Research Article

An Analytical Model for Predicting Ground Pressure under a Rigid-Flexible Tracked Vehicle on Soft Ground

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Ground pressure is a significant parameter for the mobility, tractive performance, and soil compaction. In this study, an analytical model for predicting ground pressure distribution under a rigid-flexible tracked vehicle on soft ground was developed. The model considered the primary design parameters of the tracked vehicle, soil characteristics, and soil shear. The ground pressure was not uniform, and its maximum values under the roadwheels were 90.20, 103.57, and 150.14 kPa. The ground pressure was inversely proportional to the ratio of the lengths of the flexible track and rigid grouser. An experiment was conducted to verify the analytical model. The maximum error between the measured and simulated results was smaller than 8%, thereby verifying the analytical model.

1. Introduction

Tracked vehicles have become increasingly popular in various complex terrain and special ground conditions, such as snow land, desert, swamps, jungles, mountains, and lakes, because of their high mobility. Ground pressure, which is the stress distribution in the context of the soil-track interface, is an important parameter for mobility, tractive performance, and soil compaction. Nominal ground pressure, which is obtained by dividing the vehicle weight by the soil-track contact areas, represents the ground pressure under the track on the basis of the uniform distribution. However, ground pressure is not uniform, especially in cases in which the track is flexible. In the past several decades, ground pressure has been investigated by many researchers and is found to be complex due to various factors, such as the number of roadwheels, the diameter of roadwheels, track tension, and soil characteristics. Bekker [1] proposed the pressure-sinkage formula by fitting large amounts of experimental

data. The vertical deformation of soil under load was assumed similar to soil deformation under a flat plate. The ground pressure under a flat plate was derived using the following equation:

$$p = \left(\frac{k_c}{b} + k_\phi \right) z^n, \quad (1)$$

where p is the ground pressure, b is the track width, z is the sinkage, and k_c , k_ϕ , and n are the parameters derived from the plate sinkage tests.

Mohr's theory states that materials will fail from a critical combination of normal and shearing stresses. Waldron [2] demonstrated that the Mohr theory was suitable for soil. The relationship between normal and shear stresses in the sliding surface can be described by Mohr's envelope. The envelope is given by the Coulomb formula:

$$\tau = c + p \tan \phi, \quad (2)$$

where τ is the shear stress, c is the soil cohesion, and φ is the internal friction angle.

When the track shoe cuts the soil, the soil becomes subjected to compression. As the ground pressure acts on the soil surface, the soil in this area can be considered in a Rankine passive state [3]:

$$\sigma_{pn} = \gamma_s z K_{py} + p K_{pq} + c K_{pc}, \quad (3)$$

where σ_{pn} is the ground pressure acting on the vertical surface of a grouser, γ_s is the unit weight of soil, and K_{py} , K_{pq} , and K_{pc} are coefficients related to soil strength.

Keller et al. [4–6] obtained the vertical and lateral soil stress pressures under rubber-tracked agricultural machinery via experimental data fitting. The prediction formula considered the vehicle properties, such as the number and diameter of roadwheels, but disregarded the soil characteristics. Moreover, the longitudinal (i.e., the driving direction) and the lateral (i.e., perpendicular to the driving direction) ground pressures were used as the upper stress boundary conditions to simulate the soil stresses. Zhao et al. [7] obtained the longitudinal ground pressure via experimental data fitting under the assumption that the ground pressure under the roadwheels is a quadratic cosine function and others are linear functions considering the number of roadwheels and the ground contact length. They also determined the lateral ground pressure under the assumption that the pressure is a linear function. Sun et al. [8] obtained the sinkage through the experimental data fitting. Then, the corrected model of normal and shear stresses under the flexible track was obtained using the model of Wong.

Garber and Wong [9, 10] developed an analytical method for predicting ground pressure under a tracked vehicle in a static condition by considering the vehicle parameters and soil characteristics. They analyzed the effects of initial track tension, track tension, number of roadwheels, diameter of roadwheels, suspension stiffness, and soil characteristics on the ground pressure distribution through an analytical method. Park et al. [11] introduced a mathematical model for predicting soil normal and shear stresses under a flexible tracked vehicle by considering not only the vehicle parameters and soil characteristics but also the repetitive loading. They used a traction test to verify the mathematical model. Li et al. [12] proposed an analytical method for predicting ground pressure under the rigid grouser by utilizing force balance. Yang et al. [13] corrected the model of Li by considering the soil shear and obtained the soil shear rate through the finite element method. They adopted traction experiments to verify the analytical model.

Wong and Huang [14, 15] investigated the relationship of ground pressure with initial track tension by using advanced computer simulation models, namely, TPPVTM (Tractive Performance Prediction Model for Tracked Vehicle) and NTVPM (Nepean Tracked Vehicle Performance Model). Meywerk et al. [16] adopted a coupled model of

multibody system-finite element method to obtain real-time soil deformation.

The abovementioned research methods of ground pressure are focused on fitting, mathematical analysis, and dynamic analysis, and the research objective is a flexible or rigid track. However, ground pressure has not yet been analyzed under a coupled rigid-flexible track. The rigid-flexible track system is widely used in all-terrain tracked vehicles. The objectives of current study are as follows:

- (i) To develop a mathematical model for predicting ground pressure along the longitudinal direction by using the primary design parameters of the tracked vehicle, as well as the soil characteristics and soil shear, and assuming a uniform ground pressure along the lateral direction
- (ii) To conduct an experiment that verifies and validates the theoretical model
- (iii) To analyze the influence of the lengths of the flexible track and rigid grouser on the ground pressure

2. Analytical Model

Flexible track is usually called a rubber track. A rigid track is connected by metal track shoes, and the soil can only have a convex deformation. The studied track system is different from the traditional flexible and rigid tracks; it is a type of rigid-flexible coupled track that consists of a rubber track and metal grousers. This track system can obtain a smaller ground pressure than the rigid track and a longer fatigue life than the flexible track, as shown in Figure 1. When a tracked vehicle drives on soft soil, the ground pressure distribution beneath the track depends on the soil-track interaction, as shown in Figure 2. The pressure-sinkage relationship (equation (1)), Coulomb formula (equation (2)), and Rankine passive state (equation (3)) are considered. Finite element analysis is used to analyze the entire track system. The track system in the context of its contact with the soil is divided into the track segment in contact with a roadwheel and the track segment between the roadwheels.

2.1. Analysis of the Track Segment in Contact with a Roadwheel. The shape of the track segment in contact with a roadwheel is dependent on the roadwheel diameter. Consider an arbitrary element from the track segment in contact with a roadwheel with the number i , the radius r_i , the central angle $d\theta$, and the length of the track segment $dl = r_i d\theta$, as shown in Figure 3. The force and torque equilibrium are represented by equations (4)–(6), where $T_{\alpha i}$ and $T_{\beta i}$ represent the tensions at the entry and exit angles, respectively, α_i and β_i represent the entry and exit angles of the i th roadwheel, respectively, W_i represents the vertical load of the i th roadwheel, and h represents the width of the flexible track.

FIGURE 1: Rigid-flexible coupled track.

FIGURE 2: Configuration of a track system in contact with soft ground.

FIGURE 3: Forces acting on the roadwheel in contact with a track segment.

$$T_{\beta i} \cos(\alpha_i + \beta_i) + W_i \sin \alpha_i = T_{\alpha i} + h \int_0^{r_i(\alpha_i + \beta_i)} p(l) \sin\left(\alpha_i + \frac{l}{r_i}\right) dl + h \int_0^{r_i(\alpha_i + \beta_i)} [c + p(l) \tan \varphi] \cos\left(\alpha_i + \frac{l}{r_i}\right) dl, \quad (4)$$

$$T_{\beta i} \sin(\alpha_i + \beta_i) + W_i \cos \alpha_i = h \int_0^{r_i(\alpha_i + \beta_i)} p(l) \cos\left(\alpha_i + \frac{l}{r_i}\right) dl + h \int_0^{r_i(\alpha_i + \beta_i)} [c + p(l) \tan \varphi] \sin\left(\alpha_i + \frac{l}{r_i}\right) dl, \quad (5)$$

$$T_{\beta i} r_i = h r_i \int_0^{r_i(\alpha_i + \beta_i)} [c + p(l) \tan \varphi] dl + T_{\alpha i} r_i, \quad (6)$$

For the first roadwheel ($i = 1$), $T_{\alpha 1} = T_0$ (equation (7)) and $\alpha_i = \alpha_0$ (the approach angle of the track system). The ground pressure $p(l)$ and the tension at the exit angle $T_{\beta i}$ can be calculated on the basis of equations (4)–(7) and the primary design factors, such as the vehicle weight, the track width, the number and diameter of roadwheels and

sprocket, and the torque acting on the sprocket. Then, the vertical load of the first roadwheel W_1 and shear stress $\tau(l)$ can be obtained.

For the last roadwheel ($i = 3$), $T_{\beta i} = T$ (equation (7) [17]) and $\beta_i = \beta$ (the departure angle of the track system). The ground pressure $p(l)$ and the tension at the entry

FIGURE 4: Force acting on the track segment between roadwheels.

angle $T_{\alpha i}$ can be calculated on the basis of equations (4–7) and the primary design factors. Then, the vertical weight of the last roadwheel W_3 and shear stress $\tau(l)$ can be obtained.

$$\begin{aligned} T_0 &= \left(\frac{3}{4}W - \frac{M}{r_s} \right), \\ T &= \left(\frac{3}{4}W + \frac{M}{r_s} \right), \end{aligned} \quad (7)$$

where M is the torque acting on the sprocket, T_0 is the tension at the approach angle of the track system, T is the tension at the departure angle of the track system, W is the vehicle weight, and r_s is the diameter of the sprocket.

2.2. Analysis of the Track Segment between the Roadwheels.

The shape of the track segment between the roadwheels is dependent on the interaction between the soil and track. One rigid grouser and one adjacent flexible track element Δl are used as the research objects, as shown in Figure 4. The force equilibriums are represented by equations (8) and (9):

$$\begin{aligned} T_{\beta_{i,j+1}} \cos \Delta\beta_{i,j} &= T_{\beta_{i,j}} + F_{2i,j} + F_{5i,j} + F_{3i,j} \sin \Delta\beta_{i,j} \\ &\quad + F_{4i,j} \cos \Delta\beta_{i,j}, \end{aligned} \quad (8)$$

$$T_{\beta_{i,j+1}} \sin \Delta\beta_{i,j} = F_{1i,j} + F_{3i,j} \cos \Delta\beta_{i,j} + F_{4i,j} \sin \Delta\beta_{i,j}, \quad (9)$$

where $T_{\beta_{i,j}}$ and $T_{\beta_{i,j+1}}$ represent the tensions of the j th and $j+1$ th elements in the i th segment, respectively. $\beta_{i,j}$ represents the angle between the rigid grouser and horizontal direction. $\Delta\beta_{i,j}$ represents the angle between the rigid grouser and flexible track element. $F_{1i,j}$ is the normal force

under the rigid grouser (equation (10)). $F_{2i,j}$ is the tangential force under the rigid grouser (equation (11)). $F_{3i,j}$ is the normal force under the j th flexible track element (equation (12)). $F_{4i,j}$ is the tangential force under the j th flexible track element (equation (13)). $F_{5i,j}$ is the force acting on the vertical surface of the rigid grouser (equation (14)). For the first element ($j=1$), $T_{\beta_{i,j}} = T_{\beta_i}$ and $\beta_{i,j} = \beta_i$. For the last element, $T_{\beta_{i,j}} = T_{\alpha_{i+1}}$ and $\beta_{i,j} = \alpha_{i+1}$. Hence, the ground pressure under the track between the roadwheels can be calculated.

$$F_{1i,j} = abp_{1i,j}, \quad (10)$$

where a represents the length of the rigid grouser.

$$F_{2i,j} = abp_{2i,j} = ab[c + p_{1i,j} \tan \varphi], \quad (11)$$

$$F_{3i,j} = h \int p_{3i,j}(l) dl, \quad (12)$$

$$F_{4i,j} = h \int [c + p_{3i,j}(l) \tan \varphi] dl, \quad (13)$$

$$\begin{aligned} F_{5i,j} &= b \int_0^d \frac{(\gamma_s z K_{py} + c K_{pc}) dz + c db \sin \beta_{i,j}}{\cos \beta_{i,j} + b \tan \varphi \sin \beta_{i,j}} \\ &\quad \cdot \int_0^d (\gamma_s z K_{py} + c K_{pc}) dz, \end{aligned} \quad (14)$$

where d represents the height of the grouser.

For a rigid-flexible tracked vehicle with the number of roadwheels N , the number of rigid grouser R , and the number of flexible tracks Q , when the track segments are in equilibrium conditions, the vertical and horizontal forces of the entire system are satisfied (equations (15)–(16)). Finally, the entire track system can be repeatedly analyzed via equations (1–16).

TABLE 1: Input data for the analytical model.

Parameters	Values	Units
Soil cohesive modulus (k_c)	10.21	kN/mm ⁿ⁺¹
Soil frictional modulus (k_ψ)	300.52	kN/mm ⁿ⁺²
Sinkage exponent (n)	0.53	—
Soil cohesion (c)	2.10×10^{-3}	Pa
Soil internal friction angle (ψ)	10	°
Unit weight of soil (γ_s)	7.8×10^4	kN/m ³
Weight of the track vehicle (W)	49	kN
Torque acting on the sprocket (M)	7.8	kN·m
Approach angle of the track system (α_0)	11	°
Departure angle of the track system (β)	9	°
Radius of the roadwheels (r_1, r_2, r_3)	298	mm
Radius of the sprocket (r_s)	220	mm
The grouser width (b)	440	mm
Length of the grouser (a)	74	mm
Height of the grouser (d)	55	mm
Length of the flexible track (l)	90	mm
Flexible track width (h)	600	mm
Number of roadwheel (N)	3	—
Number of rigid grouser (R)	15	—
Number of flexible track (Q)	14	—

$$\begin{aligned}
T \cos \beta_N &= T_0 \cos \alpha_1 + \sum_1^N \left(h \int_0^{r_i(\alpha_i + \beta_i)} [c + p(l) \tan \varphi] \cos \left(\alpha_i + \frac{l}{r_i} \right) dl \right) + ab \sum_1^R \sum_1^N p_{1ij} \sin(\beta_i - \Delta\beta_{ij}) \\
&+ ab \sum_1^R \sum_1^N p_{1ij} \sin(\beta_i - \Delta\beta_{ij}) + ab \sum_1^R \sum_1^N [c + p_{1ij} \tan \varphi] \cos(\beta_i - \Delta\beta_{ij}) + h \sum_1^Q \int_0^l p_{3ij}(l) \sin(\beta_i - \Delta\beta_{ij}) dl \quad (15) \\
&+ h \sum_1^Q \int_0^l (c + p_{3ij}(l) \tan \varphi) \cos(\beta_i - \Delta\beta_{ij}) dl + b \sum_1^R \int_0^d (\gamma_s z K_{py} + c K_{pc}) \sin(\beta_i - \Delta\beta_{ij}) dz, \\
T_0 \sin \alpha_1 + T \sin \beta_N &= \sum_1^N \left(h \int_0^{r_i(\alpha_i + \beta_i)} p(l) \cos \left(\alpha_i + \frac{l}{r_i} \right) dl \right) + ab \sum_1^R \sum_1^N p_{1ij} \cos(\beta_i - \Delta\beta_{ij}) + ab \sum_1^R \sum_1^N p_{1ij} \cos(\beta_i - \Delta\beta_{ij}) \\
&+ ab \sum_1^R \sum_1^N (c + p_{1ij} \tan \varphi) \sin(\beta_i - \Delta\beta_{ij}) + h \sum_1^Q \int_0^l p_{3ij}(l) \cos(\beta_i - \Delta\beta_{ij}) dl \\
&+ h \sum_1^Q \int_0^l (c + p_{3ij}(l) \tan \varphi) \sin(\beta_i - \Delta\beta_{ij}) dl + b \sum_1^R \int_0^d (\gamma_s z K_{py} + c K_{pc}) \cos(\beta_i - \Delta\beta_{ij}) dz + W. \quad (16)
\end{aligned}$$

With the front car as an example, on the basis of the analytical model for predicting ground pressure under a rigid-flexible tracked vehicle on soft ground and the input data (Table 1), the ground pressure (Figure 5) can be calculated. Moreover, the soil and vehicle parameters were obtained from Harbin First Machinery Building Group Ltd.

3. Measurements of Ground Pressure

The analytical model was verified through the ground pressure test, which was conducted at the test site of Harbin First Machinery Building Group Ltd. The test vehicle with a total weight of 148.90 kN was driven at the speed of 10 km/h. To measure the ground pressure distribution below the roadwheels, two stress sensors (diameter, 16 mm; height, 6 mm; measurement range, 0–200 kPa) were buried in the

topsoil at a depth of 100 mm. They were placed under the center line of the track (Figure 6). Figure 7 shows the comparison between the simulated and measured ground pressures.

4. Discussion

The calculation error between the simulation and measure was smaller than 10 kPa, and the maximum error was smaller than 8%. On this basis, the reasonability of the analytical model was verified and the results obtained by the simulation could be used in future research.

However, slight differences existed between the measured and simulated results and might be caused by the following reasons: first, the soil characteristics between the test and simulation differed; second, the measured results

FIGURE 5: Ground pressure distribution under the track.

FIGURE 6: Test site of ground pressure.

FIGURE 7: Simulated and measured results.

were almost larger than the simulated results, probably because the soil stiffness increased when the first track acted on the soil. Finally, the test process contained measurement errors that could not be avoided.

FIGURE 8: Comparison of different l/a values.

TABLE 2: Comparison of ground pressure under roadwheels.

l/a	Roadwheel 1 (kPa)	Roadwheel 2 (kPa)	Roadwheel 3 (kPa)
1	125.97	140.07	203.62
1.2	90.20	103.57	150.14
1.5	69.11	77.04	109.30

Figure 5 shows the ground pressure under the rigid-flexible track along the longitudinal direction. The following conclusions can be drawn from the analytical results:

- (i) The ground pressure is not uniform, and the ground pressure under the roadwheels is larger than that under the track between roadwheels, which is almost zero.
- (ii) The nominal ground pressure of the tracked vehicle, which is obtained by dividing the vehicle weight by the soil-track contact areas, is 19.90 kPa. The ground pressures under the roadwheels, which are several times more than the nominal ground pressure, are 90.20, 103.57, and 150.14 kPa. Therefore, ground pressure is a better representation for mobility, tractive performance, and soil compaction than nominal ground pressure.
- (iii) The influence of the lengths of the rigid grouser and flexible track on the ground pressure is analyzed. The original lengths of the flexible track and grouser are 90 mm and 74 mm, respectively ($l = 90$ mm, $a = 74$ mm, and $l/a \approx 1.2$). When increasing the length of the flexible track to 110 mm ($l/a \approx 1.5$) and maintaining the total length of the track constant, the ground pressure increases. When increasing the length of the grouser to 90 mm ($l/a \approx 1$) and maintaining the total length of the track constant, the ground pressure decreases (Figure 8). Table 2 presents the comparison of ground pressure under the roadwheels.

5. Conclusion

An analytical model for predicting ground pressure under a rigid-flexible tracked vehicle on soft ground was developed. The ground pressure distribution along the longitudinal direction was obtained, by considering the primary design parameters of the tracked vehicle and the soil characteristics.

The analytical model showed that the ground pressure under the roadwheels was considerably larger than that under the track between roadwheels. The maximum ground pressures under the roadwheels were 90.20, 103.57, and 150.14 kPa. The ground pressure was inversely proportional to l/a . When l/a was large, the ground pressure was small and good mobility and slight soil compaction were obtained. However, when l/a was small, the fatigue life was long.

The predicted ground pressure was relatively consistent with the values measured from the experiment. The error between the measured and simulated results was smaller than 8%. The results supported the notion that the analytical model developed for predicting ground pressure was valid. This model can provide an effective prediction of ground pressure under a rigid-flexible tracked vehicle on soft ground and a significant reference for tracked vehicles.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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