Research Article

New Waterflooding Characteristic Curves Based on Cumulative Water Injection

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When a conventional waterflooding characteristic curve (WFCC) is used to predict cumulative oil production at a certain stage, the curve depends on the predicted water cut at the predicted cutoff point, but forecasting the water cut is very difficult. For the reservoirs whose pressure is maintained by water injection, based on the water-oil phase seepage theory and the principle of material balance, the equations relating the cumulative oil production and cumulative water injection at the moderately high water cut stage and the ultrahigh water cut stage are derived and termed the Yuan-A and Yuan-B curves, respectively. And then, we theoretically analyze the causes of the prediction errors of cumulative oil production by the Yuan-A curve and give suggestions. In addition, at the ultrahigh water cut stage, the Yuan-B water cut prediction formula is established, which can predict the water cut according to the cumulative water injection and solve the difficult problem of water cut prediction. The application results show Yuan-A and Yuan-B curves are applied to forecast oil production based on cumulative water injection data obtained by the balance of injection and production, avoiding reliance on the water cut forecast and solving the problems of predicting the cumulative oil production of producers or reservoirs that have not yet shown the decline rule. Furthermore, the formulas are simple and convenient, providing certain guiding significance for the prediction of cumulative oil production and water cut for the same reservoir types.

1. Introduction

The water flooding characteristic curve (WFCC) is an important method used to predict recoverable reserves for water flooding oilfields and is widely used around the world. On the basis of the linear correlation of $\ln(k_{ro}/k_{rw})$ vs. $S_w$, Ershaghi and Omoregie [1] and Ershaghi and Abdassah [2] proposed a model for oil recovery vs. water cut. Tong [3] proposed three kinds of WFCCs based on the statistical analysis of the relationship between the actual production data of waterflooding oilfields. Chen Yuanqian [4] and Yu Qitai [5] derived different kinds of WFCCs by incorporating Craft et al.’s model and the average water saturation estimated from Buckley–Leverett [6] and Welge [7] equations. Yang [8] obtained an analytical solution of oil fractional flow and developed a diagnostic analysis tool of waterflooding reservoirs. At present, there are dozens of WFCCs, but a large number of oilfield practices have proved that the Type-A and B are efficient tools for the forecast of waterflooding performance [4, 9–11]. Type-A curve describes the relationship between cumulative water production ($W_p$) and cumulative oil production ($N_p$), whereas Type-B curve is for water-oil ratio (WOR) and $N_p$. Both of them are straight lines in a semilog coordinate system. Currently, these curves are frequently employed in waterflooding reservoirs and even recognized as the industry standard.

The $k_{ro}/k_{rw}$ vs $S_w$ correlation deviates from a straight line in a semilog plot at the stage of high water cut and ultrahigh water cut (>90%) in oilfields [12]. Aiming at this problem, Liu Shihua [13], Song Zhaojie [14], Jin Rongrong [15], Liu Zhibin [16], Wang Jiqiang [17], and so on, successively put forward new expressions of the oil-water relative permeability ratio at the stage of ultrahigh water cut and established corresponding new WFCCs. In
addition, in order to expand their application range, Zhang Jinqing [18] proposed a new curve having broadened application range. Feng Qi Hong [19] presented a unified mathematical model of $\ln(k_{ro}/k_{rw})$ vs. $S_w$ to account for different water saturations and developed two type curves to predict the waterflooding performance. Generally, they are applicable at the middle and late stages of waterflooding oilfields and only after the water cut reaches 40%–50% [20]. The WFCCs are applied to predict the recoverable reserves of reservoirs or oil wells. However, the prediction of cumulative oil production at a certain stage depends on the predicted water cut at the predicted cutoff point.

The slower rising of the water cut makes it difficult to predict at the stage of ultrahigh water cut. At present, there are three kinds of methods to predict water cut at this stage: growth curves, WFCCs, and water cut prediction modes. The main problems in the application process are as follows: (1) the frequently used growth curves, such as the Usher, logistic, and Gompertz curves [21], require the fitting of many parameters and have multiple solutions; (2) the water cut prediction mode established by Liu Peng et al. [22] based on the conventional WFCCs is not suitable at the ultrahigh water cut stage. Additionally, it is necessary to know the predicted cumulative oil production, which is difficult to predict when we forecast the water cut; and (3) with regard to the water-oil percolation principle, the water cut prediction model proposed by Yang Renfeng et al. [23] is based mainly on the linear relationship between the water saturation and logarithm of the oil-water relative permeability ratio at the medium-high water cut stage, so it is not suitable at the ultrahigh water cut stage. The water cut prediction model proposed by Zhao Yanwu et al. [24] is suitable at the ultrahigh water cut stage, but it has many fitting parameters and multiple solutions.

For the reservoirs whose pressure is maintained by water injection, based on water-oil phase seepage theory and the principle of material balance, the equations between cumulative oil production and cumulative water injection at the moderately high water cut stage and ultrahigh water cut stage are derived, termed the Yuan-A and Yuan-B curves, respectively. For water flooding reservoirs at the moderately high and ultrahigh water cut stage, under the condition of stable liquid production, the Yuan-A and Yuan-B curves are applied to forecast oil production based on cumulative water injection data obtained by the balance of injection and production, avoiding reliance on the forecast of the water cut and solving the problems of predicting the cumulative oil production of oil wells or reservoirs that have not yet shown the decline rule. This paper theoretically analyzes the causes of the prediction errors of cumulative oil production by the Yuan-A curve. In view of the errors caused by assuming a constant water cut during the derivation, the prediction accuracy of cumulative oil production can be improved by continuously updating the fitting history data. In addition, at the ultrahigh water cut stage, based on the Yuan-B and Type-Wang curves, the Yuan-B water cut prediction formula is established, which can predict the water cut according to the cumulative water injection and solve the difficult problem of water cut prediction. Then, some examples are used to illustrate the results.

2. Derivation for New WFCCs

2.1. Assumptions. (1) The formation temperature is constant; (2) gravity and capillary forces are ignored; (3) the reservoir features a weak natural aquifer; (4) the original reservoir pressure is maintained by water injection; (5) there is no loss of injection water or the loss is negligible; and (6) the percolation behavior satisfies Darcy’s law.

2.2. Derivation for the New WFCCs. During the moderately high water cut period, $K_{ro}/K_{rw}$, which is the ratio of the relative permeabilities of oil and water, has a semilogarithmic linear relationship with $S_{w}$, the water saturation at the outlet. However, the relationship deviates from the straight line and curves up after entering the ultrahigh water cut stage [20], so the new WFCCs are deduced at stages as follows.

2.2.1. The New WFCC at the Moderately High Water Cut Stage

(1) The derivation for the new WFCC at the moderately high water cut stage:

According to the literature [25], the relation between the water-oil ratio WOR and water saturation at the outlet is as follows:

$$WOR = \frac{Q_w}{Q_o} = \frac{\mu_o B_o K_{rw}}{\mu_w B_w K_{ro}} \cdot \frac{\mu}{\mu_w} \cdot \frac{dS_o}{dS_w} = \frac{\mu_o B_o}{\mu_w B_w} \cdot e^{S_w} \cdot \frac{dS_o}{dS_w},$$

where WOR is the water-oil ratio, dimensionless; $Q_w$ is the daily water production at ground conditions, $\text{m}^3/\text{d}$; $Q_o$ is the daily oil production at ground conditions, $\text{m}^3/\text{d}$; $\mu_o$ is the formation oil viscosity, mPa·s; $\mu_w$ is the formation water viscosity, mPa·s; $B_o$ is the volume factor of the formation oil, dimensionless; $B_w$ is the volume factor of formation water, dimensionless; $K_{ro}$ and $K_{rw}$ are the relative permeabilities of oil and water, respectively, dimensionless; $S_{wo}$ is the water saturation at the outlet, dimensionless; and $c$ and $d$ are constants related to the reservoir and fluid properties, respectively, dimensionless.

For a reservoir that maintains formation pressure by injecting water and that balances injection and production, the formula for the water saturation at the outlet [9] can be written as follows, and its detailed derivation is in Appendix.

$$S_{w} = 1 - S_{oi} + \frac{N_p S_{oi} - L_p S_{oi} - \left(\frac{dN_p}{dL_p}\right)}{N},$$

where $S_{w}$ is the residual oil saturation, %; $S_{oi}$ is the original oil saturation, %; $N$ is the original oil in place, $10^4 \text{m}^3$; $N_p$ is the cumulative oil production, $10^4 \text{m}^3$; and $L_p$ is the cumulative liquid production, $10^4 \text{m}^3$. 
It is known that the oil content can be written as 
\[ f_o = 1 - f_w = dN_p/dL_p, \]
so the relation formula of the water-oil ratio and surface oil content conditions can be written as follows:
\[ \text{WOR} = \frac{1 - f_o}{f_o} = \frac{f_w}{1 - f_w} = \frac{1 - (dN_p/dL_p)}{dN_p/dL_p}. \tag{3} \]
where \( f_w \) is the water cut, %, and \( f_o \) is the oil content, %.

Substituting equation (2) and (3) into equation (1) and taking the common logarithm of both sides of the equation, we can obtain
\[ \ln \frac{1 - (dN_p/dL_p)}{(dN_p/dL_p)} = \ln \frac{\mu_o B_o}{d\mu_w B_w} \]
\[ + c \left[ 1 - S_{oi} N_p \right] / N - \frac{L_p S_{oi}}{N} \left( dN_p/dL_p \right). \tag{4} \]

Differentiating with respect to \( L_p \) on both sides of equation (4), we have
\[ \frac{1}{(1 - (dN_p/dL_p))(dN_p/dL_p)} = \frac{c S_{oi} L_p}{N} \tag{5} \]
If we set \( g = 1 - dN_p/dL_p \), then equation (5) can be written as
\[ \frac{1}{g} \frac{dL_p}{dN_p} = \frac{c S_{oi} L_p}{N}. \tag{6} \]

Under the following two conditions, \( \circ \) at the ultrahigh water cut stage, \( g = 1 - dN_p/dL_p = f_w = 1 \); \( \circ \) at the stage of a small change in the water cut, which means that \( g = 1 - dN_p/dL_p = f_w = \) constant is approximately constant, equation (6) can be simplified as follows:
\[ \frac{dL_p}{dN_p} = \frac{gc S_{oi} L_p}{N}. \tag{7} \]

For reservoirs whose pressure is maintained by water injection, with a balanced cumulative injection and production, we can write
\[ L_p B_L = W_i B_w, \tag{8} \]
where \( B_L \) is the formation fluid volume factor, dimensionless, and \( W_i \) is the cumulative water injection, \( 10^4 \text{m}^3 \).

Substituting equation (8) into equation (7) and performing indefinite integral computation on the separate variables of both sides, we obtain
\[ \ln \frac{B_w}{B_L} = \frac{gc S_{oi} L_p}{N} + h, \tag{9} \]
where \( h \) is the integration constant.

We set a constant \( a = h - \ln(B_{oi}/B_i) \) and constant \( b = gc S_{oi}/N \). Then, the Yuan-A curve can be simplified as follows:
\[ \ln W_i = a + bN_p, \tag{10} \]

(2) Error analysis of forecasted cumulative oil production:

By applying the Yuan-A curve to forecast the cumulative oil production based on the cumulative water injection, the relative error in the forecasted production can be written as follows:
\[ \frac{N_{p2} - N_{p1}}{N_{p1}} = \frac{(\ln W_{i2} - a_2)/b_2 - (\ln W_{i1} - a_1)/b_1}{(\ln W_{i1} - a_1)/b_1} \]
\[ = \left( \frac{b_1 (\ln W_{i2} - a_2)}{b_2 (\ln W_{i1} - a_1)} - 1 \right) \frac{f_w (\ln W_{i2} - a)}{f_w (\ln W_{i2} - a) - 1}, \tag{11} \]

where \( N_{p1} \) is the actual cumulative oil production, \( 10^4 \text{m}^3 \); \( N_{p2} \) is the forecasted cumulative oil production, \( 10^4 \text{m}^3 \); \( f_{w1} \) is the actual water cutoff, %; \( f_{w2} \) is the forecasted water cutoff, %; \( W_{i1} \) is the actual cumulative water injection, \( 10^4 \text{m}^3 \); \( W_{i2} \) is the forecasted cumulative water injection, \( 10^4 \text{m}^3 \); \( a_1 \) and \( b_1 \) are the fitting coefficients of the Yuan-A curve at the fitting stage; and \( a_2 \) and \( b_2 \) are the fitting coefficients of Yuan-A curve at the forecast stage.

We can see from equation (11) that the forecast error in the cumulative oil production, based on the Yuan-A curve, mainly comes from the error in forecasted \( a \) and \( b \), and \( b \) is affected mainly by the water cut change. Thus, in order to ensure the prediction accuracy of cumulative oil production, we have to take into account the error caused by the change in \( a \) and \( b \) in combination with the actual reservoir performance. For the error caused by assuming that the water cut is a constant value during the formula derivation, the prediction accuracy of cumulative oil production can be improved by constantly updating the fitting of historical data.

2.2.2. The New WFCC at the Ultrahigh Water Cut Stage

(1) The derivation of the new WFCC at the ultrahigh water cut stage:

In [17], a new type of expression that can describe the relative permeability ratio of oil and water at the ultrahigh water cut stage is proposed as follows:
\[ \frac{k_{ro}}{k_{rw}} = \left( \frac{1 - S_{wud}}{m} \right)^n, \tag{12} \]
where \( S_{wud} \) can be expressed as
\[ S_{wud} = \frac{S_{wud} - S_{wuc}}{1 - S_{or} - S_{wuc}}, \tag{13} \]
where $S_{\text{wd}}$ is the normalization water saturation, dimensionless.

In the ultrahigh water cut period, the average water saturation can be replaced by the water saturation at the outlet, and the oil recovery degree can be written as

$$ R = \frac{N_p}{N} = \frac{\overline{S_w} - S_{\text{wc}}}{1 - S_{\text{wc}}} = \frac{\overline{S_w} - S_{\text{wc}}}{1 - S_{\text{wc}}}, \quad (14) $$

where $\overline{S_w}$ is the average water saturation, dimensionless, and $S_{\text{wc}}$ is the irreducible water saturation, dimensionless.

The ultimate displacement efficiency $E_D$ is written as

$$ E_D = \frac{1 - S_{\text{or}} - S_{\text{wc}}}{1 - S_{\text{wc}}}, \quad (15) $$

where $E_D$ is the ultimate displacement efficiency, dimensionless, and $S_{\text{or}}$ is the residual oil saturation, dimensionless.

The movable oil in place $N_{\text{om}}$ can be expressed as

$$ N_{\text{om}} = N E_D, \quad (16) $$

where $N_{\text{om}}$ is the movable oil in place, $10^4$ m$^3$.

Substituting equations (14)–(16) into equation (13) gives

$$ S_{\text{wd}} = \frac{(S_{\text{wc}} - S_{\text{or}}/1 - S_{\text{wc}})}{(1 - S_{\text{or}} - S_{\text{wc}}/1 - S_{\text{wc}})} = \frac{R}{E_D} = \frac{N_p}{N E_D} = \frac{N_p}{N_{\text{om}}}, \quad (17) $$

Substituting equation (12) into equation (1) gives

$$ \text{WOR} = \frac{Q_o}{Q_o} = \frac{\mu_o B_o K_{ro}}{\mu_w B_w K_{rw}} = \frac{m_o B_o}{\mu_w B_w (1 - S_{\text{wd}})}, \quad (18) $$

At the ultrahigh water cut stage, equation (3) can be simplified as follows:

$$ \text{WOR} = \frac{f_w}{1 - f_w} = \frac{1 - f_o}{f_o} \approx \frac{1}{f_o} = \left(\frac{dN_p}{dL_p}\right). \quad (19) $$

(i) Substituting equation (8) into equation (21) and taking the common logarithm of both sides of the equation, we can obtain

$$ \ln W_i = \ln \frac{N_{\text{om}} m_o B_o}{\mu_w B_w (n - 1)} - \ln \frac{B_p}{B_L} + (1 - n) \ln \left(1 - \frac{N_p}{N_{\text{om}}}ight). \quad (22) $$

Assuming constant $u = \ln[(N_{\text{om}} m_o B_o)/((\mu_w B_w (n - 1))] - \ln(B_p/B_L)$ and constant $v = 1 - n$, the Yuan-B curve can be simplified as follows:

$$ \ln W_i = u + v \ln \left(1 - \frac{N_p}{N_{\text{om}}}ight). \quad (23) $$

Equation (23) is the Yuan-B curve at the ultrahigh water cut stage.

(2) Error analysis of forecasted cumulative oil production:

Applying the Yuan-B curve, we forecast the cumulative oil production based on the cumulative water injection; the relative error in the forecasted production can be written as follows:

$$ \frac{N_{p2} - N_{pi}}{N_{pi}} = \frac{1 - e^{(\ln W_{i2} - u_2)v_2}}{1 - e^{(\ln W_{i2} - u_2)v_2}} - \frac{1 - e^{(\ln W_{i1} - u_1)v_1}}{1 - e^{(\ln W_{i1} - u_1)v_1}}, \quad (24) $$

where $u_1$ and $v_1$ are the fitting coefficients of the Yuan-B curve at the fitting stage and $u_2$ and $v_2$ are the fitting coefficients of the Yuan-B curve at the forecast stage. According to equation (23), the coefficients $u$ and $v$ are constants that do not change with the change in the cumulative oil production. As a result, if the prediction for cumulative water injection is correct, the forecasting error in cumulative oil production by using the Yuan-B curve tends to 0.

3. Comparison and Combined Solution with the Regular WFCCs

3.1. Comparison with the Regular WFCCs

Academician Tong Xianzhan [3] proposed the Type-A curve (see equation (25)) in 1978. Professor Chen Yuanqian [4] performed a theoretical derivation of equation (25) to obtain the relation formula of recoverable reserve prediction (see equation (26)). In a comparison of the Type-A curve with the Yuan-A curve, the main difference is that the former is the relation formula of cumulative water injection and cumulative oil production, while the latter is the relation formula of cumulative water production and cumulative oil production.

Wang Jiqiang et al. [17] put forward a new formula for the relative permeability ratio of oil and water in 2017 and derived a Type-Wang curve at the ultrahigh water cut stage.
(see equation (27)). The corresponding relation formula of recoverable reserve prediction is given in equation (28). In a comparison of the Yuan-B and Type-Wang curve, the main difference is that the former is the formula for cumulative water injection and cumulative oil production, while the latter is the formula for cumulative water production and cumulative oil production.

\[ \ln W_p = a_3 + b_3 N_p, \]  
\[ N_p = \ln \left( \frac{f_w (1 - f_w)}{(a_3 + \ln b_3)} \right), \]  
\[ \ln W_p = u_3 + v_3 \ln \left( 1 - \frac{N_p}{N_{om}} \right), \]  
\[ N_p = N_{om} \left( 1 - e^{(1/ \nu_3)} \right) \left[ u_3 - \ln \left( f_w N_{om} / (f_w - 1) \right) \right], \]  

where \( a_3 \) and \( b_3 \) are the fitting coefficients of the Type-A curve and \( u_3 \) and \( v_3 \) are the fitting coefficients of the Type-Wang curve.

According to equations (26) and (28), the prediction of cumulative oil production relies on the predicted water cut at the predicted cutoff point when applying the Type-A and Type-Wang curves to forecast cumulative oil production. It is very difficult to forecast the water cut, especially when the reservoir is in a period of great change in the water cut rising rate, whose sudden increase or decrease would make the water cut forecast impossible. In other words, it is difficult to forecast cumulative oil production when applying the Type-A and Type-Wang curves.

There are many advantages of using new type curves to forecast the cumulative oil production. For water flooding reservoirs at the moderately high and ultrahigh water cut stages, under the condition of stable liquid production, we can apply the Yuan-A and Yuan-B curves to forecast oil production based on cumulative water injection data obtained by the balance of injection and production, avoiding reliance on the forecast of the water cut and solving the problems of predicting the cumulative oil production of oil wells or reservoirs that have not yet shown the decline rule. In addition, those formulas are simple and convenient to use, which can help technicians substantially in oil production forecasting.

3.2. Combined Solution with the Regular WFCCs. In the medium-high water cut period, the Yuan-A curve in equation (10) and the recoverable reserve calculation (equation (26)) of the Type-A curve are combined to obtain the relation formula of the water cut and cumulative water injection, named the Yuan-A water cut prediction formula (see equation (29)). At the moderately high water cut stage, the water cut is at a certain stage, and the water cut can be predicted based on the cumulative water injection:

\[ f_w = \frac{1}{1 + e^{-\left(\left(\frac{b_3}{a_3}\right) \left( \ln W_i - a_3 \right) \ln b_3 \right)}}. \]  

At the ultrahigh water cut stage, relation formula (23) of the Yuan-B curve can be written as

\[ N_p = N_{om} \left[ 1 - e^{(-\ln W_i / \nu_3 - \ln b_3) / \nu_3} \right]. \]

Equation (30) is combined with equation (28) calculating the recoverable reserves of the Type-Wang curve, and the relationship between the water cut and cumulative water injection is obtained. This expression is named the Yuan-B water cut prediction formula (see equation (31)). The water cut can be predicted according to the cumulative water injection in an ultrahigh water cut period:

\[ f_w = \frac{1}{1 - (N_{om} / \nu_3) e^{((1/ \nu_3)(\ln W_i - a_3)/ \nu_3)}}. \]

4. Applications and Discussion

4.1. Applications and Discussion of the Numerical Model Index of the M Reservoir. According to the geological features of the M reservoir, a reservoir simulation model was built with a grid dimension of 25 × 25 × 15, a grid size in the plane of 40 meters, and a grid size in the vertical direction of 2 meters. The average porosity is 25%, and the average permeability is 750 mD. The formation oil viscosity is 1.4 mPa.s. The viscosity ratio of oil and water is 3.0. The volume of the original oil in place is 531 × 10^4 m^3. Five-spot well patterns are applied, and pressure maintenance is performed by water injection. The peak production rate is 7.5%, and the development indexes can be calculated by the reservoir numerical model (see Table 1).

4.1.1. At the Moderately High Water Cut Stage. The Yuan-A curve is applied to linearly fit \( N_p \) and \( \ln W_i \) of the M reservoir with a water cut in the range of 40%–45%, as shown in Figure 1. From Figure 1, it can be seen that the development index with a water cut in the range of 40%–55% is on the straight fitting line of the Yuan-A curve, which indicates that the stage fitting of the Yuan-A curve is good and can be used for the prediction of the stage cumulative oil production. Figure 2 shows the prediction results and error analysis of the cumulative oil production. The relative error in the predicted cumulative oil production is less than 0.5% with a water cut in the range of 40%–60%. Once the water cut is greater than 60%, the relative error increases rapidly with the water cut rising, but it is less than 5% with water cut in the range of 60%–80%, indicating that the predicted accuracy of cumulative oil production by the Yuan-A curve is high within a certain range of the water cut.

The Yuan-A curve is applied to linearly fit \( N_p \) and \( \ln W_i \) of the M reservoir with a water cut in the range of 60%–65%, and then it is used to forecast the cumulative oil production. Figure 3 shows the prediction results and error analysis of the cumulative oil production. The relative error in the predicted cumulative oil production is less than 2.0% with a water cut in the range of 66%–77%. After the water cut exceeds 77%, the relative error increases rapidly with the water cut rising, but it is less than 5% a with a water cut in the
range of 60%–86%. Therefore, the prediction accuracy of cumulative oil production can be improved by continuously updating the fitting history data in view of the error caused by the assumed conditions that the water cut is constant in the derivation process of the Yuan-A curve.

In addition, the Yuan-A curve avoids relying on the forecast of the water cut and solves the problems of predicting the cumulative oil production of oil wells or reservoirs that have not yet shown the decline rule.

With water cut in the range of 40%–45% and 50%–55%, the production indexes are fitted by the Type-A curve, and then the water cut of the $M$ reservoir in the moderately high water cut period is predicted by the Yuan-A water cut prediction formula. The prediction results and error analysis of the water cut are shown in Figures 4 and 5, and it can be seen that the relative error in the predicted water cut increases with increasing cumulative oil production. The relative error is greater than 5.0% with the fitted data with a water cut in the range of 40%–45%, and the relative error in the predicted water cut is greater than 5.0%, except for the two points with the fitted data with a water cut in the range of 50%–55%. Therefore, at the moderately high water cut stage, the Yuan-A water cut prediction formula is affected by the simplification of formula (6) in the derivation process.

<table>
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<th>Monthly oil production ($10^4$ m$^3$)</th>
<th>Monthly water production ($10^4$ m$^3$)</th>
<th>Monthly water injection ($10^4$ m$^3$)</th>
<th>Cumulative oil production ($10^4$ m$^3$)</th>
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and the prediction accuracy of the water cut is low, so it is not recommended to use this formula.

4.1.2. At the Ultrahigh Water Cut Stage. The Yuan-B curve is applied to fit $1 - \frac{N_p}{N_{om}}$ and $W_i$ of the $M$ reservoir with a water cut in the range of 91.3%–94.5% (12–15 years), as shown in Figure 6. It can be seen from Figure 6 that $1 - \frac{N_p}{N_{om}}$ and $W_i$ of the $M$ reservoir at the ultrahigh water cut stage are essentially located on the fitting curve of the Yuan-B curve.

The Yuan-B curve is used to fit the index with a water cut in the range of 91.3%–94.5%. Under the condition of stable liquid production, the cumulative oil production is predicted according to cumulative water injection data obtained by the balance of injection and production. The prediction results and error analysis of the cumulative oil production in the ultrahigh water cut period of the $M$ reservoir are shown in Figure 7. From Figure 7, we can see that the relative error in the predicted cumulative oil production increases with increasing water cut, and the max relative error is less than 1.0% with a water cut in the range of 95%–98%. The predicted accuracy of cumulative oil production by the Yuan-B curve is high in the ultrahigh water cut period.

It can be seen from predicted cumulative oil production equation (28) by the Type-Wang curve that the cumulative oil production at the prediction stage depends on the prediction results of the water cut, while when the reservoir is at the stage of an ultrahigh water cut, the water cut increase is slow, and the prediction of the water cut is difficult. Therefore, it is difficult to predict the cumulative oil production using the Type-Wang curve at this stage. For the Yuan-B curve, under the condition of stable liquid production, the cumulative oil production is predicted according to cumulative water injection data obtained by the balance of injection and production. This approach avoids relying on the forecast of the water cut and solves the problems of predicting the cumulative oil production of oil wells or reservoirs that have not yet shown the decline rule.

The Type-Wang curve is used to fit the index with a water cut in the range of 91.3%–94.5%, as shown in Figure 8. Then, the Yuan-B water cut prediction formula is used to predict the water cut of the $M$ reservoir at the ultrahigh water cut stage, and the prediction results and error analysis of the water cut are shown in Figure 9. From Figure 9, we can see that the relative error in the predicted water cut increases with increasing cumulative oil production, and the max relative error is less than 0.5% with a water cut in the range of 94.5%–98%. It shows that the predicted accuracy of the water cut by the Yuan-B water cut prediction formula is high, and it solves the difficult problem of water cut prediction in ultrahigh water cut periods.

4.2. Applications and Discussions of the $X$ Reservoir Index in the Bohai Oilfield. The $X$ reservoir is an edge water reservoir in the Bohai oilfield controlled by the structure, and the edge water energy is weak. The sedimentary faces of the reservoir are shallow water deltas. The average porosity is 34.1%, and the average permeability is 3790 mD; thus, the $X$ reservoir is a high-porosity and ultrahigh permeability reservoir. The viscosity of the formation crude oil is 21.5 mPa s, and the pressure difference between the formation and saturation pressures is 3.9 MPa. The original formation pressure is maintained by water injection, and the well pattern is an anti-seven spot. The $X$ reservoir was put into production in April 2008, and by August 2019, the reservoir water cut reached 87%.

The Yuan-A curve is applied to linearly fit $N_p$ and $\ln W_i$ of the $X$ reservoir with a water cut in the range of 40%–87%, as shown in Figure 10. From Figure 10, it can be seen that the development index with a water cut in the range of 40%–87% is on the fitting straight line of the Yuan-A curve, which indicates that stage fitting of this curve is good and can be used for the prediction of the stage cumulative oil production.

The Yuan-A curve is applied to linearly fit $N_p$ and $\ln W_i$ of the $M$ reservoir with a water cut in the range of 40%–45%
\[ y = -2.5963x + 3.1926 \]
\[ R^2 = 0.9996 \]

**Figure 2:** Prediction result analysis of cumulative oil by the Yuan-A curve in the \( M \) reservoir (fitting data with a water cut in the range of 40\%–45\%).

**Figure 3:** Prediction result analysis of cumulative oil by the Yuan-A curve in the \( M \) reservoir (fitting data with a water cut in the range of 60\%–65\%).

**Figure 4:** Prediction result analysis of the water cut by the Yuan-A water cut prediction formula in the \( M \) reservoir (fitting a water cut in the range of 40\%–45\%).

**Figure 5:** Prediction result analysis of the water cut by the Yuan-A water cut prediction formula in the \( M \) reservoir (fitting a water cut in the range of 50\%–55\%).

**Figure 6:** Data fitting of the Yuan-B curve in the \( M \) reservoir in the ultrahigh water cut period (fitting data with a water cut in the range of 91.3\%–94.5\%).

**Figure 7:** Prediction result analysis of cumulative oil production by the Yuan-B curve in the \( M \) reservoir in the ultrahigh water cut period (fitting data with a water cut in the range of 91.3\%–94.5\%).
(see Figure 11), and then it is used to forecast the cumulative oil production (see Figure 12). Figure 12 shows the prediction results and error analysis of the cumulative oil production. We can see that the relative error in the predicted cumulative oil production is less than 2.0% with a water cut in the range of 46%–60%; after the water cut exceeds 60%, the relative error increases rapidly with the water cut rising, but it is less than 5% with a water cut in the range of 60%–69%, which shows that prediction accuracy of cumulative oil production by the Yuan-A curve is high in a certain range of the water cut.

Then, logistic curve and Type-A curve (equation (26)) are used to fit the production indexes in the water cut range 40%–45% and then forecast the water cut and cumulative oil production, respectively (see Figure 13). Figure 13 shows the relative error of prediction water cut increases rapidly with the water cut rising and is less than 5.0% with a water cut in the range of 46%–50%. As a result, at the same stage, the average relative error in the predicted cumulative oil production is 4.6%, which is much bigger than the Yuan-A curve, and it increases rapidly after the water cut exceeds 55%.

When the conventional WFCC is applicable, a predicted water cut value at the cutoff point is needed when predicting cumulative oil production. If the predicted water cut is not accurate, the error in the predicted cumulative oil
production will be large. The Yuan-A curve avoids relying on the forecast of the water cut and solves the problems of predicting the cumulative oil production of oil wells or reservoirs that have not yet shown the decline rule.

5. Conclusion

(1) Based on the water-oil phase seepage theory and the principle of material balance, the equations between the cumulative oil production and cumulative water injection at the moderately high water cut stage and the ultrahigh water cut stage are derived, which are named the new Yuan-A and Yuan-B curves, respectively.

(2) For water flooding reservoirs at the moderately high and ultrahigh water cut stages, under the condition of stable liquid production, the Yuan-A and Yuan-B curves are applied to forecast oil production based on cumulative water injection data obtained by the balance of injection and production, avoiding reliance on the forecast of the water cut and solving the problems of predicting the cumulative oil production of oil wells or reservoirs that have not yet shown the decline rule.

(3) The causes of the prediction errors of cumulative oil production by the Yuan-A and Yuan-B curves are theoretically analyzed. For the Yuan-A curve, in view of the errors caused by assuming a constant water cut during the derivation, the prediction accuracy of cumulative oil production can be improved by continuously updating the fitting history data.

(4) At the ultrahigh water cut stage, based on the Yuan-B and the Wang-type curves, the Yuan-B water cut prediction formula is established, which can predict the water cut according to the cumulative water injection and solve the difficult problem of water cut prediction.

(5) The Yuan-A and Yuan-B curves, which are established based on cumulative water injection, are simple and convenient and have certain guiding significance for the prediction of the cumulative oil production and water cut for similar reservoirs.

Appendix

For a reservoir that maintains formation pressure by injecting water, when the oil-water front reaches the producers, according to the one-dimensional nonpiston waterflooding theory of Buckley–Leverett and material balance principle, we get the following equation:

\[ NB_o = N_p B_{oi} = N_o B_{oi}, \]  

where \( B_{oi} \) is the volume factor of the formation oil, dimensionless; \( N \) is original oil in place, \( 10^4 \text{ m}^3 \); \( N_p \) is the cumulative oil production, \( 10^4 \text{ m}^3 \); and \( N_o \) is remaining oil in place at a certain time after water breakthrough, \( 10^4 \text{ m}^3 \).

\( NB_o \) and \( N_o B_{oi} \) in equation (A.1) can be expressed as follows:

\[ NB_o = 100AL\phi S_{oe}, \]  

where \( A \) is the oil-bearing area, \( \text{km}^2 \); \( L \) is the oil-bearing length, \( m \); \( \phi \) is the average porosity, fraction; and \( S_{oe} \) is the original oil saturation, fraction.

\[ N_o B_{oi} = 100AL\phi S_{oe} - 100A\phi \int_{S_w}^{1-S_o} XdS_w, \]  

where \( S_{we} \) is the water saturation at the outlet, dimensionless; \( S_{oe} \) is the oil saturation at the outlet, dimensionless; \( S_w \) is the water saturation, dimensionless; and \( X \) is the distance between waterflooding front and water injector.

Using Buckley–Leverett theory (1942), we obtain

\[ X = W_i B_w \left( \frac{df_w}{dS_w} \right)_{S_w}, \]  

where \( W_i \) is the cumulative water injection, \( 10^4 \text{ m}^3 \); \( B_w \) is the volume factor of formation water, dimensionless; and \( f_w \) is the water cut, fraction.

Substituting equation (A.4) into equation (A.3) and integrating, we have

\[ N_o B_{oi} = 100AL\phi S_{oe} - W_i B_w \left( 1 - f_{we} \right), \]  

where \( f_{we} \) is the water cut at the outlet, fraction.

It is known that \( 1 - f_{we} = f_{oe} \), so equation (A.5) can be written as

\[ N_o B_{oi} = 100AL\phi S_{oe} - W_i B_w f_{oe}, \]  

where \( f_{oe} \) is the oil cut at the outlet, fraction.

Substituting equations (A.2) and (A.6) into equation (A.1), we obtain the oil saturation at the outlet:

\[ S_{oe} = S_{oi} - \frac{N_p B_{oi}}{100AL\phi} + \frac{W_i B_w f_{oe}}{100AL\phi}. \]  

Then, substituting equation (A.2) into equation (A.7), we gain
\[
S_{oc} = S_{oi} - \frac{N_pS_{oi}}{N} + \frac{W_iB_wS_{oi}}{NB_{oi}} - f_{oe}.
\]

(A.8)

Oil saturation at the outlet can be expressed as
\[
f_{oe} = \frac{Q_oB_{oi}}{Q_{oi}B_{oi} + Q_{w}B_{w}}.
\]

(A.9)

where \(Q_{iw}\) is the daily water production at ground conditions, \(m^3/d\) and \(Q_o\) is the daily oil production at ground conditions, \(m^3/d\).

Then, substituting equation (A.9) into equation (A.8), we gain
\[
S_{oc} = S_{oi} - \frac{N_pS_{oi}}{N} + \frac{W_iB_wS_{oi}}{N}\left(\frac{Q_o}{Q_{oi}B_{oi} + Q_{w}B_{w}}\right)
\]

(A.10)

For a reservoir that formation pressure and injection and production balance are maintained by injecting water, the fluid production volume is equal to the oil production volume plus water production volume, and the cumulative fluid production volume is equal to the cumulative water injection volume. Therefore, the following two equations can be written:
\[
Q_LB_L = Q_oB_{oi} + Q_wB_{w}
\]

(A.11)

where \(Q_L\) is the daily water production at ground conditions, \(m^3/d\) and \(B_L\) is the formation fluid volume factor, dimensionless.

\[
L_pB_L = N_pB_{oi} + W_pB_w = W_iB_w
\]

(A.12)

where \(L_p\) is the cumulative liquid production, \(10^4 m^3\) and \(W_p\) is the cumulative water production, \(10^4 m^3\).

Substituting equations (A.11) and (A.12) into equation (A.10), we have
\[
S_{oc} = S_{oi} - \frac{N_pS_{oi}}{N} - \frac{L_pS_{oi}}{N}\left(\frac{Q_o}{Q_L}\right)
\]

(A.13)

It is known that \(Q_{oi} = dN_p/dt\) and \(Q_L = dL_p/dt\), and equation (A.13) can be written as
\[
S_{oc} = S_{oi} - \frac{N_pS_{oi}}{N} + \frac{L_pS_{oi}}{N}\left(\frac{dN_p}{dL_p}\right)
\]

(A.14)

It is known that \(S_{wc} = 1 - S_{oc}\), so the water saturation at the outlet is obtained from equation (A.14):
\[
S_{wc} = 1 - S_{oi} + \frac{N_pS_{oi}}{N} - \frac{L_pS_{oi}}{N}\left(\frac{dN_p}{dL_p}\right)
\]

(A.15)

**Data Availability**

The data used to support the findings of this study are available from the corresponding author upon request.

**Conflicts of Interest**

The authors declare no conflicts of interest.

**Authors’ Contributions**

Zhiwang Yuan and Li Yang were responsible for conceptualization; Zhiwang Yuan contributed to methodology, writing, and original draft preparation; Zhiwang Yuan and Zhiping Li were responsible for validation; and Zhiwang Yuan, Zhiping Li, Li Yang, and Yingchun Zhang were responsible for writing the review and editing.

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**References**


