

## Research Article

# Calculation of Unloading Area of Internal Gear Pump and Optimization

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In order to obtain the calculation method of the unloading area of the internal gear pump during oil trapping, a pair of internal gears including an external gear and an internal gear was used as the research object to simulate the oil trapping process. The geometric relationship during the meshing process was established, and the unloading area expression was obtained by using the geometric pattern expansion method with the variable  $f$  as the independent variable. Guided by a mathematical model, two improved optimization schemes were proposed for the internal gear tooth profile, and the unloading area expressions  $s_{ud}$ ,  $s_{uda}$ , and  $s_{udb}$  were obtained. Taking the meshing gear pair with module 3 and number of teeth 13/19 as examples, the simulation results were very consistent with the existing literature. The reliability of the model and the feasibility of the optimization scheme are obtained based on the theoretical analysis and calculation results. This calculation method of unloading area can be applied to the same type of gear pump design in the future, providing a reference for the design of high pressure and low noise gear pumps.

## 1. Introduction

The internal gear pump has oil trapping phenomenon during the rotating work and has smaller oil trapping changes than the external gearing [1]. At present, gear pump oil trapping phenomenon is one of the key factors restricting higher performance [2]. In order to improve the design performance of the gear pump, Erturk et al. analyzed the oil trap mechanism of the hydraulic pump and found its variation rule and found that the backlash of the gear pair can affect the oil trap [3]. Sun et al. proposed the definition of backlash unloading of gear pumps and compared and verified the results [4]. In order to further reduce the trapped oil, Li and Liu calculated the unloading area of the gear pump and then performed a simulation analysis [5]. Zang et al. proposed a new method for reducing the trapped oil pressure of the gear pump [6]. Above those was an external gear pump. It is still a large difference in the structure between the internal gear and the external gear pump. There is less research on oil trapping and unloading of the internal gear pump. On internal gear pumps, Yanada et al. had

studied the phenomenon of oil trapping between gear pairs and pointed out the source of the oil trap [7]. Zhou et al. had adopted a discrete method to study conjugate gears and obtained the oil trap of this pump [8, 9]. Song et al. designed the conjugated straight-line internal gear pairs for fluid power gear machines and got the unloading curve graphically by a discretization approach [10]. Above all, research methods are of reference to internal gear pumps with involute tooth profiles, but they are not continuous in calculation and not completely accurate. In these references, the unloading area was obtained by the graphic method, and the change law of the area size with processing and parameters was analyzed, but the results of such processing have limitations. Therefore, this paper will overcome the shortcomings mentioned above and, then, use calculation and derivation methods to obtain accurate results of unloading area. The conclusions obtained can provide theoretical guidance for the model parameters and design principles of the internal gear pump and can also optimize the unloading area according to the processing and forming methods.

## 2. Oil Trapping Process Description and Calculation Basis of Internal Gear Pump

**2.1. Description of Oil Trapped Process.** A pair of backlash-free gear tooth rotation processes are used as an analysis to explain the oil trapping and unloading of the internal gear pump. In this process, the unloading groove can be designed. Although its design is more flexible and has more geometric forms, its design principles are basically the same [11]. Figure 1 shows oil trapping and unloading process of internal gear pump, the external gear  $O_1$ , and the internal gear  $O_2$  from an oil trapped area. This article uses them as an example to explain that they pass through the boundary  $ud$  of the rectangular unloading groove [12].  $p$ ,  $n_1$ , and  $n_0$  are gear joint nodes, meshing points, and backlash points. The meshing point  $n_1$  is moving along the meshing line during the rotation of the gear pair. Figure 1(a) shows the minimum oil trapped area, Figure 1(b) shows  $ud$  on the involute of gear  $O_1$  and involute of gear  $O_2$ , Figure 1(c) shows  $ud$  on the top circle of gear and gear  $O_2$  transition curve, Figure 1(d) shows  $ud$  on the tooth top circle of gear  $O_1$  and the tooth root circle of gear  $O_2$ , Figure 1(e) shows  $ud$  on tooth top circle of gear  $O_1$  and gear  $O_2$  the transition curve, Figure 1(f) shows  $ud$  involute of gear  $O_1$  and gear  $O_2$  transition curve, and Figure 1(g) shows  $ud$  on the involute of gear  $O_1$  and gear  $O_2$ .

The length from point  $p$  to point  $n_1$  is  $f$ . The length of  $f$  in Figure 1(a) is  $f_0$ , and  $f_0 = P_b/4$ . When  $f$  changes to the position of Figure 1(g) as  $n_1$  moves,  $f$  is  $f_g$ ,  $f_g = r_{b,1}(\tan \alpha_{\alpha,1} - \tan \alpha')$ . When point  $p$  moves to  $ud$  along the meshing line of the gear pair, the length of  $f$  at this time is  $f_a$  and  $f_a = B_{ud}/\cos \alpha'$ . The above equations  $p_b$ ,  $r_{b,1}$ ,  $\alpha_{\alpha,1}$ ,  $\alpha'$ , and  $B_{ud}$  are the base pitch of the gear  $O_1$ , the base circle radius, the pressure angle of the tooth top circle, the meshing angle, and the distance from  $ud$  to the line connecting the center of gear  $O_1$  and gear  $O_2$ .

**2.2. Mathematical Knowledge.** Figure 2 is an outline of an internal gear processed by a forming method. With the center of the circle  $O_2$  as the origin of the coordinates and the symmetry line of a cogging as the  $y$ -axis, a rectangular coordinate system  $xO_2y$  is established. This coordinate system can be rotated around point  $O_2$  with a rotation angle of  $\theta$ . The  $t_1t_2$  line segment is an involute equation, and the  $r_1r_2$  line segment is a transition curve. Point  $t$  can slide on the line segment  $t_1t_2$  arbitrarily, and the direction is to move from point  $t_1$  to point  $t_2$ . Point  $r$  can slide on the line segment  $r_1r_2$  arbitrarily, and the direction is to move from point  $r_1$  to point  $r_2$ . Suppose  $\alpha_x$ ,  $r_x$ ,  $x_x$ ,  $y_x$ , and  $\theta_x$  are the gear pressure angle at point  $x$ , the corresponding radius, the abscissa in the coordinate system, the ordinate in the coordinate system, and the rotation angle  $\angle t_1o_2t = \emptyset$ ,  $\angle r_1o_2t = \zeta$ . By the definition of the involute function, any two points on the involute line can be connected to the center of the circle to form a sector [13]:

$$\begin{cases} \emptyset = (\tan \alpha_t - \alpha_t) - (\tan \alpha_{t1} - \alpha_{t1}), \\ S_{inv} = \int \frac{r_t^2(\phi)}{2} d\phi. \end{cases} \quad (1)$$

Let the fan-shaped area enclosed by the line segment  $t_1t_2$  and the radius of point  $t_1$  and point  $t_2$  be expressed as  $S_{inv} \Big|_{t_1}^{t_2}$ , which can be deduced from (1),  $S_{inv} \Big|_{t_1}^{t_2} = \int ((r_t^2(\emptyset)/2)d\emptyset = (r_{b,1}^2/2) \int_{\alpha_{t1}}^{\alpha_{t2}} (\sec^4 \alpha_t - \sec^2 \alpha_t) d\alpha_t = r_{b,1}^2 ((\tan^3 \alpha_{t2} - \tan^3 \alpha_{t1})/6)$ . In rectangular coordinate system,  $\zeta = \tan^{-1}(x_r/y_r) - \tan^{-1}(x_{r1}/y_{r1})$ . Suppose that the fan-shaped area enclosed by the contour of the line segment  $r_1r_2$  and the radius of point  $r_1$  and point  $r_2$  can be expressed as  $S_{sta} \Big|_{r_1}^{r_2}$ . According to [14],  $S_{sta} = \left| \frac{r^2}{r1} (r'^2/6)(\theta_{r2}^3 - \theta_{r1}^3) - (mh_{a0}^*/2)(r' - mh_{a0}^*)(\theta_{r2} - \theta_{r1}) \right|$  the above formula,  $h_{a0}^*$  and  $m$  are the tooth height coefficient and modulus of the rack cutter for processing the gear profile and  $r'$  is the pitch circle radius.

The gears produced by the envelope method have regular geometric outer contours [15]. Figure 3(a) shows the outline of an external gear machined by generating method, and Figure 3(b) shows the profile of an internal gear machined by generating method. Let  $r_{a,1}$ ,  $r_{a,2}$ ,  $r_{f,1}$ ,  $r_{f,2}$  be the radius of the top circle and root circle of the external gear and internal gear. Let  $\angle a_1o_1a_2 = \sigma$ ,  $\angle a_5o_1a_6 = \tau$ ,  $\angle c_1o_2c_4 = \mu$ ,  $\angle c_2o_1c_3 = \lambda$ ,  $\angle c_5o_1c_6 = \omega$ , and the corresponding sector areas are  $S_{top2}$ ,  $S_{root1}$ . Then,  $S_{top2} = 0.5r_{a,1}^2\sigma$ ,  $S_{root1} = 0.5r_{f,1}^2\tau$ . Once the gear parameters are determined, the radius of the root and top circles of the gear and the angle between each gear tooth can be determined [16]; then,  $S_{top1}$  and  $S_{root1}$  are fixed values. Similarly, the sector areas corresponding to the top and root circles of the internal gear are  $S_{top2}$  and  $S_{root2}$ . Then,  $S_{top2} = 0.5r_{a,2}^2\lambda$  and  $S_{root1} = 0.5r_{f,2}^2\omega$ .

## 3. Establishing the Calculation Model of Unloading Area

**3.1. Mathematical Model.** Figure 4(a) is a pair of internal gear pairs and  $e$  is the center distance. The Cartesian coordinate systems  $x_1O_1y_1$  and  $x_2O_2y_2$  are established by using the gear circle centers  $O_1$  and  $O_2$  as the coordinate origins and the symmetry lines of the external gear teeth and internal gear grooves as the  $y$ -axis. Taking Figure 1(a) as the initial position, the rotation angles of  $O_1$  and  $O_2$  are  $\theta_1$  and  $\theta_2$ , respectively. During the rotation, the outline of the external gear, the profile of the internal gear, and  $ud$  intersect at points  $w$  and  $v$ , respectively. Among them,  $S_{\cap w o 1 m 1}$  and  $S_{\cap v o 2 n 1}$  are the radius of point  $w$ , point  $n_1$ , point  $v$ , and the area enclosed by the contour line of the gear, respectively. For ease of calculation, let  $S_1 = S_{\cap w o 2 n 1}$ ,  $S_2 = S_{\Delta o 2 v m 1}$ ,  $S_3 = S_{\Delta u v m 1}$ ,  $S_4 = S_{\cap w o 1 n 1}$ , and  $S_5 = S_{\Delta o 1 w m 1}$ . Then, fold  $O_2 v$  and  $O_2 n_1$  along  $vm_1$  to get Figure 4(b). According to the graph, the unloading area  $S_{ud}$  can be expressed as

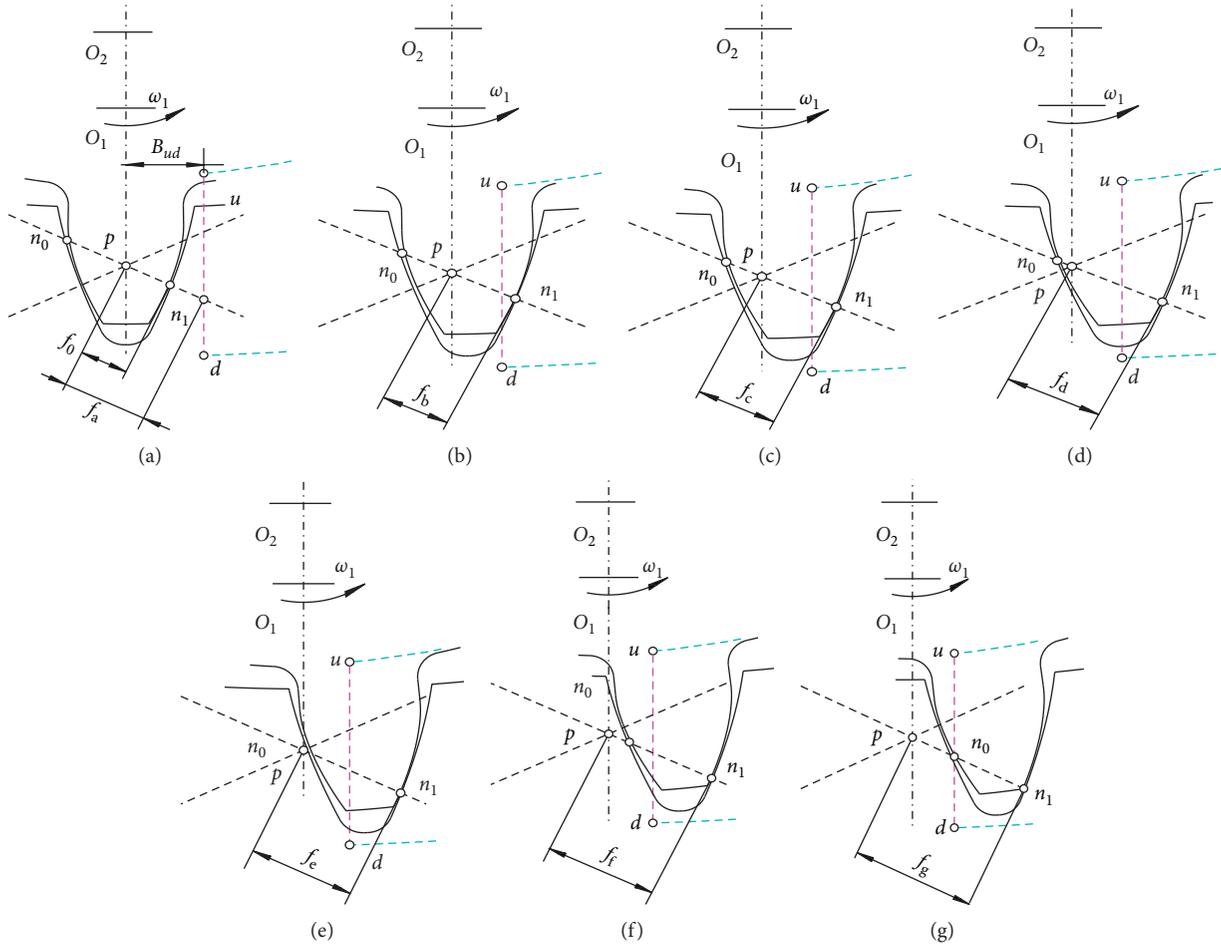


FIGURE 1: Oil trapping and unloading process of internal gear pump.

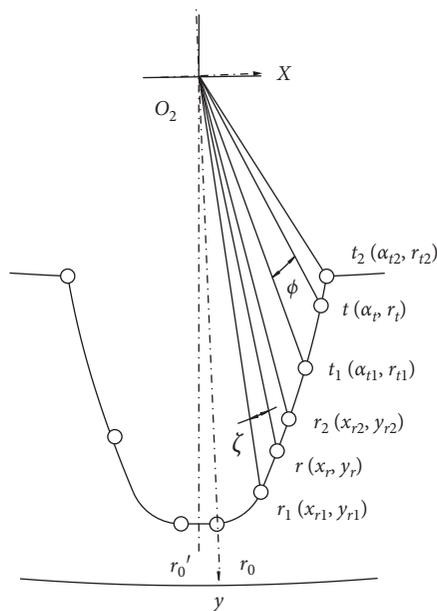


FIGURE 2: Internal gear profile.

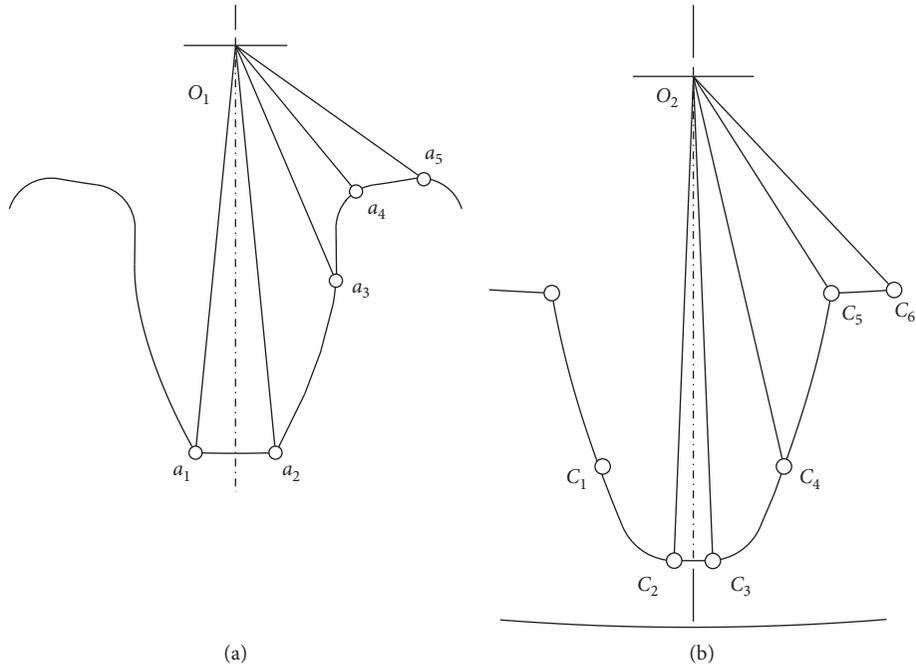


FIGURE 3: Gear profile machined by unrolling.

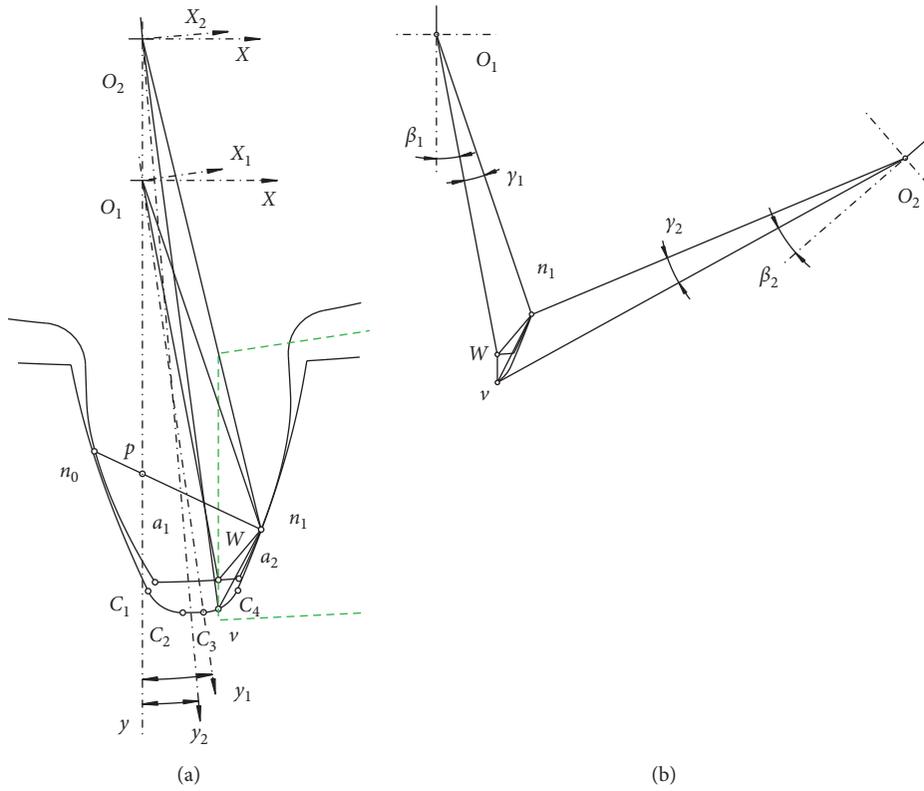


FIGURE 4: A pair of internal gears meshing.

$$S_{ud} = S_1 - S_2 + S_3 - (S_4 - S_5). \quad (2)$$

Let  $\angle po_1w = \beta_1$ ,  $\angle n_1o_1w = \gamma_1$ ,  $\angle po_2v = \beta_2$ ,  $\angle n_1o_2v = \gamma_2$ .  $r_{p,1}$ ,  $r_{p,2}$ ,  $r_{n1,1}$ ,  $r_{n1,2}$  are the pitch circle radius and the meshing point radius of  $O_1$  and  $O_2$ , respectively.  $l_{vn1}$ ,  $l_{wn1}$ , and  $l_{vw}$  are distances between points  $v$ ,  $w$ , and  $n_1$ . From the geometric relationship in the figure,

$$\begin{cases} r_{n1,1}^2 = r_{p,1}^2 + f^2 - 2r_{p,1}f \cos\left(\frac{\pi}{2} + \alpha'\right), \\ l_{wn1}^2 = r_{n1,1}^2 + r_w^2 - 2r_{n1,1}r_w \cos \beta_1, \\ r_{n1,2}^2 = r_{p,2}^2 + f^2 - 2r_{p,2}f \cos\left(\frac{\pi}{2} + \alpha'\right), \\ l_{vn1} = r_{p,2}^2 + f^2 - 2r_{p,2}f \cos\left(\frac{\pi}{2} + \alpha'\right), \\ l_{vw} = r_w \cos \gamma_2 - r_v \cos \gamma_1 - e. \end{cases} \quad (3)$$

Let

$$\begin{cases} \rho_1 = \frac{(r_w + r_{n1,1} + l_{wn1})}{2}, \\ \rho_2 = \frac{(r_v + r_{n1,2} + l_{vn1})}{2}, \\ \rho_3 = \frac{(l_{wn1} + l_{vn1} + l_{vw})}{2}. \end{cases} \quad (4)$$

From Heron's formula,

$$\begin{cases} S_2 = [\rho_2(\rho_2 - r_v)(\rho_2 - r_{n1,2})(\rho_2 - l_{vn1})]^{1/2}, \\ S_3 = [\rho_3(\rho_3 - l_{wn1})(\rho_3 - l_{vw})(\rho_3 - l_{vn1})]^{1/2}, \\ S_5 = [\rho_1(\rho_1 - r_w)(\rho_1 - r_{n1,1})(\rho_1 - l_{wn1})]^{1/2}. \end{cases} \quad (5)$$

Apply sine theorem in  $\Delta wo_1n_1$ ,  $\Delta vo_2n_1$ :

$$\begin{cases} \frac{f}{\sin(\beta_1 + \gamma_1)} = \frac{r_{p,1}}{\sin[\pi - \beta_1 - \gamma_1 - ((\pi/2) + \alpha')]}, \\ \frac{f}{\sin(\beta_2 + \gamma_2)} = \frac{r_{p,2}}{\sin[\pi - \beta_2 - \gamma_2 - ((\pi/2) + \alpha')]}. \end{cases} \quad (6)$$

Deduce

$$\begin{cases} \gamma_1 = \tan^{-1} \frac{f \cos \alpha'}{r_{p,1} + f \sin \alpha'} - \beta_1, \\ \gamma_2 = \tan^{-1} \frac{f \cos \alpha'}{r_{p,2} + f \sin \alpha'} - \beta_2. \end{cases} \quad (7)$$

Since  $\theta_1$ ,  $\theta_2$ , and  $f$  are linear functions, then from (2)–(7), only the changes of  $r_w$  and  $\beta_1$  with  $\theta_1$ ,  $r_v$  and  $\beta_2$  with  $\theta_2$  can be used to obtain the change of  $S_{ud}$ .

**3.2. External Gear Rotation Parameters.** When  $a_1$  and  $a_2$  are located on  $ud$ ,  $\beta_1$  can be expressed as  $\beta_{1,a1} = \beta_{1,a2} = \sin^{-1}(B_{ud}/r_{a,1})$ . When points  $n_0$ ,  $a_1$ ,  $a_2$ , and  $n_1$  are located on  $ud$ , these rotation angles  $\theta_1$  can be expressed as  $\theta_{1,a} = (f_a - f_0)/r_{b,1}$ ,  $\theta_{1,a1} = \beta_{1,a1} - (\sigma/2)$ ,  $\theta_{1,a2} = \beta_{1,a2} + (\sigma/2)$ ,  $\theta_{1,g} = (f_g - f_0)/r_{b,1}$ . According to the structure of the external gear, when the point  $w$  moves to  $n_0a_1$ ,  $a_2n_1$  involute segment, according to the backlash-free meshing equation [17] and Figure 3(a), it can be obtained that

$$\begin{cases} |\theta_1 - \beta_1| = \frac{\pi + 4k_1 \tan \alpha}{2z_1} - (\text{inv} \alpha_w - \text{inv} \alpha), \\ \sin \beta_1 = \frac{(B_{ud}/r_{b,1})}{\cos \alpha_w}. \end{cases} \quad (8)$$

If  $\theta_1 \geq \beta_1$ , it means that the  $w$  point is on the  $n_0a_1$  line segment. If  $\theta_1 < \beta_1$ , it means that the  $w$  point is on the  $a_2n_1$  line segment.

When the point  $w$  is on the  $a_1a_2$  line segment,  $r_w = r_{a,1}$ ,  $\beta = \sin^{-1}(B_{ud}/r_{a,1})$ .

**3.3. Internal Gear Rotation Parameters.** If  $c_1$  and  $c_4$  are located on  $ud$ ,  $\beta_2$  can be expressed as  $\beta_{2,c1} = \beta_{2,c4} = \sin^{-1}(B_{ud}/r_{f,2})$ .

When the rotation process of  $O_2$ , several special points  $n_0$ ,  $c_1$ ,  $c_2$ ,  $c_3$ ,  $c_4$ ,  $n_1$ , is located on  $ud$ , the corresponding rotation angle  $\theta_2$  changes to,

$$\begin{cases} \theta_{2,a} = \frac{f_a - f_0}{r_{b,2}}, \\ \theta_{2,c4} = \beta_{2,c4} - \frac{\mu}{2}, \\ \theta_{2,c3} = \theta_{2,c4} + \lambda, \\ \theta_{2,c2} = \theta_{2,c3} + (\mu - 2\lambda), \\ \theta_{2,c1} = \theta_{2,c2} + \lambda, \\ \theta_{2,g} = \frac{f_g - f_0}{r_{b,2}}. \end{cases} \quad (9)$$

$\theta_{2,a} \rightarrow \theta_{2,g}$  is represented, respectively, as the process from  $f$  to the end of  $\theta_2$  when  $f_a \rightarrow f_g$ .

According to the internal gear structure, when the  $v$  point moves to the  $n_0c_1$  and  $c_4n_1$  involute segments, the backlash-free meshing equation is combined with Figure 3(a):

$$\begin{cases} |\theta_2 - \beta_2| = \frac{\pi - 4k_2 \tan \alpha}{2z_2} + (\text{inv} \alpha_v - \text{inv} \alpha), \\ \sin \beta_2 = \frac{(B_{ud}/r_{b,2})}{\cos \alpha_v}. \end{cases} \quad (10)$$

If  $\theta_2 \geq \beta_2$ , it means that the  $v$  point is on the  $n_1c_4$  segment. If  $\theta_2 < \beta_2$ , it means that the  $v$  point is on the  $c_1n_0$  segment. When the  $v$  point is at the transition curve  $c_1c_2$ ,  $c_3c_4$ , as shown in the figure:

$$|\theta_2 - \beta_2| = \angle y_2o_2c_3 + \angle c_3o_2v = \frac{\mu}{2} - \lambda + \tan^{-1} \frac{x_v}{y_v}. \quad (11)$$

If  $\theta_2 \geq \beta_2$ , it means that the  $v$  point is in the  $c_3c_4$  segment. If  $\theta_2 < \beta_2$ , it means that the  $v$  point is in the  $c_1c_2$  segment.  $(x_v, y_v)$  is the coordinate value of the  $v$  point in the coordinate system of the figure.

When the  $v$  point is at the root circle  $c_2c_3$ ,  $\beta_2 = \beta_{2,c2} = \beta_{2,c3}$ , and  $r_v = r_{f,2}$ , where  $r_{f,2}$  is the radius of the  $O_2$  root circle. Therefore,

$$S_4 = \begin{cases} 0, & (\theta_1 \leq \theta_{1,a}), \\ S_{\text{inv}} \Big|_{t_1=n_1}^{t_2=w}, & (\theta_{1,a} < \theta_1 \leq \theta_{1,a_2}), \\ S_{\text{inv}1} + \frac{\theta_1 - \theta_{1,a_1}}{\sigma} S_{\text{top}1}, & (\theta_{1,a_2} < \theta_1 \leq \theta_{1,a_1}), \\ S_{\text{inv}1} + S_{\text{top}1} + S_{\text{inv}} \Big|_{t_1=n_0}^{t_2=w}, & (\theta_{1,a_1} < \theta_1 \leq \theta_{1,g}), \end{cases}$$

$$S_1 = \begin{cases} 0, & (\theta_2 \leq \theta_{2,a}), \\ S_{\text{inv}} \Big|_{t_1=n_1}^{t_2=v}, & (\theta_{2,a} < \theta_2 \leq \theta_{2,c_4}), \\ S_{\text{inv}2} + S_{\text{sta}} \Big|_{r_1=c_4}^{r_2=v}, & (\theta_{2,c_4} < \theta_2 \leq \theta_{2,c_3}), \\ S_{\text{inv}2} + S_{\text{sta}2} + \frac{\theta_2 - \theta_{2,c_3}}{0.5\mu - \lambda} S_{\text{root}2}, & (\theta_{2,c_3} < \theta_2 \leq \theta_{2,c_2}), \\ S_{\text{inv}2} + S_{\text{sta}2} + S_{\text{root}2} + S_{\text{sta}} \Big|_{r_1=c_2}^{r_2=v}, & (\theta_{2,c_2} < \theta_2 \leq \theta_{2,c_1}), \\ S_{\text{inv}2} + 2S_{\text{sta}2} + S_{\text{root}2} + S_{\text{inv}} \Big|_{t_1=n_0}^{t_2=v}, & (\theta_{2,c_1} < \theta_2 \leq \theta_{2,g}). \end{cases} \quad (12)$$

#### 4. Optimized Design

If the internal gear pair parameters and center distance are determined, then,  $S_2$ ,  $S_3$ , and  $S_5$  are determined. The design goal hopes to optimize and increase  $S_{\text{ud}}$ . According to the aforementioned calculation model of  $S_{\text{ud}}$ , it can be achieved by increasing  $S_1$  or decreasing  $S_4$ . Considering the actual

situation,  $S_1$  is easier to implement, so two optimization schemes are proposed for machining the outer contour of the internal gear with a forming tool or mold. The first method is to increase the radius of the tooth root circle; the second method is to eliminate the cogging transition curve segment on the gear profile. The improvement method is shown in Figures 5(a) and 5(b). Let the radius of the root circle of Figure 5(a) be  $r_{fa}$ , and the corresponding sector area is  $S_{\text{root}2a} = 0.5r_{fa}^2\lambda$ . During the rotation, the sector enclosed by the radii  $v$  and  $n_1$  and their contour is  $S_{1a}$ :

$$S_{1a} = \begin{cases} 0, & (\theta_2 \leq \theta_{2,a}), \\ S_{\text{inv}} \Big|_{t_1=n_1}^{t_2=v}, & (\theta_{2,a} < \theta_2 \leq \theta_{2,e_4}), \\ S_{\text{inv}2} + S_{\text{sta}} \Big|_{r_1=v}^{r_2=e_4}, & (\theta_{2,e_4} < \theta_2 \leq \theta_{2,e_3}), \\ S_{\text{inv}2} + S_{\text{sta}2} + \frac{\theta_2 - \theta_{2,e_3}}{0.5\mu - \lambda} S_{\text{root}2a}, & (\theta_{2,e_3} < \theta_2 \leq \theta_{2,e_2}), \\ S_{\text{inv}2} + S_{\text{sta}2} + S_{\text{root}2a} + S_{\text{sta}} \Big|_{r_1=v}^{r_2=e_2}, & (\theta_{2,e_2} < \theta_2 \leq \theta_{2,e_1}), \\ S_{\text{inv}2} + 2S_{\text{sta}2} + S_{\text{root}2a} + S_{\text{inv}} \Big|_{t_1=v}^{t_2=n_0}, & (\theta_{2,e_1} < \theta_2 \leq \theta_{2,g}). \end{cases} \quad (13)$$

The unloading area of the first scheme is  $S_{\text{uda}}$ .  $S_{1a}$  can be derived by replacing  $S_1$  in (2), then, obtaining  $S_{\text{uda}}$ .

Let the radius of the root circle of Figure 5(b) be  $r_{fb}$ , and its corresponding sector area is  $s_{\text{root}2b}$ ,  $s_{\text{root}2b} = 0.5r_{fb}^2\lambda$ . During its rotation, the radii of points  $v$  and  $n_1$  and their corresponding contour lines are enclosed in a fan shape, and their area is  $s_{1b}$ :

$$s_{1b} = \begin{cases} 0, & (\theta_2 \leq \theta_{2,a}), \\ s_{\text{inv}} \Big|_{t_1=n_1}^{t_2=v}, & (\theta_{2,a} < \theta_2 \leq \theta_{2,q_4}), \\ s_{\text{inv}2} + \frac{\theta_2 - \theta_{2,q_1}}{\lambda} s_{\text{root}2b}, & (\theta_{2,q_4} < \theta_2 \leq \theta_{2,q_1}), \\ s_{\text{inv}2} + s_{\text{root}2b} + s_{\text{inv}} \Big|_{t_1=v}^{t_2=n_0}, & (\theta_{2,q_1} < \theta_2 \leq \theta_{2,g}). \end{cases} \quad (14)$$

The unloading area of the second scheme is  $S_{\text{udb}}$ .  $S_{\text{udb}}$  can be derived by replacing  $S_1$  in (2) with  $S_{1b}$ .

#### 5. Simulation

Taking the parameters of medium and high pressure internal gear pumps of Fuzhou University Hydraulic Parts Factory as an example, enveloping processing, the first optimization scheme, and the second optimization scheme, the parameters are shown in Table 1.

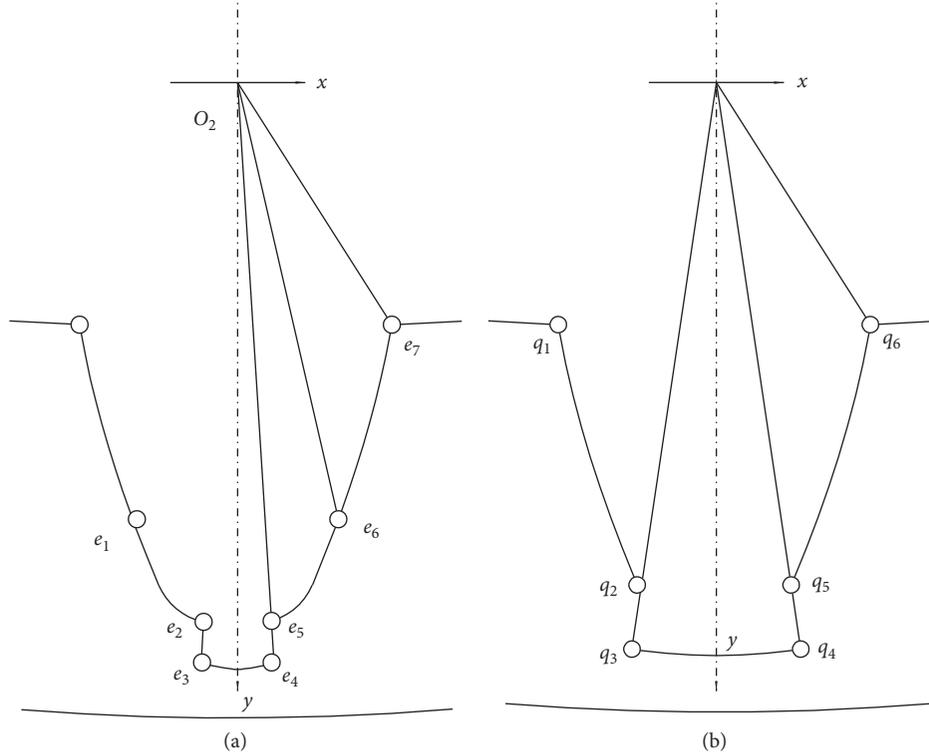


FIGURE 5: Optimizing the internal gear profile. (a) The first optimization scheme and (b) the second optimization scheme.

TABLE 1: Internal gear pump parameters.

Name of the parameter	Parameter	Value
Number of teeth	$z_1/z_2$	13/19
Modulus (mm)	$m$	3
Center distance (mm)	$e$	9.278
Pressure angle (°)	$\alpha$	20
Coefficient of rack cutter tip height	$h_{a0}^*$	1.25
External gear profile shift coefficient	$K_1$	0.432
Internal gear profile shift coefficient	$K_2$	0.55
Unloading groove boundary distance (mm)	$B_{ud}$	2.035
The first solution root radius (mm)	$r_{fa}$	32.0
The second solution root radius (mm)	$r_{fb}$	33.80

During the gear rotation, the position variable  $f$  is left-right symmetric with the position shown in Figure 1(a). MATLAB software is used to draw the unloading area, which changes with  $f$  in a meshing cycle as shown in Figure 6.

The parameters provided in Table 1 are the original data of the gear pump during the actual design and manufacture, which are consistent with the parameters under the number of different teeth ( $z_1 + z_2 = 32$ ) in [18]. From the results of the simulation of  $S_{ud}$  in Figure 6, the change law of  $S_{ud}$  is very consistent with [18] during a period in which the position variable  $f$  changes. This verifies the correctness of the calculation of the unloading area  $S_{ud}$ .

Once the gear parameters are determined, the outer contour of the gear teeth formed by the envelope method is a certain value. Therefore, the shaping method of the internal tooth contour machining method is improved.

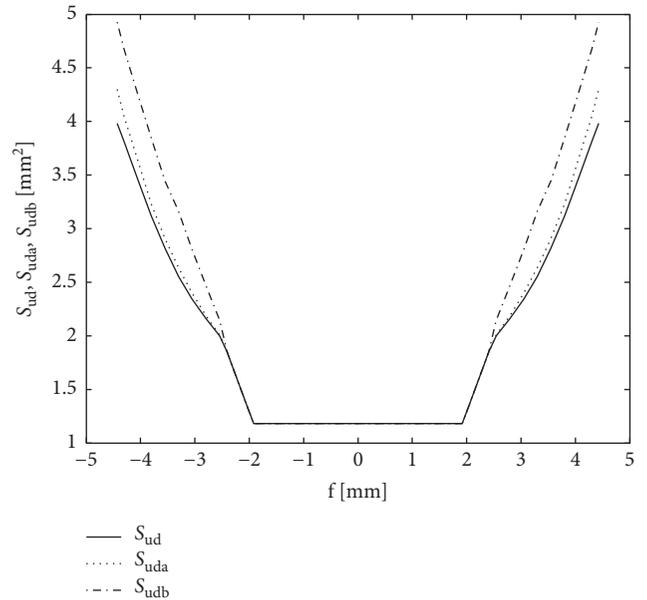


FIGURE 6: Changes in unloaded area.

From the changes of  $S_{uda}$  and  $S_{udb}$  in Figure 6, it can be seen that increasing the root circle radius of the internal gear tooth profile can obtain a larger unloading area  $S_{ud}$  during the oil trapping process. The larger the radius of the root circle, the larger  $S_{ud}$ . Therefore, the unloading area obtained by the second optimization scheme is larger.

## 6. Conclusions

- (1) The analytical method is used to accurately calculate the unloading area of the internal gear pump during oil trapping, and a calculation expression is given. This calculation result can be transplanted to a computer program, which can accurately simulate the trapped oil and unloading process of the internal gear pump. It has a guiding significance for the design of high-performance internal gear pumps.
- (2) The calculation formula of the unloading area of the internal gear profile formed by different processing methods during oil trapping is given.
- (3) The example compares the changes in the unloading area of the internal gear profile formed by the envelope method, forming tool method and die processing method during oil trapping and unloading. The larger the radius of the root circle of the internal tooth profile, the larger the unloading area, which provides a theoretical reference for the future design and improvement of the oil trapping phenomenon of the internal gear pump.

## Nomenclature

$z_1$ :	Number of external gear teeth	$t_1, t_2$ :	The limit point of the involute of the gear outline
$z_2$ :	Number of internal gear teeth	$t$ :	Sliding point from point $t_1$ to point $t_2$
$m$ :	Modulus	$t_1t_2$ :	Contour of involute
$e$ :	Distance between centers of gears $O_1$ and $O_2$	$r$ :	Sliding point from point $r_1$ to point $r_2$
$\alpha$ :	Pressure angle	$r_1, r_2$ :	The limit point of the transition curve of gear outline
$h_{a0}^*$ :	Coefficient of rack cutter tip height	$r_1r_2$ :	Contour of transition curve
$K_1$ :	External gear profile shift coefficient	$\alpha_x$ :	Gear pressure angle at point $x$
$K_2$ :	Internal gear profile shift coefficient	$r_x$ :	Circle radius at point $x$
$O_1$ :	Center of external gear	$x_x, y_x$ :	Abscissa, ordinate in the coordinate system
$O_2$ :	Center of internal gear	$\theta_x$ :	Rotation angle at point $x$
$p$ :	Gear joint node	$\emptyset$ :	Angle between line $t_1o_2$ and $t$ line $o_2t$
$n_1$ :	Meshing point	$\zeta$ :	Angle between line $r_1o_2$ and $t$ line $o_2r$
$n_0$ :	Backlash point	$s_{inv} _{t1}^2$ :	Area enclosed by $t_1t_2, t_1o_2$ , and $o_2t_2$
$ud$ :	Boundary of the rectangular unloading groove	$s_{sta} _{r1}^2$ :	Area enclosed by $r_1r_2, r_1o_2$ , and $o_2r_2$
$f$ :	Length from point $p$ to point $n_1$	$r'$ :	Pitch circle radius
$f_a$ :	Length from point $p$ to point $n_1$	$r_{a,1}, r_{a,2}$ :	Top circle of the external gear and internal gear
$f_b$ :	Length from point $p$ to point $n_1$	$r_{f,1}, r_{f,2}$ :	Root circle of the external gear and internal gear
$f_c$ :	Length from point $p$ to point $n_1$	$a_1, a_2$ :	The limit point of the addendum circle profile
$f_d$ :	Length from point $p$ to point $n_1$	$c_5, c_6$ :	The limit point of the addendum circle profile
$f_e$ :	Length from point $p$ to point $n_1$	$a_5, a_6$ :	The limit point of tooth root circle profile
$f_f$ :	Length from point $p$ to point $n_1$	$c_2, c_3$ :	The limit point of tooth root circle profile
$f_g$ :	Length from point $p$ to point $n_1$	$c_1, c_4$ :	The limit point of the transition curve of gear outline
$p_b$ :	Base pitch of external gear $O_1$	$\sigma$ :	Angle between line $a_1o_1$ and $t$ line $o_1a_2$
$r_{b,1}$ :	Base circle radius of external gear $O_1$	$\tau$ :	Angle between line $a_5o_1$ and $t$ line $o_1a_6$
$\alpha_{a,1}$ :	Pressure angle of tooth top circle	$\mu$ :	Angle between line $c_1o_2$ and $t$ line $o_2c_4$
$\alpha$ :	Meshing angle	$\lambda$ :	Angle between line $c_2o_2$ and $t$ line $o_2c_3$
$B_{ud}$ :	Distance from $ud$ to the line connecting $O_1O_2$	$\omega$ :	Angle between line $c_5o_2$ and $t$ line $o_2c_6$
$xOy$ :	Coordinate system with $O$ as origin	$s_{top1}$ :	Area enclosed by $a_1a_2, a_1o_1$ , and $o_1a_2$
$x_1O_1y_1$ :	Coordinate system with $O_1$ as origin	$s_{root1}$ :	Area enclosed by $a_5a_6, a_5o_1$ , and $o_1a_6$
$x_2O_2y_2$ :	Coordinate system with $O_2$ as origin	$s_{top2}$ :	Area enclosed by $c_5c_6, c_5o_2$ , and $o_2c_6$
$\theta$ :	$Xoy$ rotation angle around $O$	$s_{root2}$ :	Area enclosed by $c_2c_3, c_2o_2$ , and $o_2c_3$
$\theta_1$ :	$X_1o_1y_1$ rotation angle around $O_1$	$w$ :	Intersection point of $ud$ and external gear outline
$\theta_2$ :	$X_2o_2y_2$ rotation angle around $O_2$	$v$ :	Intersection point of $ud$ and internal gear outline
		$S_1, S_{\Gamma_{wo1n1}}$ :	Area enclosed by $o_1w, o_1n_1$ and the gear outline $wn_1$
		$S_4, S_{\Gamma_{vo2n1}}$ :	Area enclosed by $o_2v, o_2n_1$ and the gear outline $vn_1$
		$S_2, S_{\Delta o2vn1}$ :	Area enclosed by $o_2v, o_2n_1$ , and $vn_1$
		$S_3, S_{\Delta wvn1}$ :	Area enclosed by $wv, vn_1$ , and $wv$
		$S_5, S_{\Delta o1wn1}$ :	Area enclosed by $o_1w, o_1n_1$ , and $wn_1$
		$S_{ud}$ :	Unloading area
		$S_{uda}$ :	Unloading area
		$S_{udb}$ :	Unloading area
		$\beta_1$ :	Angle between line $po_1$ and $t$ line $o_1w$
		$\gamma_1$ :	Angle between line $n_1o_1$ and $t$ line $o_1w$
		$\beta_2$ :	Angle between line $po_2$ and $t$ line $o_2v$
		$\gamma_2$ :	Angle between line $n_1o_2$ and $t$ line $o_2v$
		$r_{p,1}$ :	Pitch circle radius of external gear $O_1$
		$r_{p,2}$ :	Pitch circle radius of internal gear $O_2$
		$r_{n1,1}$ :	Meshing point radius of $O_1$
		$r_{n1,2}$ :	Meshing point radius of $O_2$
		$l_{vn1}$ :	Distances between points $v$ and $n_1$
		$l_{wn1}$ :	Distances between points $w$ and $n_1$

$l_{vw}$ :	Distances between points $v$ and $w$
$r_w$ :	Circle radius of external gear $O_1$ at point $w$
$r_v$ :	Circle radius of internal gear $o_2$ at point $v$
$\beta_{1,a1}$ :	Angle of $\beta_1$ when point $a_1$ is located on $ud$
$\beta_{1,a2}$ :	Angle of $\beta_1$ when point $a_2$ is located on $ud$
$\rho_1, \rho_2, \rho_3$ :	Half of triangle perimeter
$\beta_{2,c1}$ :	Angle of $\beta_2$ when point $c_1$ is located on $ud$
$\beta_{2,c4}$ :	Angle of $\beta_2$ when point $c_4$ is located on $ud$
$\theta_{1,a}$ :	Angle of $\theta_1$ when point $a$ is located on $ud$
$\theta_{1,a1}$ :	Angle of $\theta_1$ when point $a_1$ is located on $ud$
$\theta_{1,a2}$ :	Angle of $\theta_1$ when point $a_2$ is located on $ud$
$\theta_{1,g}$ :	Angle of $\theta_1$ when point $g$ is located on $ud$
$\theta_{2,a}$ :	Angle of $\theta_2$ when point $a$ is located on $ud$
$\theta_{2,c4}$ :	Angle of $\theta_2$ when point $c_4$ is located on $ud$
$\theta_{2,c3}$ :	Angle of $\theta_2$ when point $c_3$ is located on $ud$
$\theta_{2,c2}$ :	Angle of $\theta_2$ when point $c_2$ is located on $ud$
$\theta_{2,c1}$ :	Angle of $\theta_2$ when point $c_1$ is located on $ud$
$\theta_{2,g}$ :	Angle of $\theta_2$ when point $g$ is located on $ud$
$r_{fa}$ :	Root circle of internal gear
$S_{root2a}$ :	Area enclosed by $o_2e_3, o_2e_4$ and the gear outline $e_3e_4$
$S_{1a}$ :	Unloading area
$r_{fb}$ :	Root circle of internal gear
$S_{root2b}$ :	Area enclosed by $o_2q_3, o_2q_4$ and the gear outline $q_3q_4$
$S_{1b}$ :	Unloading area.

## Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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