

Research Article

Theoretical Calculation and Application Test of Lift Force for Ideal Electric Asymmetric Capacitor

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The asymmetric capacitor's lift force formula can be obtained on the basis of literature review, which can almost cover all practical forms of asymmetric capacity forms. But there are still some problems we should solve. The first and foremost one is whether the formulas are correct and can they be verified in engineering practices? On the contrary, the parameter q in the formulas is normally unknown in the beginning of calculations, how can we get or reckon up it so as to use the formulas smoothly? In this paper, we set out to solve these questions.

1. Introduction

How can we solve lift force produced by a lifter formed of an asymmetric capacitor? Based on some hypothetical conditions, a formula was obtained through three methods in an ideal scenario [1, 2]. But an unknown parameter q is still contained so that numerical calculations are difficult to carry out. This paper intends to solve this problem by eliminating the unknown factor in the hope that the formula can be effortlessly put into practical application and engineering. Following that, experimental tests and practical estimations are provided to verify its validity.

In former papers [1, 2], the same result is acquired through three ways using the following equation of electric lift force of asymmetric capacitor loaded by high voltage in ideal condition:

$$f = \frac{q^2}{\varepsilon} \left(\frac{1}{S_1} - \frac{1}{S_2} \right). \quad (1)$$

An unknown variable q is still included in the above formula. In order to solve this problem thoroughly, the carried charge q should be figured out. Normally, carried charge q of capacitor is relevant to the voltage U and the capacitance C . The voltage U can be known. But the capacitance C is difficult to calculate when the capacitor is in irregular shape.

Nevertheless, the analysis of hypothetical predetermined conditions verifies that the capacitance C of the asymmetric capacity is calculable. When the small plate of asymmetric capacitor is in a slender cylinder form, its capacitance could be estimated at a cylindrical way. When the small plate of asymmetric capacitor [3–5] is in sphere form, its capacitance could be estimated at a spherical way. The result might not be ideal in the case of precision. It can still be applied to estimation in engineering assessment [6, 7]. Furthermore, the subsequent test data verified that the estimate result was fairly accurate unexpectedly.

2. Theoretical Derivation

Regarding the reason why the experimental result is more precise than expected, the analysis of the unique characteristics of the asymmetric capacitor has presented several objective reasons: (1) the distance d between two plates is more larger than the dimension of surface area S_1 of plate 1 (small plate), that is, $d \gg S_1/l$ or $d \gg \sqrt{S_1}$; (2) the area of plate 2 (large plate) is larger than that of plate 1, that is, $S_2 \gg S_1$; and (3) the voltage loaded between two plates is below the breakdown voltage that is relevant to the gap distance.

Under the initial condition, we begin to deduce capacitance of the asymmetric capacitor [8–10] and then to estimate its lift force [11, 12]. Deducing processes are as follows:

- (1) For $S_2 \gg S_1$, when high voltage is loaded on two plates, the electric field intensity on plate 1, $E_1 = q/\epsilon S_1$, is larger than that on plate 2, $E_2 = q/\epsilon S_2$, i.e., $E_1 \gg E_2$. It leads to the voltage drop $\Delta U = \Delta d \cdot E$, which mainly centralizes around plate 1. So when calculating the capacitance $C = q/\Delta U = q/\int_{U_1}^{U_2} dU$, the field intensity near plate 1 should be taken into major consideration. That is to say, the capacitance calculation can be carried out by combining the following equations (2), (3), (4), (5), and (6):

$$E = \frac{q}{\epsilon S}, \quad (2)$$

$$du = -E \cdot dr, \quad (3)$$

$$C = \frac{q}{\Delta U}, \quad (4)$$

$$r \propto \frac{S}{l} \text{ (for thin wire and board plates),} \quad (5)$$

$$r \propto \sqrt{S} \text{ (for sphere point and board plates).} \quad (6)$$

Equations (5) or (6) can be also written as

$$r = k_{sh1} \cdot \frac{S}{l}, \quad (7)$$

$$r = k_{sh2} \cdot \sqrt{S},$$

where r is the nominal dimensional size of plate 1, l is the length of plate 1, and k_{sh1} and k_{sh2} are the shape coefficient relevant to the plates' structure size.

- (2) Because the distance d between two plates is far larger than the nominal size of plate 1 r , to simplify the calculation, we assume that surface charge of plate 1 is uniformly distributed, and voltage drop of thin wire plate or spherical capacitor plate is integrated for estimating the capacitance in magnitudes. The details are shown as follows.

For a thin wire small plate capacitor, we can take

$$\begin{aligned} S_1 &= 2\pi R_1 l, \\ S_2 &= 2H_2 l, \end{aligned} \quad (8)$$

where H_2 is the width of the board plate.

Because

$$E = \frac{q}{\epsilon \cdot S}, \quad (9)$$

referring to equation (3), we get

$$du = -\frac{q}{\epsilon \cdot S} \cdot dr. \quad (10)$$

Mainly considering the electric field variation beside the thin wire, we have

$$S_{cir} = 2\pi r l. \quad (11)$$

Considering the effective fan-shaped part, we have

$$\begin{aligned} S_{fan} &= S_{cir} \cdot \int_{R_1+d}^{R_1+d+H_2} \frac{2dr}{2\pi r} \\ &= S_{cir} \cdot \frac{1}{\pi} \ln \left(\frac{R_1+d+H_2}{R_1+d} \right) \\ &= 2rl \cdot \ln \left(1 + \frac{H_2}{R_1+d} \right), \end{aligned} \quad (12)$$

$$\Rightarrow dS = dS_{fan}$$

$$\Rightarrow dr = \frac{1}{2l \ln \left(1 + \frac{H_2}{R_1+d} \right)} dS.$$

Integrating both sides of equation (10), we obtain

$$\begin{aligned}
 \Delta U &= \int_{U_1}^{U_2} du = - \int_{r_1}^{r_2} \frac{q}{\epsilon S} dr \\
 &= -\frac{q}{\epsilon} \int_{S_1}^{S_2} \frac{1}{2Sl \ln(1 + (H_2/(R_1 + d)))} dS \\
 &= -\frac{q \ln(S_2/S_1)}{2\epsilon l \ln(1 + (H_2/(R_1 + d)))} \\
 &= (q \ln[2\pi H_2 l / (2\pi R_1 l \cdot (1/\pi) \ln((R_1 + d + H_2)/(R_1 + d)))] (2\epsilon l \ln(1 + (H_2/(R_1 + d))))^{-1}, \\
 &= -((q \ln[H_2/R_1 \ln(1 + (H_2/(R_1 + d)))] (2\epsilon l \ln(1 + (H_2/(R_1 + d))))^{-1}).
 \end{aligned} \tag{13}$$

So we get the capacitance

$$C = \frac{q}{-\Delta U} = \frac{2\epsilon l \ln(1 + (H_2/(R_1 + d)))}{\ln[H_2/R_1 \ln(1 + (H_2/(R_1 + d)))]}. \tag{14}$$

For a spherical small plate capacitor, we can take

$$S_1 = 4\pi r^2. \tag{15}$$

Combining equation (2), we get

$$E = \frac{q}{\epsilon \cdot 4\pi r^2}. \tag{16}$$

Referring to equation (3), we get

$$du = -\frac{q}{\epsilon \cdot 4\pi r^2} \cdot dr. \tag{17}$$

Integrating both sides, we obtain

$$\Delta U = \int_{U_1}^{U_2} du = - \int_{R_1}^{R_1+d} \frac{q}{\epsilon \cdot 4\pi r^2} dr. \tag{18}$$

For the distance $d \gg R_1$, we have

$$\begin{aligned}
 \Delta U &= U_2 - U_1 = - \int_{R_1}^{R_1+d} \frac{q}{4\pi r^2 \epsilon} dr \\
 &= -\frac{q}{4\pi \epsilon} \left(\frac{1}{R_1} - \frac{1}{R_1 + d} \right) \approx -\frac{q}{4\pi \epsilon R_1}.
 \end{aligned} \tag{19}$$

So we get the capacitance

$$C = \frac{q}{-\Delta U} = 4\pi \epsilon R_1. \tag{20}$$

(3) We can calculate the electric lift force of asymmetric capacitor loaded by high voltage with the capacitance C .

For thin wire small plate capacitor, using equation (14),

$$\begin{aligned}
 f &= \frac{q^2}{\epsilon} \cdot \left(\frac{1}{S_1} - \frac{1}{S_2} \right) = \frac{(UC)^2}{\epsilon} \left(\frac{1}{S_1} - \frac{1}{S_2} \right) \\
 &= \frac{1}{\epsilon} \left\{ U \cdot \frac{2\epsilon l \ln(1 + (H_2/(R_1 + d)))}{\ln[H_2/R_1 \ln(1 + (H_2/(R_1 + d)))]} \right\}^2 \left(\frac{1}{2\pi R_1 l} - \frac{1}{2H_2 l} \right) \\
 &= \frac{2\epsilon U^2 \ln^2(1 + (H_2/(R_1 + d)))}{\ln^2[H_2/R_1 \ln(1 + (H_2/(R_1 + d)))]} \left(\frac{1}{\pi R_1} - \frac{1}{H_2} \right).
 \end{aligned} \tag{21}$$

This is the lift force formula about a normal lifter in thin wire asymmetric capacitor form under high voltage loaded.

For spherical small plate capacitor, using equation (20),

$$\begin{aligned}
 f &= \frac{q^2}{\varepsilon} \cdot \left(\frac{1}{S_1} - \frac{1}{S_2} \right) \\
 &= \frac{(UC)^2}{\varepsilon} \cdot \left(\frac{1}{S_1} - \frac{1}{S_2} \right) \\
 &= \frac{(U \cdot 4\pi R\varepsilon)^2}{\varepsilon} \cdot \left(\frac{1}{S_1} - \frac{1}{S_2} \right).
 \end{aligned} \tag{22}$$

Considering the condition $S_2 \gg S_1$, we have

$$f = \frac{(U \cdot 4\pi R\varepsilon)^2}{\varepsilon} \cdot \frac{1}{S_1} = U^2 \cdot 4\pi\varepsilon \cdot 4\pi R^2 \cdot \frac{1}{S_1}. \tag{23}$$

If simplifying calculation as a spherical plate, the surface area of plate 1 $S_1 = 4\pi R^2$, we can get

$$f = 4\pi\varepsilon U^2. \tag{24}$$

This is the concised formula that finally turned out, from which we can tell the maximum lift force produced by spherical asymmetric capacitor under high voltage loaded.

3. Formula Application

The formulas are exerted on two applications to test their validity. They are, respectively, lift force estimation of a electricity lifter [13, 14] and a high-voltage rising hair experiment [15, 16].

3.1. Lift Force Estimation of a Lifter. When a lifter loads with high voltage (Figure 1) $U = 30$ kV, what is the lift force produced by the lifter? The physical sizes (Figure 2) of the lifter are, respectively, length of thin wire $l = 15$ cm $\times 3 = 0.45$ m, radius of thin wire $R_1 = 0.08$ mm $= 8 \times 10^{-5}$ m, width of large board $H_2 = 30$ cm $= 0.3$ m, and gap between 2 plates $d = 40$ cm $= 0.4$ m. We take permittivity of atmosphere $\varepsilon = 8.85 \times 10^{-12}$ C²N⁻¹m⁻².

On the initial conditions, using equation (21), we have

$$\begin{aligned}
 f &= \left((2\varepsilon l U^2 \ln^2(1 + (H_2/(R_1 + d)))) (\ln^2[H_2/R_1 \ln(1 + (H_2/(R_1 + d)))])^{-1} \right) \cdot \left(\frac{1}{\pi R_1} - \frac{1}{H_2} \right) \\
 &= (2 \times 8.85 \times 10^{-12} \times 0.45 \times 30000^2 \times \ln^2(1 + (0.3/(8 \times 10^{-5} + 0.4)))) (\ln^2[0.3/8 \times 10^{-5} \times \ln(1 + (0.3/(8 \times 10^{-5} + 0.4)))])^{-1} \\
 &\quad \cdot \left(\frac{1}{\pi \times 8 \times 10^{-5}} - \frac{1}{0.3} \right) \\
 &= 0.115 \text{ N} \approx 11.7 \text{ gf}.
 \end{aligned} \tag{25}$$

That is to say, a lifter loaded with 30 kV voltage can produce a largest lift force of 11.7 gf.

3.2. Lift Force Estimation of High-Voltage Charged Conducting Sphere. As we know, when a high voltage loads on human body, our hair may be lifted up by the static electricity [17]. But there is no precise data or concrete calculating method of the length of the lifted hair by the high voltage. By equation (24), the mentioned problem can be solved. We can use the formula to quantitatively calculate the hair length lifted by the static electric field. The details are shown as follows.

3.2.1. Initial Conditions. Voltage loaded on the head $U = 100$ kV, diameter of the human head $D = 10$ cm $= 1 \times 10^{-2}$ m, average diameter of the hair $D_h = 70$ μ m $= 70 \times 10^{-6}$ m, density of the hair $\rho = 1.25$ g/cm³ $= 1.25 \times 10^3$ kg/m³, and permittivity of atmosphere $\varepsilon = 8.85 \times 10^{-12}$ C²N⁻¹m⁻².

3.2.2. Target Problem. When the voltage is loaded on the hair under the above initial conditions, what is the maximum length of the hair ($l_h = ?$) that can be lifted up?

3.2.3. Solving Process. In this case, the head and ground can be considered as the two plates of asymmetric, where the head may be regarded as a small plate and its area of sphere surface is S_1 and the distant ground as a large plate and its area of flat surface S_2 . However, $S_2 \gg S_1$, and the distance between the two plates $d \gg \sqrt{S_1}$. The voltage loaded between small plate (head) and large plate (ground) should not reach to breakdown threshold. Under this condition, we apply equation (24) to calculate the electrostatic lift force acted on hairs.

The sum of the electrostatic lift force acted on the hair is

$$\begin{aligned}
 f &= 4\pi\varepsilon U^2 \\
 &= 4 \times 3.14 \times 8.85 \times 10^{-12} \times (100 \times 10^3)^2 \\
 &= 1.11 \text{ N}.
 \end{aligned} \tag{26}$$



FIGURE 1: A lifter flying up loaded with high-voltage power.

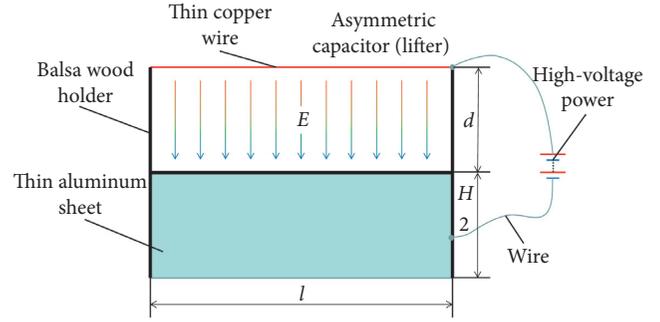


FIGURE 2: An asymmetric capacitor in lifter form.

100 kV through a conducting metallic ball, approximately 29 cm length hair floats up into air.

The surface area of the head is

$$\begin{aligned} S_1 &= \pi D^2 \\ &= 3.14 \times (10^{-2})^2 \\ &= 3.14 \times 10^{-4} \text{m}^2. \end{aligned} \quad (27)$$

The intensity of pressure supported by electrostatic force is

$$p = \frac{f}{S_1} = \frac{1.11 \times 10^6}{3.14 \times 10^{-4}} = 3.54 \times 10^3 \text{Pa}. \quad (28)$$

The average transverse area of a hair is

$$\begin{aligned} S_h &= \frac{\pi D_h^2}{4} \\ &= \frac{3.14 \times (70 \times 10^{-6})^2}{4} \\ &= 3.8 \times 10^{-9} \text{m}^2. \end{aligned} \quad (29)$$

Head surface area occupied by a hair can support a mass by the electrostatic force:

$$\begin{aligned} m_h &= \frac{G_h}{g} \\ &= \frac{3.8 \times 10^{-9} \times 3.54 \times 10^3}{9.8} \\ &= 1.4 \times 10^{-6} \text{kg}. \end{aligned} \quad (30)$$

The mass is converted into length of a hair:

$$\begin{aligned} l_h &= \frac{V_h}{S_h} \\ &= \frac{1.4 \times 10^{-6}}{1.25 \times 10^3 \times 3.8 \times 10^{-9}} \text{m} \\ &= 0.29 \text{m} \end{aligned} \quad (31)$$

Therefore, the final result is obtained: when human being's hair is loaded by high-voltage static electricity of DC

4. Explanation and Conclusion

Based on some assumptions with simplified calculation, we derived lift force formula produced by an asymmetric capacitor in different conditions, with which the assess in certain survey and qualitative research can be undertaken in spite of unsatisfying precision. The method also provides a convenient way to calculate static electricity lift capacity produced by an asymmetric capacitor or lift force of lifters. It also contributes to the parameter optimization in designing [18] a larger load force of lifter formed by an asymmetric capacitor.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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