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Research Article

Disturbance Observer-Based Integral Backstepping Control for a Two-Tank Liquid Level System Subject to External Disturbances

Xiangxiang Meng, Haisheng Yu, Herong Wu, and Tao Xu

College of Automation, Qingdao University, Qingdao, China

Correspondence should be addressed to Haisheng Yu; yu.hs@163.com

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A novel method of disturbance observer-based integral backstepping control is proposed for the two-tank liquid level system with external disturbances. The problem of external disturbances can be settled in this paper. Firstly, the mathematical model of the two-tank liquid level system is established based on fluid mechanics and principle of mass conservation. Secondly, an integral backstepping control strategy is designed in order to ensure liquid level tracking performance by making the tracking errors converge to zero in finite time. Thirdly, a disturbance observer is designed for the two-tank liquid level system with external disturbances. Finally, the validity of the proposed method is verified by simulation and experiment. By doing so, the simulation and experimental results prove that the scheme of disturbance observer-based integral backstepping control strategy can suppress external disturbances more effective than the disturbance observer-based sliding mode control method and has better dynamic and steady performance of the two-tank liquid level system.

1. Introduction

In the last decade or so, the two-tank liquid level control system has been widely used in industrial production. From the point of view of control, the liquid level system represented complex multivariable nonlinear problems, which is one of the important fields of nonlinear control theory application. The liquid level system is affected by external disturbances and the parameters are of great uncertainty, which makes the control task more complicated [1]. So, the precise control of the two-tank liquid level system is very difficult. Sliding mode control is a robust control method for uncertain systems, which maintains high robustness against various uncertainties, such as external disturbances and measurement errors. For a class of nonlinear systems with multiple input and multiple output, a second-order sliding mode control algorithm [2] was proposed, which takes the input and output of nonlinear systems as the standard form of controller design. In [3], feedback linearization and sliding mode algorithm are designed for a controller based on feedback linearization for a four-tank system. By inserting a boundary layer around the sliding surface, buffeting associated with sliding mode control can be reduced, which is far better than traditional proportional integral (PI) control. A fractional-order proportional integral-differential sliding surface sliding mode controller [4] for liquid level control in the two-tank system was proposed. In [5], for a class of uncertainty and unknown disturbance of multiple-input-multiple-output data-driven sliding mode control problem of nonlinear discrete systems, Weng and Gao used the nonparametric dynamic linearization technique and the second-order discrete sliding mode observer and proposed a discrete sliding mode control law based on the proportion integration differentiation (PID) sliding surface in order to obtain the faster transient response and the smaller steady-state tracking error. Benamor and Messaoud [6] studied the robust adaptive sliding mode control law for uncertain discrete systems with unknown time-varying time-delay input. Finally, the verification results of the control law on the actual system showed that although there was no overmodulation phenomenon, there was still chattering phenomenon. Although the advanced control algorithm can obtain better control performance in the simulation research, its structure is often complex and there are many undetermined parameters, so it is difficult to be popularized in practice, limiting its application in practice. Therefore, it is necessary to select a control strategy with relatively simple method and good control performance.

For the two-tank liquid level system, a model-based backstepping controller and an adaptive backstepping controller were designed [7]. In order to illustrate the effectiveness of the adaptive inverse control design, a detailed experimental comparison is made with the proportional integral controller. A nonlinear generalized predictive control and a backstepping algorithm were applied and tested in [8, 9]. Khalili et al. [10] have developed a robust backstepping sliding mode controller for tracking control of a 2-DOF piezoelectric micro-operating system to eliminate chattering phenomenon in the control process. Combining the finite time current observer with the adaptive backstepping control scheme, a control mechanism with high cost performance and strong robustness was obtained. The results show that the control scheme can successfully estimate the unknown current, providing a possibility for the realization of the current-free sensor controller [11]. An active queue management scheme to control network congestion in combination with $H\infty$ theory and integral backstepping technique to ensure better tracking performance and asymptotically stable of all signal probabilities in the closed-loop system is given in [12]. In [13, 14], the authors have applied nonlinear backstepping to ship control for quite a long time. And the strong coupling characteristic of underactuated system was resolved by a virtual controller [15].

In practice, more and more attention has been paid to the influence of external interference on nonlinear systems and the suppression of interference. The distributed adaptive command-filtered backstepping scheme [16] based on the neural network was presented, which can ensure that the tracking error of the container reaches the desired origin neighborhood and all signals in the closed-loop system are bounded. An adaptive output feedback control problem [17] was studied for a class of uncertain nonlinear systems with input delay and disturbance. In [18–21], the general framework of the nonlinear system can be obtained by using the disturbance observer-based control technology, and the application of this method in the industrial field was illustrated. In [22], when the static feedback cannot guarantee the closed-loop stability, the disturbance observer was allowed to feedback as a dynamic system. Liu et al. [23, 24] studied the speed tracking control problem of the synchronous motor drive system under a matched and unmatched interference, a terminal sliding mode control method, and a port control Hamiltonian control method based on nonlinear disturbance observer, which was proposed to realize the speed and current tracking control of the permanent magnet synchronous motor drive system. In addition, an adaptive sliding mode control strategy based on interference observer [25] was proposed to solve the problems of multiple actuator faults, parameter uncertainty, and external disturbances in a quadrotor helicopter.

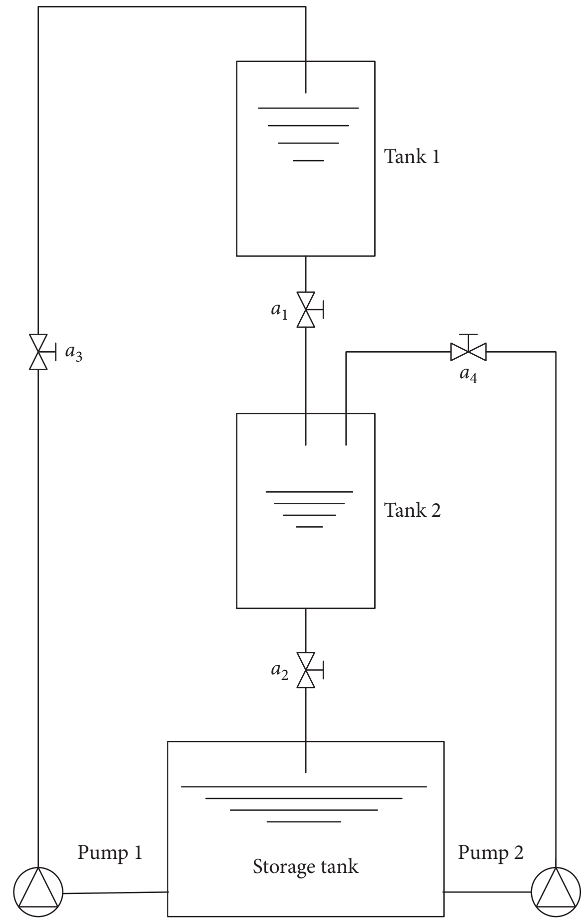

Figure 1: The schematic diagram of the two-tank process.

Table 1: Adjustable parameters of the system mode.

Parameters	Value	Unit
a_1	0.2	cm^2
a_2	0.3	cm^2
a_3	0.3	cm^2
a_4	0.3	cm^2
A_1	196	cm^2
A_2	196	cm^2

Different mechanisms compensate actuator fault, parameter uncertainty, and external disturbances, respectively, adopt an adaptive scheme to adjust actuator fault and parameter uncertainty, and design a disturbance observer to attenuate external disturbance. Moreover, the multivariable disturbance observer was proposed to improve the antidisturbance performance of traditional advanced feedback control [26]. In [27], a robust control method based on the finite time disturbance observer was proposed to track the output of the three-tank system in the presence of mismatched uncertainties. Then, a robust control strategy with time delay compensation was designed for multiple-input-multiple-output processes [28] with matching uncertainty and process delay. Based on the mixed fuzzy reasoning system, artificial hydrocarbon network was used in the

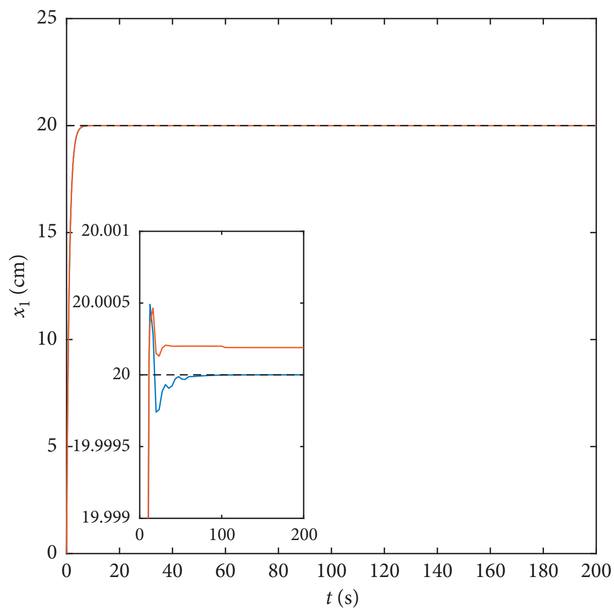

FIGURE 2: The liquid-level curve of tank 1.

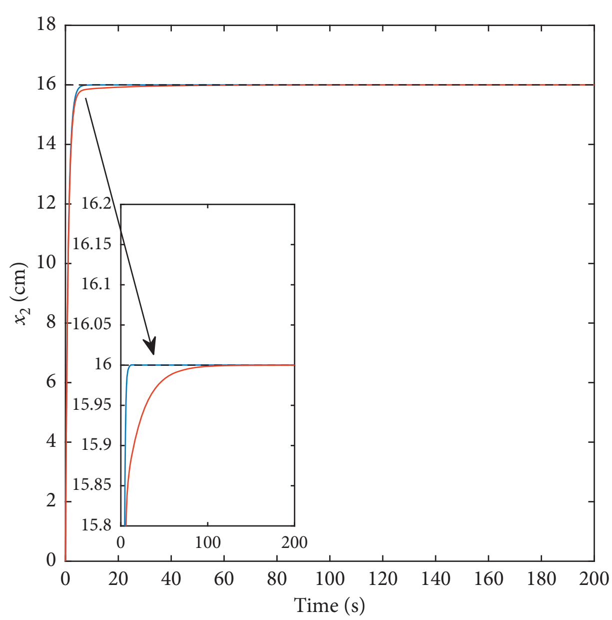

FIGURE 3: The liquid-level curve of tank 2.

TABLE 2: The sliding mode controller adjustable parameters.

Parameters	Value
m_1	1
n_1	0.1
b_1	0.05
m_2	1
n_2	0.1
b_2	0.06

TABLE 3: The integral backstepping controller adjustable parameters.

Parameters	Value
k_1	1
c_1	0.0001
k_2	1
c_2	0.0001

TABLE 4: The disturbance observer parameters.

Parameters	Value
h_1	10
h_2	−1
h_3	1
h_4	−0.01

defuzzification stage, which was called fuzzy molecular control [29]. Gouta et al. [30] proposed an adaptive control and a generalized predictive control method of the coupled two-tank system, which are to minimize the multilevel cost function defined on the prediction layer. Aiming at the problem of four-tank control [31], a set of disturbance uncertainty suppression control laws were compared and proved, whose control requirements were usually expressed in the literature in the form of a set value sequence. The uncertainty class was defined as the union of four subclasses:

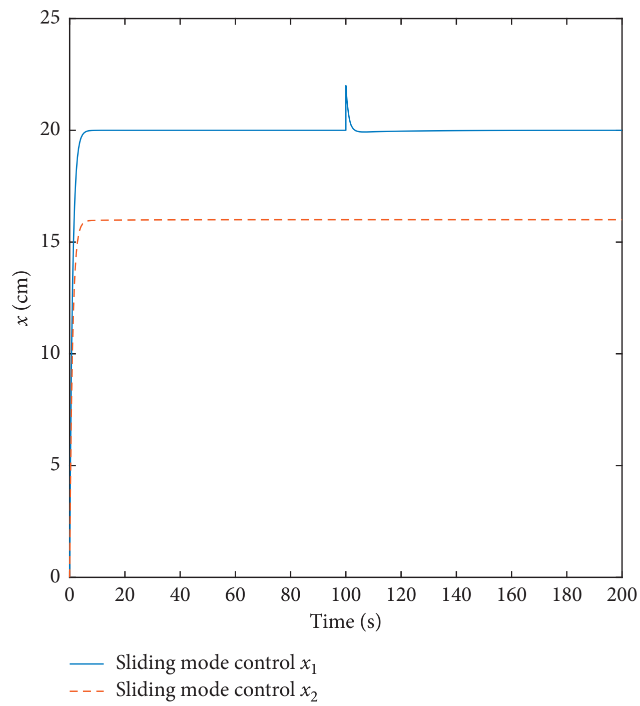

Figure 4: The liquid-level curve of tank 1 after adding disturbance.

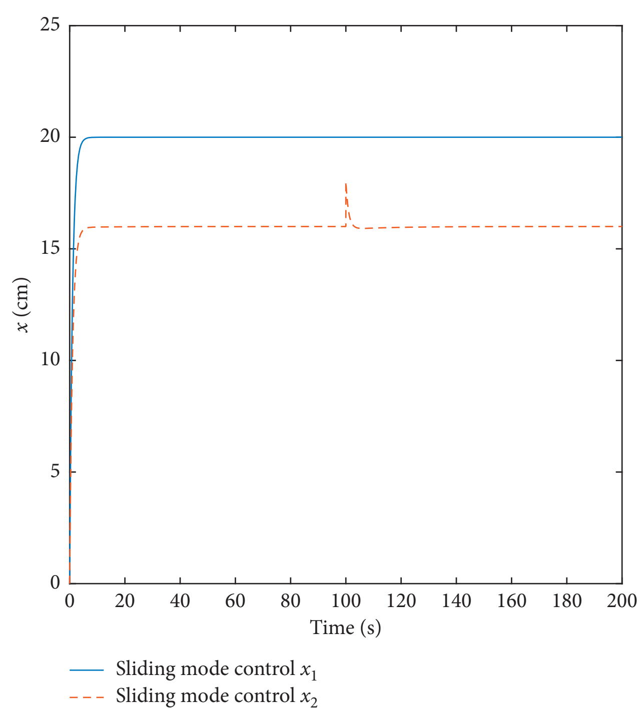

Figure 5: The liquid-level curve of tank 2 after adding disturbance.

unknown disturbance, parameter uncertainty, measurement error, and neglected dynamics. In addition to the above literature, the second-order sliding controller [32] was successfully used to adjust the liquid level of the two-tank coupling liquid level system and the computer simulation results show that the controller can adjust the liquid level with little difference in performance. A real-time implementation method of fuzzy coordinated classical proportional integral (PI) control scheme was proposed. Experimental results show that the controller structure track parameter changed quickly and had good performance when load disturbance and the set value changed

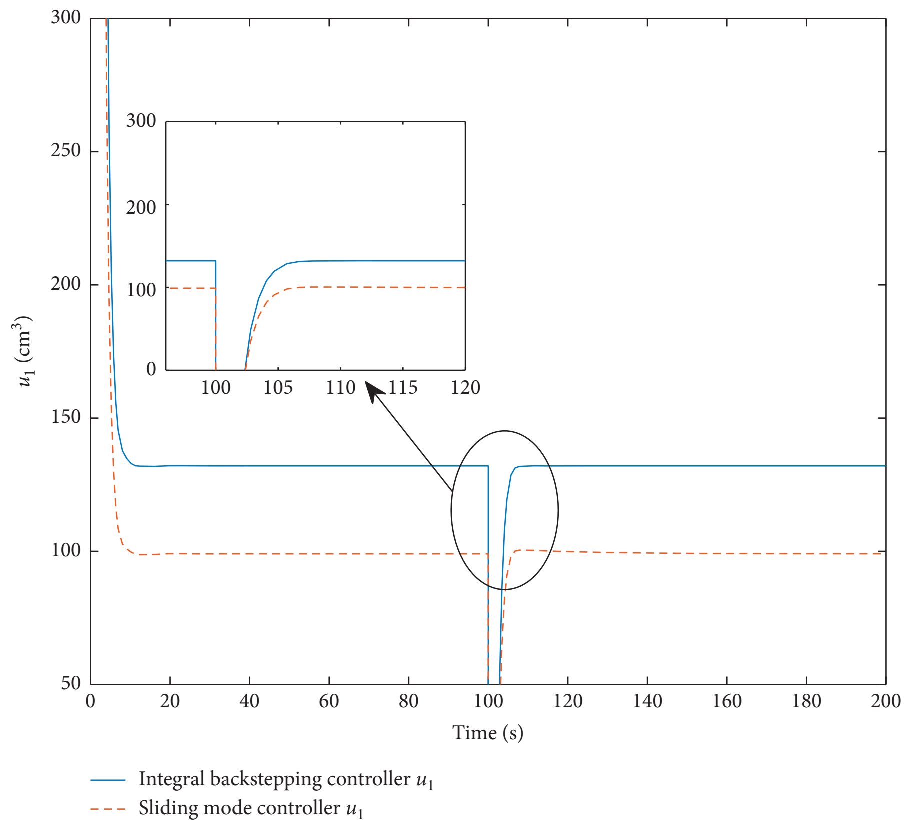

Figure 6: The input curves of controller u_1.

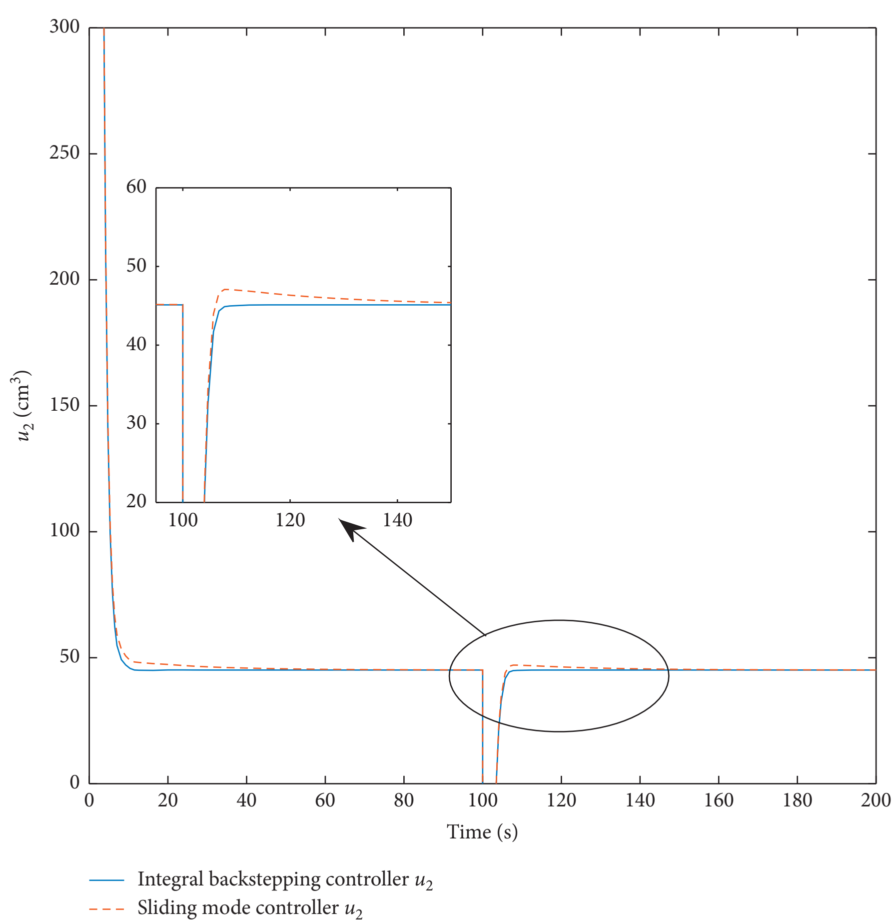

Figure 7: The input curves of controller u_2.

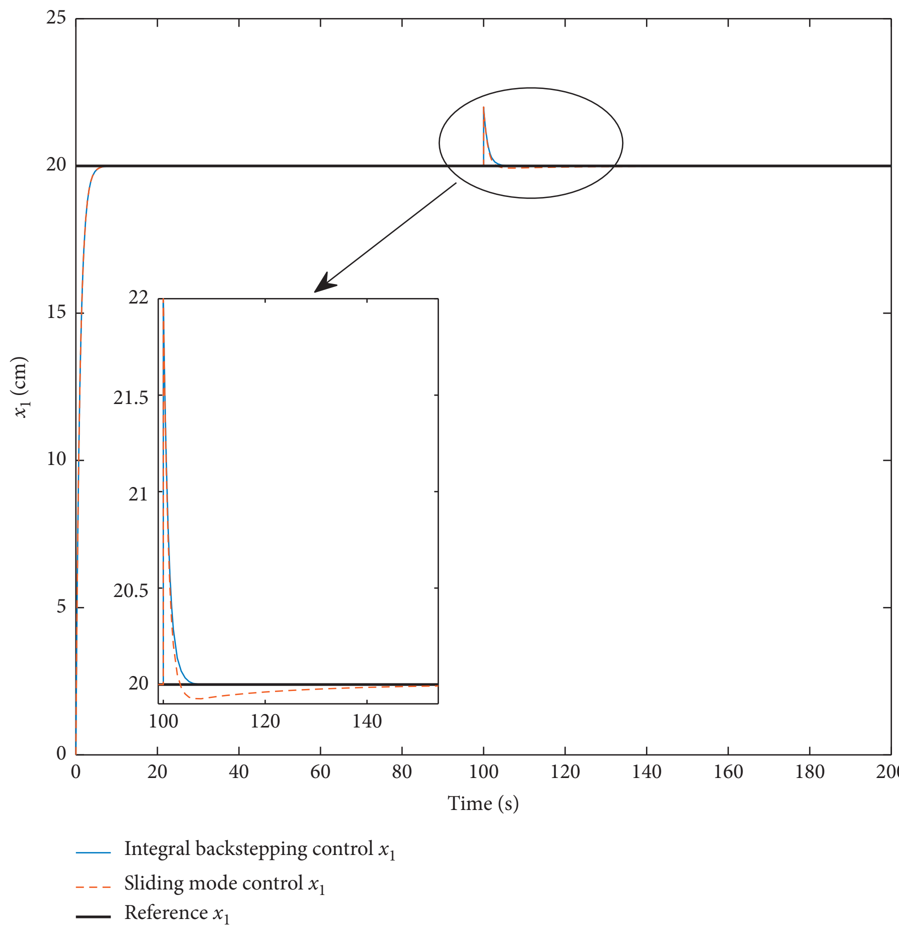

Figure 8: The liquid-level curve of tank 1 after adding disturbance.

[33]. The performance of an active disturbance rejection control method in coupling tank system control was studied [34]. The results show that compared with other controllers, the active disturbance rejection control method was effective in improving time domain measurement and suppressing interference, but it lacked experimental verification. Smida et al. [35] adopted the observation scheme combining the high-gain observer and sliding mode observer to improve the robustness of state estimation quality and to reconstruct the disturbance waveform in a better way.

In this paper, mathematical model for the two-tank liquid level system is established based on the principle of hydromechanics and the principle of mass conservation. Then, an integral backstepping control method for the two-tank liquid level system and disturbance observer is developed. Furthermore, research studies of the integral backstepping control method and the system with disturbance have been carried out on the innovative experimental platform for the complex control system of four-tank NTC-I type in quantity. Simulation and experiment results prove this suggested control strategy and disturbance attenuation strategy highly effective compared with the disturbance observer-based sliding mode control strategy of the two-tank liquid level system. Moreover, the controller has fewer adjusting parameters, simpler structure, and easier implementation. And the steady-state and dynamic performance of the proposed controller are both far better than some complex algorithms listed in the references, such as [32]. The results show that the proposed method has high-blooded dynamic and steady-state performances. In practical industrial application, the factory workshop will change the given value irregularly according to production demand. However, it is not realistic to readjust the controller parameters every time when the given value is changed. Therefore, the control method proposed in this paper has a wide range of practical application prospects.

2. System Description and Modeling

The schematic drawing in Figure 1 represents the model of a two-tank liquid level system. This system consists of two tanks, two level sensors (the level sensor is located at the top of each tank), two orifices (the orifice at the bottom of each tank), two pumps, a storage tank, and four manual valves. The two tanks have same cross sections, but the cross sections of the four manual valves are different. In this experimental device, the pump 1 feeds tank 1 and the outflow of tank 1 turns into partial input of tank 2. The pump 2 feeds

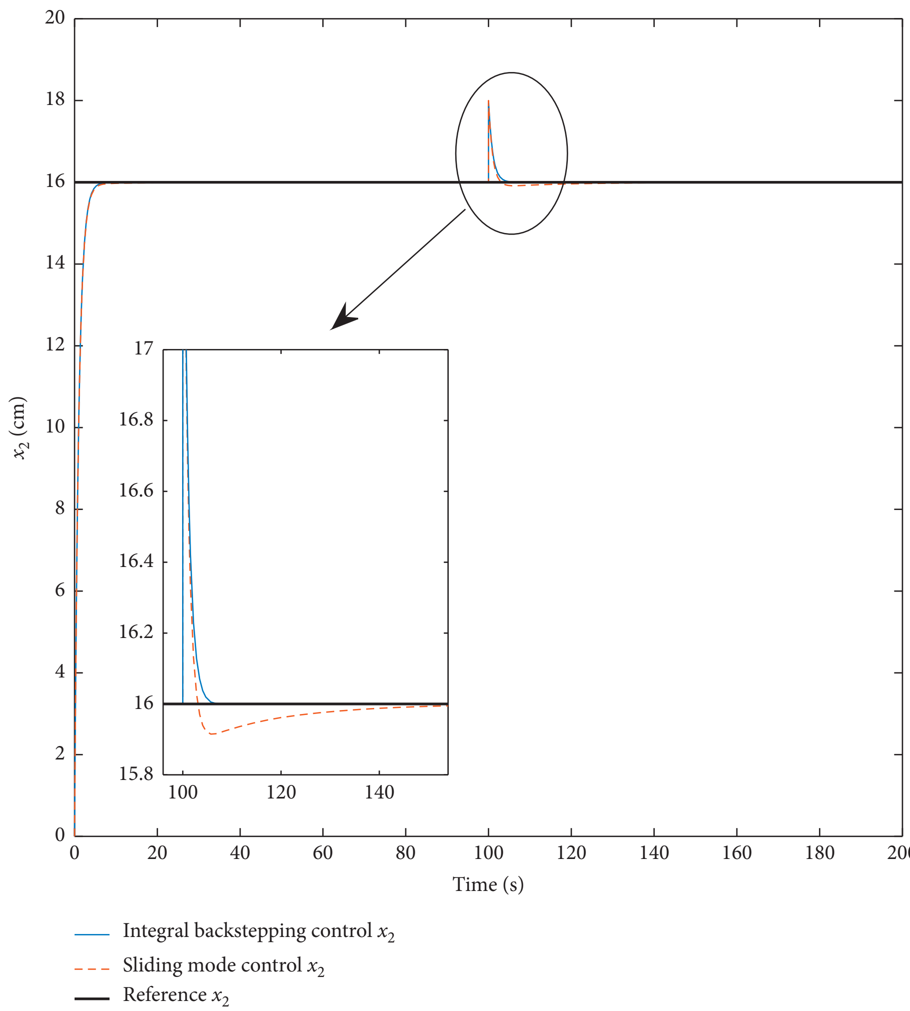

FIGURE 9: The liquid-level curve of tank 2 after adding disturbance.

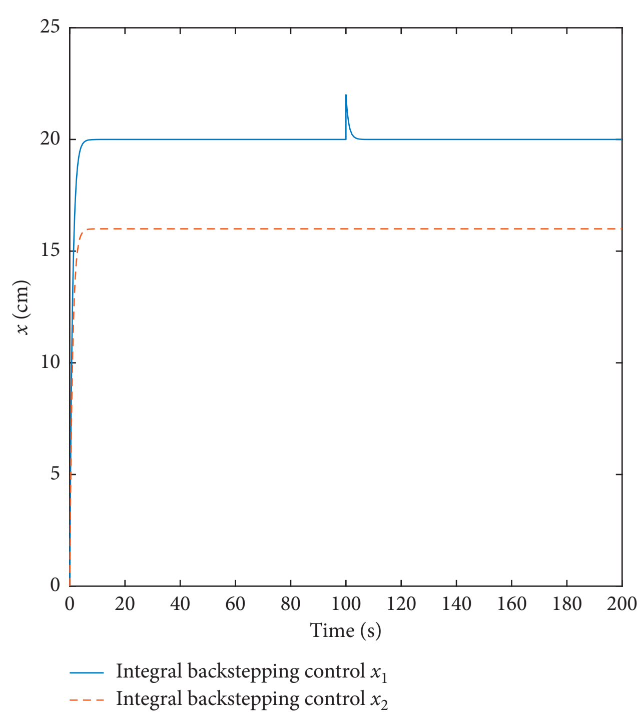

FIGURE 10: The liquid-level curve of tank 1 after adding disturbance.

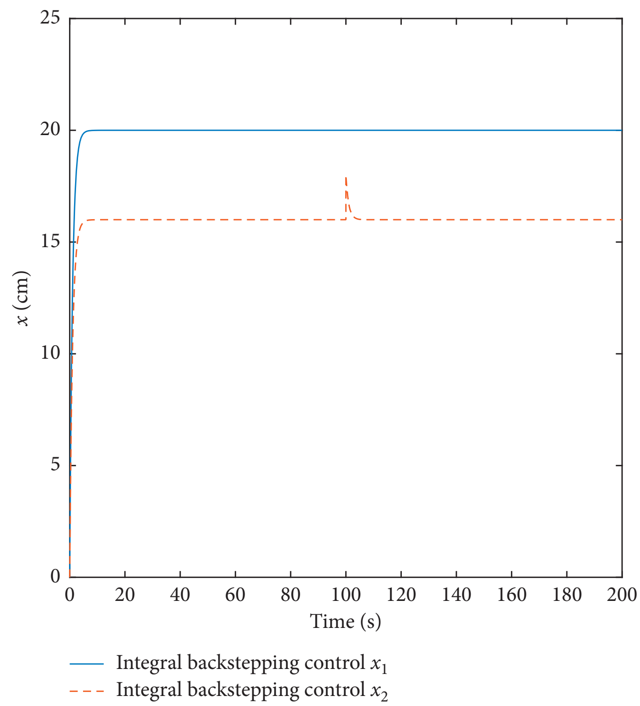

FIGURE 11: The liquid-level curve of tank 2 after adding disturbance.

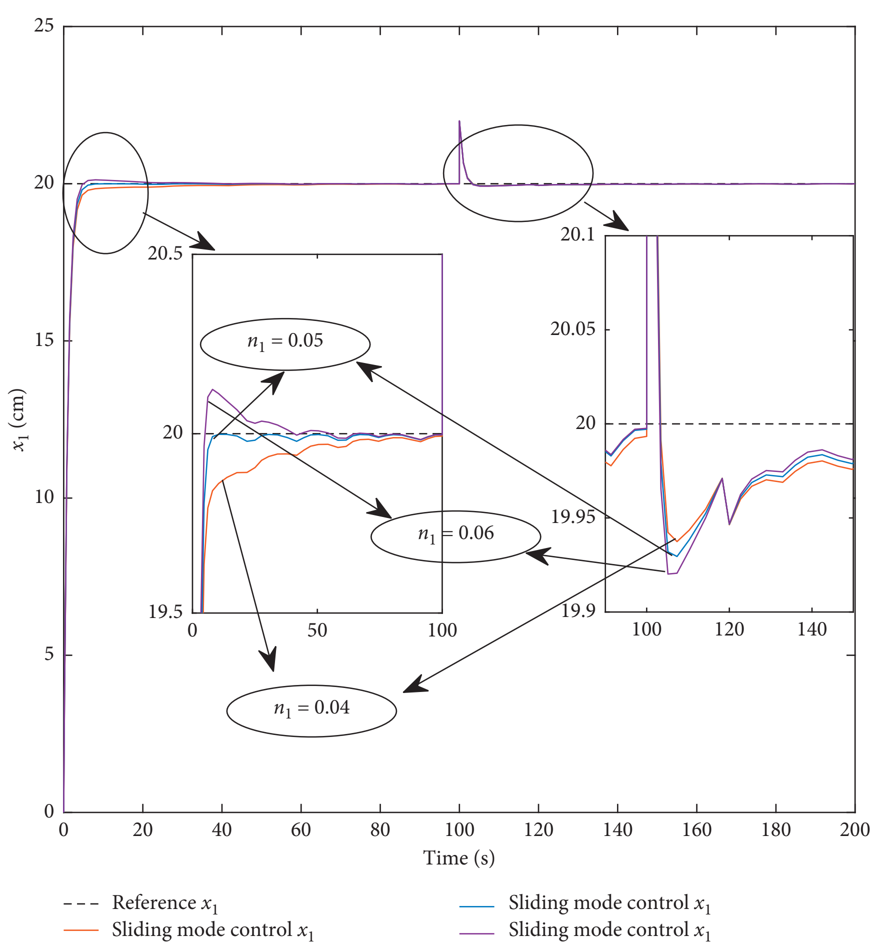

FIGURE 12: The liquid-level curve of tank 1.

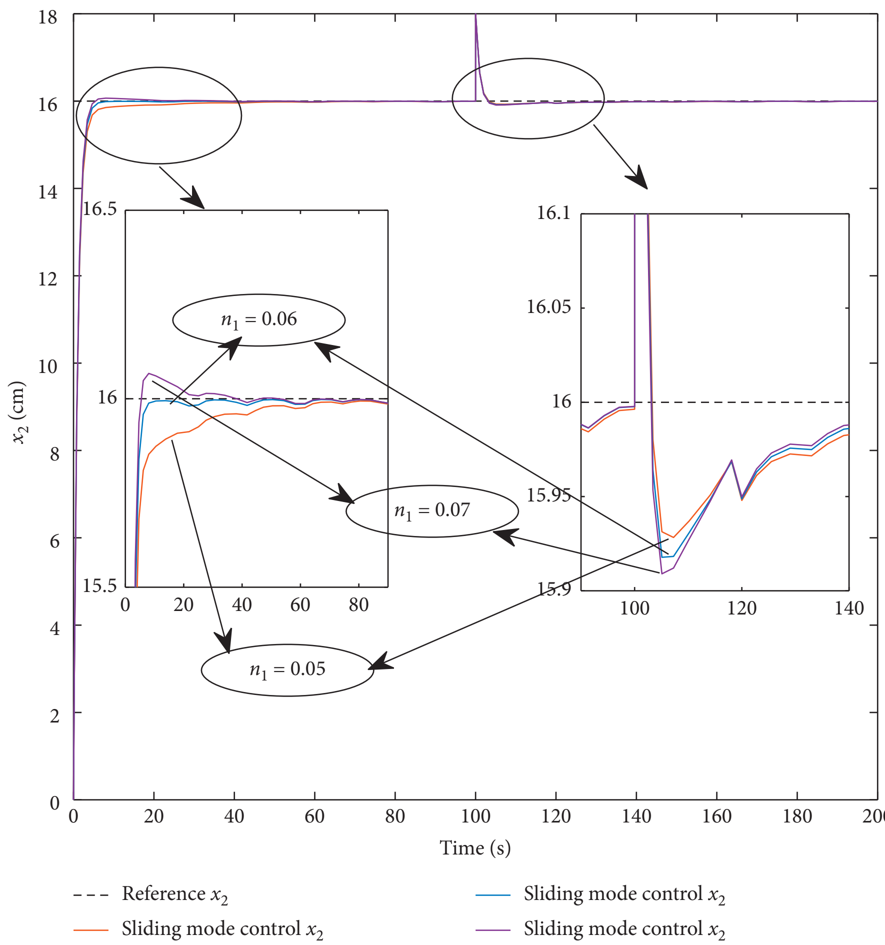

Figure 13: The liquid-level curve of tank 2.

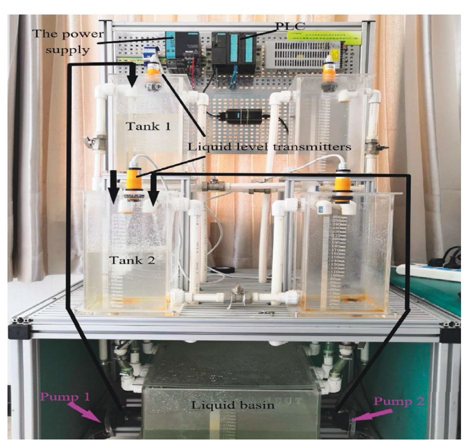

Figure 14: Experimental platform.

tank 2, and the effluent from tank 2 is discharged into the storage tank.

It can be known from the law of conservation of mass and the time rate of change of liquid in each tank is given by

$$\frac{d}{dt}\left[\rho A_i h_i(t)\right] = \rho q_{in}(t) - \rho q_{out}(t), \quad (1)$$

where ρ is density of liquid, A_i is the cross section of tank i (the unit is cm^2), a_i is the cross section of the outlet manual valve i, $h_i(t)$ is the height of liquid inside tank i (the unit is cm), $i \in \{1, 2, 3, 4\}$, the $q_{in}(t)$ is the output flow rate of electric control valve, and $q_{out}(t)$ is the output flow rate of the tank at the bottom (the unit is cm^3/s).

By the Bernoulli equation in hydrodynamics, the liquid flow velocity $q_{out}(t)$ of flowing out of the tank at the bottom

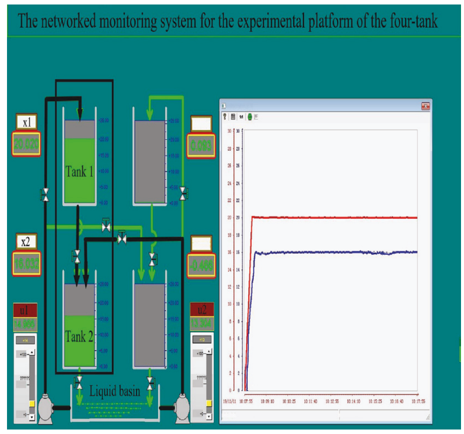

Figure 15: Wincc liquid-level monitoring interface.

Table 5: The sliding mode controller adjustable parameters.

Parameters	Value
m_1	100
n_1	10
b_1	0.01
m_2	100
n_2	10
b_2	0.01

is related to the cross-sectional area a_i of the manual control valve at the bottom of the tank and the liquid level height in the tank. So, the outflow velocity of the valve outlet the tank can be expressed as

$$q_{\text{out}}(t) = a_i\sqrt{2gh_i(t)}, \tag{2}$$

where g is the gravitational acceleration.

According to the conservation of mass principle, the differential equation of tank 1 and tank 2 can be written as follows:

$$\begin{cases} \dot{h}_1(t) = \dfrac{a_3}{A_1}q_1 - \dfrac{a_1}{A_1}\sqrt{2gh_1(t)}, \\ \dot{h}_2(t) = \dfrac{a_4}{A_2}q_2 + \dfrac{a_1}{A_2}\sqrt{2gh_1(t)} - \dfrac{a_2}{A_2}\sqrt{2gh_2(t)}, \end{cases} \tag{3}$$

where q_1 and q_2 are the output flow of the pump 1 and pump 2, respectively.

Define the system state and input as

$$\begin{cases} x_i = h_i, \\ u_i = q_i, \end{cases} \tag{4}$$

where $i \in \{1, 2\}$.

Thus, the mathematical model of the two-tank liquid level system can be expressed as

$$\begin{cases} \dot{x}_1 = -\dfrac{a_1}{A_1}\sqrt{2gx_1} + \dfrac{a_3}{A_1}u_1, \\ \dot{x}_2 = \dfrac{a_1}{A_2}\sqrt{2gx_1} - \dfrac{a_2}{A_2}\sqrt{2gx_2} + \dfrac{a_4}{A_2}u_2. \end{cases} \tag{5}$$

3. Controller Design of the Two-Tank Liquid Level System

To facilitate the calculation and interpretation of the main contents in this section, the following constants are defined:

$$\begin{aligned} A &= \frac{a_1}{A_1}\sqrt{2g}, \\ B &= \frac{a_3}{A_1}, \\ C &= \frac{a_1}{A_2}\sqrt{2g}, \\ D &= \frac{a_2}{A_2}\sqrt{2g}, \\ E &= \frac{a_4}{A_2}. \end{aligned} \tag{6}$$

Based on the situation above, the system described by equation (5) can be rewritten as follows:

$$\begin{cases} \dot{x}_1 = -A\sqrt{x_1} + Bu_1, \\ \dot{x}_2 = C\sqrt{x_1} - D\sqrt{x_2} + Eu_2. \end{cases} \tag{7}$$

3.1. The Integral Backstepping Controller Design and Stability Analysis.

The accuracy of the control target will be quantified by the liquid level tracking errors e_1 and e_2 of tank 1 and tank 2, respectively. And tracking errors are as follows:

$$\begin{cases} e_1 = x_1 - x_{1d}, \\ e_2 = x_2 - x_{2d}. \end{cases} \tag{8}$$

Then, first-order derivative of equation (8) can be written as

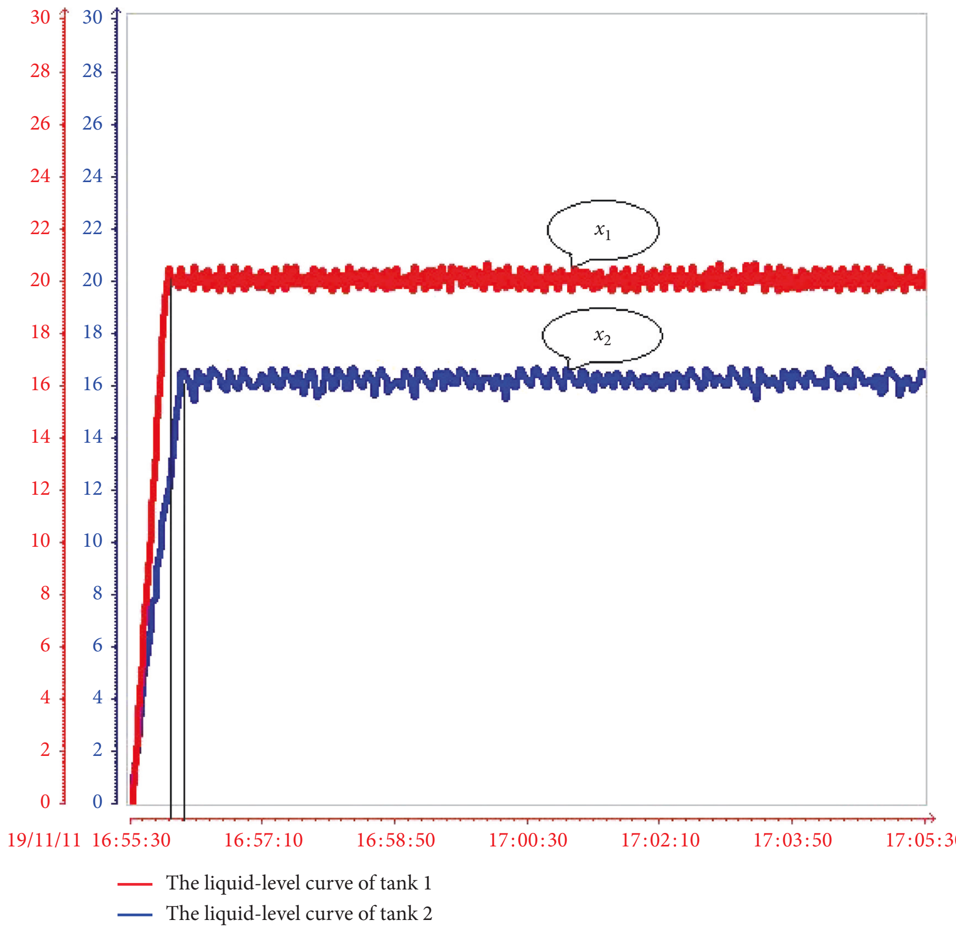

FIGURE 16: The liquid-level curves of sliding mode control.

$$\begin{cases} \dot{e}_1 = \dot{x}_1 - \dot{x}_{1d}, \\ \dot{e}_2 = \dot{x}_2 - \dot{x}_{2d}. \end{cases} \tag{9}$$

Define the first-order derivative of the errors as

$$\begin{cases} \dot{e}_1 = -k_1 e_1 - c_1 \int_0^t e_1 dt, \\ \dot{e}_2 = -k_2 e_2 - c_2 \int_0^t e_2 dt, \end{cases} \tag{10}$$

where $k_1 > 0$, $k_2 > 0$, $c_1 > 0$, and $c_2 > 0$.

Substitute equation (10) into equation (9) and end up with

$$\begin{cases} \dot{x}_1 = \dot{x}_{1d} - k_1 e_1 - c_1 \int_0^t e_1 dt, \\ \dot{x}_2 = \dot{x}_{2d} - k_2 e_2 - c_2 \int_0^t e_2 dt. \end{cases} \tag{11}$$

TABLE 6: The integral backstepping controller adjustable parameters.

Parameters	Value
k_1	0.08
c_1	10
k_2	0.12
c_2	1

By combining equations (7) and (11), the controller can be computed as

$$\begin{cases} u_1 = \dfrac{\dot{x}_{1d} + A\sqrt{x_1} - k_1 e_1 - c_1 \int_0^t e_1 dt}{B}, \\ u_2 = \dfrac{\dot{x}_{2d} + D\sqrt{x_2} - C\sqrt{x_1} - k_2 e_2 - c_2 \int_0^t e_2 dt}{E}. \end{cases} \tag{12}$$

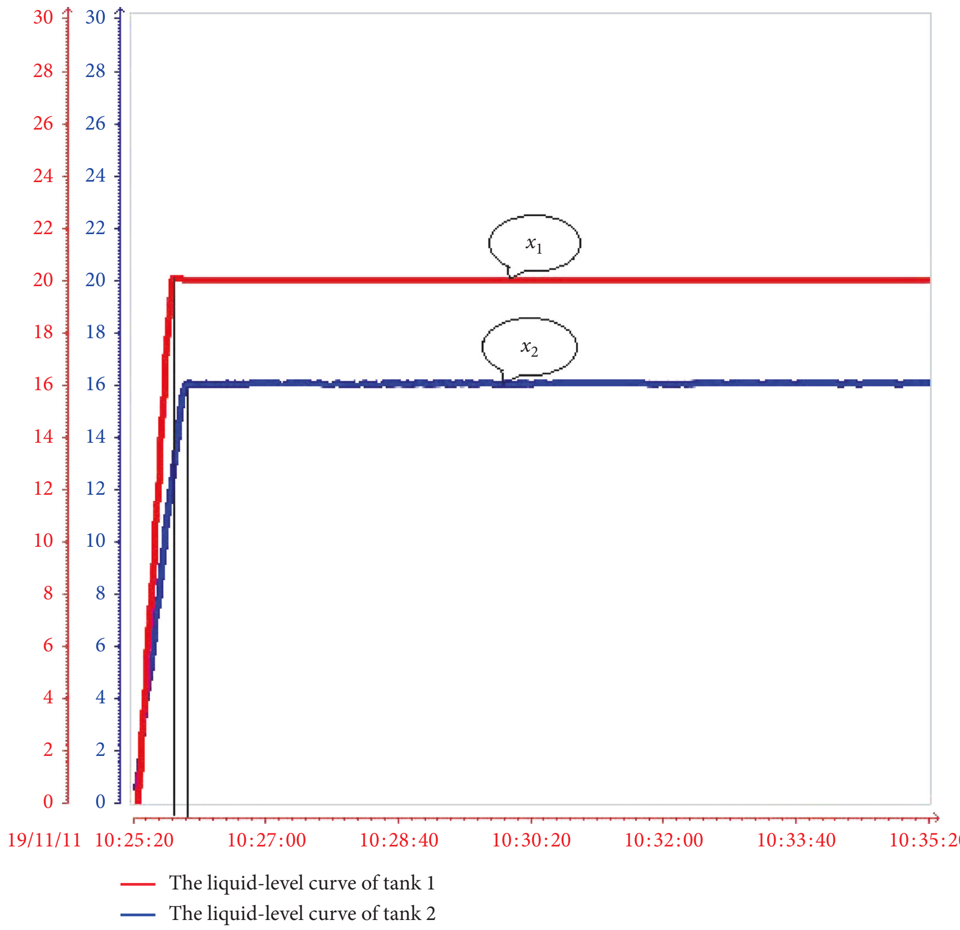

FIGURE 17: The liquid-level curves of integral backstepping control.

Hence, the integral backstepping controller can be derived from equations (6), (8), and (12). The result is

TABLE 7: The disturbance observer parameters.

Parameters	Value
h_1	1
h_2	−0.01
h_3	1
h_4	−0.01

$$\begin{cases} u_1 = \dfrac{\left(a_1\sqrt{2gx_1}/A_1\right) + \dot{x}_{1d} - k_1\left(x_1 - x_{1d}\right) - c_1\int_0^t\left(x_1 - x_{1d}\right)\mathrm{d}t}{\left(a_3/A_1\right)}, \\ u_2 = \dfrac{\left(a_2\sqrt{2gx_2}/A_2\right) - \left(a_1\sqrt{2gx_1}/A_2\right) + \dot{x}_{2d} - k_2\left(x_2 - x_{2d}\right) - c_2\int_0^t\left(x_2 - x_{2d}\right)\mathrm{d}t}{\left(a_4/A_2\right)}. \end{cases} \quad (13)$$

Select the Lyapunov function V as

$$\begin{cases} V_1 = \dfrac{1}{2}e_1^2, \\ V_2 = \dfrac{1}{2}e_2^2, \end{cases} \quad (14)$$

$$V = V_1 + V_2 = \frac{1}{2}e_1^2 + \frac{1}{2}e_2^2. \quad (15)$$

Taking the first-order time derivative for equation (15), we can obtain

$$\dot{V} = \dot{V}_1 + \dot{V}_2 = e_1\dot{e}_1 + e_2\dot{e}_2. \quad (16)$$

Then, we can apply equation (10) to equation (16) and the result of the equation (16) can be calculated as

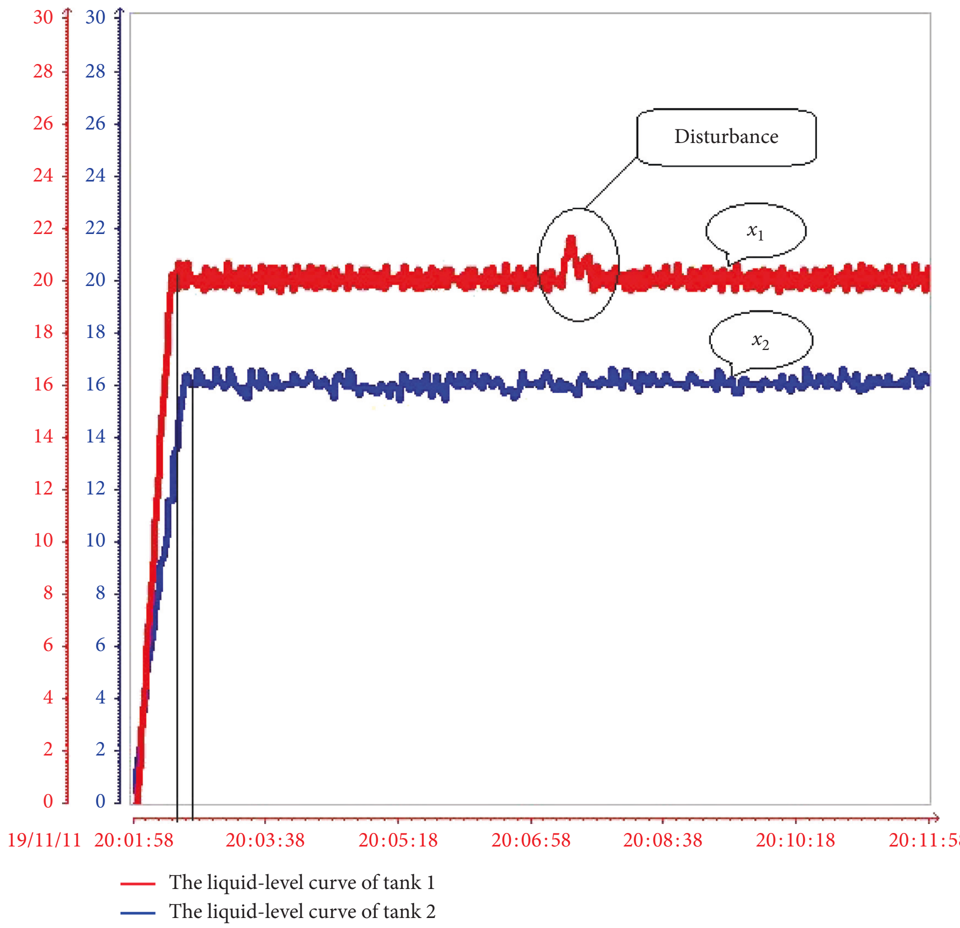

FIGURE 18: The liquid-level curves of tank 1 with disturbance on sliding mode control.

$$\dot{V} = e_1\left(-k_1 e_1 - c_1 \int_0^t e_1 dt\right) + e_2\left(-k_2 e_2 - c_2 \int_0^t e_2 dt\right)$$
$$= -k_1 e_1^2 - c_1 \int_0^t e_1^2 dt - k_2 e_2^2 - c_2 \int_0^t e_2^2 dt. \quad (17)$$

Obviously, V is positive definite and $\dot{V}$ is negative definite, and both satisfy Lyapunov stability theorem, so system (5) is asymptotically stable.

Remark 1. Assume that there exists a scalar function V of the state x, with continuous first-order derivatives such that

(i) $V(x)$ is positive definite
(ii) $\dot{V}(x)$ is negative definite
(iii) $V(x) \longrightarrow \infty$ as $\|x\| \longrightarrow \infty$

Then, the equilibrium at the x_d is globally asymptotically stable [36]. x_d is the expected value of the liquid level.

3.2. Design and Stability Analysis of the Controller with the Disturbance Observer. This paper considers a class of nonlinear systems with exogenous disturbances, which can be described as

$$\begin{cases} \dot{x}_1 = -A\sqrt{x_1} + Bu_1 + d_1, \\ \dot{x}_2 = C\sqrt{x_1} - D\sqrt{x_2} + Eu_2 + d_2, \end{cases} \quad (18)$$

where d_1 and d_2 are exogenous disturbances, and they are bounded.

$$\begin{cases} |d_1| \le D_1, \\ |d_2| \le D_2, \end{cases} \quad (19)$$

where D_1 and D_2 are known positive constants.

Construct the disturbance observers as

$$\begin{cases} \dot{\hat{x}}_1 = -A\sqrt{\hat{x}_1} + Bu_1 + \hat{d}_1 + h_1(x_1 - \hat{x}_1), \\ \dot{\hat{d}}_1 = h_2(x_1 - \hat{x}_1), \end{cases} \quad (20)$$

$$\begin{cases} \dot{\hat{x}}_2 = C\sqrt{\hat{x}_1} - D\sqrt{\hat{x}_2} + Eu_2 + \hat{d}_2 + h_3(x_2 - \hat{x}_2), \\ \dot{\hat{d}}_2 = h_4(x_2 - \hat{x}_2), \end{cases} \quad (21)$$

where h_1, h_2, h_3, and h_4 are the adjustable parameters.

With the estimated disturbance $\tilde{d}_1$ and $\tilde{d}_2$, the disturbance estimated errors are defined as

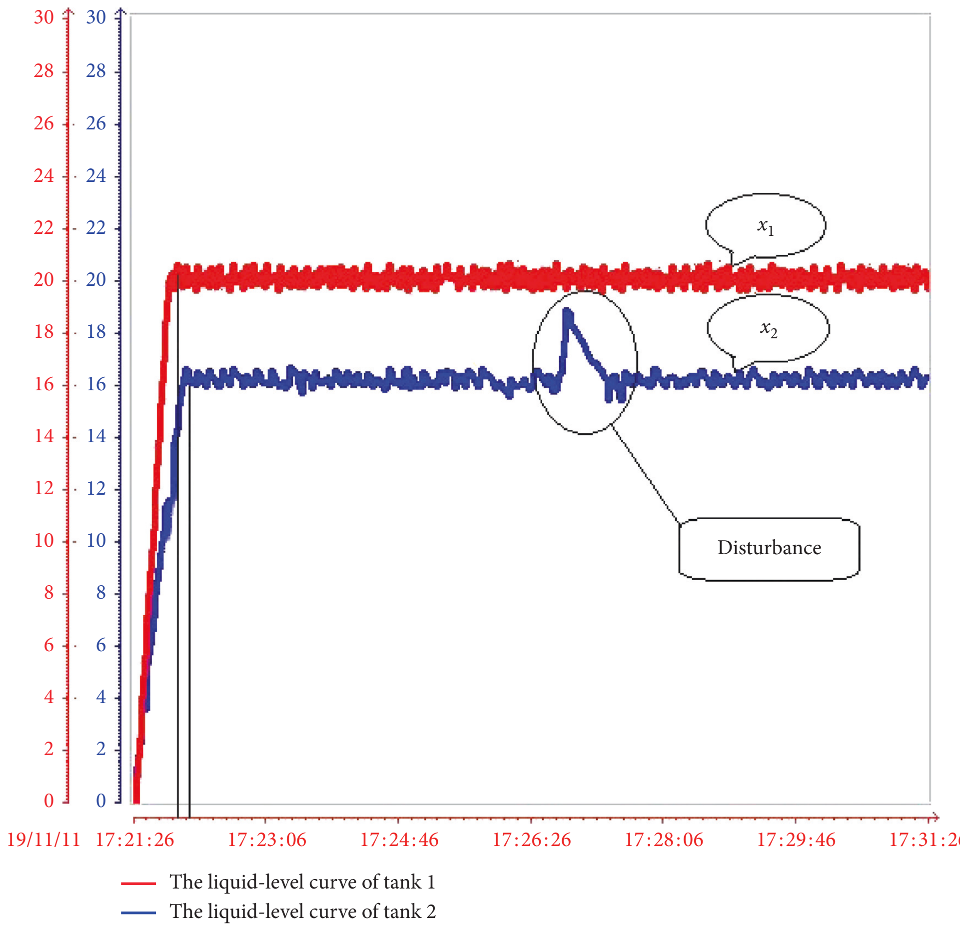

FIGURE 19: The liquid-level curves of tank 1 with disturbance on integral backstepping control.

$$\begin{cases} \tilde{x}_1 = x_1 - \hat{x}_1, \\ \tilde{d}_1 = d_1 - \hat{d}_1, \end{cases}$$
$$\begin{cases} \tilde{x}_2 = x_2 - \hat{x}_2, \\ \tilde{d}_2 = d_2 - \hat{d}_2. \end{cases} \tag{22}$$

By combining equations (19) and (21), the first-order derivative of the disturbance estimation errors can be calculate as

$$\begin{cases} \dot{\tilde{d}}_1 = \dot{d}_1 - \dot{\hat{d}}_1 = -\lambda_1 \tilde{d}_1, \\ \dot{\tilde{d}}_2 = \dot{d}_2 - \dot{\hat{d}}_2 = -\lambda_2 \tilde{d}_2, \end{cases} \tag{23}$$

there λ_1 and λ_2 are both the nonlinear disturbance observer gains.

Then, the observer error equation is given as

$$\dot{z}_i = H_i z_i. \tag{24}$$

Here,

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} \tilde{x}_1 \\ \tilde{d}_1 \end{bmatrix}, \begin{bmatrix} z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} \tilde{x}_2 \\ \tilde{d}_2 \end{bmatrix},$$
$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} \dot{\tilde{x}}_1 \\ \dot{\tilde{d}}_1 \end{bmatrix}, \begin{bmatrix} \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} \dot{\tilde{x}}_2 \\ \dot{\tilde{d}}_2 \end{bmatrix}, \tag{25}$$
$$i = 1, 2.$$

So, one can get

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} -\frac{A}{2} - h_1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix},$$

$$\begin{bmatrix} \dot{z}_3 \\ \dot{z}_4 \end{bmatrix} = \begin{bmatrix} -\frac{D}{2} - h_2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z_3 \\ z_4 \end{bmatrix},$$

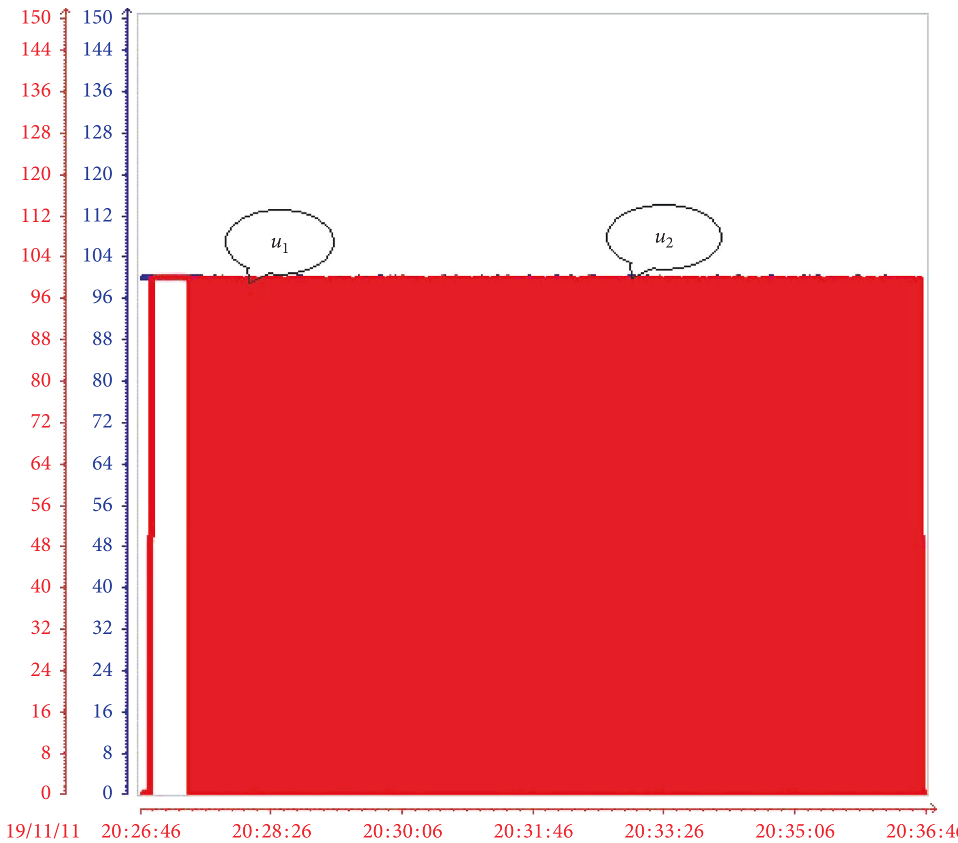

FIGURE 20: The input curves of the sliding mode controller.

$$H_1 = \begin{bmatrix} -\frac{A}{2} - h_1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$H_2 = \begin{bmatrix} -\frac{D}{2} - h_2 & 0 \\ 0 & 1 \end{bmatrix}. \quad (26)$$

It is easy to prove that the observer is asymptotically stable with proper choice of h_1, h_2, h_3, and h_4.

Remark 2. If the linearized system is strictly stable (i.e., if all eigenvalues of H are strictly in the left-half complex plane), then the equilibrium point is asymptotically stable (for the actual nonlinear system) [36].

Finally, according to equations (13), (18), (20), and (21), the controller for the nonlinear system with disturbances can be obtained as follows:

$$\begin{cases} u_1 = \dfrac{\left(a_1\sqrt{2gx_1}/A_1\right) + \dot{x}_{1d} - k_1\left(x_1 - x_{1d}\right) - c_1\int_0^t \left(x_1 - x_{1d}\right)\mathrm{d}t - \widehat{d}_1}{\left(a_3/A_1\right)}, \\ u_2 = \dfrac{\left(a_2\sqrt{2gx_2}/A_2\right) - \left(a_1\sqrt{2gx_1}/A_2\right) + \dot{x}_{2d} - k_2\left(x_2 - x_{2d}\right) - c_2\int_0^t \left(x_2 - x_{2d}\right)\mathrm{d}t - \widehat{d}_2}{\left(a_4/A_2\right)}. \end{cases} \quad (27)$$

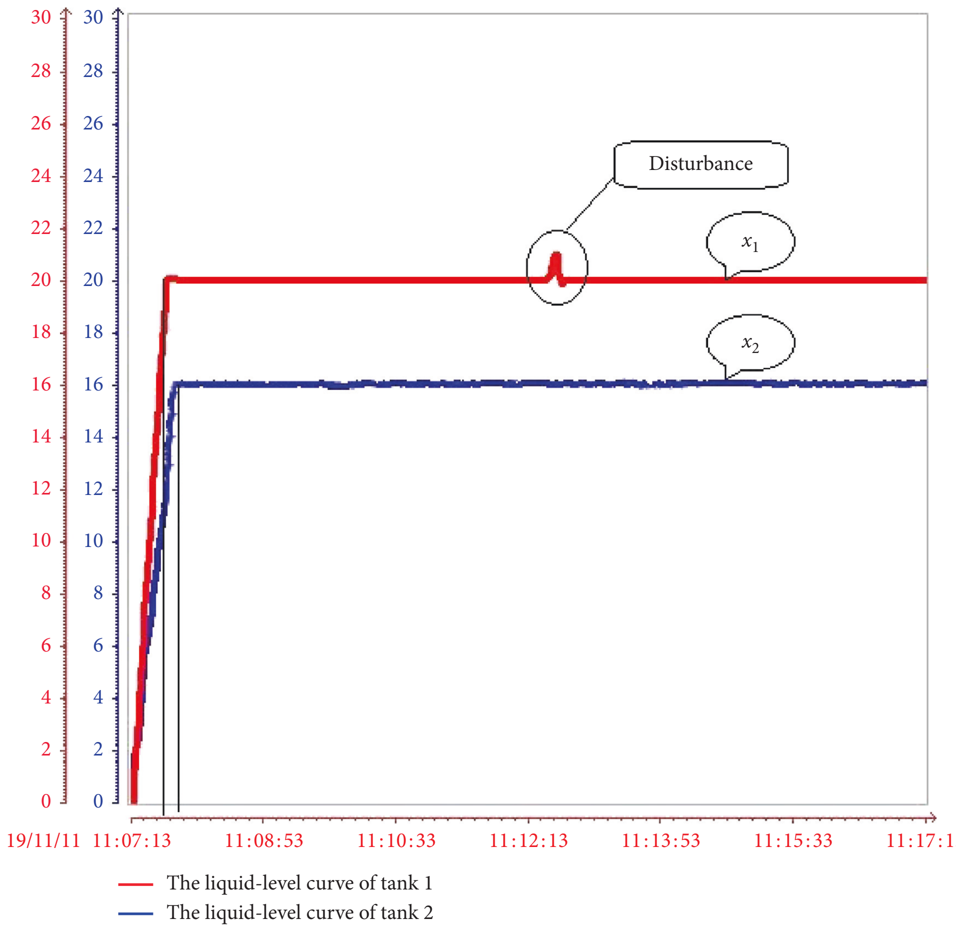

FIGURE 21: The liquid-level curves of tank 2 with disturbance on sliding mode control.

In this case, tracking error equation (8) can be rewritten as

$$\begin{cases} e_1' = x_1 - x_{1d}, \\ e_2' = x_2 - x_{2d}. \end{cases} \tag{28}$$

Consider the rate of error change, the time derivative of (28) can be expressed as

$$\begin{cases} \dot{e}_1' = \dot{x}_1 - \dot{x}_{1d}, \\ \dot{e}_2' = \dot{x}_2 - \dot{x}_{2d}. \end{cases} \tag{29}$$

Choose a Lyapunov function V' for nonlinear system (18) as

$$\begin{cases} V_1' = \frac{1}{2}e_1'^2 + \frac{1}{2}\tilde{d}_1^2, \\ V_2' = \frac{1}{2}e_2'^2 + \frac{1}{2}\tilde{d}_2^2, \end{cases} \tag{30}$$

$$V' = V_1' + V_2' = \frac{1}{2}e_1'^2 + \frac{1}{2}\tilde{d}_1^2 + \frac{1}{2}e_2'^2 + \frac{1}{2}\tilde{d}_2^2. \tag{31}$$

Then, the first-order time derivative of the selected Lyapunov function (31) is obtained as

$$\dot{V}' = \dot{V}_1' + \dot{V}_2' = e_1'\dot{e}_1' + \tilde{d}_1'\dot{\tilde{d}}_1' + e_2'\dot{e}_2' + \tilde{d}_2'\dot{\tilde{d}}_2'. \tag{32}$$

By combining equations (10) and (32), (32) can be rewritten as

$$\begin{aligned} \dot{V}' &= e_1'\left(-k_1e_1' - c_1\int_0^t e_1'\mathrm{d}t\right) + e_2'\left(-k_2e_2' - c_2\int_0^t e_2'\mathrm{d}t\right) \\ &\quad - \lambda_1\tilde{d}_1^2 - \lambda_2\tilde{d}_2^2 = -k_1e_1'^2 - c_1\int_0^t e_1'^2\mathrm{d}t - k_2e_2'^2 - c_2 \\ &\quad \cdot \int_0^t e_2'^2\mathrm{d}t - \lambda_1\tilde{d}_1^2 - \lambda_2\tilde{d}_2^2. \end{aligned} \tag{33}$$

Obviously, V' is positive definite. In addition, from equation (33), one can know that $\dot{V}'$ is negative definite. Hence, it satisfied Lyapunov stability theorem, which means system (18) is asymptotically stable.

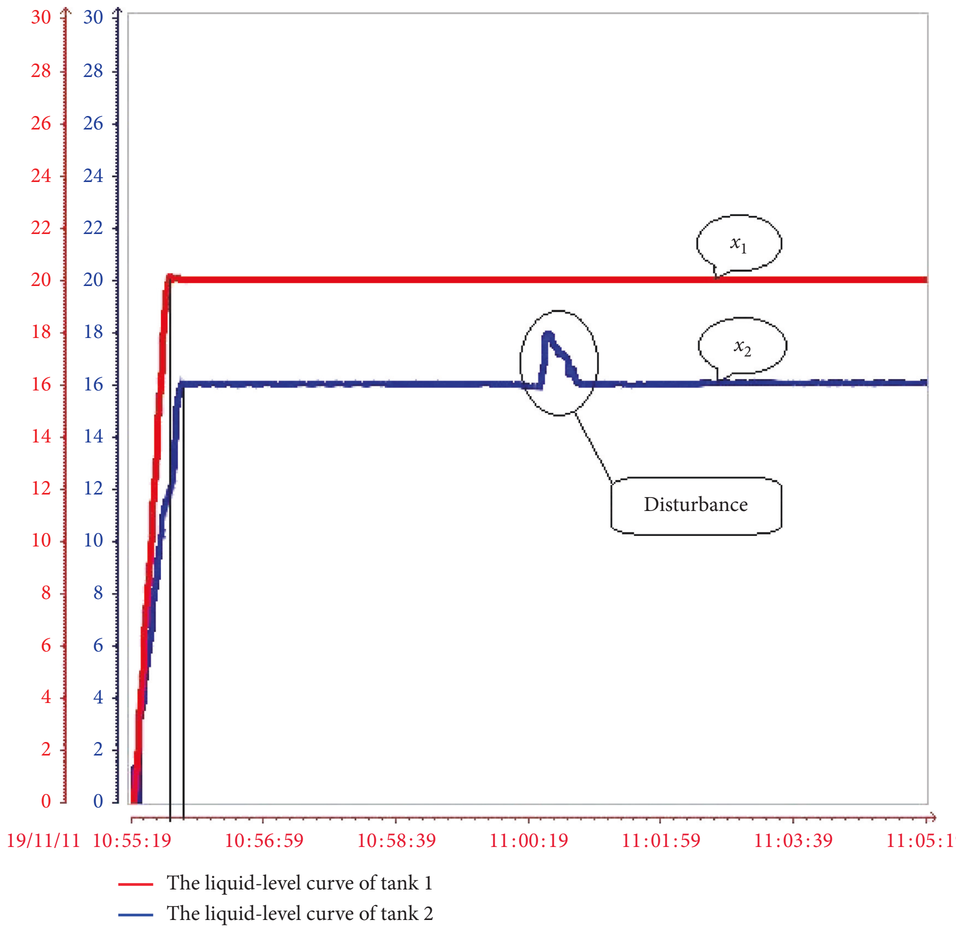

FIGURE 22: The liquid-level curves of tank 2 with disturbance on integral backstepping control.

4. Simulation Results and Analysis

In order to verify the effectiveness of the above control strategy, we compared it with the disturbance observer-based sliding mode control method.

The disturbance observer-based sliding mode controller can be described as

$$\begin{cases} u_1 = \dfrac{A_1}{a_3}\left\{-\left[m_1 \operatorname{sgn}(s_1) + (n_1 + b_1)(x_1 - x_{1d}) + n_1 b_1 \int e_1 \mathrm{d}t\right] + \dot{x}_{1d} + \dfrac{a_1\sqrt{2gx_1}}{A_1} - \widehat{d}_1\right\}, \\ u_2 = \dfrac{A_2}{a_4}\left\{-\left[m_2 \operatorname{sgn}(s_2) + (n_2 + b_2)(x_2 - x_{2d}) + n_2 b_2 \int e_2 \mathrm{d}t\right] + \dot{x}_{2d} + \dfrac{a_2\sqrt{2gx_2}}{A_2} - \dfrac{a_1\sqrt{2gx_1}}{A_2} - \widehat{d}_2\right\}, \end{cases} \tag{34}$$

where b_1, b_2, m_1, m_2, n_1, and n_2 are positive constants.

The comparative study results of the two control strategies are carried out in Simulink environment. Then, the adjustable parameters of the two-tank coupled liquid level system are given in Table 1.

The desired equilibrium points are

$$x_{1d} = 20\,\text{cm}, \quad x_{2d} = 16\,\text{cm}. \tag{35}$$

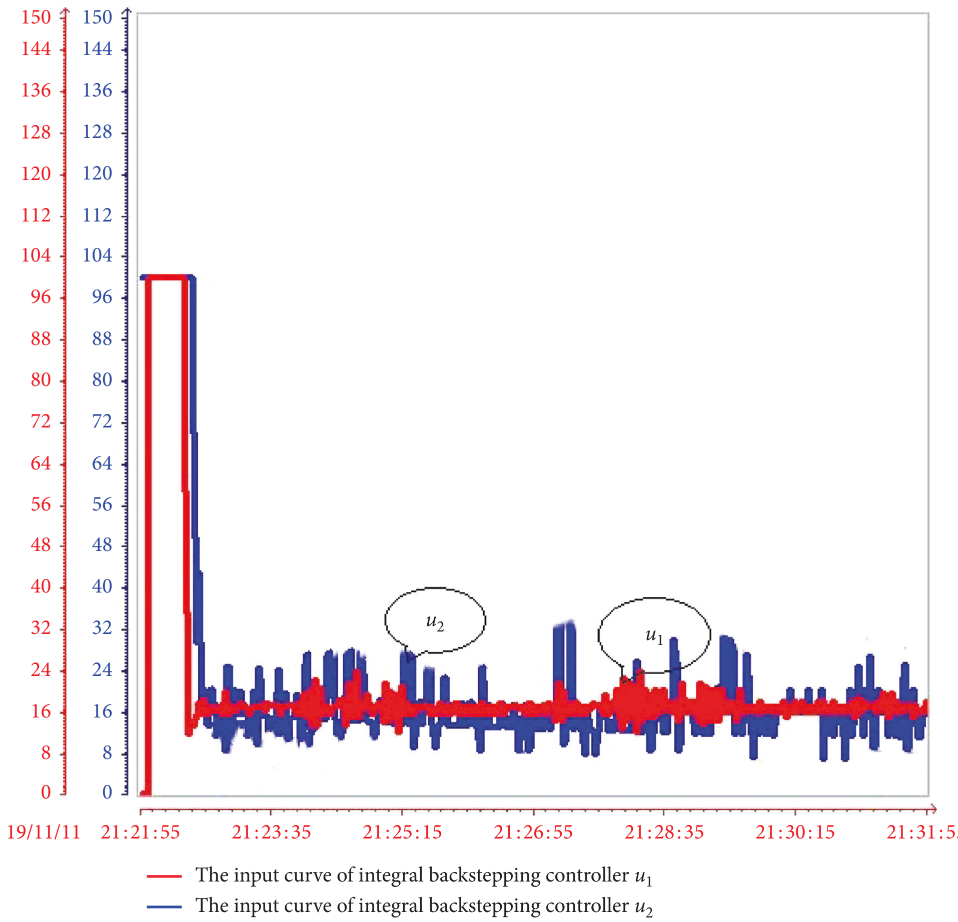

Figure 23: The input curves of the integral backstepping controller.

4.1. The Undisturbed Simulation Results

4.1.1. Sliding Mode Controller. The controlled liquid level trajectories of the two tanks are given in Figures 2 and 3, respectively. The adjustable parameters are determined, as shown in Table 2.

As shown in Figures 2 and 3, the dynamic response time of the liquid-level curve of tank 1 is short, its rise time is $t_r = 10$ s, but overshoot occurs, its overshoot is $\sigma\% = 0.0000025\%$, and there has always been a steady-state error of 0.0002 cm. Although the liquid-level curve of tank 2 did not overshoot, its dynamic stage rose slowly when approaching the equilibrium point, and the rise time is $t_r = 100$ s.

4.1.2. Integral Backstepping Controller. The controlled liquid level trajectories of the two tanks are shown in Figures 2 and 3, respectively. The adjustable parameters are determined, as shown in Table 3.

As shown in Figures 2 and 3. The rise time of liquid-level curve of tank 1 is also $t_r = 10$ s. Although overshoot occurs and overshoot is $\sigma\% = 0.0000025\%$, the steady-state error is zero. The liquid-level curve of tank 2 did not overshoot and its dynamic response time is short, and its rise time is $t_r = 10$ s.

4.2. Simulation Results with Disturbance. The simulation verification in this paper is obtained by injecting the disturbance in the form of step signal into the system without changing the controller parameters of the two-tank liquid level system. The adjustable parameters are determined, as shown in Table 4.

4.2.1. Sliding Mode Controller. The liquid-level curves and control inputs trajectories of two tanks are given in Figures 4 and 5 and Figures 6 and 7, respectively. The disturbances are added to tank 1 and tank 2 at the simulation time of 100 seconds, respectively. Simulation results show that the injection of external disturbances into a single tank has no effect on the liquid level of another tank. It can be seen from Figures 8 and 9 that the recovery time is relatively long of liquid levels in tank 1 and tank 2 after injection of external disturbance, and the adjustment time is $t_s = 50$ s.

4.2.2. Integral Backstepping Controller. The liquid-level curves and control inputs trajectories of two tanks are given in Figures 10 and 11 and Figures 6 and 7, respectively. At the simulation time of 100 seconds, the disturbances are added to tank 1 and tank 2, respectively. Then, the simulation results show that the injection of external disturbances into a single tank can also achieve almost no effect on the liquid level of

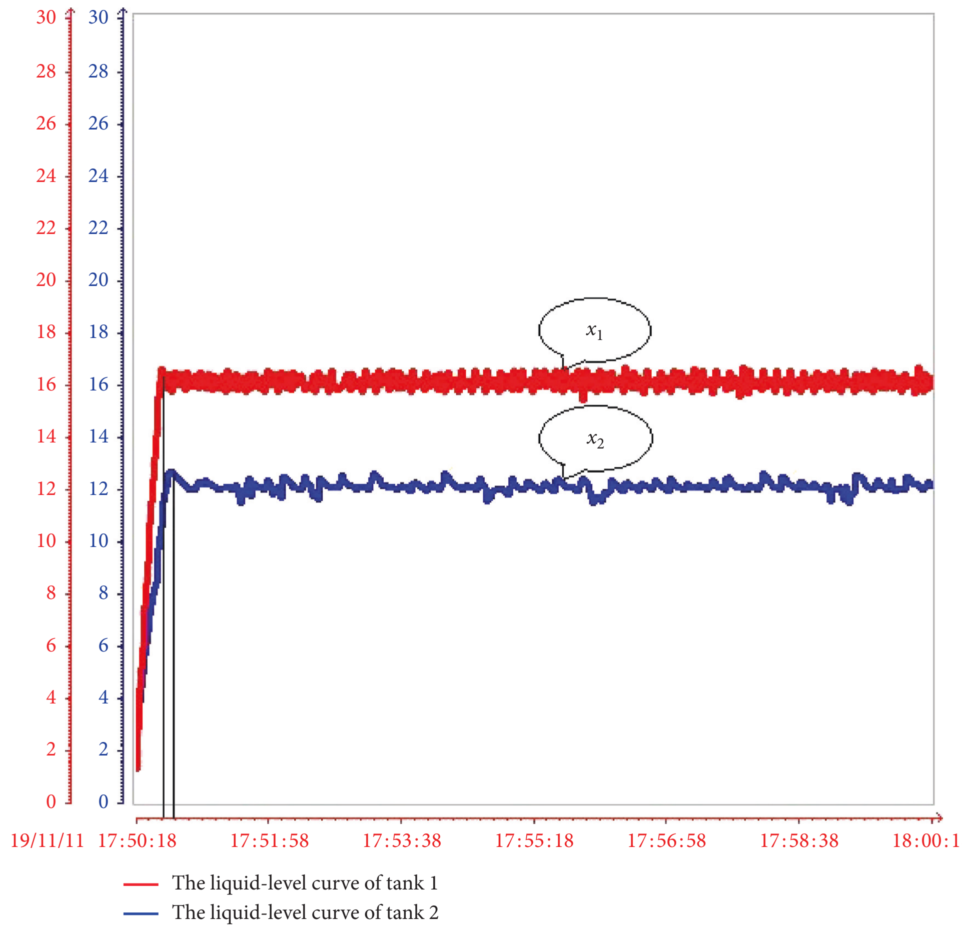

Figure 24: The liquid-level changed curves of sliding mode control.

another tank. It can be seen from Figures 8 and 9 that the recovery time is faster of liquid levels in tank 1 and tank 2 after injection of external disturbance, and the adjustment time is $t_s = 10$ s.

4.3. Comparison of Simulation Results. For the determination and optimization of the parameters, the Lyapunov stability theorem is used to determine the range of parameters, and then the specific parameter values are determined by the empirical trial and error method, such as in Figures 12 and 13.

Simulation results show that the two-tank liquid level system with sliding mode control algorithm has a slight liquid level overshoot, which not only has a certain amount of steady-state error but also takes a long time to restore stability after injection of disturbances. In the two-tank liquid level system, using the integral backstepping control algorithm based on the disturbance observer, the liquid level has a slight overshoot, but its steady-state error is zero, and it can recover to a stable value in a very short time after injecting the external disturbances. According to the above discussion, the integral backstepping control method based on the disturbance observer has good dynamic, stability, and disturbance suppression characteristics.

5. Experimental Results and Analysis

In order to verify the performances of the propose control strategy, experiments are developed on the Innovative experimental platform for the complex control system of the four-tank NTC-I type. As shown in Figures 14 and 15, they are the experimental platform and Wincc liquid-level monitoring interface, respectively. The experimental apparatus is a typical process control object in industrial process-four tank. For the object, a feedback control system based on programmable logic controller (PLC) and Matlab/simulink are constructed. The PLC part uses the Siemens CPU s7-300 module to extend the special analog input. The module collects the actual water level of the four-capacity water tank and expands the dedicated analog output module feed water. The pump provides the actual required analog voltage. Communication between Matlab and PLC/HMI industrial control is realized through OLE for process control (OPC) communication technology. The control system is integrated with some advanced system identification techniques and advanced control methods that can be applied to the controlled object of the four-tank easily. The upper computer uses a common desktop computer, and the operating system uses Microsoft's Windows XP system. Other development software used in this device includes WinCC V6.2, monitoring software of SIEMENS, down-bit STEP7 V 5.4 of PLC

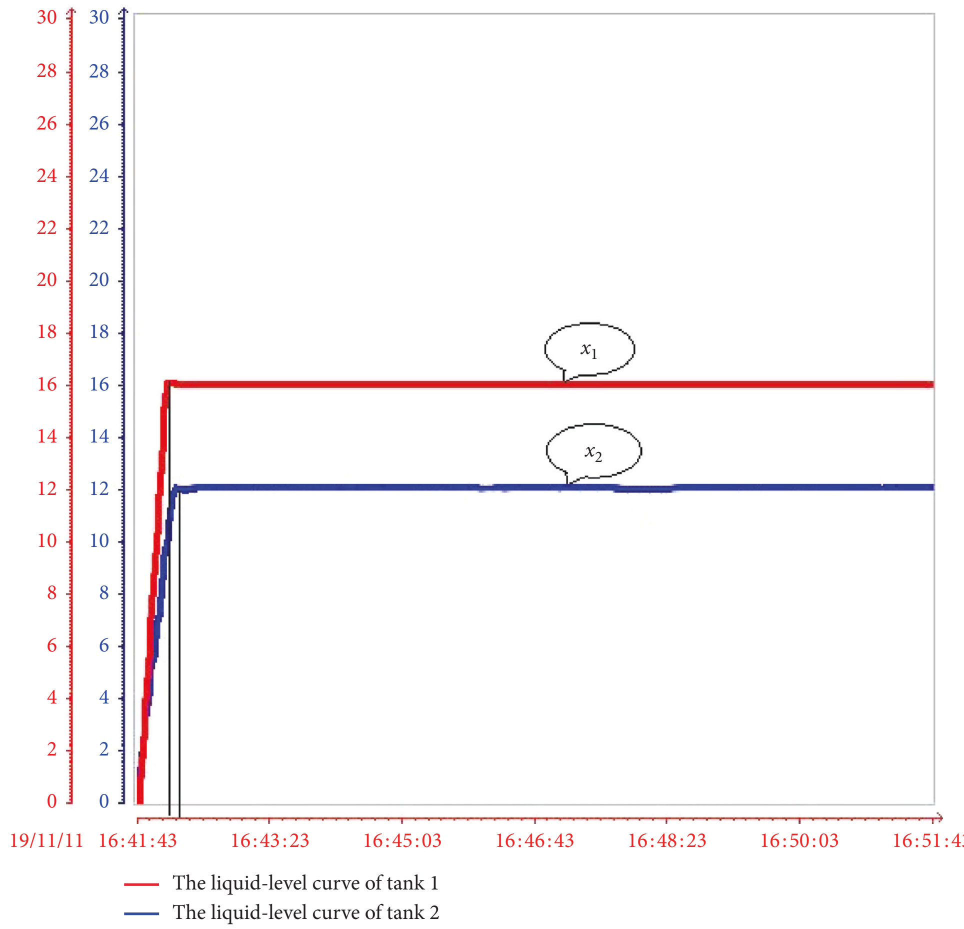

Figure 25: The liquid-level changed curves of integral backstepping control.

controller configuration programming and configuration communication mode, and OPC server configuration SIMATIC NET software. The adjustable parameters of the two-tank liquid level system are given in Table 1.

The desired equilibrium points are

$$x_{1d} = 20\,\text{cm},$$
$$x_{2d} = 16\,\text{cm}. \tag{36}$$

5.1. The Undisturbed Experimental Results

5.1.1. Sliding Mode Controller. The controller adjustable parameters are given in Table 5.

The actual controlled liquid-level trajectories of the two tanks are shown in Figure 16. The dynamic response time of the experimental curves of tank 1 and tank 2 are relatively fast, rising time is $t_r = 35$ s and $t_r = 45$ s, respectively, but there is always obvious chattering phenomenon of the liquid level, and the amplitude is ± 0.5 cm.

5.1.2. Integral Backstepping Controller. The integral backstepping controller adjustable parameters are given in Table 6.

The actual controlled liquid-level trajectories of the two tanks are given in Figure 17. Under the same condition of valve opening, the dynamic response time of tank 1 and tank 2 liquid-level curves is still very fast and the rising time is $t_r = 30$ s and $t_r = 40$ s, respectively. Moreover, there is no chattering phenomenon in the liquid-level curve under this control method and the steady-state process is very good.

5.2. Experimental Results with Disturbance. In this paper, disturbances are added to the two-tank liquid level system and the controller parameters are unchanged. The disturbance observer parameters are shown in Table 7.

5.2.1. Sliding Mode Controller. The actual controlled liquid-level curves and control input trajectories of the two tanks with disturbances are shown in Figures 18–20. A certain amount of water is injected into tank 1 at 20: 07: 20. The liquid level of tank 1 has apparent fluctuation, and the adjustment time is $t_s = 30$ s. At 17: 26: 40, a certain amount of water is injected into tank 2, the liquid level of tank 2 has apparent fluctuation, and the adjustment time is $t_s = 30$ s.

5.2.2. Integral Backstepping Controller. The actual controlled liquid-level curves and control input trajectories of the two tanks with disturbances are given in Figures 21–23. A certain amount of water is injected into tank 1 and tank 2 at 11: 12: 30 and 11: 00: 30, respectively. The liquid levels of tank 1 and tank 2 have hardly any fluctuation, and the adjustment time is $t_s = 15\,\mathrm{s}$ and $t_s = 15\,\mathrm{s}$, respectively.

As shown in Figures 24 and 25, without changing the controller parameters of the two-tank liquid level system, It still has good dynamic and steady-state performance when the set value is changed.

5.3. Comparison of Experimental Results. The sliding mode control algorithm is tested and verified on the equipment platform. It can be seen from Figures 16, 18 and 19 that the two-tank liquid level system controlled by the sliding mode algorithm always has the phenomenon of chattering, especially when the liquid level is disturbed externally. As can be seen from Figures 17, 21 and 22, the disturbance observer-based integral backstepping control method not only has no chattering phenomenon but also exhibits good steady-state performance and disturbance-suppression characteristics in the case of disturbance.

6. Conclusions

In view of the two-tank liquid level system, a methodology of disturbance observer-based integral backstepping control is proposed in this paper. According to the principle of fluid mechanics and mass conservation, the mathematical model of the two-tank liquid level system is established. As can be seen from the simulation results, when the external disturbance is added to a single tank, the liquid-level value of another tank has no influence. Furthermore, it can be known from the experiment results that the given value can be set discretionarily and reached quickly. Then, the results of simulation and experiment verified that the disturbance observer-based integral backstepping control methodology has good dynamic and static performance compared with the disturbance observer-based sliding mode control method. Moreover, the steady-state and dynamic performance of the proposed controller are both far better than some complex algorithms listed in the references. Its structure is simpler, and adjustable parameters are fewer and easier to realize. Therefore, the control method proposed in this paper has a wide range of practical application prospects. However, the controller proposed in this paper does not consider the influence of time delay on the system, and it is only limited to the two-tank water level system. Whether it is suitable for the four-tank water level system needs to be further verified.

Further, in addition to considering the influence of disturbance on the system, the influence of time delay on the system is also considered and corresponding solution is given, which is combined with the method proposed in this paper to achieve better control effect. Then, the composite control method is extended and applied to the four-tank liquid level system.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare no conflicts of interest.

Authors' Contributions

M.X.X. developed the theoretical results, wrote the paper, and performed the experiments; Y.H.S. modified the mathematical formula and grammar; W.H.R. ensured the hardware and software support; and X.T. analyzed the algorithms. All authors have read and approved the final paper.

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