Research Article

Similarity Criterion of Freezing Model Test considering Nonlinear Variation of Thermal Parameters with Temperature

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The significant differences in specific heat and thermal conductivity of ice and water lead to the changes of specific heat and thermal conductivity of soil during the freezing process. This makes it hard for the temperature field similarity criterion based on constant thermal parameters to accurately reflect the temperature field evolution of soil mass caused by nonlinearity of thermal parameters in the process. Based on heat conduction differential equation considering nonlinear changes of thermal parameters, this paper uses similarity transformation method to derive the similarity criterion of the temperature field in the frozen soil model test and arrives at the conclusion that the prototype soil and model soil should meet when the original soil is used for the model test. At the same time, given the impact of the third boundary condition on the similarity criterion, the thermal physical similarity conditions for the model soil are derived. On this basis, ABAQUS finite element software is used to numerically simulate the linear and nonlinear prototype and model temperature fields. The third boundary condition considered the temperature evolution of the characteristic points during the freezing process is analyzed. The calculation results indicate that the nonlinear thermal conductivity similarity criterion established herein can correctly reflect the evolution process of the prototype frozen soil temperature field. It is also suggested that the model soil thermal parameters are reasonably calculated. At the same time, it shows that the nonlinear freezing similarity criterion of the soil, when the third boundary condition is satisfied, has clear physical meaning and higher practical value. The research results provide a practical and reasonable parameter calculation method for the model soil preparation in the frozen soil model test and a theoretical basis and technical support for the design and implementation of the water-heat-force coupling model test on frozen soil.

1. Introduction

Model test is a science in which, on the basis of a certain geometric-physical relationship, a model is made and tested in place of a prototype so as to have results which can be used to predict the development of the prototype according to the corresponding similarity criterion [1–3]. Model test is an important means to study the occurrence mechanism and development law of complex physical phenomena, while similarity criterion is the theoretical basis for model test design [4, 5]. Derivation of the model test similarity criterion based on engineering conditions is the basis and necessary condition for improving the model test prediction accuracy.

Freezing process of soil is a complicated problem involving coupling of temperature field, moisture field, and stress field, and the evolution process of temperature field provides the basis on and the key to the study of water migration and stress changes [6, 7]. Harlan proposed a temperature field model that considers water migration and pore water phase transitions in soil. This model carries important theoretical significance for studying the variation law of water migration with time during the movement of soil freezing temperature gradient and freezing front [8]. Bao et al. established an enthalpy-based heat conduction calculation model for frozen soil based on the heat diffusion equation and then solved and analyzed the heat conduction process between soil layers [9]. Based on the transient heat

source method, Trigui et al. studied composite material with phase transition characteristics on the basis of the transient heat source method and derived its sensible heat, heat storage, and latent heat through inverse calculation [10]. From the perspective of frozen soil temperature field model test, the applicable similarity criterion is derived. Zhu et al. [11] analyzed the similarity criterion of the freezing and thawing process of frozen soil when there is no external load and carried out corresponding freezing and thawing model test based on this. Zhang et al. [12] took temperature field of single-tube frozen soil as the research object and deduced the similarity criterion of the temperature field of single-tube frozen soil. Zhang et al. [13] conducted frost heave centrifugal model tests of undisturbed soil with different water contents of 20 g and 30 g and discussed the heat transfer process of the canal centrifuge model and the impact of the moisture content of canal foundation soil in the closed system on the normal frost heave displacement of the canal.

The existing similarity criterion for the temperature field model test of frozen soil is established based on constant thermal parameters, that is, the nonlinear changes of thermal parameters such as thermal conductivity coefficient and specific heat with temperature are not considered. During the freezing process of the soil, thermal parameters such as thermal conductivity coefficient and specific heat change significantly with temperature [14–16]. Hence, the temperature field similarity criterion established based on constant thermal parameters obviously cannot accurately reflect the nonlinear freezing process of soil. In cold area engineering and artificial freezing method construction, engineering accidents caused by temperature field calculation errors are not uncommon [17, 18]. Based on the nonlinear change law of thermal parameters with negative temperature, similarity criterion for frozen soil model tests is established which not only has important theoretical significance but also means huge economic value and social benefits.

In addition, in actual engineering, as the frozen boundary of the soil is not adiabatic, there is a nonnegligible heat exchange between the frozen area and the surrounding environment or adjacent media [19, 20]. When the original soil is used for temperature field freezing model test, according to the third boundary condition, the geometric ratio of the model and the prototype must be 1 : 1, i.e., the model has to be of the same size as that of the prototype. This makes it difficult to model the prototype project, and model test conducted in the scale mode obviously loses the theoretical basis and application conditions [21–23]. Therefore, it is necessary to consider the similarity criterion of temperature field and the boundary and then derive the similarity conditions for the model soil thermal physical parameters.

Based on the nonlinear heat conduction differential equation, this paper, using similarity transformation method derives the similarity criteria for temperature field similarity in the frozen soil model test and has established the similarity relationship between the prototype and the model when original soil is used for the model test. At the same time, considering the impact of the third boundary

condition on similarity criterion of the temperature field model test, the paper also derives the conditions that the model soil should meet after material transformation in terms of thermal physical property parameters. Based on the deduced similarity criterion and the thermal property parameters of similar materials, the paper simulates numerically the linear and nonlinear frozen soil temperature fields and the third boundary condition by means of ABAQUS finite element software and then analyzes temperature evolution of characteristic points.

2. Nonlinear Heat Conduction Model and Similarity Criterion

Mathematical description of heat conduction problem involves heat conduction differential equations and boundary value conditions, and the boundary value conditions include initial conditions and boundary conditions [20]. Boundary value conditions and heat conduction mathematical model are the basis for establishing similarity criterion of nonlinear heat conduction.

2.1. Boundary Conditions of Heat Conduction Process. Boundary conditions of temperature field [23, 24] mainly fall into three categories. The first category describes the temperature of the soil boundary or the surrounding environment; the second category concerns the heat flux density on the soil boundary; the third category is about the heat exchange volume between the soil and the surrounding medium.

The existing similarity criterion often takes temperature shrink ratio of 1 in the process of establishment, which requires that the boundary temperature must be constant in heat conduction. Therefore, the criterion cannot be used to consider the heat exchange between the soil and the external environment [23]. In cold area engineering and artificial freezing method construction, freezing of the soil body is a heat conduction phenomenon under the coupling effect of the boundary value condition, so temperature field similarity criterion without considering the third boundary condition obviously cannot guarantee rationality and accuracy of the model test.

2.2. Mathematical Model of Nonlinear Heat Conduction. At present, the conventional heat conduction equation describing freezing process dismisses the change of thermal parameters with negative temperature, which is expressed as follows [24]:

$$C\rho\frac{\partial T}{\partial t} = \lambda\operatorname{div}(\operatorname{grad}T) + q_v, \quad (1)$$

where C is soil specific heat; ρ is soil density; T is soil temperature; t is freezing time; λ is soil thermal conductivity coefficient; and q_v is heat flux intensity of the cold (hot) source in the soil.

The temperature continuity and energy conservation conditions must be satisfied on the freezing front h of the soil, i.e.,

$$T_f(h(t), t) = T_u(h(t), t) = T_0, \quad (2)$$

$$\lambda_f \frac{\partial T_f}{\partial n} - \lambda_u \frac{\partial T_u}{\partial n} = Q \frac{dh(t)}{dt},$$

where T_0 is soil phase transition temperature; T_f and T_u , respectively, represent frozen soil temperature and melted soil temperature; λ_f is frozen soil thermal conductivity coefficient; n is the external normal direction of the soil boundary; λ_u is melted soil thermal conductivity coefficient; Q is the latent heat of phase change per unit volume of soil; and h is the freezing front location.

The differential equation of the nonlinear heat conduction temperature field considering thermal parameter variation with temperature is [25]

$$C(T)\rho \frac{\partial T}{\partial t} = \text{div}(\lambda(T)\text{grad}T) + q_v, \quad (3)$$

where $C(T)$ and $\lambda(T)$ are the specific heat and thermal conductivity coefficient of soil, respectively. For soil with certain dry density and water content, its thermal conductivity coefficient and specific heat involve a function related to temperature T [26]. Existing studies have shown that freezing of water in soil is concentrated in the high temperature freezing stage [27, 28], and it is difficult for one linear fitting to reflect the evolution of thermal parameters with freezing temperature. To improve fitting accuracy of the test data, a quadratic function is hereby taken to describe the variation trend of thermal parameters with temperature, namely,

$$C(T) = AT^2 + BT + C_0, \quad (4)$$

$$\lambda(T) = DT^2 + ET + \lambda_0, \quad (5)$$

where A , B , D , and E are fitting constants, whose units are $\text{kJ}/(\text{kg}\cdot^\circ\text{C}^3)$, $\text{kJ}/(\text{kg}\cdot^\circ\text{C}^2)$, $\text{W}/(\text{m}\cdot^\circ\text{C}^3)$, and $\text{W}/(\text{m}\cdot^\circ\text{C}^2)$, respectively; λ_0 is the thermal conductivity coefficient at 0°C , $\text{W}/(\text{m}\cdot^\circ\text{C})$; C_0 is the specific heat at 0°C , $\text{kJ}/(\text{kg}\cdot^\circ\text{C})$; and g_v is 0 when there is no internal heat source in the soil.

If frozen soil is isotropic body, i.e., the soil particles, unfrozen water, ice, and gas are evenly distributed in frozen soils, then the temperature continuity and energy conservation conditions must be met on the freezing front h of the soil, namely,

$$T_f(h(t), t) = T_u(h(t), t) = T_0, \quad (6)$$

$$(DT_f^2 + ET_f + \lambda_0) \frac{\partial T_f}{\partial n} - (DT_u^2 + ET_u + \lambda_0) \frac{\partial T_u}{\partial n} = Q \frac{dh(t)}{dt}. \quad (7)$$

2.3. Nonlinear Heat Conduction Similarity Criterion. The similarity criterion is derived based on the similarity transformation method. First, define the similar transformation of each physical quantity in formulas (3) and (7), namely,

$$\frac{A_p}{A_m} = C_A,$$

$$\frac{B_p}{B_m} = C_B,$$

$$\frac{D_p}{D_m} = C_D,$$

$$\frac{E_p}{E_m} = C_E,$$

$$\frac{C_{0p}}{C_{0m}} = C_{C_0},$$

$$\frac{\lambda_{0p}}{\lambda_{0m}} = C_{\lambda_0},$$

$$\frac{t_p}{t_m} = C_t,$$

$$\frac{q_{vp}}{q_{vm}} = C_{q_v},$$

$$\frac{x_p}{x_m} = \frac{y_p}{y_m} = \frac{z_p}{z_m} = \frac{h_p}{h_m} = C_l,$$

$$\frac{T_p}{T_m} = \frac{T_{0p}}{T_{0m}} = \frac{T_{fp}}{T_{fm}} = \frac{T_{up}}{T_{um}} = C_T,$$

$$\frac{Q_p}{Q_m} = C_Q,$$

$$\frac{\rho_p}{\rho_m} = C_\rho.$$

In the formula, the subscript “ p ” represents the physical quantity of the prototype; the subscript “ m ” represents the physical quantity of the model; C_A , C_B , C_D , and C_E are shrink ratios of the corresponding coefficients in formulas (4) and (5); C_{C_0} is the soil specific heat shrink ratio; C_{λ_0} is the thermal conductivity shrink ratio; C_t is the time shrink ratio; C_{q_v} is the heat flux intensity shrink ratio of cold (hot) source; C_l is the geometric shrink ratio; C_T is the temperature shrink ratio; C_Q is the latent heat shrink ratio; C_ρ is the soil density shrink ratio. Substitute equation (8) into equations (3) and (7), then

$$\begin{aligned}
& \frac{C_T C_\rho}{C_t} (C_A (C_T)^2 A_m T_m^2 + C_B C_T B_m T_m + C_{C_0} C_{0m}) \rho_m \frac{\partial T_m}{\partial t_m} \\
&= \frac{C_T}{C_l^2} \operatorname{div}((C_D (C_T)^2 D_m T_m^2 + C_E C_T E_m T_m + C_{\lambda_0} \lambda_{0m}) \operatorname{grad} T_m) \\
&+ C_{qv} q_{vm} (C_D (C_T)^2 D_m T_{fm}^2 + C_E C_T E_m T_{fm} + C_{\lambda_0} \lambda_{0m}) \frac{C_T}{C_l} \frac{\partial T_{fm}}{\partial n_m} \\
&- (C_D (C_T)^2 D_m T_{um}^2 + C_E C_T E_m T_{um} + C_{\lambda_0} \lambda_{0m}) \frac{C_T}{C_l} \frac{\partial T_{um}}{\partial n_m} = \frac{C_Q C_l}{C_t} Q_m \frac{dh_m(t_m)}{dt_m}.
\end{aligned} \tag{9}$$

According to principle of similarity, we describe similar physical phenomena with the same differential equation, and their single-valued conditions are similar [14]. By comparing the balance equations of the prototype and the model, 7 similarity criteria are derived after sorting, namely,

$$\begin{aligned}
\pi_1 &= \frac{A(T)^2}{C_0}, \\
\pi_2 &= \frac{BT}{C_0}, \\
\pi_3 &= \frac{D(T)^2}{\lambda_0}, \\
\pi_4 &= \frac{ET}{\lambda_0}, \\
\pi_5 &= \frac{\lambda_0 \tau}{L^2 \rho C_0}, \\
\pi_6 &= \frac{q_v \tau}{C_0 T \rho}, \\
\pi_7 &= \frac{C_0 T \rho}{Q}.
\end{aligned} \tag{10}$$

$$\begin{aligned}
\frac{C_A (C_T)^2}{C_{C_0}} &= 1, \\
\frac{C_B C_T}{C_{C_0}} &= 1, \\
\frac{C_D (C_T)^2}{C_{\lambda_0}} &= 1, \\
\frac{C_E C_T}{C_{\lambda_0}} &= 1, \\
\frac{C_{\lambda_0} C_t}{(C_l)^2 C_\rho C_{C_0}} &= 1, \\
\frac{C_{qv} C_t}{C_{C_0} C_T C_\rho} &= 1, \\
\frac{C_{C_0} C_T C_\rho}{C_Q} &= 1.
\end{aligned} \tag{11}$$

In the existing soil freezing model test, the original soil is often used as the model material, that is, $C_A = C_B = C_{C_0} = C_D = C_E = C_{\lambda_0} = 1$. According to $C_B C_T = C_{C_0}$, the temperature shrinkage ratio C_T is 1. That is, the initial temperature and the first boundary condition of each point in the model are the same as those of the corresponding point in the prototype. From

$$\frac{C_{\lambda_0} C_t}{(C_l)^2 C_\rho C_{C_0}} = 1, \tag{12}$$

3. Similarity Condition Analysis

Based on the 7 derived similarity criteria, the similarity index formula can be further derived, namely,

it can be known that

$$C_t = C_l^2 C_p, \quad (13)$$

where C_t is the time shrink ratio; C_l is the geometric shrink ratio; and C_p is the soil density.

In actual engineering, due to fragmentation of the soil and the cold source conditions during the test, the soil heat transfer process cannot be strictly carried out with temperature shrink ratio C_T equal to 1. That is, the impact of the third boundary condition on temperature field should be taken into account. If the temperature shrink ratio C_T is not 1, according to the index formula of the similarity ratio, there is $C_A \neq C_B \neq C_{C0} \neq C_D \neq C_E \neq C_{\lambda0} \neq 1$. That is, it is unreasonable to use original soil for the model test. Hence, it is necessary to seek materials with similar characteristics as the thermophysical properties of the original soil for the model test by material transformation.

The relationship between the specific heat, thermal conductivity coefficient, and temperature of the prototype soil as indicated by the quadratic function is

$$\begin{aligned} C_p(T_p) &= A_p T_p^2 + B_p T_p + C_{0p}, \\ \lambda_p(T_p) &= D_p T_p^2 + E_p T_p + \lambda_{0p}, \end{aligned} \quad (14)$$

where A_p , B_p , D_p , and E_p are constants; T_p ($^{\circ}\text{C}$) is the prototype soil temperature; C_{0p} ($\text{kJ}/(\text{kg}\cdot^{\circ}\text{C})$) is the specific heat of the prototype soil at 0°C ; and λ_{0p} ($\text{W}/(\text{m}\cdot^{\circ}\text{C})$) is the thermal conductivity coefficient of the prototype soil at 0°C .

If it is assumed that the temperature shrink ratio is C_T , the specific heat shrink ratio of soil is C_{C0} and the thermal conductivity coefficient shrink ratio of soil is $C_{\lambda0}$; then,

$$\begin{aligned} A_m &= \frac{A_p C_T^2}{C_{C0}}, \\ B_m &= \frac{B_p C_T}{C_{C0}}, \\ C_{0m} &= \frac{C_{0p}}{C_{C0}}, \\ D_m &= \frac{D_p C_T^2}{C_{\lambda0}}, \\ E_m &= \frac{E_p C_T}{C_{\lambda0}}, \\ \lambda_{0m} &= \frac{\lambda_{0p}}{C_{\lambda0}}. \end{aligned} \quad (15)$$

Therefore, the specific heat and thermal conductivity coefficient of the model soil are

$$\begin{aligned} C_m(T_m) &= \frac{A_p C_T^2}{C_{C0}} T_m^2 + \frac{B_p C_T}{C_{C0}} T_m + \frac{C_{0p}}{C_{C0}}, \\ \lambda_m(T_m) &= \frac{D_p C_T^2}{C_{\lambda0}} T_m^2 + \frac{E_p C_T}{C_{\lambda0}} T_m + \frac{\lambda_{0p}}{C_{\lambda0}}, \end{aligned} \quad (16)$$

where T_m ($^{\circ}\text{C}$) is the model soil temperature.

According to $(C_{C0} C_T C_p / C_Q) = 1$, model soil latent heat per unit volume should meet $Q_m = (Q_p / C_{C0} C_T C_p)$. According to $(C_{\lambda0} C_t / (C_l)^2 C_p C_{C0}) = 1$, time shrink ratio is $C_t = (C_{C0} C_p (C_l)^2 / C_{\lambda0})$.

Similarity criterion for the third boundary condition is

$$\pi = \frac{\alpha L}{\lambda}, \quad (17)$$

where α is the convection heat transfer coefficient between the soil and the environment. The corresponding similarity index formula is

$$\frac{C_{\alpha} C_l}{C_{\lambda}} = 1, \quad (18)$$

where C_{α} is the convection heat transfer coefficient shrink ratio.

After considering the impact of the third boundary condition, the specific heat and thermal conductivity of the model soil should be

$$\begin{aligned} C_m(T_m) &= \frac{A_p C_T^2}{C_{C0}} T_m^2 + \frac{B_p C_T}{C_{C0}} T_m + \frac{C_{0p}}{C_{C0}}, \\ \lambda_m(T_m) &= \frac{D_p C_T^2}{C_{\alpha} C_l} T_m^2 + \frac{E_p C_T}{C_{\alpha} C_l} T_m + \frac{\lambda_{0p}}{C_{\alpha} C_l}. \end{aligned} \quad (19)$$

According to $(C_{C0} C_T C_p / C_Q) = 1$, model soil latent heat per unit volume should meet $Q_m = (Q_p / C_{C0} C_p C_T)$. According to $(C_{\lambda0} C_t / (C_l)^2 C_p C_{C0}) = 1$ and $(C_{\alpha} C_l / C_{\lambda0}) = 1$, time shrink ratio is $C_t = (C_{C0} C_p C_l / C_{\alpha})$.

4. Verification of Similarity Criterion and Model Soil Similarity Condition

To test the derived similarity criterion of the nonlinear freezing temperature field of the soil and the conditions that the model soil should meet in terms of thermal physical properties after considering the third boundary condition, a series of soil freezing model tests were designed and implemented. The original soil thermal conductivity coefficient and change law of specific heat with temperature were measured, and then by establishing soil temperature field models with different shrink ratios, the derived similarity criterion and similarity relationship between the model soil and the original soil considering the third boundary condition were tested.

4.1. Determination of Thermal Conductivity Coefficient and Specific Heat. Take a silty clay sample with remodeling density of $1910 \text{ kg}/\text{m}^3$ and water content of 33%. The specific heat and thermal conductivity coefficient of clay samples are measured by means of calorimetry and probe method under different negative temperatures. The resulting specific heat and thermal conductivity coefficient changes with temperature are shown in Figure 1.

FIGURE 1: Variation curves of specific heat and thermal conductivity with temperature.

TABLE 1: Calculation of latent heat of soil at the negative temperature stage.

Temperature (°C)	Unfrozen content (%)	Latent heat gain (kJ·m ⁻³)
0	33.0	—
-0.5	18.2	71052
-1.0	13.8	21201
-1.5	9.6	20246
-2.0	6.5	14898
-5.0	4.0	12033
-10.0	3.7	1528
-15.0	3.6	573

TABLE 2: Numerical simulation test design.

Model number	Model scale	Material properties	Type	Boundary conditions considered
1	1.0	Native soil	Nonlinearity	1st, 2nd
2	0.5	Native soil	Nonlinearity	1st, 2nd
3	1.0	Native soil	Linear	1st, 2nd

It can be seen from Figure 1 that, as the temperature decreases, the soil sample specific heat gradually decreases and the thermal conductivity coefficient increases gradually, which is consistent with laws in the existing research [27, 28]. According to the previous fitting formulas (4) and (5), the change law of specific heat and thermal conductivity coefficients with negative temperature are quadratic fitted, and the resulting expression is

$$\begin{aligned} C(T) &= 3 \times 10^{-7}T^2 + 0.0256T + 1.7468, \\ \lambda(T) &= -0.0003T^2 - 0.0246T + 1.6892. \end{aligned} \quad (20)$$

The latent heat calculation method provided by literature [24] is

$$Q_p = \rho_d L (w_{T_1} - w_{T_2}) = \frac{\rho L (w_{T_1} - w_{T_2})}{1 + w_0}, \quad (21)$$

where Q (kJ/m³) is the latent heat per unit volume of soil; ρ_d (kg/m³) and ρ (kg/m³) represent soil dry density and soil density, respectively; L (kJ/kg) is the water latent heat of phase change; w_{T_1} and w_{T_2} are the unfrozen water content in

TABLE 3: Values of similarity constants.

Parameter	Similarity constant
C_A	1
C_B	1
C_{C0}	1
C_D	1
C_E	1
$C_{\lambda 0}$	1
C_t	4
C_T	1
C_l	2
C_p	1
C_Q	1

frozen soil under negative temperature of T_1 and T_2 , respectively; and w_0 indicates the initial water content.

According to the unfrozen water content test method provided in literature [29], unfrozen water content of frozen soil samples was obtained under different negative temperatures. According to equation (21), phase change latent heat of frozen soil can be obtained for different negative

TABLE 4: Values of model test parameters.

Model number	Model scale $l \times h$ (m $\times$ m)	Freezing time (h)	Cold source temperature ($^{\circ}\text{C}$)	Initial soil temperature ($^{\circ}\text{C}$)	Ambient temperature ($^{\circ}\text{C}$)	Soil density ($\text{kg}\cdot\text{m}^{-3}$)
1	2×0.50	200				
2	1×0.25	50	-30	7	7	1910
3	2×0.50	200				

TABLE 5: Numerical simulation test design.

Model number	Model scale	Material properties	Type	Boundary conditions considered
4	1.0	Native soil	Nonlinearity	1st, 2nd, 3rd
5	0.5	Model soil	Nonlinearity	1st, 2nd, 3rd

temperature intervals. The calculation results are shown in Table 1.

4.2. Freezing Test When the Temperature Shrink Ratio Is 1. To test the validity of the established nonlinear heat conduction similarity criterion and the soil model established with the third boundary condition considered, the corresponding numerical model is established by ABAQUS finite element software. The adopted thermal parameter model and boundary conditions are shown in Table 2.

It can be seen from Table 2 that Model 1, Model 2, and Model 3 are soil freezing test plans with a temperature shrink ratio C_T at 1, that is, the impact of the third boundary condition is excluded. The difference between Model 1 and Model 3 is that Model 3 is linear, that is, it does not consider thermal parameter variation with negative temperature. According to the similarity criterion, the similarity constants of Model 1 and Model 2 can be determined, as shown in Table 3. The dimensions and test parameters of each model are shown in Table 4.

The numerical simulation of one-dimensional freezing temperature field is carried out by means of ABAQUS finite element software according to the parameters determined in Table 3. Model 1 is a $2\text{ m} \times 0.5\text{ m}$ rectangle. At the lower boundary of the rectangular model, a constant temperature cold source of -20°C is set. As can be seen from Table 1, when the temperature is reduced to -5°C , the phase change latent heat of the frozen soil sample has very small increment. To optimize the model calculation while guaranteeing calculation accuracy, the method adopted in [28] sets the latent heat release range of frozen soil at $[0^{\circ}\text{C} \sim -2^{\circ}\text{C}]$, and the phase change latent heat value is set at $139430\text{ kJ}/\text{m}^3$. The specific heat and thermal conductivity coefficient of the soil are set according to the values given in Figure 1. Model 2 is a $1\text{ m} \times 0.25\text{ m}$ rectangle. The same soil body as Model 1 is taken, and it has the same boundary and cold source conditions as Model 1. Model 3 has the same size and material with Model 1, but when setting the soil thermal parameters, the variation of specific heat and thermal conductivity coefficient with temperature is not considered, and the average value in Figure 1 is taken as its thermal conductivity coefficient and specific heat, that is, in Model 3, the thermal conductivity coefficient is $1.82\text{ W}/(\text{m}\cdot^{\circ}\text{C})$ and the specific heat is $1.58\text{ kJ}/(\text{kg}\cdot^{\circ}\text{C})$.

TABLE 6: Values of similarity constants.

Parameter	Similarity constant
C_A	1
C_B	2
C_D	1
C_E	2
$C_{\lambda 0}$	4
C_t	4
C_T	2
C_l	2
C_p	1
C_Q	8
C_{α}	2

4.3. Freezing Test When the Temperature Shrink Ratio C_T Is not 1. According to the previous derivation, the similarity conditions established with consideration to the third boundary condition are verified, and the relevant parameters are shown in Table 5.

Model 4 and Model 5 consider the impact of the third boundary condition, that is, heat exchange between the soil interface and the external environment is allowed. Model 4 is the same as model 1 in terms of size and material properties, but when the upper condition is set, No. 4 upper boundary condition is set to be convective heat transfer. Model 5 has a size shrink ratio $C_l = 2$ and temperature shrink ratio $C_T = 2$ compared with Model 4, and the remaining boundary settings are the same as in Model 4. Because there is reduced scale in size and cold source between Model 4 and Model 5 and convective heat transfer exists between the boundary and the environment, Model 5 needs to be tested using model soil in place of original soil. If it is assumed that the convection heat transfer coefficient shrink ratio C_{α} is 2, according to the similarity criterion of the third boundary condition, the thermal conductivity coefficient shrink ratio $C_{\lambda 0}$ is 4. If it is assumed that the specific heat shrink ratio C_{C0} is 4, the conditions for the model soil thermal physical parameters are deduced according to the above conclusion, and the specific heat and thermal conductivity coefficient of the model soil are derived as

$$\begin{aligned} C_m(T_m) &= 3 \times 10^{-7} T_m^2 + 0.0128 T_m + 0.4367, \\ \lambda_m(T_m) &= -0.0003 T_m^2 - 0.0123 T_m + 0.4223. \end{aligned} \quad (22)$$

TABLE 7: Values of model test parameters.

Model number	Model size $l \times h$ (m $\times$ m)	Freezing time (h)	Cold source temperature ($^{\circ}\text{C}$)	Initial soil temperature ($^{\circ}\text{C}$)	Ambient temperature ($^{\circ}\text{C}$)	Soil density ($\text{kg}\cdot\text{m}^{-3}$)	Latent heat ($\text{kJ}\cdot\text{m}^{-3}$)	Convective heat transfer coefficient ($\text{W}\cdot\text{m}^{-1}\text{C}^{-1}$)
4	2×0.5	200	-20	7	7	1910	139430	10
5	1×0.25	50	-10	3.5	3.5	1910	17428.75	5

FIGURE 2: Layout of temperature measurement points.

FIGURE 3: Comparison of different model temperature measurement points 1 and 2.

According to the derived similarity criterion, the similarity constant between Model 4 and Model 5 is determined, as shown in Table 6. The dimensions and test parameters of each model are shown in Table 7.

5. Results Analysis

To test the validity of the established similarity criterion and model soil parameters, the stable development process of three characteristic points of the model test is extracted on the basis of the previous numerical simulation results. The temperature measurement points are arranged, as shown in Figure 2.

The calculation results of No. 1 and No. 2 measurement points in Model 1, Model 2, and Model 3 are sorted out, with results shown in Figure 3. The temperature evolution process of No. 3 measurement point is shown in Table 8.

It can be seen from Figure 3 and Table 8 that certain errors exist in temperature curves of Model 1 and Model 3. Existing literature also indicates that prediction accuracy is higher when frozen soil temperature field considers thermal parameter variation with temperature [28–31]. That is, similarity criterion derived by nonlinear theory has higher prediction accuracy. By multiplying Model 2 by the time shrink ratio C_b , it can be found that the corresponding time has equal temperature value, and the temperature shrink ratio between Model 1 and Model 2 is 1. This suggests that when theoretical similarity criterion and original soil are used for model test, the similarity relationship between the prototype and the model is established.

The calculation results of No. 1 and No. 2 measurement points in Model 4 and Model 5 are sorted out, with results shown in Figure 4. In addition, the temperature of No. 3 measurement point in Model 4 and Model 5 is shown in Table 9 for different time periods.

TABLE 8: Temperature of measurement point 3 in three models.

Model 1		Model 2		Model 3	
Time (h)	Temperature (°C)	Time (h)	Temperature (°C)	Time (h)	Temperature (°C)
0	7.00	0	7.00	0	7.00
40	0.26	10	0.26	40	-0.01
80	-1.56	20	-1.56	80	-1.15
120	-6.14	30	-6.14	120	-5.71
160	-8.65	40	-8.65	160	-7.98
200	-14.95	50	-14.95	200	-13.60

FIGURE 4: Comparison of different model temperature measurement points 1 and 2.

TABLE 9: Temperature of measurement point 3 in different models.

Model 4		Model 5	
Time (h)	Temperature (°C)	Time (h)	Temperature (°C)
0	7.00	0	3.50
40	0.92	10	0.46
80	-1.11	20	-0.56
120	-5.21	30	-2.60
160	-6.06	40	-3.03
200	-6.56	50	-3.28

It can be seen from Figure 4 and Table 9 that Model 4 and Model 5 have similar temperature fields, that is, the temperature value of corresponding time involves a twofold relationship. This indicates that the model soil thermal parameters given herein are reasonable when the model soil after transformation is taken for the model test.

In actual engineering, heat exchange between the frozen soil and the outside world is inevitable, and the constancy of heat source form is also difficult to ensure. Hence, when original soil is taken for temperature field model test, certain errors are inevitable between the predicted value and that in engineering. It is more reasonable to consider the presence of similarity criterion in heat exchange between the model and the environment, which requires that the original soil and the model soil have similar thermal parameters under different material temperatures, that is, when performing temperature field model test, material transformation is a must for the model test of frozen soil temperature field.

6. Conclusions

After considering material nonlinearity and the third boundary condition, the similarity criterion of the freezing model test and the calculation method of the model soil thermal parameters are derived. The research results provide directions for the preparation of frozen model soil, which is expected to provide theoretical basis and technical support for the design and implementation of frozen soil model tests. The main research conclusions are drawn as follows:

- (1) The nonlinear freezing of soil and the thermal parameter variation with negative temperature determine that the impact of material nonlinearity on the heat conduction equation and similarity criterion must be considered when designing the frozen soil model test.
- (2) The heat exchange between the soil and the external environment during the freezing process determines that study on frozen soil heat conduction must consider impact of the third boundary condition, that is, model soil with a similar relationship with the original soil is required for the temperature field model test.
- (3) Considering the similarity criterion of heat exchange between the model and the environment, material similarity under different temperatures should be met, that is, the similarity criterion and similarity materials of the frozen soil model test must be determined by material transformation.

- (4) From the perspective of constitution and composition of porous medium, the model soil showing certain similar relationship with the original soil is configured according to the specific heat conduction so that it can be applied to the model test of frozen soil temperature field. This should be an important research direction for the study on similar problems of frozen soil temperature field.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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