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Research Article

Adaptive Vector Nonsingular Terminal Sliding Mode Control for a Class of n-Order Nonlinear Dynamical Systems with Uncertainty

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This paper proposed an adaptive vector nonsingular terminal sliding mode control (NTSMC) algorithm for the finite-time tracking control of a class of n-order nonlinear dynamical systems with uncertainty. The adaptive vector NTSMC incorporates a vector design idea and novel adaptive updating laws based on the commonly used NTSMC, which consider the common existence of the degree-of-freedom (DOF) directional differences and eliminate the chattering problem. The closed-loop stability of the n-order nonlinear dynamical systems under the adaptive vector NTSMC is demonstrated using Lyapunov direct method. Simulations performed on a two-degree-of-freedom (DOF) manipulator are provided to illustrate the effectiveness and advantages of the proposed adaptive vector NTSMC by comparing with the common NTSMC.

1. Introduction

Sliding mode control (SMC), which provides invariance to uncertainty, is one of the effective and efficient nonlinear robust control schemes [1, 2]. It has been successfully implemented in many systems, such as induction motor [3], Stewart platform [4], car-like mobile robots [5], and PMSM speed regulation system [6, 7]. Essentially, two basic components cause the characteristics of the SMC: a driving effort that forces the system states to reach and stay on a stable hyperplane, and a sliding surface achieves the desired error dynamics [8]. Commonly, the linear sliding hyperplane assures asymptotic stability of the system in the sliding mode but cannot make the system state errors converge to zero at a finite time.

To achieve the finite-time convergence of the system state errors, terminal SMC (TSMC) has been derived by introducing a nonlinear sliding mode to provide faster convergence than the linear hyperplane-based sliding mode [8, 9]. However, TSMC has a singularity problem [10]; accordingly, nonsingular TSMC (NTSMC) has been successfully developed to avoid the singularity [10, 11]. Thus, owing to its advantages of insensitive to uncertainty, finite-time convergence, TSMC, and NTSMC, as variant schemes of SMC, have attracted great attention [12–14] and have also been widely adopted in both linear and nonlinear uncertain systems [15–19].

In the past decade, the appropriate selection of the uncertainty upper bound of the TSMC/NTSMC has been a hot topic, which may cause serious chattering problem as a large one or existence of tracking errors as a low one. To tackle the problem of upper bound design, the adaptive algorithms have been widely investigated in recent years [20–29]. Since the fuzzy logic and neural networks are universal function approximators, the adaptive TSMC/NTSMC schemes which integrated the unknown dynamic-learning algorithms using the function approximators have developed quickly [20–23]. Moreover, other adaptive updating laws have also been proposed to improve the control performances and have been adopted in the multiple motion axis systems [24], DC-DC buck converters [25], uncertain nonlinear SISO systems [26], nonlinear differential inclusion systems [27], and the electromechanical actuator [28], and so on. However, the uncertainty upper bound of the previous adaptive schemes without the function approximators is designed as a constant, which cannot analyze the common existence of the DOF directional different characteristics. Fortunately, few available control schemes without the finite-convergence performance, whose upper bounds of the uncertainty are designed as vector numbers, have been proposed for spacecraft formation flying [29], a class of MIMO nonlinear systems [30].

Motivated by the above discussion, this paper considers the finite-time stabilization for a class of n-order nonlinear dynamical systems with unknown uncertainty upper bound. The main contributions of this work can be summarized as follows: (i) the upper bound of the uncertainty is designed as a vector, which can analyze the DOF directional different characteristics; (ii) novel adaptive updating laws for the vector upper bound of the uncertainty are derived to improve the performance of usually NTSMC; and (iii) the closed-loop stability of the n-order nonlinear dynamical systems under the proposed adaptive vector NTSMC is demonstrated using Lyapunov direct method.

2. Preliminaries

The kinematic and dynamic equations of a class of n-order nonlinear dynamical systems can be described as follows:

$$\dot{x}_1 = \mathbf{f}_1(\mathbf{x}_1, \mathbf{x}_2), \tag{1}$$

$$\dot{x}_2 = \mathbf{f}_2(\mathbf{x}_1, \mathbf{x}_2) + \mathbf{G}(\mathbf{x}_1, \mathbf{x}_2)\mathbf{u}, \tag{2}$$

where $\mathbf{x}_1 \in \Re^n$ and $\mathbf{x}_2 \in \Re^n$ represent the system states; $\mathbf{u} \in \Re^n$ is the control vector; $\mathbf{f}_1(\mathbf{x}_1, \mathbf{x}_2) \in \Re^n$ and $\mathbf{f}_2(\mathbf{x}_1, \mathbf{x}_2) \in \Re^n$ are smooth vector functions; and $\mathbf{G}(\mathbf{x}_1, \mathbf{x}_2) \in \Re^{n\times n}$ denotes a nonsingular matrix. The class of n-order nonlinear dynamical system model can be used to describe many physical systems, such as robot manipulators, spacecraft [31], and conventional mechanical systems.

Considering the uncertainty, such as unmodeled dynamics, parameter variation, and external disturbances, the dynamics of the n-order uncertain nonlinear dynamical system can be rewritten as

$$\dot{x}_2 = \mathbf{f}_{2,0} + \Delta_{f2} + \left(\mathbf{G}_0 + \Delta_g\right)\mathbf{u}, \tag{3}$$

where $\mathbf{f}_{2,0}$ and $\mathbf{G}_0$ denote the estimated terms and $\Delta\mathbf{f}_2$ and $\Delta\mathbf{G}$ are the uncertain terms.

(3) can be further rearranged as

$$\dot{\mathbf{x}}_2 = \mathbf{f}_{2,0} + \mathbf{G}_0\mathbf{u} + \boldsymbol{\delta}_{\text{total},n}, \tag{4}$$

$$\boldsymbol{\delta}_{\text{total},n} = \Delta_{f2} + \Delta_g\mathbf{u}. \tag{5}$$

By recalling that the n-order uncertain nonlinear dynamical system is considered (described by (1), (4), and (5)), the following properties can be obtained.

Assumption 1. $\boldsymbol{\delta}_{\text{total},n}$ denotes the uncertainties and disturbances satisfying

$$\left\|\boldsymbol{\delta}_{\text{total},n}\right\| \le b_{1,n} + b_{2,n}\left\|\mathbf{x}_1\right\| + b_{3,n}\left\|\mathbf{x}_2\right\|^2, \tag{6}$$

where $b_{1,n}$, $b_{2,n}$, and $b_{3,n}$ are the positive numbers and $\| \|$denotes the norm operation.

Remark 1. The mathematics model to describe the physical systems is simplified, which is hard to employ the accuracy parameters to consider the uncertainties such as the unmodeled dynamics, parameter variation, and external disturbances. Generally, the boundary of the total uncertainty is assumed to be a constant [12, 14], which is usually enlarged to satisfy the assumption. In practice, the uncertainty is related to the state of the physical systems, such as the motion command of the robots. Therefore, Assumption 1 proposed a more feasible uncertainty boundary considering the influence of the system states, which includes the general constant total uncertainty boundary assumption.

Remark 2. In practice, it is probably the existence of the following conditions: (i) there are serious discrepancies between the dynamical characteristics of the different DOF directions; (ii) some DOF directions have special tracking performance demands. Besides, the system state elements ($x_{1,i}$ and $x_{2,i}$) commonly represent special physical meanings, such as the DOF directional values and control command directional values, which can be directly or indirectly transformed as the DOF directional values. For instance, the DOF directional dynamical performances of the typical Stewart platform are different, especially for the heave direction. Therefore, it is an urgent work to propose a vector NTSMC scheme to consider different design demands or dynamical characteristics corresponding to the special DOF directions.

In this paper, we aim to design a control scheme to obtain the satisfactory tracking performances of the n-order uncertain nonlinear dynamical systems with uncertainty (described by (1), (4), and (5)) and make the tracking error of the nonlinear systems converge to zero in finite time.

3. Control Development

In this section, two trajectory tracking controllers are proposed for the n-order uncertain nonlinear dynamical systems with uncertainty (described by (1), (4), and (5)), which can achieve the finite-time convergence of the system state errors.

3.1. NTSMC for Nonlinear Systems. The commonly utilized NTSMC algorithm proposed by Feng et al. has been successfully employed in the rigid manipulators and also been extended to the n-order nonlinear dynamical systems [11]. However, in [11], the stability analysis of the NTSMC for the n-order nonlinear dynamical systems has not been mentioned, and the controller is available for the trajectory stabilization of the n-order nonlinear dynamical systems. To overcome this problem, the corresponding NTSMC scheme for the tracking control of the n-order nonlinear dynamical systems is introduced in this section.

To aid the subsequent control development, we define the vector $\mathbf{V}_{\text{vec}}(\cdot) \in \mathfrak{R}^{n\times 1}$ and matrix $\mathbf{D}_{\text{diag}}(\cdot) \in \mathfrak{R}^{n\times n}$ as follows:

$$\mathbf{V}_{\text{vec}}(z_i) = [z_1, \ldots, z_n]^{\text{T}}, \tag{7}$$

$$\mathbf{D}_{\text{diag}}(z_i) = \text{diag}(z_1, \ldots, z_n), \tag{8}$$

where $\text{diag}(\cdot)$ represents the diagonal matrix function.

Based on the definition of (7) and (8), the sliding surface considered different DOF directional design demands which can be formulated as

$$\mathbf{s} = \boldsymbol{\varepsilon}_1 + \boldsymbol{\Lambda}\mathbf{V}_{\text{vec}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i}\right), \tag{9}$$

$$\boldsymbol{\varepsilon}_1 = \mathbf{x}_1 - \mathbf{x}_{1,d}, \tag{10}$$

$$\boldsymbol{\Lambda} = \mathbf{D}_{\text{diag}}(\lambda_i), \tag{11}$$

where $\mathbf{x}_{1,d}$ is the desired system state; $(\cdot)_i$ means the ith element of $(\cdot)$; p_i, q_i, and λ_i $(i = 1, \ldots, n)$ are positive numbers; and the condition $(1 < p_i/q_i < 2)$ must be satisfied to achieve the nonsingularity of the NTSMC control [11].

The proposed NTSMC scheme can be formulated as

$$\begin{aligned}\mathbf{u}_{\text{antsmc},n} = -\left(\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\mathbf{G}_0\right)^{+}\Bigg(&\boldsymbol{\Psi}\|\mathbf{s}\|\left\|\mathbf{D}_{\text{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\right\|\left(b_{1,n} + b_{2,n}\|\mathbf{x}_1\| + b_{3,n}\|\mathbf{x}_2\|^2\right)\\ &+ \frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_1}\mathbf{f}_1 + \frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\mathbf{f}_{2,0} + \Upsilon^{-1}\boldsymbol{\Lambda}^{-1}\mathbf{V}_{\text{vec}}\left(\dot{\varepsilon}_{1,i}^{2-p_i/q_i}\right) - \ddot{x}_{1,d}\Bigg),\end{aligned} \tag{12}$$

where

$$\Upsilon = \mathbf{D}_{\text{diag}}\left(\frac{p_i}{q_i}\right), \tag{13}$$

$$\boldsymbol{\Psi} = \mathbf{V}_{\text{vec}}\left(\frac{\left[s_i\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right]^{\text{T}}}{\left|s_i\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right|^2}\right). \tag{14}$$

Now, we are in a position to state the NTSMC control scheme for n-order nonlinear dynamical systems.

Theorem 1. *Given the n-order uncertain nonlinear dynamical systems of (1) and (4), the system tracking error $\boldsymbol{\varepsilon}_1$ will converge to zero in finite time under the NTSMC scheme (designed as (9) and (14)) if Assumption 1 holds.*

Proof of Theorem 1. The proof proceeding is divided into two parts: firstly, the finite-time convergence of the sliding surface of NTSMC (9) is proved based on Lyapunov method; secondly, the convergence time of the tracking error is calculated.

Step 1. To this end, the following Lyapunov-like function candidate is adopted:

$$V_{\text{ntsmc},n,i} = \frac{1}{2}s_i^{\text{T}}s_i. \tag{15}$$

Differentiating $V_{\text{ntsmc},n,i}$ with respect to time and substituting (9) and (10), we have

$$\begin{aligned}\dot{V}_{\text{ntsmc},n,i} &= s_i^{\text{T}}\dot{s}_i = s_i^{\text{T}}\left(\dot{\varepsilon}_1 + \Upsilon\boldsymbol{\Lambda}\mathbf{D}_{\text{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\varepsilon_1\right)_i\\ &= s_i^{\text{T}}\left(\dot{\varepsilon}_1 + \Upsilon\boldsymbol{\Lambda}\mathbf{D}_{\text{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\left(x_1 - \ddot{x}_{1,d}\right)\right)_i.\end{aligned} \tag{16}$$

According to system models (1) and (4), (16) can be further formulated as

$$\dot{V}_{\text{ntsmc},n,i} = s_i^{\text{T}}\left(\dot{\varepsilon}_1 + \Upsilon\boldsymbol{\Lambda}\mathbf{D}_{\text{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\left(\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_1}\mathbf{f}_1 + \frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\left(\mathbf{f}_{2,0} + \mathbf{G}_0\mathbf{u} + \boldsymbol{\delta}_{\text{total},n}\right) - x_{1,d}\right)\right)_i. \tag{17}$$

Replacing the control command $\mathbf{u}$ by $\mathbf{u}_{\text{antsmc},n}$ (12), (17) can be rearranged as

$$\begin{aligned}\dot{V}_{\text{ntsmc},n,i} &= s_i^{\mathrm{T}}\Bigg(\dot{\varepsilon}_1 + \Upsilon\mathbf{\Lambda}\mathbf{D}_{\text{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\Bigg(\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_1}\mathbf{f}_1 - \ddot{x}_{1,d} + \frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\Bigg(\mathbf{f}_{2,0} + \boldsymbol{\delta}_{\text{total,n}} + \mathbf{G}_0\Bigg(-\left(\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\mathbf{G}_0\right)^{+}\Bigg(\mathbf{\Psi}\|\mathbf{s}\|\left\|\mathbf{D}_{\text{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\right\|\left(b_{1,\text{n}} + b_{2,\text{n}}\|\mathbf{x}_1\| + b_{3,\text{n}}\|\mathbf{x}_2\|^2\right)\\
&\quad + \frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_1}\mathbf{f}_1 + \frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\mathbf{f}_{2,0} + \Upsilon^{-1}\mathbf{\Lambda}^{-1}\mathbf{V}_{\text{vec}}\left(\dot{\varepsilon}_{1,i}^{2-p_i/q_i}\right) - x_{1,d}\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)_i\\
&= s_i^{\mathrm{T}}\left(\dot{\varepsilon}_1 + \Upsilon\mathbf{\Lambda}\mathbf{D}_{\text{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\left(\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\boldsymbol{\delta}_{\text{total},n} - \mathbf{\Psi}\|\mathbf{s}\|\left\|\mathbf{D}_{\text{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\right\|\left(b_{1,n} + b_{2,n}\|\mathbf{x}_1\| + b_{3,\text{n}}\|\mathbf{x}_2\|^2\right) - \Upsilon^{-1}\mathbf{\Lambda}^{-1}\mathbf{V}_{\text{vec}}\left(\dot{\varepsilon}_{1,i}^{2-p_i/q_i}\right)\right)\right)_i.\end{aligned} \tag{18}$$

Consider the fact that

$$\Upsilon\mathbf{\Lambda}\mathbf{D}_{\text{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\Upsilon^{-1}\mathbf{\Lambda}^{-1}\mathbf{V}_{\text{vec}}\left(\dot{\varepsilon}_{1,i}^{2-p_i/q_i}\right) = \dot{\varepsilon}_1. \tag{19}$$

So, (18) can be given as follows by substituting (11), (13), and (19):

$$\begin{aligned}&= s_i^{\mathrm{T}}\left(\Upsilon\mathbf{\Lambda}\mathbf{D}_{\text{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\left(\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\boldsymbol{\delta}_{\text{total},n} - \mathbf{\Psi}\|\mathbf{s}\|\left\|\mathbf{D}_{\text{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\right\|\left(b_{1,n} + b_{2,n}\|\mathbf{x}_1\| + b_{3,n}\|\mathbf{x}_2\|^2\right)\right)\right)_i\\
&= s_i^{\mathrm{T}}\left(\mathbf{D}_{\text{diag}}\left(\frac{p_i}{q_i}\right)\mathbf{D}_{\text{diag}}(\lambda_i)\mathbf{D}_{\text{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\boldsymbol{\delta}_{\text{total},n}\right)_i\\
&\quad - s_i^{\mathrm{T}}\left(\mathbf{D}_{\text{diag}}\left(\frac{p_i}{q_i}\right)\mathbf{D}_{\text{diag}}(\lambda_i)\mathbf{D}_{\text{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\mathbf{\Psi}\|\mathbf{s}\|\left\|\mathbf{D}_{\text{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\right\|\left(b_{1,n} + b_{2,n}\|\mathbf{x}_1\| + b_{3,n}\|\mathbf{x}_2\|^2\right)\right)_i\\
&= s_i^{\mathrm{T}}\frac{p_i}{q_i}\lambda_i\left(\mathbf{D}_{\text{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\boldsymbol{\delta}_{\text{total},n}\right)_i\\
&\quad - s_i^{\mathrm{T}}\left(\mathbf{D}_{\text{diag}}\left(\frac{p_i}{q_i}\right)\mathbf{D}_{\text{diag}}(\lambda_i)\mathbf{D}_{\text{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\mathbf{\Psi}\|\mathbf{s}\|\left\|\mathbf{D}_{\text{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\right\|\left(b_{1,n} + b_{2,n}\|\mathbf{x}_1\| + b_{3,n}\|\mathbf{x}_2\|^2\right)\right)_i\\
&< \frac{p_i}{q_i}\lambda_i\|\mathbf{s}\|\left\|\mathbf{D}_{\text{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\right\|\left\|\boldsymbol{\delta}_{\text{total},n}\right\| - \frac{p_i}{q_i}\lambda_i s_i^{\mathrm{T}}\left(\mathbf{D}_{\text{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\mathbf{\Psi}\|\mathbf{s}\|\left\|\mathbf{D}_{\text{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\right\|\left(b_{1,n} + b_{2,n}\|\mathbf{x}_1\| + b_{3,n}\|\mathbf{x}_2\|^2\right)\right)_i.\end{aligned} \tag{20}$$

Substituting (14) into (20) yields

$$\begin{aligned}\dot{V}_{\text{ntsmc},n,i} &< -\frac{p_i}{q_i}\lambda_i\|\mathbf{s}\|\left\|\mathbf{D}_{\text{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\right\|\\
&\quad\cdot\left(b_{1,n} + b_{2,n}\|\mathbf{x}_1\| + b_{3,n}\|\mathbf{x}_2\|^2 - \left\|\boldsymbol{\delta}_{\text{total},n}\right\|\right).\end{aligned} \tag{21}$$

From (21), if Assumption 1 holds, we can conclude that $\dot{V}_{\text{ntsmc},n,i} < 0$. By LaSalle's invariant principle [9], we have $s_i(t) \longrightarrow 0$ for any initial state $(\mathbf{x}_{1,0}, \mathbf{x}_{2,0})$.

Step 2. According to (15) and (21), we have

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}s_i^{\mathrm{T}}s_i < -\frac{p_i}{q_i}\lambda_i\|\mathbf{s}\|\left\|\mathbf{D}_{\mathrm{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\right\| \cdot\left(b_{1,n}+b_{2,n}\|\mathbf{x}_1\|+b_{3,n}\|\mathbf{x}_2\|^2-\|\boldsymbol{\delta}_{\mathrm{total},n}\|\right). \tag{22}$$

Then, we define the following equation when $s_i \neq 0$:

$$\min\left(\frac{p_i}{q_i}\lambda_i\left\|\mathbf{D}_{\mathrm{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\right\|\left(b_{1,n}+b_{2,n}\|\mathbf{x}_1\| + b_{3,n}\|\mathbf{x}_2\|^2-\|\boldsymbol{\delta}_{\mathrm{total},n}\|\right)\right)=\eta_{1,i}>0, \tag{23}$$

where $\eta_{1,i}$ is a positive number and $\min(\cdot)$ is the minimum of $(\cdot)$.

Based on (23), (22) can be rewritten as

$$\frac{1}{2}\frac{\mathrm{d}}{\mathrm{d}t}s_i^{\mathrm{T}}s_i < -\eta_{1,i}\|\mathbf{s}\| < -\eta_{1,i}|s_i|. \tag{24}$$

So, if $s_i \neq 0$, the system states will reach the sliding mode $s_i = 0$ within the finite time $t_{r,i}$, which satisfies

$$t_{r,i} \leq -\frac{|s_i(0)|}{\eta_{1,i}}. \tag{25}$$

When the sliding mode $s_i = 0$ is achieved, the convergence time $t_{s,i}$ that is taken to travel from $\varepsilon_{1,i}(t_{r,i}) \neq 0$ to $\varepsilon_{1,i}(t_{r,i}+t_{s,i}) = 0$ is calculated by

$$t_{s,i} = -\lambda_i^{q_i/p_i}\int_{\varepsilon_{1,i}(t_{r,i})}^{0}\varepsilon_{1,i}^{-q_i/p_i}\mathrm{d}\varepsilon_{1,i} = \lambda_i^{q_i/p_i}\left(\frac{p_i}{p_i-q_i}\right)\left|\varepsilon_{1,i}^{1-q_i/p_i}(t_{r,i})\right|. \tag{26}$$

Therefore, the total finite time $t_{f,i}$ of the system states can be formulated by

$$t_{f,i} = t_{r,i} + t_{s,i}. \tag{27}$$

Moreover, the total finite time t_f for all of the system state elements can be given by

$$t_f = \max\left(t_{f,i}\right) = \max\left(t_{r,i}+t_{s,i}\right). \tag{28}$$

Hence, the asymptotic stability in finite time of the nonlinear dynamical systems under NTSMC has been proved.

Remark 3. The improved NTSMC in the previous section solves the following problems: (i) available for the trajectory tracking and trajectory stabilization; (ii) the asymptotic stability in finite time of the nonlinear dynamical systems under NTSMC has been proved. However, there are still some conservatisms of the proposed NTSMC: (i) the upper bound of the uncertainty is chosen as a constant value, but the actual uncertain upper boundary is unknown; (ii) the discrepancies between the dynamical characteristics of different DOF directions cannot be considered owing to the constant number designing of the upper bound.

Thus, an adaptive vector NTSMC algorithm for the n-order nonlinear dynamical systems is derived in the following section.

3.2. Adaptive Vector NTSMC for Nonlinear Systems

Assumption 2. To further consider the influences of different dynamical characteristics corresponding to the DOF directions, the total uncertainty is assumed to be bounded as (29), where $b_{1,n,i}$, $b_{2,n,i}$, and $b_{3,n,i}$ are positive numbers, and $|\cdot|$ means the absolute operation.

$$\left\|\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\right\|\left(b_{1,n,i}+b_{2,n,i}\|\mathbf{x}_1\|+b_{3,n,i}\|\mathbf{x}_2\|^2\right) > \left|\left(\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\boldsymbol{\delta}_{\mathrm{total},n}\right)_i\right|. \tag{29}$$

Remark 4. Assumption 1 considers the influence of the system states to induce the uncertainty boundary, which is beneficial to reduce the chattering of the SMC controller. For MIMO systems, the uncertainty boundary can be further reduced by considering different dynamical characteristics corresponding to the DOF directions, which is verified in the literature [4] to propose a novel sliding mode controller for the Stewart platform.

Then, the adaptive vector NTSMC can be formulated by

$$\mathbf{u}_{\mathrm{avntsmc},n} = -\left(\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\mathbf{G}_0\right)^{+}\left(\boldsymbol{\kappa}\|\mathbf{s}\|\left\|\mathbf{D}_{\mathrm{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\right\|\left\|\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\right\|\widehat{\Gamma}_{\mathrm{adaptive},n} + \frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_1}\mathbf{f}_1 + \frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\mathbf{f}_{2,0} + \Upsilon^{-1}\boldsymbol{\Lambda}^{-1}\mathbf{V}_{\mathrm{vec}}\left(\dot{\varepsilon}_{1,i}^{2-p_i/q_i}\right) - \ddot{x}_{1,d}\right), \tag{30}$$

$$\boldsymbol{\kappa} = \mathbf{D}_{\mathrm{diag}}\left(\frac{\left[s_i\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right]^{\mathrm{T}}}{\left|s_i\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right|^2}\right), \tag{31}$$

where $\widehat{\Gamma}_{\mathrm{adaptive},n} \in \mathfrak{R}^{n\times 1}$ represents the estimated upper bound, which is designed as a time-varying vector.

Moreover, the updating laws are chosen as

$$\widehat{\Gamma}_{\mathrm{adaptive},n,i} = \widehat{b}_{1,n,i} + \widehat{b}_{2,n,i}\|\mathbf{x}_1\| + \widehat{b}_{3,n,i}\|\mathbf{x}_2\|^2, \tag{32}$$

$$\dot{\widehat{b}}_{1,n,i} = d_{1,n,i}\frac{p_i}{q_i}\lambda_i\|\mathbf{s}\|\left\|\mathbf{D}_{\mathrm{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\right\|\left\|\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\right\|, \tag{33}$$

$$\dot{\widehat{b}}_{2,n,i} = d_{2,n,i}\frac{p_i}{q_i}\lambda_i\|\mathbf{s}\|\left\|\mathbf{D}_{\mathrm{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\right\|\left\|\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\right\|\|\mathbf{x}_1\|, \tag{34}$$

$$\dot{\widehat{b}}_{3,n,i} = d_{3,n,i}\frac{p_i}{q_i}\lambda_i\|\mathbf{s}\|\left\|\mathbf{D}_{\mathrm{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\right\|\left\|\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\right\|\|\mathbf{x}_2\|^2. \tag{35}$$

Therefore, the adaptive vector NTSMC scheme for the n-order nonlinear dynamical systems can be summarized as follows.

Theorem 2. *Given the n-order uncertain nonlinear dynamical systems of (1) and (4), the system tracking error* $\boldsymbol{\varepsilon}_1$ *will converge to zero in finite time under the adaptive vector NTSMC scheme (designed as (9) and (30)–(35)) if Assumption 2 holds.*

Proof of Theorem 2. The proof proceeding is divided into two parts: firstly, the finite-time convergence of the sliding surface of adaptive vector NTSMC (30) is proved based on Lyapunov method; secondly, the convergence time of the tracking error is calculated.

Step 1. Consider the following Lyapunov function:

$$V_{\mathrm{avntsmc},n,i} = \frac{1}{2}\left(s_i^{\mathrm{T}} s_i + \frac{1}{d_{1,n,i}}\tilde{b}_{1,n,i}^{\mathrm{T}}\tilde{b}_{1,n,i} + \frac{1}{d_{2,n,i}}\tilde{b}_{2,n,i}^{\mathrm{T}}\tilde{b}_{2,n,i} + \frac{1}{d_{3,n,i}}\tilde{b}_{3,n,i}^{\mathrm{T}}\tilde{b}_{3,n,i}\right), \tag{36}$$

and the mismatch between the actual and estimated value of $b_{1,n,i}$, $b_{2,n,i}$, and $b_{3,n,i}$ can be given by

$$\tilde{b}_{1,n,i} = b_{1,n,i} - \hat{b}_{1,n,i}, \tag{37}$$

$$\tilde{b}_{2,n,i} = b_{2,n,i} - \hat{b}_{2,n,i}, \tag{38}$$

$$\tilde{b}_{3,n,i} = b_{3,n,i} - \hat{b}_{3,n,i}. \tag{39}$$

Taking the time derivative of (36) and substituting (37)–(39), we can obtain

$$\dot{V}_{\mathrm{avntsmc},n,i} = s_i^{\mathrm{T}}\dot{s}_i - \frac{1}{d_{1,n,i}}\tilde{b}_{1,n,i}^{\mathrm{T}}\dot{\hat{b}}_{1,n,i} - \frac{1}{d_{2,n,i}}\tilde{b}_{2,n,i}^{\mathrm{T}}\dot{\hat{b}}_{2,n,i} - \frac{1}{d_{3,n,i}}\tilde{b}_{3,n,i}^{\mathrm{T}}\dot{\hat{b}}_{3,n,i}. \tag{40}$$

Similar to the proof process of Theorem 1, (40) can be further rearranged by substituting (9), (11), (13), (19), and (30):

$$\begin{aligned}
\dot{V}_{\mathrm{avntsmc},n,i} &= s_i^{\mathrm{T}}\left(\dot{\varepsilon}_1 + \Upsilon\boldsymbol{\Lambda}\mathbf{D}_{\mathrm{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\left(\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_1}\mathbf{f}_1 + \frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\left(\mathbf{f}_{2,0} + \mathbf{G}_0\mathbf{u}_{\mathrm{avntsmc},n} + \boldsymbol{\delta}_{\mathrm{total},n}\right) - \ddot{x}_{1,d}\right)\right)_i \\
&\quad - \frac{1}{d_{1,n,i}}\tilde{b}_{1,n,i}^{\mathrm{T}}\dot{\hat{b}}_{1,n,i} - \frac{1}{d_{2,n,i}}\tilde{b}_{2,n,i}^{\mathrm{T}}\dot{\hat{b}}_{2,n,i} - \frac{1}{d_{3,n,i}}\tilde{b}_{3,n,i}^{\mathrm{T}}\dot{\hat{b}}_{3,n,i} \\
&= \frac{p_i}{q_i}\lambda_i s_i^{\mathrm{T}}\left(\mathbf{D}_{\mathrm{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\boldsymbol{\delta}_{\mathrm{total},n}\right)_i - \frac{p_i}{q_i}\lambda_i s_i^{\mathrm{T}}\left(\mathbf{D}_{\mathrm{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\boldsymbol{\kappa}\|\mathbf{s}\|\left\|\mathbf{D}_{\mathrm{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\right\|\left\|\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\right\|\hat{\Gamma}_{\mathrm{adaptive},n}\right)_i \\
&\quad - \frac{1}{d_{1,n,i}}\tilde{b}_{1,n,i}^{\mathrm{T}}\dot{\hat{b}}_{1,n,i} - \frac{1}{d_{2,n,i}}\tilde{b}_{2,n,i}^{\mathrm{T}}\dot{\hat{b}}_{2,n,i} - \frac{1}{d_{3,n,i}}\tilde{b}_{3,n,i}^{\mathrm{T}}\dot{\hat{b}}_{3,n,i}.
\end{aligned} \tag{41}$$

Then, (41) can be rewritten as

$$\begin{aligned}
\dot{V}_{\mathrm{avntsmc},n,i} &< \frac{p_i}{q_i}\lambda_i\|\mathbf{s}\|\left\|\mathbf{D}_{\mathrm{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\right\|\left|\left(\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\boldsymbol{\delta}_{\mathrm{total},n}\right)_i\right| - \frac{p_i}{q_i}\lambda_i s_i^{\mathrm{T}}\left(\mathbf{D}_{\mathrm{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\boldsymbol{\kappa}\|\mathbf{s}\|\left\|\mathbf{D}_{\mathrm{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\right\|\left\|\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\right\|\hat{\Gamma}_{\mathrm{adaptive},n}\right)_i \\
&\quad - \frac{1}{d_{1,n,i}}\tilde{b}_{1,n,i}^{\mathrm{T}}\dot{\hat{b}}_{1,n,i} - \frac{1}{d_{2,n,i}}\tilde{b}_{2,n,i}^{\mathrm{T}}\dot{\hat{b}}_{2,n,i} - \frac{1}{d_{3,n,i}}\tilde{b}_{3,n,i}^{\mathrm{T}}\dot{\hat{b}}_{3,n,i}.
\end{aligned} \tag{42}$$

Substituting (31) into (42) gives

$$\begin{aligned}
\dot{V}_{\mathrm{avntsmc},n,i} &< -\frac{p_i}{q_i}\lambda_i\|\mathbf{s}\|\left\|\mathbf{D}_{\mathrm{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\right\|\left(\left\|\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\right\|\hat{\Gamma}_{\mathrm{adaptive,n},i} - \left|\left(\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\boldsymbol{\delta}_{\mathrm{total},n}\right)_i\right|\right) \\
&\quad - \frac{1}{d_{1,n,i}}\tilde{b}_{1,n,i}^{\mathrm{T}}\dot{\hat{b}}_{1,n,i} - \frac{1}{d_{2,n,i}}\tilde{b}_{2,n,i}^{\mathrm{T}}\dot{\hat{b}}_{2,n,i} - \frac{1}{d_{3,n,i}}\tilde{b}_{3,n,i}^{\mathrm{T}}\dot{\hat{b}}_{3,n,i}.
\end{aligned} \tag{43}$$

Substituting (32)–(35) and (37)–(39) into (43) yields

$$\begin{aligned}
&\dot{V}_{\mathrm{avntsmc},n,i}\\
&= -\frac{p_i}{q_i}\lambda_i\|\mathbf{s}\|\left\|\mathbf{D}_{\mathrm{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\right\|\left(\left\|\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\right\|\left(\hat{b}_{1,n,i}+\hat{b}_{2,n,i}\|\mathbf{x}_1\|+\hat{b}_{3,n,i}\|\mathbf{x}_2\|^2\right)-\left|\left(\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\boldsymbol{\delta}_{\mathrm{total},n}\right)_i\right|\right)\\
&\quad -\frac{p_i}{q_i}\lambda_i\|\mathbf{s}\|\left\|\mathbf{D}_{\mathrm{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\right\|\left\|\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\right\|\left(\tilde{b}_{1,n,i}^{\mathrm{T}}+\tilde{b}_{2,n,i}^{\mathrm{T}}\|\mathbf{x}_1\|+\tilde{b}_{3,n,i}^{\mathrm{T}}\|\mathbf{x}_2\|^2\right)\\
&= -\frac{p_i}{q_i}\lambda_i\|\mathbf{s}\|\left\|\mathbf{D}_{\mathrm{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\right\|\left(\left\|\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\right\|\left(b_{1,n,i}+b_{2,n,i}\|\mathbf{x}_1\|+b_{3,n,i}\|\mathbf{x}_2\|^2\right)-\left|\left(\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\boldsymbol{\delta}_{\mathrm{total},n}\right)_i\right|\right).
\end{aligned} \tag{44}$$

Based on (44) and LaSalle's invariant principle [9], we have $s_i(t) \longrightarrow 0$ for any initial state $(\mathbf{x}_{1,0}, \mathbf{x}_{2,0})$ if Assumption 2 holds.

Step 2. We define the following equation when $s_i \neq 0$:

$$\min\left(\frac{p_i}{q_i}\lambda_i\left\|\mathbf{D}_{\mathrm{diag}}\left(\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right)\right\|\left(\left\|\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\right\|\left(b_{1,n,i}+b_{2,n,i}\|\mathbf{x}_1\| + b_{3,n,i}\|\mathbf{x}_2\|^2\right)-\left|\left(\frac{\partial \mathbf{f}_1}{\partial \mathbf{x}_2}\boldsymbol{\delta}_{\mathrm{total},n}\right)_i\right|\right)\right) = \eta_{2,i} > 0, \tag{45}$$

where $\eta_{2,i}$ is a positive number.

Therefore, the finite time for the ith system state element $t_{f,i}$ and total system states t_f under the adaptive vector NTSMC can be calculated by

$$\begin{aligned}
t_{s,i} &= -\lambda_i^{q_i/p_i}\int_{\varepsilon_{1,i}(t_{r,i})}^{0}\varepsilon_{1,i}^{-q_i/p_i}\mathrm{d}\varepsilon_{1,i}\\
&= \lambda_i^{q_i/p_i}\left(\frac{p_i}{p_i-q_i}\right)\left|\varepsilon_{1,i}^{1-q_i/p_i}\left(t_{r,i}\right)\right|,\\
t_{r,i} &\le -\frac{|s_i(0)|}{\eta_{2,i}},\\
t_{f,i} &= t_{r,i}+t_{s,i},\\
t_f &= \max\left(t_{f,i}\right) = \max\left(t_{r,i}+t_{s,i}\right).
\end{aligned} \tag{46}$$

Hence, the asymptotic stability in finite time of the nonlinear dynamical systems under adaptive vector NTSMC has been proved.

Remark 5. The advantages of the proposed adaptive vector NTSMC can be summarized as follows: (i) the vector design idea of the uncertain upper bound can analyze the DOF directional different characteristics; (ii) novel adaptive updating laws can online adjust the upper bound of the uncertainty, which can extensively eliminate the chattering problem of the SMC schemes.

Remark 6. To eliminate the chattering problem of the SMC, the following equations are adopted to replace $\mathbf{\Psi}$ (14) and $\boldsymbol{\kappa}$ (31) in Theorem 1 and Theorem 2:

$$\begin{aligned}
\mathbf{\Psi} &= \mathbf{V}_{\mathrm{vec}}\left(\frac{\left[s_i\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right]^{\mathrm{T}}}{\left(\left|s_i\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right|+\varsigma_i\right)^2}\right),\\
\boldsymbol{\kappa} &= \mathbf{D}_{\mathrm{diag}}\left(\frac{\left[s_i\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right]^{\mathrm{T}}}{\left(\left|s_i\dot{\varepsilon}_{1,i}^{p_i/q_i-1}\right|+\varsigma_i\right)^2}\right),
\end{aligned} \tag{47}$$

where ς_i is a positive number.

4. Application to the 2-DOF Manipulator

This section presents a comparison study of performance between the adaptive vector NTSMC and the NTSMC with the application to a 2-DOF robot manipulator.

The dynamics of a 2-DOF robot manipulator can be formulated using the Euler–Lagrange equations as follows [32, 33]:

$$\begin{aligned}
&\mathbf{M}(\mathbf{q})\ddot{q} + \mathbf{C}(\mathbf{q},\dot{q})\dot{q} + \mathbf{D}\dot{q} + \mathbf{g}(\mathbf{q}) = \boldsymbol{\tau},\\
&\mathbf{M}(\mathbf{q}) = \begin{bmatrix} k_1 + 2k_2\cos\left(q_{\mathrm{rob},2}\right) & k_3 + k_2\cos\left(q_{\mathrm{rob},2}\right)\\ k_3 + k_2\cos\left(q_{\mathrm{rob},2}\right) & k_3\end{bmatrix},\\
&\mathbf{C}(\mathbf{q},\dot{q}) = \begin{bmatrix} -k_2 sin\left(q_{\mathrm{rob},2}\right)\dot{q}_{\mathrm{rob},2} & -k_2 sin\left(q_{\mathrm{rob},2}\right)\dot{q}_{\mathrm{rob},2}\\ k_2 sin\left(q_{\mathrm{rob},2}\right)\dot{q}_{\mathrm{rob},1} & 0\end{bmatrix},\\
&\mathbf{D} = \mathbf{D}_{\mathrm{diag}}(0,\ldots,0),\\
&\mathbf{g}(\mathbf{q}) = \begin{bmatrix} k_4 sin\left(q_{\mathrm{rob},1}\right) + k_5 sin\left(q_{\mathrm{rob},1}+q_{\mathrm{rob},2}\right)\\ k_5 sin\left(q_{\mathrm{rob},1}+q_{\mathrm{rob},2}\right)\end{bmatrix},
\end{aligned} \tag{48}$$

where k_1, k_2, k_3, k_4, and k_5 (SI units) are 8.77, 0.51, 0.76, 74.48, and 6.174, respectively.

Based on the previous description, the NTSMC and adaptive vector NTSMC available for the two-DOF robot manipulator can be formulated as follows:

Sliding surface:

$$\mathbf{s}_{\text{rob}} = \boldsymbol{\varepsilon}_{\text{rob}} + \mathbf{D}_{\text{diag}}(\lambda_i)\mathbf{V}_{\text{vec}}\left(\dot{\varepsilon}_{\text{rob},i}^{p_i/q_i}\right). \tag{49}$$

NTSMC:

$$\boldsymbol{\tau}_{\text{ntsmc,rob}} = -\mathbf{M}_0\left(\begin{array}{l}\mathbf{V}_{\text{vec}}\left(\dfrac{\left[s_{\text{rob},i}\dot{\varepsilon}_{rob,i}^{p_i/q_i-1}\right]^{\text{T}}}{\left|s_{\text{rob},i}\dot{\varepsilon}_{rob,i}^{p_i/q_i-1}\right|^2}\right)\|\mathbf{s}_{\text{rob}}\|\left\|\mathbf{D}_{\text{diag}}\left(\dot{\varepsilon}_{rob,i}^{p_i/q_i-1}\right)\right\|\left(b_{1,\text{rob}} + b_{2,\text{rob}}\|\mathbf{q}\| + b_{3,\text{rob}}\|\dot{q}\|^2\right) + \mathbf{M}_0^{-1}\left(-\mathbf{C}_0(\mathbf{q},\dot{q})\dot{q}_d - \mathbf{D}_0\dot{q}_d - \mathbf{g}_0(\mathbf{q})\right)\\ + \mathbf{D}_{\text{diag}}^{-1}\left(\dfrac{p_i}{q_i}\right)\mathbf{D}_{\text{diag}}^{-1}(\lambda_i)\mathbf{V}_{\text{vec}}\left(\dot{\varepsilon}_{rob,i}^{2-p_i/q_i}\right) - \ddot{q}_d\end{array}\right). \tag{50}$$

Adaptive vector NTSMC:

$$\begin{aligned}\boldsymbol{\tau}_{\text{avntsmc,rob}} &= -\mathbf{M}_0\left(\begin{array}{l}\mathbf{D}_{\text{diag}}\left(\dfrac{\left[s_{\text{rob},i}\dot{\varepsilon}_{rob,i}^{p_i/q_i-1}\right]^{\text{T}}}{\left|s_{\text{rob},i}\dot{\varepsilon}_{rob,i}^{p_i/q_i-1}\right|^2}\right)\|\mathbf{s}_{\text{rob}}\|\left\|\mathbf{D}_{\text{diag}}\left(\dot{\varepsilon}_{rob,i}^{p_i/q_i-1}\right)\right\|\widehat{\Gamma}_{\text{adaptive,rob}} + \mathbf{M}_0^{-1}\left(-\mathbf{C}_0(\mathbf{q},\dot{q})\dot{q}_d - \mathbf{D}_0\dot{q}_d - \mathbf{g}_0(\mathbf{q})\right)\\ + \mathbf{D}_{\text{diag}}^{-1}\left(\dfrac{p_i}{q_i}\right)\mathbf{D}_{\text{diag}}^{-1}(\lambda_i)\mathbf{V}_{\text{vec}}\left(\dot{\varepsilon}_{rob,i}^{2-p_i/q_i}\right) - \ddot{q}_d\end{array}\right),\\
\widehat{\Gamma}_{\text{adaptive,rob},i} &= \widehat{b}_{1,\text{rob},i} + \widehat{b}_{2,\text{rob},i}\|\mathbf{q}\| + \widehat{b}_{3,\text{rob},i}\|\dot{q}\|^2,\\
\dot{\widehat{b}}_{1,\text{rob},i} &= d_{1,\text{rob},i}\frac{p_i}{q_i}\lambda_i\|\mathbf{s}_{\text{rob}}\|\left\|\mathbf{D}_{\text{diag}}\left(\dot{\varepsilon}_{rob,i}^{p_i/q_i-1}\right)\right\|,\\
\dot{\widehat{b}}_{2,\text{rob},i} &= d_{2,\text{rob},i}\frac{p_i}{q_i}\lambda_i\|\mathbf{s}_{\text{rob}}\|\left\|\mathbf{D}_{\text{diag}}\left(\dot{\varepsilon}_{rob,i}^{p_i/q_i-1}\right)\right\|\|\mathbf{q}\|,\\
\dot{\widehat{b}}_{3,\text{rob},i} &= d_{3,\text{rob},i}\frac{p_i}{q_i}\lambda_i\|\mathbf{s}_{\text{rob}}\|\left\|\mathbf{D}_{\text{diag}}\left(\dot{\varepsilon}_{rob,i}^{p_i/q_i-1}\right)\right\|\|\dot{q}\|^2.\end{aligned} \tag{51}$$

The desired and the initial position trajectories ($\mathbf{q}_d$, $\mathbf{q}_0$, and $\dot{\mathbf{q}}_0$) for links 1 and 2 are selected as

$$\begin{aligned}\mathbf{q}_d &= [\sin(\pi t), \sin(\pi t)]^{\text{T}},\\ \mathbf{q}_0 &= [0.5, 0.5]^{\text{T}},\\ \dot{\mathbf{q}}_0 &= [2.2, 0]^{\text{T}}.\end{aligned} \tag{52}$$

Considering the influences of the uncertainty and disturbance, the estimated parameters $k_{1,0}$, $k_{2,0}$, $k_{3,0}$, $k_{4,0}$, and $k_{5,0}$ of k_1, k_2, k_3, k_4, and k_5 are assumed to be 8.77, 0.51, 0.76, 97, and 6.174. Besides, the sampling period is set as 0.5 ms, and the saturation value of torque is set as 80 Nm. For fair comparison, the control parameters (except the uncertainty upper bound designing) of NTSMC and adaptive vector NTSMC are chosen as the same: $\lambda_1 = \lambda_2 = 1$, $p_1 = p_2 = 9$, $q_1 = q_2 = 5$, and $\varsigma_1 = \varsigma_2 = 0.001$. Moreover, to illustrate the effectiveness and advantages of the proposed adaptive vector NTSMC, three NTSMC with different upper bounds are given as follows:

NTSMC1:

$$0.8 + 1.8\|\mathbf{q}\| + 0.09\|\dot{\mathbf{q}}\|^2. \tag{53}$$

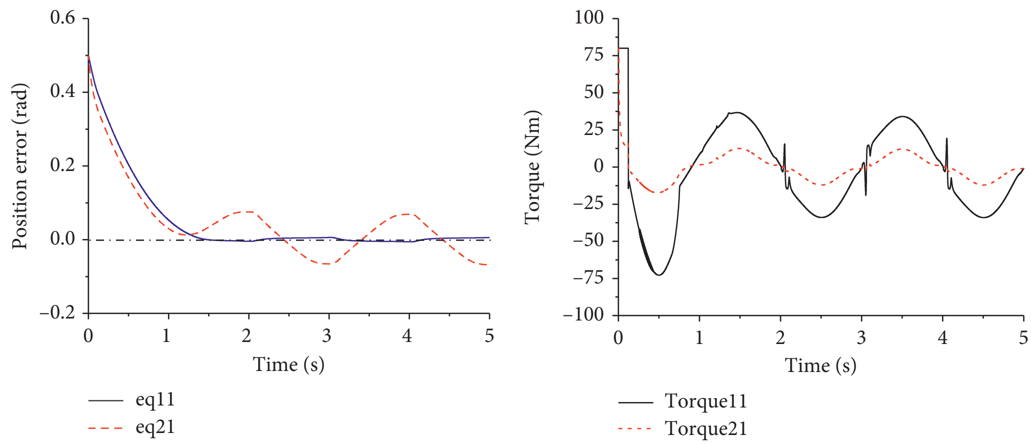
NTSMC2:

$$2.4 + 5.4\|\mathbf{q}\| + 0.27\|\dot{\mathbf{q}}\|^2. \tag{54}$$

NTSMC3:

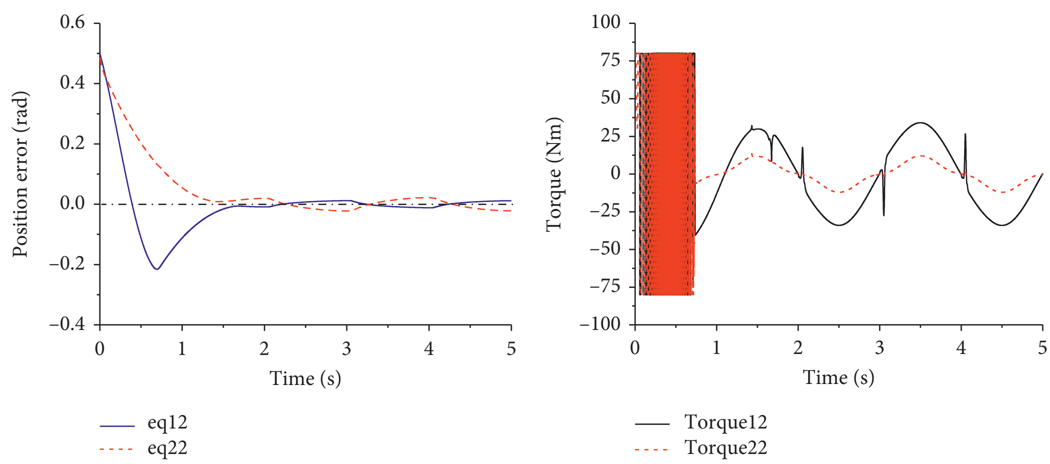
$$7.2 + 16.2\|\mathbf{q}\| + 0.8\|\dot{\mathbf{q}}\|^2. \tag{55}$$

Adaptive vector NTSMC:



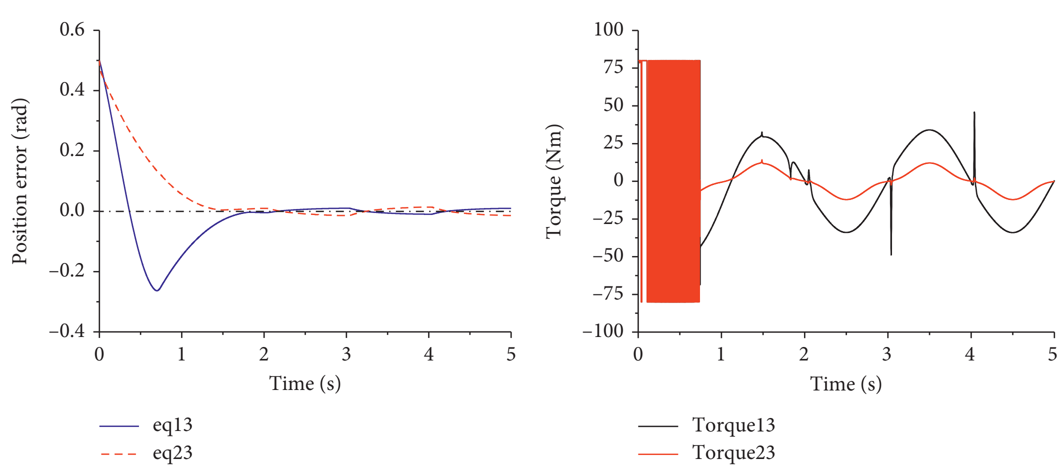
(a) (b)

Figure 1: Position tracking errors and input torque of the n-order nonlinear dynamical systems under NTSMC1.



(a) (b)

Figure 2: Position tracking errors and input torque of n-order nonlinear dynamical systems under NTSMC2.



(a) (b)

Figure 3: Position tracking errors and input torque of n-order nonlinear dynamical systems under NTSMC3.

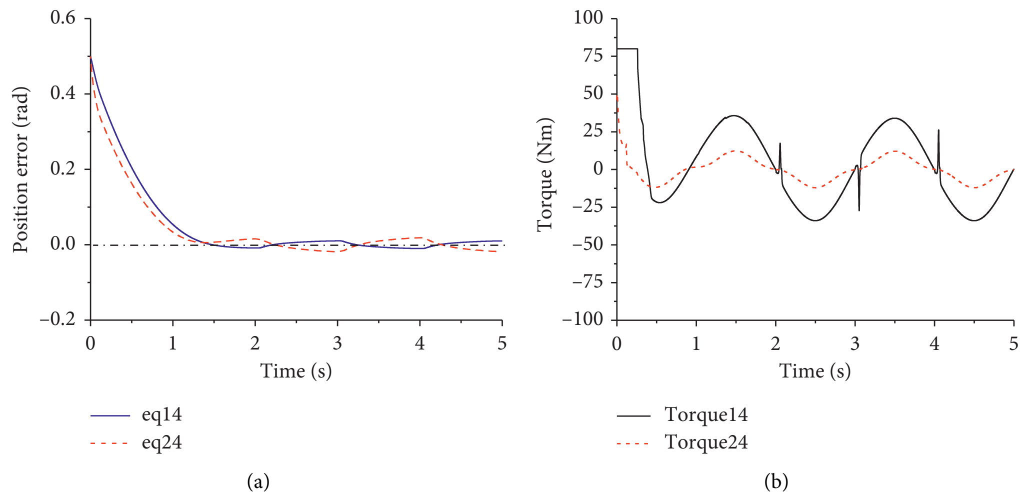


FIGURE 4: Position tracking errors and input torque of n-order nonlinear dynamical systems under NTSMC4.

$$
\begin{aligned}
d_{1,\mathrm{rob},1} &= 10,\\
d_{1,\mathrm{rob},2} &= 1.8,\\
d_{2,\mathrm{rob},1} &= 17,\\
d_{2,\mathrm{rob},2} &= 2,\\
d_{3,\mathrm{rob},1} &= 0.001,\\
d_{3,\mathrm{rob},2} &= 0.001,\\
\widehat{b}_{1,\mathrm{rob},1,0} &= 1,\\
\widehat{b}_{1,\mathrm{rob},2,0} &= 0.1,\\
\widehat{b}_{2,\mathrm{rob},1,0} &= 1,\\
\widehat{b}_{2,\mathrm{rob},2,0} &= 0.1,\\
\widehat{b}_{3,\mathrm{rob},1,0} &= 0.1,\\
\widehat{b}_{3,\mathrm{rob},2,0} &= 0.1,
\end{aligned}
\tag{56}
$$

where $\widehat{b}_{1,\mathrm{rob},1,0}$, $\widehat{b}_{1,\mathrm{rob},2,0}$, $\widehat{b}_{2,\mathrm{rob},1,0}$, $\widehat{b}_{2,\mathrm{rob},2,0}$, $\widehat{b}_{3,\mathrm{rob},1,0}$, and $\widehat{b}_{3,\mathrm{rob},2,0}$ are the initial values of $\widehat{b}_{1,\mathrm{rob},1}$, $\widehat{b}_{1,\mathrm{rob},2}$, $\widehat{b}_{2,\mathrm{rob},1}$, $\widehat{b}_{2,\mathrm{rob},2}$, $\widehat{b}_{3,\mathrm{rob},1}$, and $\widehat{b}_{3,\mathrm{rob},2}$.

Figures 1–4 illustrate the tracking errors and input torque under the NTSMC1, NTSMC2, NTSMC3, and adaptive vector NTSMC, where "eq*ij*" and "torque*ij*" mean the tracking error and input torque of the *i*th link under the *j*th control scheme, respectively. NTSMC1, NTSMC2, NTSMC3, and adaptive vector NTSMC represent the 1st, 2nd, 3rd, and 4th control schemes.

It can be seen that the tracking errors of the nonlinear system under the NTSMC2, NTSMC3, and the adaptive vector NTSMC can be converged to the acceptable level in finite time (about 1.5 s), but the tracking error of the second link under NTSMC1 is obvious even in the stabilization period (about 0.07 rad after 1.5 s). Besides, the tracking errors of the first link have amplitude overshoot (0.22 rad for NTSMC2 and 0.26 rad for NTSMC3) under NTSMC2 and NTSMC3 in the initial 1.5 s. From the comparison, it can be clearly seen that the proposed adaptive vector NTSMC can obtain a much faster transient and better tracking performance in comparison with the NTSMC. Moreover, considering the input torque of the controllers, the chattering problems under NTSMC2 and NTSMC3 are serious, and the input commands of NTSMC1 and adaptive vector NTSMC are smooth. As a result, from the simulations, we can conclude that the proposed adaptive vector NTSMC can solve the problem of finite-time tracking without chattering phenomenon.

To clearly show the effectiveness and advantages of the proposed adaptive vector NTSMC, we simply explain the results illustrated in Figures 1–4. Firstly, the vector upper bound analyzes different dynamical characteristics of the two-DOF manipulator (the peak torque of the first link and the second link is about 36 Nm and 12 Nm, respectively). Secondly, the initial position errors of the dynamical system usually cause serious chattering in the initial time for the SMC schemes, which also was resolved by introducing of novel adaptive updating laws for the upper bound.

5. Conclusions

In this paper, we considered the finite-time tracking problem of n-link nonlinear dynamical systems with uncertainty. The well-known NTSMC for the robot manipulator was extended to the general n-link nonlinear dynamical systems, and the corresponding rigorous stability analysis of the NTSMC for the nonlinear systems was also established based on the Lyapunov method. Moreover, a novel adaptive vector NTSMC is further proposed by replacing the constant upper bound with an adaptive vector bound, and its asymptotic stability has also been analyzed. Furthermore, the convergence times of the NTSMC and adaptive vector NTSMC are calculated for the nonlinear system. Finally, simulations performed on a two-DOF manipulator demonstrate the effectiveness and advantages of the proposed adaptive vector NTSMC in comparison with the NTSMC. The developed controller offers an alternative approach to a large class of nonlinear systems, which considers different dynamical characteristics and special design demands and eliminates the chattering problem existing in the common SMC schemes.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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