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Research Article

Fixed-Time Flocking Problem of a Cucker–Smale Model

Fen Nie and Yicheng Liu

College of Liberal Arts and Sciences, National University of Defense Technology, Changsha 410073, China

Correspondence should be addressed to Yicheng Liu; liuyc2001@hotmail.com

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In this paper, we studied a Cucker–Smale model with continuous non-Lipschitz protocol. The methodology presented in the current work is based on the explicit construction of a Lyapunov functional. By using the fixed-time control technology, we show that the flocking can occur in fixed time if the communication rate function is locally Lipschitz continuous and has a lower bound, and we can obtain the estimation of the converging time which is independent of the initial states of agents. Theoretical results are supported by numerical simulations.

1. Introduction

Collective motions refers to an orderly movement organized by agents with limited environmental information and simple rules. In recent years, the study on collective motions has gained increasing interest in robotics, control theory, economics, and social sciences [1, 2]. Several mathematical models have been proposed [3–6] to characterize the mechanism of collective flocking motion without central direction. Among others, the celebrated Cucker–Smale model [5] provided a framework to explain the self-organizing behavior in various complex systems, and the model is given by the following ODE system:

$$\begin{cases} \dfrac{\mathrm{d}x_i}{\mathrm{d}t} = v_i, t > 0, \quad i = 1, 2, \cdots, N, \\ \dfrac{\mathrm{d}v_i}{\mathrm{d}t} = k \displaystyle\sum_{j=1}^{N} a_{ij}\left(\left\| x_j - x_i \right\|\right)\left(v_j - v_i\right), \end{cases} \tag{1}$$

subject to the initial configuration $(x_i(0), v_i(0)) = (x_{i0}, v_{i0})$, where N denotes the number of particles, k measures the interaction strength, x_i and v_i denote the position and velocity of the i th particle at the time t, and a_{ij} measures the influence intensity quantified by the pairwise influence of particle j on the alignment of particle i. It is a function of the distance of two particles defined as

$$a_{ij}^{\mathrm{CS}}\left(\left\| x_j - x_i \right\|\right) = \frac{\psi\left(\left\| x_j - x_i \right\|\right)}{N}, \tag{2}$$

where $\psi(r) = (1/(1 + r^2)^{\beta})$ is called the influence function, $\beta \geq 0$. In [5], the authors showed that the unconditional flocking occurs when $\beta < (1/2)$, while the conditional flocking occurs under some restricted conditions on the initial data setting when $\beta \geq (1/2)$. In [7], the authors extended the conclusions of unconditional flocking to $\beta \leq (1/2)$ by the energy method. In [8], the authors introduced a nonsymmetric influence function and took into account relative distance between agents instead of the distance between agents:

$$a_{ij}^{\mathrm{MT}}\left(\left\| x_j - x_i \right\|\right) = \frac{\psi\left(\left\| x_j - x_i \right\|\right)}{\sum_{1 \leq k \leq N} \psi\left(\left\| x_k - x_i \right\|\right)}. \tag{3}$$

Based on the notion of active sets, a sufficient condition for flocking was derived. Recently, there are many extensive observations and improvements to the Cucker–Smale model. See, for examples, time delay is introduced in [9–12], collision avoidance is considered in [13–16], and hierarchical structure is involved in [12, 17–20]. However, the flocking phenomenon described in the most previous works is an asymptotic behaviour, which means that the flocking can only occur when time approaches to infinity. Then, a natural question is that whether the system undergoes flocking behaviours within a finite time? In fact, under some occasional perturbations, individuals in bird flocks or fish schools can return back to ordered group motion after adjusting their states in a short time. Recently, there are few contributions to the Cucker–Smale model by using finite-time control theory. In [21], when the communication rate function is locally Lipschitz continuous and has a lower bound, the authors obtain the finite-time flocking by constructing a Lyapunov functional. In [22], the authors modified the Cucker–Smale model with continuous non-Lipschitz protocol. When the influence function has a singular interval, the system will undergo a flocking evolution in finite time, and the minimum distance between agents in the flocking evolution process is greater than the control parameter. Although finite-time flocking performance has favourable properties, the estimation of convergence time usually depends on initial states of networked particles. It will restrict the applications in practice if the initial conditions are unavailable previously. In the similar performance, there are lots of works in the field of fix-time consensus [23–26]. However, there is little work about the fixed-time flocking performance of the Cucker–Smale model.

The main purpose of this article is to investigate the fixed-time flocking performance of a Cucker–Smale model. The remaining of this paper is organized as follows. In Section 2, a Cucker–Smale model with continuous non-Lipschitz protocol is presented, and some useful preliminaries are also given in this section. In Section 3, the sufficient conditions for fixed-time flocking are established, and the numerical simulations are provided to validate the theoretical results in Section 4. Finally, the conclusions are drawn in Section 5.

2. Problem Statement and Preliminaries

In this section, we consider an N-agent model with nonlinear terms. Let $x_i = (x_i^1, x_i^2, \ldots, x_i^d) \in \mathbb{R}^d$, $v_i = (v_i^1, v_i^2, \ldots, v_i^d) \in \mathbb{R}^d$ denote the position and velocity of the i th agent at the time t, respectively. The modified Cucker–Smale model in this paper can be described by the following equations:

$$\begin{cases} \dfrac{\mathrm{d}x_i}{\mathrm{d}t} = v_i, t > 0, \quad i = 1, 2, \cdots, N, \\ \dfrac{\mathrm{d}v_i}{\mathrm{d}t} = \dfrac{k_1}{N} \sum_{j=1}^{N} \psi\left(\left\|x_j - x_i\right\|\right) \mathrm{sig}\left(v_j - v_i\right)^p + \dfrac{k_2}{N} \sum_{j=1}^{N} \psi\left(\left\|x_j - x_i\right\|\right) \mathrm{sig}\left(v_j - v_i\right)^q, \end{cases} \tag{4}$$

subject to initial configuration

$$\left(x_i(0), v_i(0)\right) = \left(x_{i0}, v_{i0}\right), \tag{5}$$

where p and q are two constants with $0 < p < 1 < q$. k_1 and k_2 measure the interaction strengths, ψ is defined in (2), and

$$\mathrm{sig}\left(v_j - v_i\right)^p = \left\{\mathrm{sgn}\left(v_j^1 - v_i^1\right)\left|v_j^1 - v_i^1\right|^p, \ldots, \mathrm{sgn}\left(v_j^d - v_i^d\right)\left|v_j^d - v_i^d\right|^p\right\}, \tag{6}$$

where $\mathrm{sgn}(\cdot)$ is the signum function:

$$\mathrm{sgn}(s) = \begin{cases} 1, & s > 0, \\ 0, & s = 0, \\ -1, & s < 0. \end{cases} \tag{7}$$

At this stage, we list the following lemmas, which play an important role in the proof of the main results.

Lemma 1 (special case with $k = 1$ in [27]). *Consider the following equation:*

$$\begin{aligned} \dot{x} &= f(t, x), \\ x(0) &= x_0, \end{aligned} \tag{8}$$

where $x \in \mathbb{R}^n$ and $f: \mathbb{R}_+ \times \mathbb{R}^n \longrightarrow \mathbb{R}^n$ is a nonlinear continuous function. Assume the origin is an equilibrium point of (8). If there exists a continuous radially unbounded function $H: \mathbb{R}^n \longrightarrow \mathbb{R}_+ \cup \{0\}$ such that

(i) $H(z) = 0 \Leftrightarrow z = 0$.

(ii) If there are some positive constants ϑ, δ, a, and b such that $0 < a < 1 < b$ and the inequality

$$\dot{H}(z(t)) \leq -\vartheta H^a(z(t)) - \delta H^b(z(t)), \tag{9}$$

holds for any solution $z(t)$ of (8), then the origin is globally fixed-time stable, and $H(t) \equiv 0$ if

$$t \geq \frac{1}{\vartheta(1-a)} + \frac{1}{\delta(b-1)}. \tag{10}$$

Lemma 2 (see [28]). *Let $y \in \mathbb{R}^n$ and $0 < r < s$; then, the following inequalities hold:*

$$\begin{aligned} \left(\sum_{i=1}^{n}\left|y_i\right|^s\right)^{1/s} &\leq \left(\sum_{i=1}^{n}\left|y_i\right|^r\right)^{1/r}, \\ \left(\frac{1}{n}\sum_{i=1}^{n}\left|y_i\right|^s\right)^{1/s} &\geq \left(\frac{1}{n}\sum_{i=1}^{n}\left|y_i\right|^r\right)^{1/r}. \end{aligned} \tag{11}$$

3. Sufficient Conditions for Fixed-Time Flocking

In this section, we shall show that systems (4) and (5) with continuous non-Lipschitz protocol have fixed-time flocking. First, we first introduce the definition of the fixed-time flocking.

Definition 1. Systems (4) and (5) are said to reach fixed-time flocking if and only if the systems satisfy the following two conditions:

(i) Velocity alignment: the velocity fluctuations go to zero in the fixed-time T; the time function T is called the convergence time independent of the initial values.

$$\left\| v_i - v_j \right\| = 0, \quad \forall t \geq T \text{ for } i, j = 1, 2, \cdots, N. \tag{12}$$

(ii) Forming a group: the position fluctuations are uniformly bounded in time t:

$$\sup_{0 \leq t \leq \infty} \left\| x_i - x_j \right\| < \infty, \quad \text{for } i, j = 1, 2, \cdots, N. \tag{13}$$

Theorem 1. *Consider the Cucker–Smale model ((4) and (5)) and assume that the communication rate function ψ is locally Lipschitz continuous with a lower bound, that is, there exists $\psi^* > 0$ such that $\inf_{s>0} \psi(s) \geq \psi^*$. Then, systems (4) and (5) reach fixed-time flocking. And the convergence time is given by*

$$T \leq T^* \doteq \frac{2}{\vartheta(1-p)} + \frac{2}{\delta(q-1)}, \tag{14}$$

where $\vartheta = k_1 \psi^* 2^{(p+1)/2} N^{(p-1)/2}$ and $\delta = k_2 \psi^* 2^{(q+1)/2} d^{(1-q)/2}$.

Proof. Firstly, we consider macroscopic variables:

$$x_c = \frac{1}{N} \sum_{i=1}^{N} x_i, v_c = \frac{1}{N} \sum_{i=1}^{N} v_i. \tag{15}$$

By the symmetry of the indices, we have

$$\sum_{i=1}^{N} \frac{\mathrm{d} v_i}{\mathrm{d} t} = \frac{k_1}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} \psi\left(\left\| x_j - x_i \right\|\right) \operatorname{sig}\left(v_j - v_i\right)^p + \frac{k_2}{N} \sum_{i=1}^{N} \sum_{j=1}^{N} \psi\left(\left\| x_j - x_i \right\|\right) \operatorname{sig}\left(v_j - v_i\right)^q = 0. \tag{16}$$

Hence, the explicit dynamics for the macroscopic variables is given as

$$\begin{aligned} \frac{\mathrm{d} x_c}{\mathrm{d} t} &= v_c, \\ \frac{\mathrm{d} v_c}{\mathrm{d} t} &= 0, \end{aligned} \tag{17}$$

which implies that

$$\begin{aligned} v_c(t) &= v(0), \\ x_c(t) &= x_c(0) + t v_c(0), \\ t &\geq 0. \end{aligned} \tag{18}$$

Introducing the fluctuations $(\widehat{x}_i, \widehat{v}_i)$,

$$\begin{aligned} \widehat{x}_i &\doteq x_i - x_c, \\ \widehat{v}_i &\doteq v_i - v_c. \end{aligned} \tag{19}$$

Then, systems (4) and (5) can be written as

$$\begin{cases} \dfrac{\mathrm{d} \widehat{x}_i}{\mathrm{d} t} = \widehat{v}_i, t > 0, \quad i = 1, 2, \cdots, N, \\ \dfrac{\mathrm{d} \widehat{v}_i}{\mathrm{d} t} = \dfrac{k_1}{N} \displaystyle\sum_{j=1}^{N} \psi\left(\left\| \widehat{x}_j - \widehat{x}_i \right\|\right) \operatorname{sig}\left(\widehat{v}_j - \widehat{v}_i\right)^p + \dfrac{k_2}{N} \sum_{j=1}^{N} \psi\left(\left\| \widehat{x}_j - \widehat{x}_i \right\|\right) \operatorname{sig}\left(\widehat{v}_j - \widehat{v}_i\right)^q, \end{cases} \tag{20}$$

with the initial value

$$(\widehat{x}_i(0), \widehat{v}_i(0)) = (\widehat{x}_{i0}, \widehat{v}_{i0}). \tag{21}$$

For convenience, we remove the hat out of the variables and also use (x_i, v_i) instead of $(\widehat{x}_i, \widehat{v}_i)$. It is easy to see that

$$\sum_{i=1}^{N} x_i = 0, \sum_{i=1}^{N} v_i = 0. \tag{22}$$

Take the candidate Lyapunov function

$$V(t) \doteq \sum_{i=1}^{N} \|v_i\|^2, \; X(t) \doteq \sum_{i=1}^{N} \|x_i\|^2. \tag{23}$$

Then, we have

$$\sum_{1 \le i,j \le N} \left\|v_j - v_i\right\|^2 = 2N \sum_{i=1}^{N} \|v_i\|^2 - 2\left\langle \sum_{i=1}^{N} v_i, \sum_{j=1}^{N} v_j \right\rangle = 2NV, \tag{24}$$

$$\sum_{1 \le i,j \le N} \left\|x_j - x_i\right\|^2 = 2N \sum_{i=1}^{N} \|x_i\|^2 - 2\left\langle \sum_{i=1}^{N} x_i, \sum_{j=1}^{N} x_j \right\rangle = 2NX. \tag{25}$$

It is easy to see that the velocity difference of all individuals will tend to zero in fixed time if the function $V(t)$ tends to 0 in fixed time. And the diameter of a group is bounded if the function $X(t)$ is bound.

From Lemma 2, using the fact $\inf_{s>0} \psi(s) \ge \psi^*$, we see that

$$\begin{aligned}
\left|\frac{dV}{dt}\right| &= \left|\frac{d}{dt} \sum_{i=1}^{N} \|v_i\|^2\right| = \left|2 \sum_{i=1}^{N} \langle v_i, \dot{v}_i \rangle\right| \\
&= \frac{2k_1}{N} \left\langle \sum_{1 \le i,j \le N} \psi\left(\left\|x_j - x_i\right\|\right) \operatorname{sig}\left(v_j - v_i\right)^p, v_i \right\rangle \\
&\quad + \frac{2k_2}{N} \left\langle \sum_{1 \le i,j \le N} \psi\left(\left\|x_j - x_i\right\|\right) \operatorname{sig}\left(v_j - v_i\right)^q, v_i \right\rangle \\
&= -\frac{k_1}{N} \left\langle \sum_{1 \le i,j \le N} \psi\left(\left\|x_j - x_i\right\|\right) \operatorname{sig}\left(v_j - v_i\right)^p, \left(v_j - v_i\right) \right\rangle \\
&\quad - \frac{k_2}{N} \left\langle \sum_{1 \le i,j \le N} \psi\left(\left\|x_j - x_i\right\|\right) \operatorname{sig}\left(v_j - v_i\right)^q, \left(v_j - v_i\right) \right\rangle \\
&\le -\frac{k_1}{N} \psi^* \sum_{1 \le i,j \le N} \sum_{k=1}^{d} \left|v_{jk} - v_{ik}\right|^{p+1} - \frac{k_2}{N} \psi^* \sum_{1 \le i,j \le N} \sum_{k=1}^{d} \left|v_{jk} - v_{ik}\right|^{q+1}.
\end{aligned} \tag{26}$$

Note that $0 < p < 1 < q$. Then, employing Lemma 2, one can easily obtain

$$\begin{aligned}
\left(\sum_{k=1}^{d} \left|v_{jk} - v_{ik}\right|^{p+1}\right)^{1/(p+1)} &\ge \left(\sum_{k=1}^{d} \left|v_{jk} - v_{ik}\right|^2\right)^{1/2} = \left\|v_j(t) - v_i(t)\right\|, \\
\left(\frac{1}{d} \sum_{k=1}^{d} \left|v_{jk} - v_{ik}\right|^{q+1}\right)^{1/(q+1)} &\ge \left(\frac{1}{d} \sum_{k=1}^{d} \left|v_{jk} - v_{ik}\right|^2\right)^{1/2}.
\end{aligned} \tag{27}$$

Thus, we have

$$\begin{aligned}
\sum_{k=1}^{d} \left|v_{jk} - v_{ik}\right|^{p+1} &\ge \left\|v_j(t) - v_i(t)\right\|^{p+1}, \\
\sum_{k=1}^{d} \left|v_{jk} - v_{ik}\right|^{q+1} &\ge d\left(\frac{1}{d} \sum_{k=1}^{d} \left|v_{jk} - v_{ik}\right|^2\right)^{(q+1)/2} = d^{(1-q)/2} \left\|v_j(t) - v_i(t)\right\|^{q+1}.
\end{aligned} \tag{28}$$

Let $s = 1$ and $r = ((p+1)/2)$; by applying (11) and (24) to the processing inequality, we show that

$$\sum_{i,j=1}^{N} \left(\left\|v_j(t) - v_i(t)\right\|^2\right)^{(p+1)/2} \ge \left(\sum_{i,j=1}^{N} \left\|v_j(t) - v_i(t)\right\|^2\right)^{(p+1)/2} = (2NV)^{(p+1)/2}. \tag{29}$$

Similarly, let $s = ((q+1)/2)$ and $r = 1$; by applying (11) and (24), we get that

$$\sum_{i,j=1}^{N}\left(\left\|v_j(t)-v_i(t)\right\|^2\right)^{(q+1)/2} \geq N^{(1-q)/2}\left(\sum_{i,j=1}^{N}\left\|v_j(t)-v_i(t)\right\|^2\right)^{(q+1)/2} = N^{(1-q)/2}(2NV)^{(q+1)/2}. \quad (30)$$

Hence, we conclude

$$\frac{\mathrm{d}V}{\mathrm{d}t} \leq -k_1\psi^* 2^{(p+1)/2}N^{(p-1)/2}V^{(p+1)/2} - k_2\psi^* 2^{(q+1)/2}d^{(1-q)/2}V^{(q+1)/2}. \quad (31)$$

Finally, from Lemma 1, when $a = ((p+1)/2)$ and $b = ((q+1)/2)$, we have

$$V(t) \equiv 0, \quad t \geq T, \quad (32)$$

and the convergence time independent of the initial values is estimated by

$$\begin{aligned} T \leq T^* &\doteq \frac{1}{\vartheta(1-((p+1)/2))} + \frac{1}{\delta(((q+1)/2)-1)} \\ &= \frac{2}{\vartheta(1-p)} + \frac{2}{\delta(q-1)}, \end{aligned} \quad (33)$$

where $\vartheta = k_1\psi^* 2^{(p+1)/2}N^{(p-1)/2}$ and $\delta = k_2\psi^* 2^{(q+1)/2}d^{(1-q)/2}$. Thus, from (24), we achieve

$$v_i(t) \equiv 0, \quad \forall t \geq T,\ i = 1, 2, \cdots, N. \quad (34)$$

This implies that condition (i) of the definition of fixed-time flocking holds.

Now, we prove condition (ii) of the definition of fixed-time flocking is also true. It is necessary to show that the function $X(t)$ is bounded.

It follows from (31) that $V(t)$ is a nonincreasing function with respect to t. That is, when $t > 0$, $V(0) \geq V(t) \geq 0$. By using the triangle inequality and Cauchy–Schwarz inequality, we have

$$\frac{\mathrm{d}X}{\mathrm{d}t} = 2\sum_{i=1}^{N}\langle x_i, v_i\rangle \leq 2\sum_{i=1}^{N}\|x_i\|\|v_i\| \leq 2X^{1/2}V^{1/2}. \quad (35)$$

Integrating the differential inequality (35) from 0 to t yields that

$$X^{1/2}(t) \leq X^{1/2}(0) + \int_0^t V^{1/2}(s)\mathrm{d}s. \quad (36)$$

If $t < T$, then it is deduced from (36) that

$$X^{1/2}(t) \leq X^{1/2}(0) + \int_0^T V^{1/2}(t)\mathrm{d}t \leq X^{1/2}(0) + V^{1/2}(0)T < \infty. \quad (37)$$

If $t > T$, then it is deduced from (31) and (36) that

$$\begin{aligned} X^{1/2}(t) &\leq X^{1/2}(0) + \int_0^T V^{1/2}(t)\mathrm{d}t + \int_T^t V^{1/2}(t)\mathrm{d}t \\ &\leq X^{1/2}(0) + \int_0^T V^{1/2}(t)\mathrm{d}t \\ &\leq X^{1/2}(0) + V^{1/2}(0)T < \infty. \end{aligned} \quad (38)$$

Thus,

$$\sup_{0\leq t\leq\infty}\left\|x_i - x_j\right\|^2 < \infty, \quad \text{for } i, j = 1, 2, \cdots, N. \quad (39)$$

This completes the proof. □

Remark 1. Compared to [21], we added the term

$$\frac{k_2}{N}\sum_{j=1}^{N}\psi\left(\left\|x_j - x_i\right\|\right)\mathrm{sig}\left(v_j - v_i\right)^q, \quad (40)$$

to the control protocol; the advantage of Theorem 1 is that the convergence time is independent of the initial states of agents which is estimated by

$$\begin{aligned} T \leq T^* &\doteq \frac{2}{k_1\psi^* 2^{(p+1)/2}N^{(p-1)/2}(1-p)} \\ &+ \frac{2}{k_2\psi^* 2^{(q+1)/2}d^{(1-q)/2}(q-1)}. \end{aligned} \quad (41)$$

However, in [21], the convergence time is estimated by

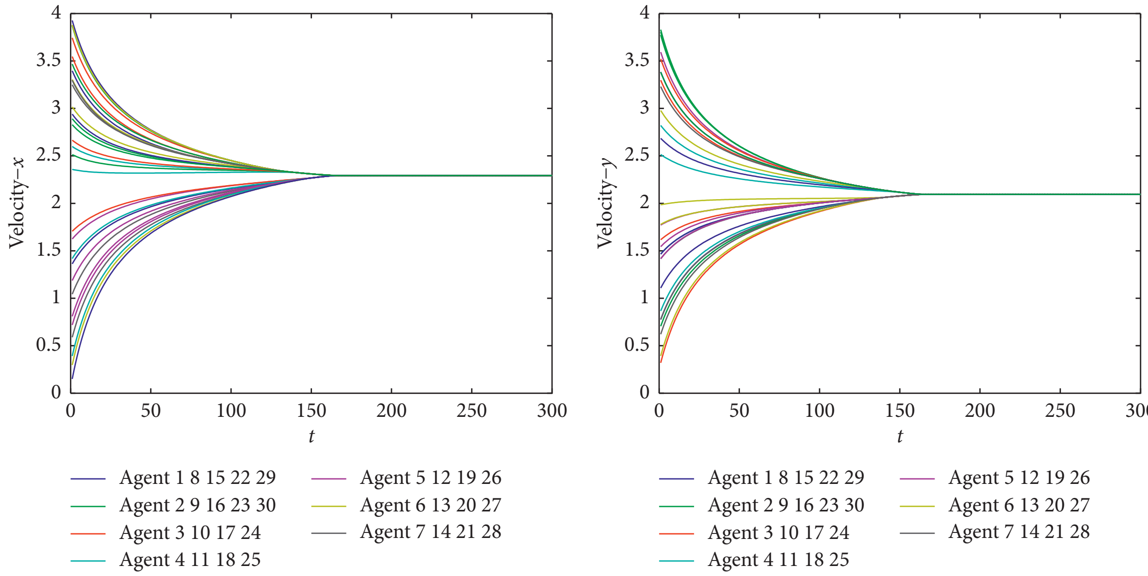
$$T \leq T^* \doteq \frac{2\left(\sum_{i=1}^{N}\|v_i(0)\|^2\right)^{(1-p)/2}N^{-((1+p)/2)}}{\psi^* 2^{(1+p)/2}(1-p)}, \quad (42)$$

which is formulated by the initial speed of all agents.

4. Simulations

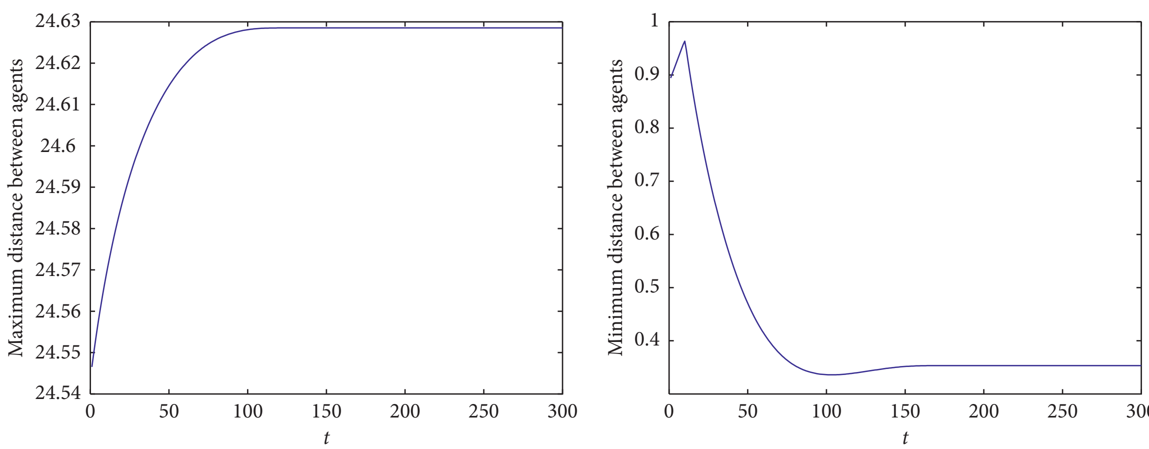
In this section, we choose some special initial values and parameters to verify our results. Let $N = 30$, $p = 0.2$, $q = 2$, $k_1 = 1$, $k_2 = 2$, $d = 2$, Using Euler algorithm, step length $h = 0.01$, and

$$\psi(r) = \begin{cases} \dfrac{1}{(1+r^2)^{\beta}}, & \text{if } r \leq r^*, \\[2ex] \dfrac{1}{(1+(r^*)^2)^{\beta}}, & \text{if } r > r^*, \end{cases} \quad (43)$$



(a) (b)

Figure 1: The velocity of agents.



(a) (b)

Figure 2: The maximum (a) and minimum (b) distance between agents.

where $\beta = 0.2$, $r^* = 3$, and $\psi^* = 0.6310$. Using the formula in Theorem 1, we see that $T^* = 10.9824$ for the random initial position generated on $[0, 20]$ and random velocity on $[0, 4]$.

Then, the following simulation results (Figures 1 and 2) are obtained. In Figure 1, the x, y direction velocity of the all agents is presented, and the velocity of all agents converges to the same value after about $T = 1.5$. Moreover, in Figure 2, it shows that the maximum distance between all agents is stable after $T = 1.5$. In the process of forming fixed-time flocking, the minimum distance among all agents is about 0.3359 for given random initial values.

5. Conclusion

In this paper, we investigated the flocking problem of a modified Cucker–Smale model with continuous non-Lipschitz protocol. By using a Lyapunov functional, we show that the flocking can occur in fixed time if communication rate function is locally Lipschitz continuous and with a lower bound. The main results demonstrate that the flocking converging time is independent of the initial states of agents. Theoretical results are supported by numerical simulations; at the same time, we observe that the minimum distance of the agent is only 0.339, which is very dangerous. Therefore, avoiding the collision problem will guide significance to our further research studies.

Data Availability

The simulation data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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