathcal{M}_2)$

4. q-Rung Orthopair Fuzzy Soft Rough Averaging (q-ROFS_tRA) Aggregation Operator

This section is devoted to the analysis of q-ROFS_tR averaging aggregation operators such as q-ROFS_tRWA, q-ROFS_tROWA, and q-ROFS_tRHA operators. We will present the fundamental properties of these operators in detail.

4.1. q-Rung Orthopair Fuzzy Soft Rough Weighted Averaging (q-ROFS_tRWA) Operator

Definition 11. Let $\mathcal{L}_{c_j}(\mathcal{M}_i) = (\mathcal{L}_{c_j}(\mathcal{M}_i), \overline{\mathcal{L}_{c_j}(\mathcal{M}_i)})$ ($i = 1, 2, \dots, m, j = 1, 2, \dots, n$) be the collection of q-ROFS_tRNs. Let the weight vectors $t = (t_1, t_2, \dots, t_m)^T$ and $v = (v_1, v_2, \dots, v_n)^T$ for experts s_i and parameters c_j with $\sum_{i=1}^m t_i = 1$, $\sum_{j=1}^n v_j = 1$, and $0 \leq t_i, v_j \leq 1$, respectively. The q-ROFS_tRWA operator is defined as

$$\begin{aligned} & \text{q-ROFS}_t\text{RWA}(\mathcal{L}_{c_1}(\mathcal{M}_1), \dots, \mathcal{L}_{c_n}(\mathcal{M}_n)) \\ &= \left(\oplus_{j=1}^n v_j \left(\oplus_{i=1}^m t_i \underline{\mathcal{L}_{c_j}}(\mathcal{M}_i) \right), \oplus_{j=1}^n v_j \left(\oplus_{i=1}^m t_i \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i) \right) \right). \end{aligned} \quad (14)$$

In view of the above definition, Theorem 1 illustrates the aggregated result for q-ROFS_tRWA.

Theorem 1. Let $\mathcal{L}_{c_j}(\mathcal{M}_i) = (\mathcal{L}_{c_j}(\mathcal{M}_i), \overline{\mathcal{L}_{c_j}(\mathcal{M}_i)})$ ($i = 1, 2, \dots, m, j = 1, 2, \dots, n$) be the collection of q-ROFS_tRNs. Let the weight vectors $t = (t_1, t_2, \dots, t_m)^T$ and $v = (v_1, v_2, \dots, v_n)^T$ for experts s_i and parameters c_j with $\sum_{i=1}^m t_i = 1$, $\sum_{j=1}^n v_j = 1$ and $0 \leq t_i, v_j \leq 1$, respectively. Then, q-ROFS_tRWA operator is given as

$$\begin{aligned} & \text{q-ROFS}_t\text{RWA}(\mathcal{L}_{c_1}(\mathcal{M}_1), \dots, \mathcal{L}_{c_n}(\mathcal{M}_n)) = \left[\oplus_{j=1}^n v_j \left(\oplus_{i=1}^m t_i \underline{\mathcal{L}_{c_j}}(\mathcal{M}_i) \right), \oplus_{j=1}^n v_j \left(\oplus_{i=1}^m t_i \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i) \right) \right] \\ &= \left[\left\{ \sqrt[q]{1 - \prod_{j=1}^n \left(\prod_{i=1}^m \left(1 - \underline{\beta}_{ij}^q \right)^{t_i} \right)^{v_j}}, \prod_{j=1}^n \left(\prod_{i=1}^m \frac{t_i}{\underline{\vartheta}_{ij}} \right)^{v_j} \right\}, \left\{ \sqrt[q]{1 - \prod_{j=1}^n \left(\prod_{i=1}^m \left(1 - \overline{\beta}_{ij}^q \right)^{t_i} \right)^{v_j}}, \prod_{j=1}^n \left(\prod_{i=1}^m \frac{t_i}{\overline{\vartheta}_{ij}} \right)^{v_j} \right\} \right]. \end{aligned} \quad (15)$$

Proof. To get the required proof, we will use the method of mathematical induction.

As by operational law,

$$\begin{aligned} & \mathcal{L}_{c_1}(\mathcal{M}_1) \oplus \mathcal{L}_{c_1}(\mathcal{M}_2) = ((\underline{\beta}_{11}, \underline{\vartheta}_{11}) \oplus (\underline{\beta}_{12}, \underline{\vartheta}_{12}), (\overline{\beta}_{11}, \overline{\vartheta}_{11}) \oplus (\overline{\beta}_{12}, \overline{\vartheta}_{12})) \\ &= \left[\left(\sqrt[q]{\underline{\beta}_{11}^q + \underline{\beta}_{12}^q + \underline{\beta}_{11}^q \underline{\beta}_{12}^q}, \underline{\vartheta}_{11} \underline{\vartheta}_{12} \right), \left(\sqrt[q]{\overline{\beta}_{11}^q + \overline{\beta}_{12}^q + \overline{\beta}_{11}^q \overline{\beta}_{12}^q}, \overline{\vartheta}_{11} \overline{\vartheta}_{12} \right) \right], \\ & \alpha \mathcal{L}_{c_1}(\mathcal{M}_1) = \left[\left(\sqrt[q]{1 - \left(1 - \underline{\beta}_{11}^q \right)^\alpha}, \underline{\vartheta}_{11}^\alpha \right), \left(\sqrt[q]{1 - \left(1 - \overline{\beta}_{11}^q \right)^\alpha}, \overline{\vartheta}_{11}^\alpha \right) \right]. \end{aligned} \quad (16)$$

Suppose the result is true for $m = 2$ and $n = 2$; that is,

Now, consider

$$\begin{aligned} & q - \text{ROFS}_t \text{RWA} \left(\mathcal{L}_{c_j}(\mathcal{M}_i), \mathcal{L}_{c_j}(\mathcal{M}_i) \right) \\ &= \left(\oplus_{j=1}^2 v_j \left(\oplus_{i=1}^2 t_i \underline{\mathcal{L}_{c_j}}(\mathcal{M}_i) \right), \oplus_{j=1}^2 v_j \left(\oplus_{i=1}^2 t_i \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i) \right) \right). \end{aligned} \quad (17)$$

$$\begin{aligned} q - \text{ROFS}_t \text{RWA} \left(\mathcal{L}_{c_j}(\mathcal{M}_i), \mathcal{L}_{c_j}(\mathcal{M}_i) \right) &= \left[\oplus_{j=1}^2 v_j \left(\oplus_{i=1}^2 t_i \underline{\mathcal{L}_{c_j}}(\mathcal{M}_i) \right), \oplus_{j=1}^2 v_j \left(\oplus_{i=1}^2 t_i \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i) \right) \right] \\ &= \left[\left\{ v_1 \left(t_1 \underline{\mathcal{L}_{c_1}}(\mathcal{M}_1) \oplus t_2 \underline{\mathcal{L}_{c_1}}(\mathcal{M}_2) \right) \oplus v_2 \left(t_1 \underline{\mathcal{L}_{c_2}}(\mathcal{M}_1) \oplus t_2 \underline{\mathcal{L}_{c_2}}(\mathcal{M}_2) \right) \right\}, \right. \\ &\quad \left. \left\{ v_1 \left(t_1 \overline{\mathcal{L}_{c_1}}(\mathcal{M}_1) \oplus t_2 \overline{\mathcal{L}_{c_1}}(\mathcal{M}_2) \right) \oplus v_2 \left(t_1 \overline{\mathcal{L}_{c_2}}(\mathcal{M}_1) \oplus t_2 \overline{\mathcal{L}_{c_2}}(\mathcal{M}_2) \right) \right\} \right], \\ q - \text{ROFS}_t \text{RWA} \left(\mathcal{L}_{c_j}(\mathcal{M}_i), \mathcal{L}_{c_j}(\mathcal{M}_i) \right) &= \left[\left\{ \sqrt[q]{1 - \prod_{j=1}^2 \left(\prod_{i=1}^2 \left(1 - \underline{\beta}_{ij}^q \right)^{t_i} \right)^{v_j}}, \prod_{j=1}^2 \left(\prod_{i=1}^2 \underline{\vartheta}_{ij}^{t_i} \right)^{v_j} \right\}, \right. \\ &\quad \left. \left\{ \left(\sqrt[q]{1 - \prod_{j=1}^2 \left(\prod_{i=1}^2 \left(1 - \overline{\beta}_{ij}^q \right)^{t_i} \right)^{v_j}}, \prod_{j=1}^2 \left(\prod_{i=1}^2 \overline{\vartheta}_{ij}^{t_i} \right)^{v_j} \right) \right\} \right]. \end{aligned} \quad (18)$$

The result is true for $m = 2$ and $n = 2$.

Now, consider the result is for $n = k_1$ and $m = k_2$:

$$\begin{aligned} & q - \text{ROFS}_t \text{RWA} \left(\mathcal{L}_{c_j}(\mathcal{M}_i), \mathcal{L}_{c_j}(\mathcal{M}_i), \dots, \mathcal{L}_{c_{k_1}}(\mathcal{M}_{k_2}) \right) \\ &= \left[\left\{ \sqrt[q]{1 - \prod_{j=1}^{k_1} \left(\prod_{i=1}^{k_2} \left(1 - \underline{\beta}_{ij}^q \right)^{t_i} \right)^{v_j}}, \prod_{j=1}^{k_1} \left(\prod_{i=1}^{k_2} \underline{\vartheta}_{ij}^{t_i} \right)^{v_j} \right\}, \left\{ \left(\sqrt[q]{1 - \prod_{j=1}^{k_1} \left(\prod_{i=1}^{k_2} \left(1 - \overline{\beta}_{ij}^q \right)^{t_i} \right)^{v_j}}, \prod_{j=1}^{k_1} \left(\prod_{i=1}^{k_2} \overline{\vartheta}_{ij}^{t_i} \right)^{v_j} \right) \right\} \right]. \end{aligned} \quad (19)$$

Suppose the result holds for $n = k_1 + 1$ and $m = k_2 + 1$, so we have

$$\begin{aligned} & q - \text{ROFS}_t \text{RWA} \left[\left(\mathcal{L}_{c_j}(\mathcal{M}_i), \mathcal{L}_{c_j}(\mathcal{M}_i), \dots, \mathcal{L}_{c_{k_1}}(\mathcal{M}_{k_2}) \right), \mathcal{L}_{c_{k_1+1}}(\mathcal{M}_{k_2+1}) \right] \\ &= \left[\oplus_{j=1}^{k_1} v_j \left(\oplus_{i=1}^{k_2} t_i \underline{\mathcal{L}_{c_j}}(\mathcal{M}_i) \right) \oplus v_{k_1+1} \left(t_{k_2+1} \underline{\mathcal{L}_{c_{k_1+1}}}(\mathcal{M}_{k_2+1}) \right), \oplus_{j=1}^{k_1} v_j \left(\oplus_{i=1}^{k_2} t_i \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i) \right) \oplus v_{k_1+1} \left(t_{k_2+1} \overline{\mathcal{L}_{c_{k_1+1}}}(\mathcal{M}_{k_2+1}) \right) \right] \\ &= \left[\left\{ \sqrt[q]{1 - \prod_{j=1}^{k_1+1} \left(\prod_{i=1}^{k_2+1} \left(1 - \underline{\beta}_{ij}^q \right)^{t_i} \right)^{v_j}}, \prod_{j=1}^{k_1+1} \left(\prod_{i=1}^{k_2+1} \underline{\vartheta}_{ij}^{t_i} \right)^{v_j} \right\}, \left\{ \left(\sqrt[q]{1 - \prod_{j=1}^{k_1+1} \left(\prod_{i=1}^{k_2+1} \left(1 - \overline{\beta}_{ij}^q \right)^{t_i} \right)^{v_j}}, \prod_{j=1}^{k_1+1} \left(\prod_{i=1}^{k_2+1} \overline{\vartheta}_{ij}^{t_i} \right)^{v_j} \right) \right\} \right]. \end{aligned} \quad (20)$$

This implies the result is true for $n = k_1 + 1$ and $m = k_2 + 1$. Therefore, the result holds for all $m, n \geq 1$.

Since it is clear that $\underline{\mathcal{L}_{c_j}}(\mathcal{M}_i)$ and $\overline{\mathcal{L}_{c_j}}(\mathcal{M}_i)$ are q-ROFNs, by Definition 7, we have that $\oplus_{j=1}^n v_j \left(\oplus_{i=1}^m t_i \underline{\mathcal{L}_{c_j}}(\mathcal{M}_i) \right)$ and $\oplus_{j=1}^n v_j \left(\oplus_{i=1}^m t_i \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i) \right)$ are

also q-ROFNs. Therefore, $q - \text{ROFS}_t \text{RWA} \left(\mathcal{L}_{c_1}(\mathcal{M}_1), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m) \right)$ is also a q-ROFS_tRN in approximation space $(\mathcal{T}, \mathcal{E}, \mathcal{L})$. $\square$

Example 2. Let $Y = \{s_1, s_2, s_3\}$ be a set and $\mathcal{M} = \{c_1, c_2\} \subseteq \mathcal{E}$ a set of parameters with weight vector $t = (0.25, 0.3, 0.45)^T$ for

s_i ($i = 1, 2, 3$) and $v = (0.55, 0.45)$ for c_j ($j = 1, 2$). Then, q-ROFS_tRNs is given in Table 3:

$$\begin{aligned}
 & \text{q-ROFS}_t\text{RWA}(\mathcal{L}_{c_1}(\mathcal{M}_1), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)) = \left[\oplus_{j=1}^2 v_j \left(\oplus_{i=1}^3 t_i \underline{\mathcal{L}_{c_j}}(\mathcal{M}_i) \right), \oplus_{j=1}^2 v_j \left(\oplus_{i=1}^3 t_i \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i) \right) \right] \\
 & = \left[\left\{ \sqrt[3]{1 - \left(\left[(1 - 0.9^3)^{0.25} (1 - 0.7^3)^{0.25} (1 - 0.29^3)^{0.45} \right]^{0.55} \left[(1 - 0.55^3)^{0.25} (1 - 0.92^3)^{0.25} (1 - 0.4^3)^{0.45} \right]^{0.45}} \right.} \right. \\
 & \quad \left. \left(0.3^{0.25} 0.1^{0.3} 0.25^{0.45} \right)^{0.55} \left(0.2^{0.25} 0.3^{0.3} 0.85^{0.45} \right)^{0.45} \right\}, \\
 & \quad \left\{ \sqrt[3]{1 - \left(\left[(1 - 0.4^3)^{0.25} (1 - 0.2^3)^{0.25} (1 - 0.65^3)^{0.45} \right]^{0.55} \left[(1 - 0.76^3)^{0.25} (1 - 0.6^3)^{0.25} (1 - 0.88^3)^{0.45} \right]^{0.45}} \right.} \\
 & \quad \left. \left(0.4^{0.25} 0.75^{0.3} 0.15^{0.45} \right)^{0.55} \left(0.14^{0.25} 0.3^{0.3} 0.12^{0.45} \right)^{0.45} \right\} \right] \\
 & = [(0.831432, 0.255487), (0.72581, 0.26258)].
 \end{aligned} \tag{21}$$

From the analysis of Theorem 1, some related characteristics of q-ROFS_tRWA operator are given as follows:

Theorem 2. Let $\mathcal{L}_{c_j}(\mathcal{M}_i) = (\mathcal{L}_{c_j}(\mathcal{M}_i), \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i))$ ($i = 1, 2, \dots, m, j = 1, 2, \dots, n$) be the collection of q-ROFS_tRNs. Let the weight vectors $t = (t_1, t_2, \dots, t_m)^T$ and $v = (v_1, v_2, \dots, v_n)^T$ for experts s_i and parameters c_j with $\sum_{i=1}^m t_i = 1$, $\sum_{j=1}^n v_j = 1$ and $0 \leq t_i, v_j \leq 1$, respectively. Then, the following properties hold for q-ROFS_tRWA operator:

(i) (Idempotency): if $\mathcal{L}_{c_j}(\mathcal{M}_i) = \mathcal{P}_c(\mathcal{N})$ (for all $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$), where $\mathcal{P}_c(\mathcal{M}) = (\underline{\mathcal{P}_c}(\mathcal{M}), \overline{\mathcal{P}_c}(\mathcal{M})) = ((\underline{b}, \underline{d}), (\overline{b}, \overline{d}))$, then

$$\begin{aligned}
 & \text{q-ROFS}_t\text{RWA}(\mathcal{L}_{c_1}(\mathcal{M}_1), \mathcal{L}_{c_2}(\mathcal{M}_2), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)) \\
 & = \mathcal{P}_c(\mathcal{M}).
 \end{aligned} \tag{22}$$

(ii) (Boundedness): let $(\mathcal{L}_{c_j}(\mathcal{M}_i))^- = (\min_j \min_i \mathcal{L}_{c_j}(\mathcal{M}_i), \max_j \max_i \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i))$ and $(\mathcal{L}_{c_j}(\mathcal{M}_i))^+ = (\max_j \max_i \mathcal{L}_{c_j}(\mathcal{M}_i), \min_j \min_i \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i))$. Then,

$$\begin{aligned}
 & \left(\mathcal{L}_{c_j}(\mathcal{M}_i) \right)^- \leq \text{q-ROFS}_t\text{RWA}(\mathcal{L}_{c_1}(\mathcal{M}_1), \mathcal{L}_{c_2}(\mathcal{M}_2), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)) \leq \left(\mathcal{L}_{c_j}(\mathcal{M}_i) \right)^+.
 \end{aligned} \tag{23}$$

(iii) (Monotonicity): let $\mathcal{P}_{c_j}(\mathcal{N}_i) = (\mathcal{P}_{c_j}(\mathcal{N}_i), \overline{\mathcal{P}_{c_j}}(\mathcal{N}_i))$ ($i = 1, 2, \dots, m, j = 1, 2, \dots, n$) be another collection of q-ROFS_tRNs such that $\underline{\mathcal{P}_{c_j}}(\mathcal{N}_i) \leq \underline{\mathcal{L}_{c_j}}(\mathcal{M}_i)$ and $\overline{\mathcal{P}_{c_j}}(\mathcal{N}_i) \leq \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i)$. Then,

$$\begin{aligned}
 & \text{q-ROFS}_t\text{RWA}(\mathcal{P}_{c_1}(\mathcal{N}_1), \mathcal{P}_{c_2}(\mathcal{N}_2), \dots, \mathcal{P}_{c_n}(\mathcal{N}_m)) \\
 & \leq \text{q-ROFS}_t\text{RWA}(\mathcal{L}_{c_1}(\mathcal{M}_1), \mathcal{L}_{c_2}(\mathcal{M}_2), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)).
 \end{aligned} \tag{24}$$

(iv) (Shift invariance): let $\mathcal{P}_c(\mathcal{N}) = (\underline{\mathcal{P}_c}(\mathcal{N}), \overline{\mathcal{P}_c}(\mathcal{N})) = ((\underline{b}, \underline{d}), (\overline{b}, \overline{d}))$ be any other q-ROFS_tRN. Then,

$$\begin{aligned}
 & \text{q-ROFS}_t\text{RWA}(\mathcal{L}_{c_1}(\mathcal{M}_1) \oplus \mathcal{P}_c(\mathcal{N}), \mathcal{L}_{c_2}(\mathcal{M}_2) \oplus \mathcal{P}_c(\mathcal{N}), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m) \oplus \mathcal{P}_c(\mathcal{N})) \\
 & = \text{q-ROFS}_t\text{RWA}(\mathcal{L}_{c_1}(\mathcal{M}_1), \mathcal{L}_{c_2}(\mathcal{M}_2), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)) \oplus \mathcal{P}_c(\mathcal{N}).
 \end{aligned} \tag{25}$$

(v) (Homogeneity): for any real number $\lambda > 0$,

$$\begin{aligned}
 & \text{q-ROFS}_t\text{RWA}(\lambda \mathcal{L}_{c_1}(\mathcal{M}_1), \lambda \mathcal{L}_{c_2}(\mathcal{M}_2), \dots, \lambda \mathcal{L}_{c_n}(\mathcal{M}_m)) \\
 & = \lambda \text{q-ROFS}_t\text{RWA}(\mathcal{L}_{c_1}(\mathcal{M}_1), \mathcal{L}_{c_2}(\mathcal{M}_2), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)).
 \end{aligned} \tag{26}$$

Remark 2

- (a) If $q = 1$, so, in this case, the developed q-ROFS_tRWA operator reduces to IFS_tRWA operator
- (b) If $q = 2$, so, in this case, the developed q-ROFS_tRWA operator reduces to PFS_tRWA operator

TABLE 3: Tabular representation of $\mathcal{L}_{c_j}(\mathcal{M}_i) = (\underline{\mathcal{L}_{c_j}}(\mathcal{M}_i), \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i))$.

$\mathcal{L}$	c_1	c_2
s_1	((0.9, 0.3)), (0.8, 0.4)	((0.55, 0.2)), (0.76, 0.14)
s_2	((0.7, 0.1), (0.2, 0.75))	((0.92, 0.3), (0.6, 0.3))
s_3	((0.92, 0.25), (0.65, 0.15))	((0.4, 0.85), (0.88, 0.12))

(c) If the soft parameter c_1 is only one (means $n = 1$), then, in this case, the developed q-ROFS_tROWA operator reduces to q-ROFRWA operator

4.2. q-Runge Orthopair Fuzzy Soft Rough Ordered Weighted Averaging (q-ROFS_tROWA) Operator. This subsection presents the detailed study of q-ROFS_tROWA operator and some of its desirable characteristics such as Idempotency, Boundedness, and Monotonicity. The basic advantage of q-ROFS_tROWG operator is to weight the ordered position of the q-ROFVs instead of weighting the values themselves.

Definition 12. Let $\mathcal{L}_{c_j}(\mathcal{M}_i) = (\underline{\mathcal{L}_{c_j}}(\mathcal{M}_i), \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i))$ ($i = 1, 2, \dots, m, j = 1, 2, \dots, n$) be the collection of q-ROFS_tRNs. Let the weight vectors $t = (t_1, t_2, \dots, t_m)^T$ and $v = (v_1, v_2, \dots, v_n)^T$ for experts s_i and parameters c_j with

TABLE 4: Tabular representation of $\mathcal{L}_{\delta c_j}(\mathcal{M}_i) = (\underline{\mathcal{L}_{\delta c_j}}(\mathcal{M}_i), \overline{\mathcal{L}_{\delta c_j}}(\mathcal{M}_i))$.

$\mathcal{L}$	c_1	c_2
s_1	((0.9, 0.2), (0.8, 0.4))	((0.92, 0.3), (0.6, 0.3))
s_2	((0.92, 0.25), (0.65, 0.15))	((0.55, 0.2), (0.76, 0.14))
s_3	((0.7, 0.1), (0.2, 0.75))	((0.4, 0.85), (0.88, 0.12))

$\sum_{i=1}^m t_i = 1, \sum_{j=1}^n v_j = 1$, and $0 \leq t_i, v_j \leq 1$, respectively. The q-ROFS_tROWA operator is defined as

$$\begin{aligned} & \text{q-ROFS}_t\text{ROWA}(\mathcal{L}_{c_1}(\mathcal{M}_1), \dots, \mathcal{L}_{c_n}(\mathcal{M}_n)) \\ &= \left(\oplus_{j=1}^n v_j \left(\oplus_{i=1}^m t_i \underline{\mathcal{L}_{\delta c_j}}(\mathcal{M}_i) \right), \oplus_{j=1}^n v_j \left(\oplus_{i=1}^m t_i \overline{\mathcal{L}_{\delta c_j}}(\mathcal{M}_i) \right) \right). \end{aligned} \quad (27)$$

In view of the above definition, the aggregated result for q-ROFS_tROWA is described in Theorem 3.

Theorem 3. Let $\mathcal{L}_{c_j}(\mathcal{M}_i) = (\underline{\mathcal{L}_{c_j}}(\mathcal{M}_i), \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i))$ ($i = 1, 2, \dots, m, j = 1, 2, \dots, n$) be the collection of q-ROFS_tRNs. Let the weight vectors $t = (t_1, t_2, \dots, t_m)^T$ and $v = (v_1, v_2, \dots, v_n)^T$ for experts s_i and parameters c_j with $\sum_{i=1}^m t_i = 1, \sum_{j=1}^n v_j = 1$, and $0 \leq t_i, v_j \leq 1$, respectively. Then, q-ROFS_tROWA operator is given as

$$\begin{aligned} & \text{q-ROFS}_t\text{ROWA}(\mathcal{L}_{c_1}(\mathcal{M}_1), \dots, \mathcal{L}_{c_n}(\mathcal{M}_n)) = \left(\oplus_{j=1}^n v_j \left(\oplus_{i=1}^m t_i \underline{\mathcal{L}_{\delta c_j}}(\mathcal{M}_i) \right), \oplus_{j=1}^n v_j \left(\oplus_{i=1}^m t_i \overline{\mathcal{L}_{\delta c_j}}(\mathcal{M}_i) \right) \right) \\ &= \left[\left(\sqrt[q]{1 - \prod_{j=1}^n \left(\prod_{i=1}^m \left(1 - \frac{q}{\beta_{\delta ij}} \right)^{t_i} \right)^{v_j}}, \prod_{j=1}^n \left(\prod_{i=1}^m \frac{t_i}{\vartheta_{\delta ij}} \right)^{v_j} \right), \left(\sqrt[q]{1 - \prod_{j=1}^n \left(\prod_{i=1}^m (1 - \beta_{\delta ij}^q)^{t_i} \right)^{v_j}}, \prod_{j=1}^n \left(\prod_{i=1}^m \vartheta_{\delta ij}^{t_i} \right)^{v_j} \right) \right], \end{aligned} \quad (28)$$

where $\mathcal{L}_{\delta c_j}(\mathcal{M}_i) = (\underline{\mathcal{L}_{\delta c_j}}(\mathcal{M}_i), \overline{\mathcal{L}_{\delta c_j}}(\mathcal{M}_i))$ denotes the largest value of the permutation from i th row and j th column of the collection $i \times j$ q-ROFS_tRNs $\mathcal{L}_{c_j}(\mathcal{M}_i) = (\underline{\mathcal{L}_{c_j}}(\mathcal{M}_i), \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i))$.

Example 3. Consider Table 3 of Example 2, for the collection q-ROFS_tRNs $\mathcal{L}_{c_j}(\mathcal{M}_i) = (\underline{\mathcal{L}_{c_j}}(\mathcal{M}_i), \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i))$ and the new ordered of tabular representation of $\mathcal{L}_{c_j}(\mathcal{M}_i)$ through score function is given in Table 4.

Now, $\text{q-ROFS}_t\text{ROWA}(\mathcal{L}_{c_1}(\mathcal{M}_1), \dots, \mathcal{L}_{c_n}(\mathcal{M}_n)) = [\oplus_{j=1}^2 v_j (\oplus_{i=1}^3 t_i \underline{\mathcal{L}_{\delta c_j}}(\mathcal{M}_i)), \oplus_{j=1}^2 v_j (\oplus_{i=1}^3 t_i \overline{\mathcal{L}_{\delta c_j}}(\mathcal{M}_i))]$

$$\begin{aligned} & \text{q-ROFS}_t\text{ROWA}(\mathcal{L}_{c_1}(\mathcal{M}_1), \dots, \mathcal{L}_{c_n}(\mathcal{M}_n)) \\ &= ((0.838595, 0.261642), (0.727318, 0.255189)). \end{aligned} \quad (29)$$

From the analysis of Theorem 3, the following desirable properties hold for q-ROFS_tROWA operator.

Theorem 4. Let $\mathcal{L}_{c_j}(\mathcal{M}_i) = (\underline{\mathcal{L}_{c_j}}(\mathcal{M}_i), \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i))$ ($i = 1, 2, \dots, m, j = 1, 2, \dots, n$) be the collection of q-ROFS_tRNs. Let the weight vectors $t = (t_1, t_2, \dots, t_m)^T$ and $v =$

$(v_1, v_2, \dots, v_n)^T$ for experts s_i and parameters c_j with $\sum_{i=1}^m t_i = 1, \sum_{j=1}^n v_j = 1$, and $0 \leq t_i, v_j \leq 1$, respectively. Then, the following properties hold for q-ROFS_tROWA operator:

(i) (Idempotency): if $\mathcal{L}_{\delta c_j}(\mathcal{M}_i) = \mathcal{P}_c(\mathcal{N})$ (for all $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$), where $\mathcal{P}_c(\mathcal{M}) = (\underline{\mathcal{P}_c}(\mathcal{M}), \overline{\mathcal{P}_c}(\mathcal{M})) = ((\underline{b}, \underline{d}), (\overline{b}, \overline{d}))$, then

$$\text{q-ROFS}_t\text{ROWA}(\mathcal{L}_{c_1}(\mathcal{M}_1), \mathcal{L}_{c_2}(\mathcal{M}_2), \dots, \mathcal{L}_{c_n}(\mathcal{M}_n)) = \mathcal{P}_c(\mathcal{M}). \quad (30)$$

(ii) (Boundedness): let $(\underline{\mathcal{L}_{\delta c_j}}(\mathcal{M}_i))^- = (\min_i \min_j \underline{\mathcal{L}_{\delta c_j}}(\mathcal{M}_i), \max_j \max_i \underline{\mathcal{L}_{\delta c_j}}(\mathcal{M}_i))$ and $(\underline{\mathcal{L}_{\delta c_j}}(\mathcal{M}_i))^+ = (\max_j \max_i \underline{\mathcal{L}_{\delta c_j}}(\mathcal{M}_i), \min_j \min_i \underline{\mathcal{L}_{\delta c_j}}(\mathcal{M}_i))$. Then,

$$\begin{aligned} & \left(\underline{\mathcal{L}_{\delta c_j}}(\mathcal{M}_i) \right)^- \leq \text{q-ROFS}_t\text{ROWA}(\mathcal{L}_{c_1}(\mathcal{M}_1), \\ & \mathcal{L}_{c_2}(\mathcal{M}_2), \dots, \mathcal{L}_{c_n}(\mathcal{M}_n)) \leq \left(\underline{\mathcal{L}_{\delta c_j}}(\mathcal{M}_i) \right)^+. \end{aligned} \quad (31)$$

(iii) (Monotonicity): let $\mathcal{P}_{c_j}(\mathcal{N}_i) = (\underline{\mathcal{P}_{c_j}}(\mathcal{N}_i), \overline{\mathcal{P}_{c_j}}(\mathcal{N}_i))$ ($i = 1, 2, \dots, m, j = 1, 2, \dots, n$) be another

collection of q -ROFS_{*t*}RNs such that $\underline{\mathcal{P}}_{c_j}(\mathcal{N}_i) \leq \underline{\mathcal{L}}_{c_j}(\mathcal{M}_i)$ and $\overline{\mathcal{P}}_{c_j}(\mathcal{N}_i) \leq \overline{\mathcal{L}}_{c_j}(\mathcal{M}_i)$. Then,

$$q\text{-ROFS}_t\text{ROWA}(\mathcal{P}_{c_1}(\mathcal{M}_1), \mathcal{P}_{c_2}(\mathcal{M}_2), \dots, \mathcal{P}_{c_n}(\mathcal{M}_m)) \leq q\text{-ROFS}_t\text{ROWA}(\mathcal{L}_{c_1}(\mathcal{M}_1), \mathcal{L}_{c_2}(\mathcal{M}_2), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)). \quad (32)$$

(iv) (Shift invariance): let $\mathcal{P}_c(\mathcal{N}) = (\mathcal{P}_c(\mathcal{N}), \overline{\mathcal{P}}_c(\mathcal{N})) = ((\underline{b}, \underline{d}), (\overline{b}, \overline{d}))$ be any other q -ROFS_{*t*}RN. Then,

$$\begin{aligned} & q\text{-ROFS}_t\text{ROWA}(\mathcal{L}_{c_1}(\mathcal{M}_1) \oplus \mathcal{P}_c(\mathcal{N}), \mathcal{L}_{c_2}(\mathcal{M}_2) \\ & \oplus \mathcal{P}_c(\mathcal{N}), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m) \oplus \mathcal{P}_c(\mathcal{N})) \\ & = q\text{-ROFS}_t\text{ROWA}(\mathcal{L}_{c_1}(\mathcal{M}_1), \mathcal{L}_{c_2}(\mathcal{M}_2), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)) \\ & \oplus \mathcal{P}_c(\mathcal{N}). \end{aligned} \quad (33)$$

(v) (Homogeneity): for any real number $\lambda > 0$,

$$\begin{aligned} & q\text{-ROFS}_t\text{ROWA}(\lambda \mathcal{L}_{c_1}(\mathcal{M}_1), \lambda \mathcal{L}_{c_2}(\mathcal{M}_2), \dots, \\ & \lambda \mathcal{L}_{c_n}(\mathcal{M}_m)) = \lambda q\text{-ROFS}_t\text{ROWA} \\ & (\mathcal{L}_{c_1}(\mathcal{M}_1), \mathcal{L}_{c_2}(\mathcal{M}_2), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)). \end{aligned} \quad (34)$$

Remark 3

(a) If $q = 1$, then the developed q -ROFS_{*t*}ROWA operator reduces to IFS_{*t*}ROWA operator

(b) If $q = 2$, then the developed q -ROFS_{*t*}ROWA operator reduces to PFS_{*t*}ROWA operator

(c) If the soft parameter c_1 is one (means $n = 1$), then the developed q -ROFS_{*t*}ROWA operator reduces to q -ROFROWA operator

4.3. q -Rung Orthopair Fuzzy Soft Rough Hybrid Averaging (q -ROFS_{*t*}RHA) Operator. From the analysis of Definitions 11 and 12, it is clear that the q -ROFS_{*t*}RWA operator weights only the q -ROFVs, while q -ROFS_{*t*}ROWA operator weights the ordered position of the q -ROFVs instead of weighting the values themselves. To overcome this limitation and motivated by the idea of combining weighted averaging and the ordered weighted averaging by using the combined notion of soft rough set, we present q -ROFS_{*t*}RHA operator, which weights both the given q -ROFV and its ordered position. The basic desirable properties of the developed operator are presented in detail.

Definition 13. Let $\mathcal{L}_{c_j}(\mathcal{M}_i) = (\underline{\mathcal{L}}_{c_j}(\mathcal{M}_i), \overline{\mathcal{L}}_{c_j}(\mathcal{M}_i))$ ($i = 1, 2, \dots, m, j = 1, 2, \dots, n$) be the collection of q -ROFS_{*t*}RNs. Let the weight vectors $k = (k_1, k_2, \dots, k_m)^T$ and $l = (l_1, l_2, \dots, l_n)^T$ for experts s_i and parameters c_j with $\sum_{i=1}^m t_i = 1$, $\sum_{j=1}^n v_j = 1$, and $0 \leq k_i, l_j \leq 1$. Consider $t = (t_1, t_2, \dots, t_m)^T$ and $v = (v_1, v_2, \dots, v_n)^T$ as the associated weight vectors of experts s_i and parameters c_j with $\sum_{i=1}^m t_i = 1$, $\sum_{j=1}^n v_j = 1$, and $0 \leq t_i, v_j \leq 1$, respectively. The q -ROFS_{*t*}RHA operator is defined as

$$q\text{-ROFS}_t\text{RHA}(\mathcal{L}_{c_1}(\mathcal{M}_1), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)) = \left(\oplus_{j=1}^n v_j \left(\oplus_{i=1}^m t_i \underline{\mathcal{L}}_{\delta c_j}^* (\mathcal{M}_i) \right), \oplus_{j=1}^n v_j \left(\oplus_{i=1}^m t_i \overline{\mathcal{L}}_{\delta c_j}^* (\mathcal{M}_i) \right) \right). \quad (35)$$

From the above definition, the aggregated result for q -ROFS_{*t*}RHA operator is described in Theorem 5.

Theorem 5. Let $\mathcal{L}_{c_j}(\mathcal{M}_i) = (\underline{\mathcal{L}}_{c_j}(\mathcal{M}_i), \overline{\mathcal{L}}_{c_j}(\mathcal{M}_i))$ ($i = 1, 2, \dots, m, j = 1, 2, \dots, n$) be the collection of q -ROFS_{*t*}RNs. Let the weight vectors $k = (k_1, k_2, \dots, k_m)^T$ and $l = (l_1, l_2, \dots, l_n)^T$ of experts s_i and parameters c_j with $\sum_{i=1}^m t_i = 1$, $\sum_{j=1}^n v_j = 1$, and $0 \leq k_i, l_j \leq 1$. Consider the associated weight vectors $t = (t_1, t_2, \dots, t_m)^T$ and $v = (v_1, v_2, \dots, v_n)^T$ of experts s_i and parameters c_j with $\sum_{i=1}^m t_i = 1$, $\sum_{j=1}^n v_j = 1$, and $0 \leq t_i, v_j \leq 1$, respectively. Then, q -ROFS_{*t*}RHA operator is given as

$$\begin{aligned} & q\text{-ROFS}_t\text{RHA}(\mathcal{L}_{c_1}(\mathcal{M}_1), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)) = \left(\oplus_{j=1}^n v_j \left(\oplus_{i=1}^m t_i \underline{\mathcal{L}}_{\delta c_j}^* (\mathcal{M}_i) \right), \oplus_{j=1}^n v_j \left(\oplus_{i=1}^m t_i \overline{\mathcal{L}}_{\delta c_j}^* (\mathcal{M}_i) \right) \right) \\ & = \left[\left(\sqrt[q]{1 - \prod_{j=1}^n \left(\prod_{i=1}^m \left(1 - \frac{q}{\beta_{\delta ij}^*} \right)^{t_i} \right)^{v_j}}, \prod_{j=1}^n \left(\prod_{i=1}^m \frac{t_i}{\vartheta_{\delta ij}^*} \right)^{v_j} \right), \left(\sqrt[q]{1 - \prod_{j=1}^n \left(\prod_{i=1}^m (1 - \overline{\beta_{\delta ij}^*}^q)^{t_i} \right)^{v_j}}, \prod_{j=1}^n \left(\prod_{i=1}^m \overline{\vartheta_{\delta ij}^*}^{t_i} \right)^{v_j} \right) \right], \end{aligned} \quad (36)$$

where $\mathcal{L}_{\delta c_j}^*(\mathcal{M}_i) = nkl \mathcal{L}_{c_j} = (nkl \underline{\mathcal{L}}_{c_j}(\mathcal{M}_i), nkl \overline{\mathcal{L}}_{c_j}(\mathcal{M}_i))$ denotes the largest value of the permutation from i th row and

j th column of the collection $i \times j$ q -ROFS_{*t*}RNs $\mathcal{L}_{c_j}(\mathcal{M}_i) = (\underline{\mathcal{L}}_{c_j}(\mathcal{M}_i), \overline{\mathcal{L}}_{c_j}(\mathcal{M}_i))$ and n represents the balancing coefficient.

Example 4. Consider Table 3 of Example 2, for the collection q -ROFS_tRNs $\mathcal{L}_{c_j}(\mathcal{M}_i) = (\underline{\mathcal{L}_{c_j}}(\mathcal{M}_i), \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i))$ with $k = (0.33, 0.37, 0.3)^T$ and $l = (0.42, 0.58)^T$ as the weight vectors of experts s_i and parameters c_j . Consider

$t = (0.36, 0.34, 0.3)^T$ and $v = (0.55, 0.45)^T$ as the associated weight vectors of experts s_i and parameters c_j . The tabular representation of $\mathcal{L}_{\delta c_j}^*(\mathcal{M}_i)$ through operation law and score function is given in Tables 5 and 6. Now,

$$\begin{aligned} q\text{-ROFS}_t\text{RHA}(\mathcal{L}_{c_1}(\mathcal{M}_1), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)) &= \left(\oplus_{j=1}^2 v_j \left(\oplus_{i=1}^3 t_i \underline{\mathcal{L}_{\delta c_j}^*}(\mathcal{M}_i) \right), \oplus_{j=1}^2 v_j \left(\oplus_{i=1}^3 t_i \overline{\mathcal{L}_{\delta c_j}^*}(\mathcal{M}_i) \right) \right), \\ q\text{-ROFS}_t\text{RHA}(\mathcal{L}_{c_1}(\mathcal{M}_1), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)) &= [(0.701609, 0.129765), (0.600425, 0.122969)]. \end{aligned} \quad (37)$$

From the analysis of Theorem 5, the following characteristics hold in q -ROFS_tRHA operator.

Theorem 6. Let $\mathcal{L}_{c_j}(\mathcal{M}_i) = (\underline{\mathcal{L}_{c_j}}(\mathcal{M}_i), \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i))$ ($i = 1, 2, \dots, m, j = 1, 2, \dots, n$) be the collection of q -ROFS_tRNs. Let the weight vectors $k = (k_1, k_2, \dots, k_m)^T$ and $l = (l_1, l_2, \dots, l_n)^T$ of experts s_i and parameters c_j with $\sum_{i=1}^m t_i = 1$, $\sum_{j=1}^n v_j = 1$, and $0 \leq k_i, l_j \leq 1$. Consider the associated weight vectors $t = (t_1, t_2, \dots, t_m)^T$ and $v = (v_1, v_2, \dots, v_n)^T$ of experts s_i and parameters c_j with $\sum_{i=1}^m t_i = 1$, $\sum_{j=1}^n v_j = 1$, and $0 \leq t_i, v_j \leq 1$, respectively. Then, the following properties hold for q -ROFS_tRHA operator:

(i) (Idempotency): if $\mathcal{L}_{\delta c_j}^*(\mathcal{M}_i) = \mathcal{P}_c(\mathcal{N})$ (for all $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$) where $\mathcal{P}_c(\mathcal{M}) = (\underline{\mathcal{P}_c}(\mathcal{M}), \overline{\mathcal{P}_c}(\mathcal{M})) = ((\underline{b}, \underline{d}), (\overline{b}, \overline{d}))$, then

$$q\text{-ROFS}_t\text{RHA}(\mathcal{L}_{c_1}(\mathcal{M}_1), \mathcal{L}_{c_2}(\mathcal{M}_2), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)) = \mathcal{P}_c(\mathcal{M}). \quad (38)$$

(ii) (Boundedness): let $(\mathcal{L}_{\delta c_j}^*(\mathcal{M}_i))^- = (\min_j \min_i \underline{\mathcal{L}_{\delta c_j}^*}(\mathcal{M}_i), \max_j \max_i \overline{\mathcal{L}_{\delta c_j}^*}(\mathcal{M}_i))$ and $(\mathcal{L}_{\delta c_j}^*(\mathcal{M}_i))^+ = (\max_j \max_i \underline{\mathcal{L}_{\delta c_j}^*}(\mathcal{M}_i), \min_j \min_i \overline{\mathcal{L}_{\delta c_j}^*}(\mathcal{M}_i))$. Then,

$$\begin{aligned} (\mathcal{L}_{\delta c_j}^*(\mathcal{M}_i))^- &\leq q\text{-ROFS}_t\text{RHA}(\mathcal{L}_{c_1}(\mathcal{M}_1), \mathcal{L}_{c_2}(\mathcal{M}_2), \dots, \\ &\mathcal{L}_{c_n}(\mathcal{M}_m)) \leq (\mathcal{L}_{\delta c_j}^*(\mathcal{M}_i))^+. \end{aligned} \quad (39)$$

(iii) (Monotonicity): let $\mathcal{P}_{c_j}(\mathcal{N}_i) = (\underline{\mathcal{P}_{c_j}}(\mathcal{N}_i), \overline{\mathcal{P}_{c_j}}(\mathcal{N}_i))$ ($i = 1, 2, \dots, m, j = 1, 2, \dots, n$) be another collection of q -ROFS_tRNs such that $\underline{\mathcal{P}_{c_j}}(\mathcal{N}_i) \leq \underline{\mathcal{L}_{c_j}}(\mathcal{M}_i)$ and $\overline{\mathcal{P}_{c_j}}(\mathcal{N}_i) \leq \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i)$. Then,

$$\begin{aligned} q\text{-ROFS}_t\text{RHA}(\mathcal{P}_{c_1}(\mathcal{M}_1), \mathcal{P}_{c_2}(\mathcal{M}_2), \dots, \mathcal{P}_{c_n}(\mathcal{M}_m)) \\ \leq q\text{-ROFS}_t\text{RHA}(\mathcal{L}_{c_1}(\mathcal{M}_1), \mathcal{L}_{c_2}(\mathcal{M}_2), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)). \end{aligned} \quad (40)$$

(iv) (Shift invariance): let $\mathcal{P}_c(\mathcal{N}) = (\underline{\mathcal{P}_c}(\mathcal{N}), \overline{\mathcal{P}_c}(\mathcal{N})) = ((\underline{b}, \underline{d}), (\overline{b}, \overline{d}))$ be any other q -ROFS_tRN. Then,

$$\begin{aligned} q\text{-ROFS}_t\text{RHA}(\mathcal{L}_{c_1}(\mathcal{M}_1) \oplus \mathcal{P}_c(\mathcal{N}), \mathcal{L}_{c_2}(\mathcal{M}_2) \\ \oplus \mathcal{P}_c(\mathcal{N}), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m) \oplus \mathcal{P}_c(\mathcal{N})) = q\text{-ROFS}_t\text{RHA} \\ (\mathcal{L}_{c_1}(\mathcal{M}_1), \mathcal{L}_{c_2}(\mathcal{M}_2), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)) \oplus \mathcal{P}_c(\mathcal{N}). \end{aligned} \quad (41)$$

(v) (Homogeneity): for any real number $\lambda > 0$,

$$\begin{aligned} q\text{-ROFS}_t\text{RHA}(\lambda \mathcal{L}_{c_1}(\mathcal{M}_1), \lambda \mathcal{L}_{c_2}(\mathcal{M}_2), \dots, \lambda \mathcal{L}_{c_n}(\mathcal{M}_m)) \\ = \lambda q\text{-ROFS}_t\text{RHA}(\mathcal{L}_{c_1}(\mathcal{M}_1), \mathcal{L}_{c_2}(\mathcal{M}_2), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)). \end{aligned} \quad (42)$$

Remark 4

- (a) If $q = 1$, then q -ROFS_tRHA operator reduces to IFS_tRHA operator
- (b) If $q = 2$, then q -ROFS_tRHA operator reduces to PFS_tRHA operator
- (c) If soft parameter c_1 is one (means $n = 1$), then q -ROFS_tRHA operator reduces to q -ROFRHA operator
- (d) If $kl = ((1/n), (1/n), \dots, (1/n))^T$, then the proposed q -ROFS_tRHA operator reduces to q -ROFS_tRWA operator
- (e) If $tv = ((1/n), (1/n), \dots, (1/n))^T$, then the proposed q -ROFS_tRHA operator reduces to q -ROFS_tROWA operator

5. q-Rung Orthopair Fuzzy Soft Rough Geometric (q-ROFS_tRG) Aggregation Operator

This section is devoted to the study of q -ROFS_tR geometric aggregation operators such as q -ROFS_tRWG, q -ROFS_tROWG, and q -ROFS_tRHG operators. We will present the fundamental properties of these operators in detail.

TABLE 5: Tabular representation by using operation law for $\mathcal{L}_{\delta c_j}^*(\mathcal{M}_i) = nkl\mathcal{L}_{c_j} = (nkl\mathcal{L}_{c_j}(\mathcal{M}_i), nkl\overline{\mathcal{L}_{c_j}}(\mathcal{M}_i))$.

$\mathcal{L}$	c_1	c_2
s_1	((0.7483, 0.1247), (0.6366, 0.1663))	((0.4629, 0.1148), (0.6561, 0.0804))
s_2	((0.5624, 0.0466), (0.1551, 0.3497))	((0.8533, 0.1931), (0.5254, 0.1931))
s_3	((0.7574, 0.0945), (0.4853, 0.0567))	((0.3238, 0.4437), (0.7661, 0.0626))

TABLE 6: Tabular representation after using score function $\mathcal{L}_{\delta c_j}^*(\mathcal{M}_i) = (\mathcal{L}_{\delta c_j}^*(\mathcal{M}_i), \overline{\mathcal{L}_{\delta c_j}^*}(\mathcal{M}_i))$.

$\mathcal{L}$	c_1	c_2
s_1	((0.7483, 0.1247), (0.6366, 0.1663))	((0.8533, 0.1931), (0.5254, 0.1931))
s_2	((0.7574, 0.0945), (0.4853, 0.0567))	((0.3238, 0.4437), (0.7661, 0.0626))
s_3	((0.5624, 0.0466), (0.1551, 0.3497))	((0.4629, 0.1148), (0.6561, 0.0804))

5.1. q -Rung Orthopair Fuzzy Soft Rough Weighted Geometric (q -ROFS_tRWG) Operator

Definition 14. Let $\mathcal{L}_{c_j}(\mathcal{M}_i) = (\mathcal{L}_{c_j}(\mathcal{M}_i), \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i))$ ($i = 1, 2, \dots, m, j = 1, 2, \dots, n$) be the collection of q -ROFS_tRNs.

Let the weight vectors $t = (t_1, t_2, \dots, t_m)^T$ and $v = (v_1, v_2, \dots, v_n)^T$ of experts s_i and parameters c_j with $\sum_{i=1}^m t_i = 1$, $\sum_{j=1}^n v_j = 1$, and $0 \leq t_i, v_j \leq 1$, respectively. The q -ROFS_tRWG operator is defined as

$$q\text{-ROFS}_t\text{RWG}(\mathcal{L}_{c_1}(\mathcal{M}_1), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)) = \left[\oplus_{j=1}^n \left\{ \oplus_{i=1}^m \left(\mathcal{L}_{c_j}(\mathcal{M}_i) \right)^{t_i} \right\}^{v_j}, \oplus_{j=1}^n \left\{ \oplus_{i=1}^m \left(\overline{\mathcal{L}_{c_j}}(\mathcal{M}_i) \right)^{t_i} \right\}^{v_j} \right]. \quad (43)$$

Based on the above definition the aggregated result for q -ROFS_tRWG operator is given in Theorem 7.

Theorem 7. Let $\mathcal{L}_{c_j}(\mathcal{M}_i) = (\mathcal{L}_{c_j}(\mathcal{M}_i), \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i))$ ($i = 1, 2, \dots, m, j = 1, 2, \dots, n$) be the collection of

q -ROFS_tRNs. Let the weight vectors $t = (t_1, t_2, \dots, t_m)^T$ and $v = (v_1, v_2, \dots, v_n)^T$ of experts s_i and parameters c_j with $\sum_{i=1}^m t_i = 1$, $\sum_{j=1}^n v_j = 1$, and $0 \leq t_i, v_j \leq 1$, respectively. Then, q -ROFS_tRWG operator is given as

$$\begin{aligned} q\text{-ROFS}_t\text{RWG}(\mathcal{L}_{c_1}(\mathcal{M}_1), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)) &= \left[\oplus_{j=1}^n \left\{ \oplus_{i=1}^m \left(\mathcal{L}_{c_j}(\mathcal{M}_i) \right)^{t_i} \right\}^{v_j}, \oplus_{j=1}^n \left\{ \oplus_{i=1}^m \left(\overline{\mathcal{L}_{c_j}}(\mathcal{M}_i) \right)^{t_i} \right\}^{v_j} \right] \\ &= \left[\left\{ \prod_{j=1}^n \left(\prod_{i=1}^m \beta_{ij}^{t_i} \right)^{v_j}, \sqrt[q]{1 - \prod_{j=1}^n \left(\prod_{i=1}^m \left(1 - \beta_{ij}^q \right)^{t_i} \right)^{v_j}} \right\}, \left\{ \prod_{j=1}^n \left(\prod_{i=1}^m \overline{\beta}_{ij}^{t_i} \right)^{v_j}, \sqrt[q]{1 - \prod_{j=1}^n \left(\prod_{i=1}^m \left(1 - \overline{\beta}_{ij}^q \right)^{t_i} \right)^{v_j}} \right\} \right]. \end{aligned} \quad (44)$$

Since it is clear that $\mathcal{L}_{c_j}(\mathcal{M}_i)$ and $\overline{\mathcal{L}_{c_j}}(\mathcal{M}_i)$ are q -ROFNs, by Definition 7, we have that

$\oplus_{j=1}^n \left\{ \oplus_{i=1}^m \left(\mathcal{L}_{c_j}(\mathcal{M}_i) \right)^{t_i} \right\}^{v_j}$ and $\oplus_{j=1}^n \left\{ \oplus_{i=1}^m \left(\overline{\mathcal{L}_{c_j}}(\mathcal{M}_i) \right)^{t_i} \right\}^{v_j}$ are also q -ROFNs. Therefore,

q -ROFS_tRWG($\mathcal{L}_{c_1}(\mathcal{M}_1), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)$) is also a q -ROFS_tRN in approximation space $(\mathcal{T}, \mathcal{E}, \mathcal{L})$.

Example 5. Consider Table 3 of Example 2, for the collection q -ROFS_tRNs $\mathcal{L}_{c_j}(\mathcal{M}_i) = (\underline{\mathcal{L}_{c_j}}(\mathcal{M}_i), \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i))$. Then, the aggregated result for q -ROFS_tRWG is given as

$$\begin{aligned} q\text{-ROFS}_t\text{RWG}(\mathcal{L}_{c_1}(\mathcal{M}_1), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)) &= \left[\oplus_{j=1}^2 \left\{ \oplus_{i=1}^3 \left(\underline{\mathcal{L}_{c_j}}(\mathcal{M}_i) \right)^{t_i} \right\}^{v_j}, \oplus_{j=1}^2 \left\{ \oplus_{i=1}^3 \left(\overline{\mathcal{L}_{c_j}}(\mathcal{M}_i) \right)^{t_i} \right\}^{v_j} \right] \\ &= \left[\left\{ (0.9^{0.25} 0.7^{0.3} 0.92^{0.45})^{0.55} (0.55^{0.25} 0.92^{0.3} 0.4^{0.45})^{0.45}, \right. \right. \\ &\quad \left. \sqrt[3]{1 - \left(\left[(1 - 0.3^3)^{0.25} (1 - 0.1^3)^{0.25} (1 - 0.25^3)^{0.45} \right]^{0.55} \left[(1 - 0.2^3)^{0.25} (1 - 0.3^3)^{0.25} (1 - 0.85^3)^{0.45} \right]^{0.45}} \right\}, \right. \\ &\quad \left. \left\{ (0.8^{0.25} 0.2^{0.3} 0.65^{0.45})^{0.55} (0.76^{0.25} 0.6^{0.3} 0.88^{0.45})^{0.45}, \right. \right. \\ &\quad \left. \left. \sqrt[3]{1 - \left(\left[(1 - 0.4^3)^{0.25} (1 - 0.75^3)^{0.25} (1 - 0.15^3)^{0.45} \right]^{0.55} \left[(1 - 0.14^3)^{0.25} (1 - 0.3^3)^{0.25} (1 - 0.12^3)^{0.45} \right]^{0.45}} \right\} \right\} \right] \\ &= [(0.715607, 0.509925), (0.573442, 0.484819)]. \end{aligned} \quad (45)$$

From the analysis of Theorem 7, the following hold for q -ROFS_tRWG operator.

Theorem 8. Let $\mathcal{L}_{c_j}(\mathcal{M}_i) = (\underline{\mathcal{L}_{c_j}}(\mathcal{M}_i), \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i))$ ($i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$) be the collection of q -ROFS_tRNs. Let the weight vectors $t = (t_1, t_2, \dots, t_m)^T$ and $v = (v_1, v_2, \dots, v_n)^T$ of experts s_i and parameters c_j with $\sum_{i=1}^m t_i = 1$, $\sum_{j=1}^n v_j = 1$, and $0 \leq t_i, v_j \leq 1$, respectively. Then, the following properties hold for q -ROFS_tRWG operator:

(i) (Idempotency): if $\mathcal{L}_{c_j}(\mathcal{M}_i) = \mathcal{P}_c(\mathcal{N})$ (for all $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$) where $\mathcal{P}_c(\mathcal{M}) = (\underline{\mathcal{P}_c}(\mathcal{M}), \overline{\mathcal{P}_c}(\mathcal{M})) = ((\underline{b}, \underline{d}), (\overline{b}, \overline{d}))$, then

$$\begin{aligned} q\text{-ROFS}_t\text{RWG}(\mathcal{L}_{c_1}(\mathcal{M}_1), \mathcal{L}_{c_2}(\mathcal{M}_2), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)) \\ = \mathcal{P}_c(\mathcal{M}). \end{aligned} \quad (46)$$

(ii) (Boundedness): let $(\mathcal{L}_{c_j}(\mathcal{M}_i))^- = (\min_j \min_i \underline{\mathcal{L}_{c_j}}(\mathcal{M}_i), \max_j \max_i \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i))$ and $(\mathcal{L}_{c_j}(\mathcal{M}_i))^+ = (\max_j \max_i \underline{\mathcal{L}_{c_j}}(\mathcal{M}_i), \min_j \min_i \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i))$. Then,

$$\begin{aligned} (\mathcal{L}_{c_j}(\mathcal{M}_i))^- \leq q\text{-ROFS}_t\text{RWG}(\mathcal{L}_{c_1}(\mathcal{M}_1), \mathcal{L}_{c_2}(\mathcal{M}_2), \dots, \\ \mathcal{L}_{c_n}(\mathcal{M}_m)) \leq (\mathcal{L}_{c_j}(\mathcal{M}_i))^+. \end{aligned} \quad (47)$$

(iii) (Monotonicity): let $\mathcal{P}_{c_j}(\mathcal{N}_i) = (\underline{\mathcal{P}_{c_j}}(\mathcal{N}_i), \overline{\mathcal{P}_{c_j}}(\mathcal{N}_i))$ ($i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$) be another collection of q -ROFS_tRNs such that $\underline{\mathcal{P}_{c_j}}(\mathcal{N}_i) \leq \underline{\mathcal{L}_{c_j}}(\mathcal{M}_i)$ and $\overline{\mathcal{P}_{c_j}}(\mathcal{N}_i) \leq \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i)$. Then,

$$\begin{aligned} q\text{-ROFS}_t\text{RWG}(\mathcal{P}_{c_1}(\mathcal{M}_1), \mathcal{P}_{c_2}(\mathcal{M}_2), \dots, \mathcal{P}_{c_n}(\mathcal{M}_m)) \\ \leq q\text{-ROFS}_t\text{RWG}(\mathcal{L}_{c_1}(\mathcal{M}_1), \mathcal{L}_{c_2}(\mathcal{M}_2), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)). \end{aligned} \quad (48)$$

(iv) (Shift invariance): let $\mathcal{P}_c(\mathcal{N}) = (\underline{\mathcal{P}_c}(\mathcal{N}), \overline{\mathcal{P}_c}(\mathcal{N})) = ((\underline{b}, \underline{d}), (\overline{b}, \overline{d}))$ be any other q -ROFS_tRN. Then,

$$\begin{aligned} q\text{-ROFS}_t\text{RWG}(\mathcal{L}_{c_1}(\mathcal{M}_1) \oplus \mathcal{P}_c(\mathcal{N}), \mathcal{L}_{c_2}(\mathcal{M}_2) \\ \oplus \mathcal{P}_c(\mathcal{N}), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m) \oplus \mathcal{P}_c(\mathcal{N})) \\ = q\text{-ROFS}_t\text{RWG}(\mathcal{L}_{c_1}(\mathcal{M}_1), \mathcal{L}_{c_2}(\mathcal{M}_2), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)) \\ \oplus \mathcal{P}_c(\mathcal{N}). \end{aligned} \quad (49)$$

(v) (Homogeneity): for any real number $\lambda > 0$,

$$\begin{aligned} q\text{-ROFS}_t\text{RWG}(\lambda \mathcal{L}_{c_1}(\mathcal{M}_1), \lambda \mathcal{L}_{c_2}(\mathcal{M}_2), \dots, \lambda \mathcal{L}_{c_n}(\mathcal{M}_m)) \\ = \lambda q\text{-ROFS}_t\text{RWG}(\mathcal{L}_{c_1}(\mathcal{M}_1), \mathcal{L}_{c_2}(\mathcal{M}_2), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)). \end{aligned} \quad (50)$$

Remark 5

- (a) If $q = 1$, then q -ROFS_tRWG operator reduces to IFS_tRWG operator
- (b) If $q = 2$, then q -ROFS_tRWG operator reduces to PFS_tRWG operator
- (c) If soft parameter c_1 is one (means $n = 1$), then q -ROFS_tRWG operator reduces to q -ROFRWG operator

5.2. *q*-Rung Orthopair Fuzzy Soft Rough Ordered Weighted Geometric (*q*-ROFS_{*t*}ROWG) Operator. Here, we will put forward the detailed study of *q*-ROFS_{*t*}ROWG operator and some of its fundamental properties. The *q*-ROFS_{*t*}ROWG operator weights the ordered position of the *q*-ROFVs instead of weighting the values themselves.

Definition 15. Let $\mathcal{L}_{c_j}(\mathcal{M}_i) = (\mathcal{L}_{c_j}(\mathcal{M}_i), \overline{\mathcal{L}_{c_j}(\mathcal{M}_i)})$ ($i = 1, 2, \dots, m, j = 1, 2, \dots, n$) be the collection of *q*-ROFS_{*t*}RNs. Let the weight vectors $t = (t_1, t_2, \dots, t_m)^T$ and $v = (v_1, v_2, \dots, v_n)^T$ of experts s_i and parameters c_j with $\sum_{i=1}^m t_i = 1$, $\sum_{j=1}^n v_j = 1$, and $0 \leq t_i, v_j \leq 1$, respectively. The *q*-ROFS_{*t*}ROWG operator is defined as

$$q\text{-ROFS}_t\text{ROWG}(\mathcal{L}_{c_1}(\mathcal{M}_1), \dots, \mathcal{L}_{c_n}(\mathcal{M}_n)) = \left[\oplus_{j=1}^n \left\{ \oplus_{i=1}^m \left(\mathcal{L}_{\delta_{c_j}}(\mathcal{M}_i) \right)^{t_i} \right\}^{v_j}, \oplus_{j=1}^n \left\{ \oplus_{i=1}^m \left(\overline{\mathcal{L}_{\delta_{c_j}}(\mathcal{M}_i)} \right)^{t_i} \right\}^{v_j} \right]. \quad (51)$$

In view of the above definition, the aggregated result for *q*-ROFS_{*t*}ROWG operator is described in Theorem 9.

Theorem 9. Let $\mathcal{L}_{c_j}(\mathcal{M}_i) = (\mathcal{L}_{c_j}(\mathcal{M}_i), \overline{\mathcal{L}_{c_j}(\mathcal{M}_i)})$ ($i = 1, 2, \dots, m, j = 1, 2, \dots, n$) be the collection of

q-ROFS_{*t*}RNs. Let the weight vectors $t = (t_1, t_2, \dots, t_m)^T$ and $v = (v_1, v_2, \dots, v_n)^T$ of experts s_i and parameters c_j with $\sum_{i=1}^m t_i = 1$, $\sum_{j=1}^n v_j = 1$, and $0 \leq t_i, v_j \leq 1$, respectively. Then, *q*-ROFS_{*t*}ROWG operator is given as

$$\begin{aligned} q\text{-ROFS}_t\text{ROWG}(\mathcal{L}_{c_1}(\mathcal{M}_1), \dots, \mathcal{L}_{c_n}(\mathcal{M}_n)) &= \left[\oplus_{j=1}^n \left\{ \oplus_{i=1}^m \left(\mathcal{L}_{\delta_{c_j}}(\mathcal{M}_i) \right)^{t_i} \right\}^{v_j}, \oplus_{j=1}^n \left\{ \oplus_{i=1}^m \left(\overline{\mathcal{L}_{\delta_{c_j}}(\mathcal{M}_i)} \right)^{t_i} \right\}^{v_j} \right] \\ &= \left[\left\{ \prod_{j=1}^n \left(\prod_{i=1}^m \beta_{\delta_{ij}}^{t_i} \right)^{v_j}, \sqrt[q]{1 - \prod_{j=1}^n \left(\prod_{i=1}^m \left(1 - \beta_{\delta_{ij}}^q \right)^{t_i} \right)^{v_j}} \right\}, \left\{ \prod_{j=1}^n \left(\prod_{i=1}^m \overline{\beta_{\delta_{ij}}^{t_i}} \right)^{v_j}, \sqrt[q]{1 - \prod_{j=1}^n \left(\prod_{i=1}^m \left(1 - \overline{\beta_{\delta_{ij}}^q} \right)^{t_i} \right)^{v_j}} \right\} \right], \end{aligned} \quad (52)$$

where $\mathcal{L}_{\delta_{c_j}}(\mathcal{M}_i) = (\mathcal{L}_{\delta_{c_j}}(\mathcal{M}_i), \overline{\mathcal{L}_{\delta_{c_j}}(\mathcal{M}_i)})$ denotes the largest value of the permutation from *i*th row and *j*th column of the collection $i \times j$ *q*-ROFS_{*t*}RNs, $\mathcal{L}_{c_j}(\mathcal{M}_i) = (\mathcal{L}_{c_j}(\mathcal{M}_i), \overline{\mathcal{L}_{c_j}(\mathcal{M}_i)})$.

Remark 6

- If $q = 1$, then *q*-ROFS_{*t*}ROWG operator reduces to IFS_{*t*}ROWG operator
- If $q = 2$, then *q*-ROFS_{*t*}ROWG operator reduces to PFS_{*t*}ROWG operator
- If the soft parameter c_1 is one (means $n = 1$), then *q*-ROFS_{*t*}ROWG operator reduces to *q*-ROFROWG operator

5.3. *q*-Rung Orthopair Fuzzy Soft Rough Hybrid Geometric (*q*-ROFS_{*t*}RHG) Operator. From the analysis of Definitions 14 and 15, it is clear that the *q*-ROFS_{*t*}ROWG operator weights

only the *q*-ROFVs, while *q*-ROFS_{*t*}ROWG operator weights the ordered position of the *q*-ROFVs instead of weighting themselves. Motivated by the idea of combining weighted geometric and the ordered weighted geometric by using the combined notion of soft rough set, we present *q*-ROFS_{*t*}RHG operator, which weights both the given *q*-ROFV and its ordered position. The basic desirable properties of the developed operator are presented in detail.

Definition 16. Let $\mathcal{L}_{c_j}(\mathcal{M}_i) = (\mathcal{L}_{c_j}(\mathcal{M}_i), \overline{\mathcal{L}_{c_j}(\mathcal{M}_i)})$ ($i = 1, 2, \dots, m, j = 1, 2, \dots, n$) be the collection of *q*-ROFS_{*t*}RNs. Let the weight vectors $k = (k_1, k_2, \dots, k_m)^T$ and $l = (l_1, l_2, \dots, l_n)^T$ of experts s_i and parameters c_j with $\sum_{i=1}^m t_i = 1$, $\sum_{j=1}^n v_j = 1$, and $0 \leq k_i, l_j \leq 1$. Consider the associated weight vectors $t = (t_1, t_2, \dots, t_m)^T$ and $v = (v_1, v_2, \dots, v_n)^T$ of experts s_i and parameters c_j with $\sum_{i=1}^m t_i = 1$, $\sum_{j=1}^n v_j = 1$, and $0 \leq t_i, v_j \leq 1$, respectively. The *q*-ROFS_{*t*}RHG operator is defined as

$$q\text{-ROFS}_t\text{RHG}(\mathcal{L}_{c_1}(\mathcal{M}_1), \dots, \mathcal{L}_{c_n}(\mathcal{M}_n)) = \left[\oplus_{j=1}^n \left\{ \oplus_{i=1}^m \left(\mathcal{L}_{\delta_{c_j}}^*(\mathcal{M}_i) \right)^{t_i} \right\}^{v_j}, \oplus_{j=1}^n \left\{ \oplus_{i=1}^m \left(\overline{\mathcal{L}_{\delta_{c_j}}^*(\mathcal{M}_i)} \right)^{t_i} \right\}^{v_j} \right]. \quad (53)$$

From the above definition, the aggregated value for q-ROFS_tRHG operator is described in Theorem 11.

Theorem 10. Let $\mathcal{L}_{c_j}(\mathcal{M}_i) = (\underline{\mathcal{L}_{c_j}}(\mathcal{M}_i), \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i))$ ($i = 1, 2, \dots, m, j = 1, 2, \dots, n$) be the collection of q-ROFS_tRNs. Let the weight vectors $k = (k_1, k_2, \dots, k_m)^T$ and $l = (l_1, l_2, \dots, l_n)^T$ of experts s_i and parameters c_j with

$$\begin{aligned} \text{q-ROFS}_t\text{RHG}(\mathcal{L}_{c_1}(\mathcal{M}_1), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)) &= \left[\oplus_{j=1}^n \left\{ \oplus_{i=1}^m \left(\underline{\mathcal{L}_{\delta c_j}}^*(\mathcal{M}_i) \right)^{t_i} \right\}^{v_j}, \oplus_{j=1}^n \left\{ \oplus_{i=1}^m \left(\overline{\mathcal{L}_{\delta c_j}}^*(\mathcal{M}_i) \right)^{t_i} \right\}^{v_j} \right] \\ &= \left[\left\{ \prod_{j=1}^n \left(\prod_{i=1}^m \beta_{\delta ij}^{*t_i} \right)^{v_j}, \sqrt[q]{1 - \prod_{j=1}^n \left(\prod_{i=1}^m \left(1 - \frac{q}{\delta_{ij}} \right)^{t_i} \right)^{v_j}} \right\}, \left\{ \prod_{j=1}^n \left(\prod_{i=1}^m \beta_{\delta ij}^{*t_i} \right)^{v_j}, \sqrt[q]{1 - \prod_{j=1}^n \left(\prod_{i=1}^m \left(1 - \frac{q}{\delta_{ij}} \right)^{t_i} \right)^{v_j}} \right\} \right], \end{aligned} \quad (54)$$

where $\mathcal{L}_{\delta c_j}^*(\mathcal{M}_i) = (\mathcal{L}_{c_j})^{nkl} = ((\underline{\mathcal{L}_{c_j}}(\mathcal{M}_i))^{nkl}, (\overline{\mathcal{L}_{c_j}}(\mathcal{M}_i))^{nkl})$ denotes the largest value of the permutation from i th row and j th column of the collection $i \times j$ q-ROFS_tRNs $\mathcal{L}_{c_j}(\mathcal{M}_i) = (\underline{\mathcal{L}_{c_j}}(\mathcal{M}_i), \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i))$ and n represents the balancing coefficient.

$\sum_{i=1}^m t_i = 1, \sum_{j=1}^n v_j = 1$, and $0 \leq k_i, l_j \leq 1$. Consider the associated weight vectors $t = (t_1, t_2, \dots, t_m)^T$ and $v = (v_1, v_2, \dots, v_n)^T$ of experts s_i and parameters c_j with $\sum_{i=1}^m t_i = 1, \sum_{j=1}^n v_j = 1$, and $0 \leq t_i, v_j \leq 1$, respectively. Then, q-ROFS_tRHG operator is given as

Example 6. Consider Tables 2, 4, and 5 of Examples 2 and 4, for the collection q-ROFS_tRNs $\mathcal{L}_{c_j}(\mathcal{M}_i) = (\underline{\mathcal{L}_{c_j}}(\mathcal{M}_i), \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i))$ with $k = (0.33, 0.37, 0.3)^T$ and $l = (0.42, 0.58)^T$ is the weight vectors of experts s_i and parameters c_j . Consider $t = (0.36, 0.34, 0.3)^T$ and $v = (0.55, 0.45)^T$ as the associated weight vectors of experts s_i and parameters c_j . Then, the aggregated result for $\mathcal{L}_{\delta c_j}^*(\mathcal{M}_i)$ is given as

$$\begin{aligned} \text{q-ROFS}_t\text{RHG}(\mathcal{L}_{c_1}(\mathcal{M}_1), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)) &= \left[\oplus_{j=1}^2 \left\{ \oplus_{i=1}^3 \left(\underline{\mathcal{L}_{\delta c_j}}(\mathcal{M}_i) \right)^{t_i} \right\}^{v_j}, \oplus_{j=1}^2 \left\{ \oplus_{i=1}^3 \left(\overline{\mathcal{L}_{\delta c_j}}(\mathcal{M}_i) \right)^{t_i} \right\}^{v_j} \right], \\ \text{q-ROFS}_t\text{RHG}(\mathcal{L}_{c_1}(\mathcal{M}_1), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)) &= ((0.602629, 0.250898), (0.479995, 0.210973)). \end{aligned} \quad (55)$$

From the analysis of Theorem 11, the properties of q-ROFS_tRHG operator are given below.

Theorem 11. Let $\mathcal{L}_{c_j}(\mathcal{M}_i) = (\underline{\mathcal{L}_{c_j}}(\mathcal{M}_i), \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i))$ ($i = 1, 2, \dots, m, j = 1, 2, \dots, n$) be the collection of q-ROFS_tRNs. Let the weight vectors $k = (k_1, k_2, \dots, k_m)^T$ and $l = (l_1, l_2, \dots, l_n)^T$ of experts s_i and parameters c_j with $\sum_{i=1}^m t_i = 1, \sum_{j=1}^n v_j = 1$, and $0 \leq k_i, l_j \leq 1$. Consider the associated weight vectors $t = (t_1, t_2, \dots, t_m)^T$ and $v = (v_1, v_2, \dots, v_n)^T$ of experts s_i and parameters c_j with $\sum_{i=1}^m t_i = 1, \sum_{j=1}^n v_j = 1$, and $0 \leq t_i, v_j \leq 1$, respectively. Then, q-ROFS_tRHG operator holds as follows:

(i) (Idempotency): if $\mathcal{L}_{\delta c_j}^*(\mathcal{M}_i) = \mathcal{P}_c(\mathcal{N})$ (for all $i = 1, 2, \dots, m$ and $j = 1, 2, \dots, n$) where $\mathcal{P}_c(\mathcal{M}) = (\underline{\mathcal{P}_c}(\mathcal{M}), \overline{\mathcal{P}_c}(\mathcal{M})) = ((\underline{b}, \underline{d}), (\overline{b}, \overline{d}))$, then

$$\text{q-ROFS}_t\text{RHG}(\mathcal{L}_{c_1}(\mathcal{M}_1), \mathcal{L}_{c_2}(\mathcal{M}_2), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)) = \mathcal{P}_c(\mathcal{M}). \quad (56)$$

(ii) (Boundedness): let $(\underline{\mathcal{L}_{\delta c_j}^*}(\mathcal{M}_i))^- = (\min_j \min_i \underline{\mathcal{L}_{\delta c_j}^*}(\mathcal{M}_i), \max_j \max_i \underline{\mathcal{L}_{\delta c_j}^*}(\mathcal{M}_i))$ and $(\underline{\mathcal{L}_{\delta c_j}^*}(\mathcal{M}_i))^+ = (\max_j \max_i \underline{\mathcal{L}_{\delta c_j}^*}(\mathcal{M}_i), \min_j \min_i \underline{\mathcal{L}_{\delta c_j}^*}(\mathcal{M}_i))$. Then,

$(\mathcal{M}_i)^+ = (\max_j \max_i \underline{\mathcal{L}_{\delta c_j}^*}(\mathcal{M}_i), \min_j \min_i \overline{\mathcal{L}_{\delta c_j}^*}(\mathcal{M}_i))$. Then,

$$\begin{aligned} (\underline{\mathcal{L}_{\delta c_j}^*}(\mathcal{M}_i))^- &\leq \text{q-ROFS}_t\text{RHG}(\mathcal{L}_{c_1}(\mathcal{M}_1), \mathcal{L}_{c_2}(\mathcal{M}_2), \dots, \\ &\mathcal{L}_{c_n}(\mathcal{M}_m)) \leq (\underline{\mathcal{L}_{\delta c_j}^*}(\mathcal{M}_i))^+. \end{aligned} \quad (57)$$

(iii) (Monotonicity): let $\mathcal{P}_{c_j}(\mathcal{N}_i) = (\underline{\mathcal{P}_{c_j}}(\mathcal{N}_i), \overline{\mathcal{P}_{c_j}}(\mathcal{N}_i))$ ($i = 1, 2, \dots, m, j = 1, 2, \dots, n$) be another collection of q-ROFS_tRNs such that $\mathcal{P}_{c_j}(\mathcal{N}_i) \leq \underline{\mathcal{L}_{c_j}}(\mathcal{M}_i)$ and $\overline{\mathcal{P}_{c_j}}(\mathcal{N}_i) \leq \overline{\mathcal{L}_{c_j}}(\mathcal{M}_i)$. Then,

$$\begin{aligned} \text{q-ROFS}_t\text{RHG}(\mathcal{P}_{c_1}(\mathcal{M}_1), \mathcal{P}_{c_2}(\mathcal{M}_2), \dots, \mathcal{P}_{c_n}(\mathcal{M}_m)) \\ \leq \text{q-ROFS}_t\text{RHG}(\mathcal{L}_{c_1}(\mathcal{M}_1), \mathcal{L}_{c_2}(\mathcal{M}_2), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)). \end{aligned} \quad (58)$$

(iv) (Shift invariance): let $\mathcal{P}_c(\mathcal{N}) = (\underline{\mathcal{P}_c}(\mathcal{N}), \overline{\mathcal{P}_c}(\mathcal{N})) = ((\underline{b}, \underline{d}), (\overline{b}, \overline{d}))$ be any other q-ROFS_tRN. Then,

$$\begin{aligned}
& q - \text{ROFS}_t\text{RHG}(\mathcal{L}_{c_1}(\mathcal{M}_1) \oplus \mathcal{P}_c(\mathcal{N}), \mathcal{L}_{c_2}(\mathcal{M}_2) \\
& \oplus \mathcal{P}_c(\mathcal{N}), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m) \oplus \mathcal{P}_c(\mathcal{N})) = q - \text{ROFS}_t\text{RHG} \\
& (\mathcal{L}_{c_1}(\mathcal{M}_1), \mathcal{L}_{c_2}(\mathcal{M}_2), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)) \oplus \mathcal{P}_c(\mathcal{N}).
\end{aligned} \quad (59)$$

(v) (Homogeneity): for any real number $\lambda > 0$,

$$\begin{aligned}
& q - \text{ROFS}_t\text{RHG}(\lambda \mathcal{L}_{c_1}(\mathcal{M}_1), \lambda \mathcal{L}_{c_2}(\mathcal{M}_2), \dots, \\
& \lambda \mathcal{L}_{c_n}(\mathcal{M}_m)) = \lambda q - \text{ROFS}_t\text{RHG}(\mathcal{L}_{c_1}(\mathcal{M}_1), \mathcal{L}_{c_2}(\mathcal{M}_2), \dots, \mathcal{L}_{c_n}(\mathcal{M}_m)).
\end{aligned} \quad (60)$$

Remark 7

- (a) If $q = 1$, then $q - \text{ROFS}_t\text{RHG}$ operator reduces to IFS_tRHG operator
- (b) If the value of rung $q = 2$, then $q - \text{ROFS}_t\text{RHG}$ operator reduces to PFS_tRHG operator
- (c) If the soft parameter c_1 is one (means $n = 1$), then $q - \text{ROFS}_t\text{RHG}$ operator reduces to $q - \text{ROFRHG}$ operator
- (d) If $kl = ((1/n), (1/n), \dots, (1/n))^T$, then the proposed $q - \text{ROFS}_t\text{RHG}$ operator reduces to $q - \text{ROFS}_t\text{RWG}$ operator
- (e) If $tv = ((1/n), (1/n), \dots, (1/n))^T$, then the proposed $q - \text{ROFS}_t\text{RHG}$ operator reduces to $q - \text{ROFS}_t\text{ROWG}$ operator

6. MCDM Based on Soft Rough Aggregation Operator by Using q-ROF Information

MCDM has a high potential and discipline process to improve and evaluate multiple conflicting criteria in all areas of decision-making. In this competitive environment, an enterprise needs a more accurate and more repaid response to change customer needs. So, MCDM has the ability to handle successfully the evaluation process of multiple contradictory criteria. For an intelligent decision, experts analyze each and every character of an alternative and, then, they take the decision. Further, we will present the model for MCDM and their basic steps of construction by utilizing the proposed aggregation operators under $q - \text{ROF}$ soft rough information.

Suppose that $Y = \{s_1, s_2, s_3, \dots, s_p\}$ be the initial set of various alternatives and $\mathcal{E} = \{c_1, c_2, c_3, \dots, c_n\}$ be the set of n parameters. Consider $\mathcal{Z} = \{z_1, z_2, z_3, \dots, z_m\}$ as the set of m professional experts of this area who present their assessment expertise for each alternative s_k ($k = 1, 2, \dots, p$) corresponding to n parameters. Let the weight vectors $t = (t_1, t_2, \dots, t_m)^T$ of experts z_i and $v = (v_1, v_2, \dots, v_n)^T$ of parameters c_j with $\sum_{i=1}^m t_i = 1$, $\sum_{j=1}^n v_j = 1$, and $0 \leq t_i, v_j \leq 1$, respectively. The professional experts express their preference evaluation for alternative s_k with respect to parameter c_j in the form of $q - \text{ROFS}_t\text{RNs}$. The collective preference information given by the professionals is managed in $q - \text{ROFS}_t\text{R}$ decision matrix, which is $\mathbb{M} = [\mathcal{L}_{c_j}(\mathcal{M}_i)]_{n \times m}$

where $\mathcal{M} \subseteq \mathcal{E}$. Further, using the proposed models aggregate, the preferred choices of experts are to get the aggregated results $\mathcal{R}_k$ ($k = 1, \dots, p$) for each alternative s_k against their parameter c_j . Finally, they utilize the score function on the aggregated results $\mathcal{R}_k = [(\underline{\beta}, \underline{\gamma}), (\overline{\beta}, \overline{\gamma})]$ and rank all the results in a specific order to get the most desirable option. The stepwise decision algorithm for the investigated operators:

Step (i): the professional experts express their preference evaluation for alternative s_k with respect to parameter c_j in the form of $q - \text{ROFS}_t\text{RNs}$. Then, they collect the preference information given by the professionals and manage them in $q - \text{ROFS}_t\text{R}$ decision matrix, which is $\mathbb{M} = [\mathcal{L}_{c_j}(\mathcal{M}_i)]_{n \times m}$ where $\mathcal{M} \subseteq \mathcal{E}$.

Step (ii): apply the presented aggregation operators of each decision matrix $\mathbb{M} = [\mathcal{L}_{c_j}(\mathcal{M}_i)]_{n \times m}$ for each alternative s_k ($k = 1, 2, \dots, p$) against parameter c_j to get the aggregated results $\mathcal{R}_k = [(\underline{\beta}, \underline{\gamma}), (\overline{\beta}, \overline{\gamma})]$.

Step (iii): calculate the score value of aggregated results $\mathcal{R}_k = [(\underline{\beta}, \underline{\gamma}), (\overline{\beta}, \overline{\gamma})]$ for each object s_k .

Step (iv): rank the score value of $\mathcal{R}_k$ in a specific order to get the optimum option of professional experts.

7. Numerical Example

In this section, we will initiate an illustrative example to prove the quality and excellency of the developed operators. Let the Higher Education Commission (HEC) in Pakistan plans to introduce a selection board of four high potential and professional professors $\mathcal{Z} = \{z_1, z_2, z_3, z_4\}$ from home and abroad to select the most desirable applicant. Out of many applicants, three applicants $Y = \{s_1, s_2, s_3\}$ were called for interviews. The interview mainly judges the applicants against some parameters $\mathcal{M} = \{c_1 = \text{academic level}, c_2 = \text{development potential}, c_3 = \text{professional ethics}, c_4 = \text{research productivity}\} \subseteq \mathcal{E}$. Let the weight vector $t = (0.3, 0.28, 0.24, 0.18)^T$ for professional experts z_i ($i = 1, \dots, 4$) and $v = (0.32, 0.17, 0.31, 0.2)^T$ be the weight vectors for parameters c_j ($j = 1, 2, 3$), respectively. The professional experts express their preference evaluation for candidate s_k with respect to parameter c_j in the form of $q - \text{ROFS}_t\text{RNs}$. Finally, follow the following steps by utilizing the proposed models to select the most desirable and suitable applicant s_k .

7.1. Aggregation Results Rendered by the $q - \text{ROFS}_t\text{RWA}$ Method

Step (i): the professional experts express their preference evaluation for alternative s_k with respect to parameter c_j in the form of $q - \text{ROFS}_t\text{RNs}$. Then, they collect the preference information given by the professionals and manage them in $q - \text{ROFS}_t\text{R}$ decision matrix, which is $\mathbb{M} = [\mathcal{L}_{c_j}(\mathcal{M}_i)]_{n \times m}$ where $\mathcal{M} \subseteq \mathcal{E}$ which is given in Tables 7–9.

Step (ii): apply the presented $q - \text{ROFS}_t\text{RWA}$ aggregation operators on each decision matrix

TABLE 7: q-ROFS_tR matrix for candidate s_1 .

	c_1	c_2	c_3	c_4
z_1	[(0.7, 0.2), (0.8, 0.1)]	[(0.65, 0.25), (0.3, 0.6)]	[(0.82, 0.18), (0.6, 0.4)]	[(0.5, 0.2), (0.4, 0.1)]
z_2	[(0.6, 0.1), (0.5, 0.3)]	[(0.5, 0.1), (0.7, 0.15)]	[(0.3, 0.2), (0.2, 0.7)]	[(0.2, 0.3), (0.6, 0.2)]
z_3	[(0.4, 0.5), (0.6, 0.2)]	[(0.75, 0.2), (0.4, 0.1)]	[(0.65, 0.3), (0.7, 0.25)]	[(0.5, 0.4), (0.1, 0.5)]
z_4	[(0.5, 0.3), (0.3, 0.7)]	[(0.6, 0.4), (0.9, 0.1)]	[(0.78, 0.22), (0.45, 0.4)]	[(0.8, 0.1), (0.3, 0.1)]

TABLE 8: q-ROFS_tR matrix for candidate s_2 .

	c_1	c_2	c_3	c_4
z_1	[(0.6, 0.3), (0.9, 0.1)]	[(0.2, 0.4), (0.6, 0.1)]	[(0.5, 0.2), (0.9, 0.1)]	[(0.6, 0.2), (0.7, 0.2)]
z_2	[(0.4, 0.25), (0.3, 0.5)]	[(0.5, 0.15), (0.7, 0.3)]	[(0.77, 0.1), (0.6, 0.35)]	[(0.4, 0.3), (0.5, 0.1)]
z_3	[(0.3, 0.6), (0.65, 0.25)]	[(0.66, 0.2), (0.8, 0.17)]	[(0.8, 0.15), (0.55, 0.2)]	[(0.7, 0.1), (0.3, 0.6)]
z_4	[(0.5, 0.15), (0.55, 0.2)]	[(0.8, 0.1), (0.4, 0.5)]	[(0.62, 0.3), (0.9, 0.1)]	[(0.2, 0.4), (0.5, 0.3)]

TABLE 9: q-ROFS_tR matrix for candidate s_3 .

	c_1	c_2	c_3	c_4
z_1	[(0.8, 0.13), (0.8, 0.1)]	[(0.5, 0.2), (0.6, 0.1)]	[(0.4, 0.1), (0.7, 0.2)]	[(0.7, 0.1), (0.6, 0.3)]
z_2	[(0.5, 0.16), (0.4, 0.2)]	[(0.8, 0.12), (0.3, 0.4)]	[(0.6, 0.2), (0.4, 0.3)]	[(0.5, 0.2), (0.4, 0.1)]
z_3	[(0.4, 0.5), (0.7, 0.3)]	[(0.5, 0.4), (0.5, 0.12)]	[(0.2, 0.4), (0.8, 0.14)]	[(0.3, 0.4), (0.2, 0.5)]
z_4	[(0.3, 0.2), (0.5, 0.15)]	[(0.6, 0.25), (0.7, 0.3)]	[(0.8, 0.18), (0.75, 0.1)]	[(0.6, 0.2), (0.8, 0.1)]

$\mathbb{M} = [\mathcal{L}_{c_j}(\mathcal{M}_i)]_{n \times m}$ for each alternative s_k ($k = 1, 2, 3$) against parameter c_j to get the aggregated results $\mathcal{R}_k = [(\underline{\beta}, \underline{\vartheta}), (\overline{\beta}, \overline{\vartheta})]$; that is,

$$\begin{aligned}\mathcal{R}_1 &= [(0.640506, 0.218382), (0.600249, 0.254431)], \\ \mathcal{R}_2 &= [(0.607809, 0.217229), (0.730645, 0.202415)], \\ \mathcal{R}_3 &= [(0.606198, 0.198551), (0.649417, 0.183108)].\end{aligned}\quad (61)$$

Step (iii): calculate the score value of aggregated results $\mathcal{R}_k = [(\underline{\beta}, \underline{\vartheta}), (\overline{\beta}, \overline{\vartheta})]$ for each object s_k ; that is,

$$\begin{aligned}SC(\mathcal{R}_1) &= 0.226074, \\ SC(\mathcal{R}_2) &= 0.298025, \\ SC(\mathcal{R}_3) &= 0.241341.\end{aligned}\quad (62)$$

Step (iv): rank the score value of $\mathcal{R}_k$ in a specific order to get the optimum option of professional experts; that is,

$$SC(\mathcal{R}_2) > SC(\mathcal{R}_3) > SC(\mathcal{R}_1). \quad (63)$$

Therefore, from the above analysis, it is observed that s_2 is a more suitable and desirable candidate against the given position.

7.2. Aggregation Results Rendered by the q-ROFS_tRWG Method

Step (i): similar to above.

Step (ii): apply the presented q-ROFS_tRWG aggregation operators on each decision matrix

$\mathbb{M} = [\mathcal{L}_{c_j}(\mathcal{M}_i)]_{n \times m}$ for each alternative s_k ($k = 1, 2, 3$) against parameter c_j to get the aggregated results $\mathcal{R}_k = [(\underline{\beta}, \underline{\vartheta}), (\overline{\beta}, \overline{\vartheta})]$; that is,

$$\begin{aligned}\mathcal{R}_1 &= [(0.540892, 0.29117), (0.446263, 0.448329)], \\ \mathcal{R}_2 &= [(0.50166, 0.321735), (0.598613, 0.331553)], \\ \mathcal{R}_3 &= [(0.491017, 0.291113), (0.545527, 0.264822)].\end{aligned}\quad (64)$$

Step (iii): calculate the score value of the proposed q-ROFS_tRWG aggregated results $\mathcal{R}_k = [(\underline{\beta}, \underline{\vartheta}), (\overline{\beta}, \overline{\vartheta})]$ for each object s_k ; that is,

$$\begin{aligned}SC(\mathcal{R}_1) &= 0.06616, \\ SC(\mathcal{R}_2) &= 0.135502, \\ SC(\mathcal{R}_3) &= 0.118744.\end{aligned}\quad (65)$$

Step (iv): rank the score value of $\mathcal{R}_k$ in a specific order to get the optimum option of professional experts; that is,

$$SC(\mathcal{R}_2) > SC(\mathcal{R}_3) > SC(\mathcal{R}_1). \quad (66)$$

Therefore, from the above analysis, it is observed that s_2 is a more suitable and desirable candidate against the given position.

7.3. Aggregation Results Rendered by the q-ROFS_tROWA Method

Step (i): similar to above.

Step (ii):

$$\begin{aligned}\mathcal{R}_1 &= [(0.657933, 0.217542), (0.621125, 0.234448)], \\ \mathcal{R}_2 &= [(0.612743, 0.216413), (0.744755, 0.193923)], \\ \mathcal{R}_3 &= [(0.621268, 0.203392), (0.677814, 0.180796)].\end{aligned}\quad (67)$$

Step (iii): $SC(\mathcal{R}_1) = 0.250625$, $SC(\mathcal{R}_2) = 0.312857$, $SC(\mathcal{R}_3) = 0.26844$.

Step (iv): $SC(\mathcal{R}_2) > SC(\mathcal{R}_3) > SC(\mathcal{R}_1)$.

Therefore, from the above analysis, it is observed that s_2 is a more suitable and desirable candidate against the given position.

7.4. Aggregation Results Rendered by the q -ROFS_tROWG Method

Step (i): similar to above.

Step (ii):

$$\begin{aligned}\mathcal{R}_1 &= [(0.562833, 0.293072), (0.473363, 0.418793)], \\ \mathcal{R}_2 &= [(0.517595, 0.319441), (0.614971, 0.321818)], \\ \mathcal{R}_3 &= [(0.49826, 0.296096), (0.583543, 0.259781)].\end{aligned}\quad (68)$$

Step (iii): $SC(\mathcal{R}_1) = 0.09287$, $SC(\mathcal{R}_2) = 0.152658$, $SC(\mathcal{R}_3) = 0.139459$.

Step (iv): $SC(\mathcal{R}_2) > SC(\mathcal{R}_3) > SC(\mathcal{R}_1) \Rightarrow$

Therefore, from the above analysis, it is observed that s_2 is a more suitable and desirable candidate against the given position.

7.5. Aggregation Results Rendered by the q -ROFS_tRHA Method

Step (i): similar to above.

Step (ii): apply the presented q -ROFS_tRHA aggregation operators on each decision matrix $\mathbb{M} = [\mathcal{L}_{\delta c_j}^*(\mathcal{M}_i)]_{n \times m}$ for each alternative s_k ($k = 1, 2, 3$) against parameter c_j to get the aggregated results $\mathcal{R}_k = [(\underline{\beta}_\delta^*, \underline{\vartheta}_\delta^*), (\overline{\beta}_\delta^*, \overline{\vartheta}_\delta^*)]$, with $k = (0.25, 0.29, 0.3, 0.16)^T$ and $l = (0.27, 0.23, 0.32, 0.18)^T$ being the weight vectors of experts s_i and parameters c_j . Then, the aggregated result for $\mathcal{L}_{\delta c_j}^*(\mathcal{M}_i)$ is given as

$$\begin{aligned}\mathcal{R}_1 &= [(0.432025, 0.669215), (0.408108, 0.691652)], \\ \mathcal{R}_2 &= [(0.417457, 0.662671), (0.503394, 0.6509)], \\ \mathcal{R}_3 &= [(0.392067, 0.655437), (0.442525, 0.635251)].\end{aligned}\quad (69)$$

Step (iii): $SC(\mathcal{R}_1) = -0.24099$, $SC(\mathcal{R}_2) = -0.18323$, $SC(\mathcal{R}_3) = -0.1955$.

Step (iv): $SC(\mathcal{R}_2) > SC(\mathcal{R}_3) > SC(\mathcal{R}_1)$.

Therefore, from the above analysis, it is observed that s_2 is a more suitable and desirable candidate against the given position.

7.6. Aggregation Results Rendered by the q -ROFS_tRHG Method

Step (i): similar to above.

Step (ii): apply the presented q -ROFS_tRHG aggregation operators on each decision matrix $\mathbb{M} = [\mathcal{L}_{\delta c_j}^*(\mathcal{M}_i)]_{n \times m}$ for each alternative s_k ($k = 1, 2, 3$) against parameter c_j to get the aggregated results $\mathcal{R}_k = [(\underline{\beta}_\delta^*, \underline{\vartheta}_\delta^*), (\overline{\beta}_\delta^*, \overline{\vartheta}_\delta^*)]$, with $k = (0.25, 0.29, 0.3, 0.16)^T$ and $l = (0.27, 0.23, 0.32, 0.18)^T$ being the weight vectors of experts s_i and parameters c_j . Then, the aggregated result for $\mathcal{L}_{\delta c_j}^*(\mathcal{M}_i)$ is given as

$$\begin{aligned}\mathcal{R}_1 &= [(0.365745, 0.709626), (0.308693, 0.751049)], \\ \mathcal{R}_2 &= [(0.332518, 0.713922), (0.407739, 0.706165)], \\ \mathcal{R}_3 &= [(0.318639, 0.693983), (0.36813, 0.673448)].\end{aligned}\quad (70)$$

Step (iii): $SC(\mathcal{R}_1) = -0.35133$, $SC(\mathcal{R}_2) = -0.30573$, $SC(\mathcal{R}_3) = -0.27871$.

Step (iv): $SC(\mathcal{R}_3) > SC(\mathcal{R}_2) > SC(\mathcal{R}_1)$.

Therefore, from the above analysis, it is observed that s_3 is a more suitable and desirable candidate against the given position. The higher the score value, the more optimist that value and the smaller the score value, the more pessimist that value. From the ranking results of the above proposed operators, it is clear that averaging operators' results are more optimist than geometric operators.

7.7. Comparative Study. To present the applicability and efficiency of the developed approach with some other existing methods based on IFS, PFS, and q -ROFS methods by using the same illustrative example. A comparative study has been made based on different aggregation operators (see [4, 5, 23, 27, 28, 54, 60, 61]). For this purpose, different parameters of the above numerical example are aggregated by utilizing the proposed aggregation operators having weight vectors $v = (0.32, 0.17, 0.31, 0.2)^T$, and their collective aggregated decision matrix for each candidate s_i ($i = 1, 2, 3$) is given in Table 10. Now, by using the information of the evaluated matrix, a comparative study of the investigated models with some existing aggregation operators is presented in Table 11. From Table 11, it is observed that the methods presented in [4, 5, 23, 27, 28, 54, 60, 61] are only capable of solving the q -ROFV of the form (β, ϑ) but are not capable of solving the q -ROFS_tRV of the form $[(\underline{\beta}, \underline{\vartheta}), (\overline{\beta}, \overline{\vartheta})]$. Thus, from the existing methods, it is clear that these existing methods have a lack of rough information and they are not capable of solving and ranking the given illustrative example. Therefore, from this

TABLE 10: Aggregated matrix for candidate.

	s_1	s_2	s_3
z_1	[(0.719, 0.201), (0.651, 0.208)]	[(0.538, 0.256), (0.849, 0.115)]	[(0.670, 0.122), (0.713, 0.154)]
z_2	[(0.471, 0.154), (0.534, 0.320)]	[(0.602, 0.179), (0.550, 0.298)]	[(0.616, 0.171), (0.387, 0.222)]
z_3	[(0.598, 0.350), (0.582, 0.229)]	[(0.678, 0.226), (0.629, 0.260)]	[(0.370, 0.430), (0.682, 0.225)]
z_4	[(0.703, 0.230), (0.616, 0.287)]	[(0.605, 0.211), (0.733, 0.205)]	[(0.650, 0.201), (0.703, 0.137)]

TABLE 11: Comparative analysis of different methods.

Methods	Score values			Ranking
	$\mathcal{R}_1$	$\mathcal{R}_2$	$\mathcal{R}_3$	
IFWA [4]	Unapproachable			$\times$
IFS _i WA [54]	Unapproachable			$\times$
PFS _i WA [60]	Unapproachable			$\times$
q-ROFWA [23]	Unapproachable			$\times$
q-ROFHOWAGA [61]	Unapproachable			$\times$
q-ROFS _i WA [60]	Unapproachable			$\times$
Cq-ROFWA [27]	Unapproachable			$\times$
q-RONFWA [28]	Unapproachable			$\times$
q-ROFS _i RWA (proposed)	0.226074, 0.298025, 0.241341			$\mathcal{R}_2 \succ \mathcal{R}_3 \succ \mathcal{R}_1$
q-ROFS _i ROWA (proposed)	0.250625, 0.312857, 0.26844			$\mathcal{R}_2 \succ \mathcal{R}_3 \succ \mathcal{R}_1$
q-ROFS _i RHA (proposed)	-0.24099, -0.18323, -0.1955			$\mathcal{R}_2 \succ \mathcal{R}_3 \succ \mathcal{R}_1$
IFWG [5]	Unapproachable			$\times$
IFS _i WG [54]	Unapproachable			$\times$
q-ROFWG [23]	Unapproachable			$\times$
Cq-ROFWG [27]	Unapproachable			$\times$
q-RONFWG [28]	Unapproachable			$\times$
q-ROFS _i RWG (proposed)	0.06616, 0.135502, 0.118744			$\mathcal{R}_2 \succ \mathcal{R}_3 \succ \mathcal{R}_1$
q-ROFS _i ROWG (proposed)	0.09287, 0.152658, 0.139459			$\mathcal{R}_2 \succ \mathcal{R}_3 \succ \mathcal{R}_1$
q-ROFS _i RHG (proposed)	-0.35133, -0.30573, -0.27871			$\mathcal{R}_3 \succ \mathcal{R}_2 \succ \mathcal{R}_1$

analysis, it is clear that the developed methods are more superior and capable than the existing methods.

8. Conclusion

MCDM has a high potential and discipline process to improve and evaluate multiple conflicting criteria in all areas of decision-making. For an intelligent decision, the experts analyze each and every character of an alternative and then they take the decision. For an intelligent and successful decision, the experts require a careful preparation and analysis of each and every character for an alternative and then they can take a good decision if they are armed with all the data and information that they need. The dominant notions of fuzzy sets, S_i Ss, and rough sets generalized the classical set theory to cope with uncertain information. Molodtsov investigated the pioneer notion of S_i S which is free from the inherent complexity which the contemporary theories faced. It is observed that S_i S has too close relation with fuzzy sets and rough sets. The S_i S theory is an effective

mathematical tool for handling the uncertain, ambiguous, and imprecise data. The aim of our work is to investigate the hybrid concept of S_i S and rough set with the notion of q-ROFS to obtain the new notion of q-ROFS_iRS. In addition, some averaging aggregation operators such as q-ROFS_iRWA, q-ROFS_iROWA, and q-ROFS_iRHA are presented. Then, important properties of these investigated averaging operators are given in detail. Moreover, we investigated the geometric aggregation operators such as q-ROFS_iRWG, q-ROFS_iROWG, and q-ROFS_iRHG and proposed the basic desirable characteristics of investigated geometric operators. The technique for MCDM and stepwise algorithm for decision-making by utilizing the proposed approaches are demonstrated clearly. Finally, a numerical example for the developed approach is presented and a comparative study of the investigated models with some existing methods is brought to light in detail which shows that the proposed models are more effective and superior than existing approaches.

8.1. Future Work. In the future, we intend to further discuss the following topics:

- The investigation of q-ROF-entropy of soft rough sets
- The investigation of the picture and spherical fuzzy information by using soft rough sets
- Applying other decision-making methodology based q-ROFS_iRS to solve the MCDM problem
- The discussions of other applied methods in information systems

Abbreviations

IFS:	Intuitionistic fuzzy set
PFS:	Pythagorean fuzzy set
q-ROFS:	q-Rung orthopair fuzzy set
S_i S:	Soft set
q-ROFS _i RS:	q-ROF soft rough set
q-ROFS _i RWA:	q-ROF soft rough weighted averaging
q-ROFS _i ROWA:	q-ROF soft rough ordered weighted averaging
q-ROFS _i RHA:	q-ROF soft rough hybrid averaging
q-ROFS _i RWG:	q-ROF soft rough weighted geometric
q-ROFS _i ROWG:	q-ROF soft rough ordered weighted geometric
q-ROFS _i RHG:	q-ROF soft rough hybrid geometric
MCDM:	Multicriteria decision-making
MemG:	Membership grade
NMemG:	Nonmembership grade

IFWA:	IF weighted averaging
IFOWA:	IF ordered weighted averaging
IFHA:	IF hybrid averaging
IFWG:	IF weighted geometric
IFOWG:	IF ordered weighted geometric
IFHG:	IF hybrid geometric
q-ROFWA:	q-ROF weighted averaging
q-ROFWG:	q-ROF weighted geometric
IFS_iS :	IF soft set
IFS_iWA :	IF soft weighted averaging
IFS_iWG :	IF soft weighted geometric
$q-ROFS_iS$:	q-Rung orthopair fuzzy soft set
$q-ROFS_iWA$:	q-ROF soft weighted averaging
$q-ROFS_iOWA$:	q-ROF soft ordered weighted averaging
$q-ROFS_iHA$:	q-ROF soft hybrid averaging
$q-ROFS_iN$:	q-ROF soft number
$q-ROFS_iRN$:	q-ROF soft rough number.

Data Availability

The data used in this article are artificial and hypothetical, and anyone can use these data before prior permission by just citing this article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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