Research Article

Synchronous Behavior for Memristive Synapse-Connected Chay Twin-Neuron Network and Hardware Implementation

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Synchronous behavior can be responsible for the function or dysfunction of a neural network. To employ a memristor with threshold memductance as a bidirectional synapse, a memristive synapse-connected Chay twin-neuron network is constructed. This paper numerically presents the synchronous behavior for four representative firing activities in the memristive twin-neuron network by utilizing time-domain waveforms, synchronized transition states (STs), and mean synchronization errors (MSEs). Indeed, the synchronous behaviors are truly related to the coupling strength and initial condition of the memristor. Besides, utilizing the powerful XC7Z020 FPGA, a digitally circuit-implemented electro-neuron and the memristive synapse-connected Chay twin-neuron network are made. Thereafter, the four representative firing activities and their STs are experimentally captured to further confirm the numerical simulations.

1. Introduction

Biological system propagates and handles neural signal via abundant collective behaviors [1–5]. As one of the most outstanding collective behaviors in neuronal network, synchronous behavior is important for the realization of signal propagation, information handling, and other activities [6–8]. Synchronous behavior for neuron-coupled models abstracted from the biological system has attracted much interest due to their simplicity and without loss of generality [9–15]. Numerous investigations of synchronous behavior in gap junction-coupled Huber–Braun neurons [9], gap junction-coupled or noise-sustained FitzHugh–Nagumo neurons [10–12], feedback unidirectional-coupled Hindmarsh–Rose neurons [7, 13], and chemical synaptic-coupled Hindmarsh–Rose neurons [14] have been explored in theoretical and numerical levels. Besides, excessive synchronization can be achieved through stimulating neuron by external control signal, which can be employed to cure some disorder diseases of Alzheimer's, epilepsy, Parkinson's, and hand shaking [15–17].

Based on the law of electromagnetic induction theory, Lv et al. have come up with the idea that the memristor can be employed as a connected synapse to describe transmembrane current generated by

the membrane potentials' difference between two coupled neurons [18, 19]. When two coupled neurons own different initial conditions, i.e., different neuron membrane potentials, electromagnetic induction current can be generated to exchange the charged ions across the membrane. In this way, the neuron membrane potentials are altered and synchronization could be achieved [7]. The memristor is a novel circuit device with its memductance really depending on the input and with memory effect [20], which is regarded as a valid candidate for neuronal synapse [21]. Thus, it is interesting to find the emergence of synchronous behavior for memristor synapse-coupled two neurons having different membrane potentials between them [22–25]. The coexisting firing activities and synchronous behaviors associated to initial condition of the memristor in a memristive synapse-connected Morris–Lecar double neurons' network are explored [22]. Besides, the coexisting firing activities and initial-associating bifurcation behaviors, as well as the extreme events are disclosed in two adjacent Hindmarsh–Rose neurons connected by flux-controlled memristor [26]. However, the memristor initial condition-associated synchronous behavior still needs further investigation.

Three-dimensional (3D) Chay neuron model is a more physiologically excitable three-variable neuron model with simple mathematical form and rich essential features of electrical activities [27]. In essence, the 3D Chay neuron

model has the feature of a fast-slow structure to represent the membrane potential and ionic events in excitable membranes as in [28]. The 3D Chay neuron model can give birth to abundant firing activities of periodic and chaotic bursting/spiking behaviors [29], as well as the bifurcation scenarios for stochastic resonance are disclosed [30]. Besides, GWN and magneto-acoustical simulation-induced multiple firing activities are explored by numerical simulations [31–33]. From the viewpoint of ions' effect mechanisms in biological neuron, the outward current carried by K^+ ions and the inward one by Na^+ and Ca^{2+} ions play key role in living biological activities. Thus, beyond all these investigations, by selecting four sets of the maximal conductances for the two ions currents, four representative firing activities are revealed. Besides, the synchronization for these four representative firing activities in coupled Chay neuron models are barely disclosed, which hinders the process of unveiling living biological activities.

The theoretical explorations and numerical simulations are classical methods to investigate neural dynamics, but hardware implementations and experimental measurements have become increasingly promising for diverse neuron-based engineering applications [34–43]. To date, on the benefit of real-time modification, easy software control, and adjustment, field programmable gate array (FPGA) has been employed to verify the dynamical behaviors of Hindmarsh–Rose neuron model [35, 36], Wilson neuron model [37], Morris–Lecar neuron model [38], and FitzHugh–Nagumo neuron model [39] in the hardware level. One of another remarkable benefit is that the FPGA-based experiments can assign the desired initial conditions in software accurately and easily, which is conducive to

investigate initial condition-associated electrical activities in the hardware level [40]. To consider the virtue of FPGA-based realization method, there should be important significance to develop various digital electronic neurons for utilizing in hardware level synthesizers for large-scale neuromorphic circuits. Besides, the digitally circuit-implemented neuron network can effectively imitate the synchronous behavior in the network, which can promote the integrated circuit design and diverse neuron-based engineering applications [22]. To our best knowledge, there is no comprehensive exploration about the Chay neuron model in this valuable approach.

The arrangement of this paper is well designed as follows. The representative firing activities in a 3D Chay neuron model are briefly reviewed by employing numerical plots of phase trajectories and time-domain waveforms. Besides, a memristive synapse-connected Chay twin-neuron network is built in Section 2. Section 3 explores synchronous behavior in the memristive Chay twin-neuron network by utilizing time-domain waveforms, STSs, and MSEs. A FPGA-based electronic neuron and the memristive synapse-connected Chay twin-neuron network are realized and the hardware tests are executed to verify the numerically simulated ones in Section 4. Eventually, the conclusion is drawn.

2. Memristive Synapse-Connected Chay Twin-Neuron Network

2.1. Minireview for 3D Chay Neuron Model. The 3D Chay neuron model is an effective candidate in the numerical simulation for electrical activities in single biological neuron [27], which is described as

$$\begin{aligned}\frac{dV}{dt} &= g_I m_\infty^3 h_\infty (V_I - V) + g_{K,V} n^4 (V_K - V) + g_{K,C} \frac{C}{1+C} (V_K - V) + g_L (V_L - V), \\ \frac{dn}{dt} &= \frac{(n_\infty - n)}{\tau_n}, \\ \frac{dC}{dt} &= \rho [m_\infty^3 h_\infty (V_C - V) - k_C C],\end{aligned}\tag{1}$$

where the three dynamic variables V , n , and C correspond to the membrane potential, open probability of the voltage-sensitive potassium ions channel, and intracellular concentration of calcium ions, respectively. The model

parameter in (1) is assigned and shown in Table 1. Besides, $\gamma_\infty = \alpha_\gamma / (\alpha_\gamma + \beta_\gamma)$ is unified to the expressions of m_∞ , h_∞ , and n_∞ , in which γ presents the alphabets m , h , and n . Herein, the expression formats for α_m , β_m , α_h , β_h , α_n , β_n , and τ_n are

$$\begin{aligned}
\alpha_m &= \frac{0.1(25 + V)}{(1 - e^{-(V+25/10)})}, \\
\beta_m &= 4e^{-(V+50/18)}, \\
\alpha_h &= 0.07e^{-(V+50/20)}, \\
\beta_h &= \frac{1}{(1 + e^{-(V+20/10)})}, \\
\alpha_n &= \frac{0.01(20 + V)}{(1 - e^{-(V+20/10)})}, \\
\beta_n &= 0.125e^{-(V+30/80)}, \\
\tau_n &= \frac{1}{[r_n(\alpha_n + \beta_n)]}.
\end{aligned} \tag{2}$$

On account of the fast-slow effect, the bursting and spiking behaviors are two representative firing activities generated in this neuron model [27]. By using MATLAB ODE45-based method with time step 10^{-3} s, fixing the initial conditions (0.1 mV, 0.1, 0.1 nmol/L), and employing the model parameters in Table 1, note that “MaxStep” and “RelTol” in MATLAB-based ODE45 algorithm should be assigned as 10^{-3} s and 10^{-4} s, respectively.

The firing activities related to the reversal potentials [30, 32], external stimulus [31, 33], and time kinetic constant [41] have been investigated in numerical survey. Differently, by selecting four sets of model parameters for the maximal conductance g_I of mixed Na^+ - Ca^{2+} channel and maximal conductance $g_{K,V}$ of potassium ions channel, four representative types of firing activities including periodic and

chaotic bursting and chaotic spiking behaviors are revealed by utilizing the trajectories in the C - V plane and the time-domain waveforms of the membrane potential V and intracellular concentration C of calcium ions, as shown in Figure 1. Figures 1(a) and 1(d) exhibit two kinds of periodic bursting behaviors, Figure 1(b) demonstrates chaotic bursting behavior with disordered spikes, and Figure 1(c) illustrates conventional chaotic spiking behavior. The bifurcation mechanisms for the two kinds of periodic bursting behaviors are different. Those are periodic fold/fold bursting in Figure 1(a) and periodic fold/homoclinic bursting in Figure 1(d) [42, 43]. Figure 2(b) shows the chaotic fold/homoclinic bursting behaviors, and Figure 1(c) denotes the chaotic spiking activity. The synchronous behaviors in a memristive synapse-connected Chay twin-neuron network under the four sets of model parameters will be disclosed in the next sections, respectively.

2.2. Modeling for Memristive Synapse-Connected Twin-Neuron Network. Support two identical neurons are bidirectionally coupled by memristor synapse to represent the electromagnetic induction effect induced with the membrane potential differences between them. Herein, the memristor with threshold memductance $W(\varphi) = \tanh(\varphi)$ and the membrane potential difference $V_M = V_1 - V_2$ are employed [26], so the induced current I_M and memristor synapse characteristic model can be described as

$$\begin{aligned}
I_M &= W(\varphi)V_M = \tanh(\varphi)(V_1 - V_2), \\
\dot{\varphi} &= V_M = V_1 - V_2,
\end{aligned} \tag{3}$$

with two identical Chay neurons coupled by the threshold memristor, and a memristive synapse-connected Chay twin-neuron network can be described as

$$\begin{aligned}
\frac{dV_1}{dt} &= g_1 m_{1\infty}^3 h_{1\infty} (V_I - V_1) + g_{K,V} n_1^4 (V_K - V_1) + g_{K,C} \frac{C_1}{1 + C_1} (V_K - V_1) + g_L (V_L - V_1) + k \tanh \varphi (V_1 - V_2), \\
\frac{dn_1}{dt} &= \frac{(n_{1\infty} - n_1)}{\tau_{1n}}, \\
\frac{dC_1}{dt} &= \rho [m_{1\infty}^3 h_{1\infty} (V_C - V_1) - k_C C_1], \\
\frac{dV_2}{dt} &= g_1 m_{2\infty}^3 h_{2\infty} (V_I - V_2) + g_{K,V} n_2^4 (V_K - V_2) + g_{K,C} \frac{C_2}{1 + C_2} (V_K - V_2) + g_L (V_L - V_2) - k \tanh \varphi (V_1 - V_2), \\
\frac{dn_2}{dt} &= \frac{(n_{2\infty} - n_2)}{\tau_{2n}}, \\
\frac{dC_2}{dt} &= \rho [m_{2\infty}^3 h_{2\infty} (V_C - V_2) - k_C C_2], \\
\frac{d\varphi}{dt} &= V_1 - V_2,
\end{aligned} \tag{4}$$

TABLE 1: Parameters' values and significations of the 3D Chay neuron model.

Parameters	Significations	Values
V_I	Reversal potentials for mixed Na^+ - Ca^{2+} ions	100 mV
V_K	Reversal potentials for K^+ ions	-75 mV
V_L	Reversal potentials for leakage ions	-40 mV
V_C	Reversal potentials for Ca^{2+} ions	100 mV
g_I	Maximal conductance of mixed Na^+ - Ca^{2+} channel	1925 mS/cm ²
$g_{K,V}$	Maximal conductance of K^+ channel	1700 mS/cm ²
$g_{K,C}$	Maximal conductance of Ca^{2+} -sensitive K^+ channel	12 mS/cm ²
g_L	Maximal conductance of leakage channel	7 mS/cm ²
τ_n	Time kinetic constant of the fast variable	230 ms
k_C	Rate constant for the efflux of intracellular Ca^{2+} ions	3.3/18 ms ⁻¹
ρ	Time constant determines the changing rate C	0.27 ms

FIGURE 1: Trajectories in the $C - V$ plane and time-domain waveforms of V and C for different g_I and $g_{K,V}$. (a) Periodic bursting behavior for $g_I = 1250$ mS/cm² and $g_{K,V} = 1700$ mS/cm²; (b) chaotic bursting behavior for $g_I = 1850$ mS/cm² and $g_{K,V} = 1700$ mS/cm²; (c) chaotic spiking behavior for $g_I = 1925$ mS/cm² and $g_{K,V} = 1700$ mS/cm²; (d) periodic bursting behavior with 5 spikes per burst for $g_I = 1800$ mS/cm² and $g_{K,V} = 1650$ mS/cm².

where V_1 and V_2 are the membrane potential, n_1 and n_2 are probability of the voltage-sensitive K^+ channel for the two identical Chay neuron models, C_1 and C_2 are the intracellular concentration of calcium ions for them, respectively, and $k \tanh \phi(V_1 - V_2)$ stands for the memristor synapse-coupled item with k as positive to feature the coupling strength. In (4), the coupling strength k can control the effect of memristor synapse. Actually, the memristor synapse-connected item just inputs additive current leading to that the membrane potentials which can be regulated. Also, (4) explained the mechanism for memristive feedback control. Besides, $y_{i\infty} = \alpha_{iy}/(\alpha_{iy} + \beta_{iy})$ can be unified to the explications of $m_{i\infty}$, $h_{i\infty}$, and $n_{i\infty}$, in which y stands for m , h , and n and $i = 1, 2$. The explicit expressions for α_{im} , β_{im} , α_{ih} , β_{ih} , α_{in} , β_{in} , and τ_{in} are

$$\begin{aligned}
 \alpha_{im} &= \frac{0.1(25 + V_i)}{(1 - e^{-(V_i + 25/10)})}, \quad \beta_{im} = 4e^{-(V_i + 50/18)}, \\
 \alpha_{ih} &= 0.07e^{-(V_i + 50/20)}, \quad \beta_{ih} = \frac{1}{(1 + e^{-(V_i + 20/10)})}, \\
 \alpha_{in} &= \frac{0.01(20 + V_i)}{(1 - e^{-(V_i + 20/10)})}, \quad \beta_{in} = 0.125e^{-(V_i + 30/80)}, \\
 \tau_{in} &= \frac{1}{[r_n(\alpha_{in} + \beta_{in})]}.
 \end{aligned} \tag{5}$$

FIGURE 2: Synchronous behavior for the memristive twin-neuron network with various k and φ_0 , where the lefts are time-domain waveforms and the rights are STSs. (a) $k = 1$ and $\varphi_0 = -2$ mWb; (b) $k = 3$ and $\varphi_0 = -2$ mWb; (c) $k = 4$ and $\varphi_0 = -2$ mWb; (d) $k = 4$ and $\varphi_0 = 1$ mWb.

Thus, the synchronous behaviors for the memristive synapse-connected Chay twin-neuron network can be disclosed by system (4). Following on this, we mainly focus on exploring the memristor initial condition and coupling strength associated synchronous behaviors in the proposed memristive network.

3. Synchronous Behaviors for the Chay Twin-Neuron Network

The exploration of synchronous behaviors for the Chay twin-neuron network is performed by MATLAB numerical simulations. Herein, we mainly focus on the two coupled neurons with only difference initial membrane potentials between them. Thus, the Chay twin-neuron network is triggered by initial conditions (0.1 mV, 0, 0.1 nmol/L, 1 mV, 0, 0.1 nmol/L, φ_0) without loss of the generality, within which the memristor initial condition φ_0 is tunable.

The electrical activity is chaotic spiking under the model parameters in Table 1, for each neuron in the memristive synapse-connected Chay twin-neuron network. The synchronized transition states (STSs) are plotted at the right in Figures 2(a)–2(d). There exist no errors between the two membrane potentials with a line in the V_1 - V_2 plane, which means that the two neurons are in sync. Otherwise, the relatively large errors between the two membrane potentials indicate the two neurons out of synchronization. When $\varphi_0 = -2$ mWb, the difference between the two membrane potentials become smaller with $k = 1$ and 3 increasing and disappear with $k = 4$, as shown in Figures 2(a)–2(c). This manifests that the two coupled neurons are asymptotically synchronized with increasing k . However, when $k = 4$ with

increasing φ_0 as -2 mWb and 1 mWb, the difference between the two membrane potentials become larger, as shown in Figures 2(c) and 2(d). It is indicated that the two coupled neurons are loose their synchronization. Summarily, by increasing the coupling strength k or decreasing memristor initial condition φ_0 , the synchronization can be realized and the difference between the two membrane potentials is becoming smaller in the memristive synapse-connected Chay twin-neuron network. The mechanism for this process is that the coupling memristor exchanges the magnetic flux. Thus, the induced current is the carrier to drive the two Chay neurons in sync. Otherwise, the two connected Chay neurons are loss synchronization under tiny electromagnetic induction outputs.

To fully explore the synchronous behavior associated to the coupling strength k and initial condition φ_0 of the memristor synapse simultaneously, mean synchronization error (MSE) E is employed and defined as

$$E = \frac{1}{N} \sum_{j=1}^N \frac{\sqrt{[V_1(j) - V_2(j)]^2 + [n_1(j) - n_2(j)]^2 + [C_1(j) - C_2(j)]^2}}{\sqrt{V_1(j)^2 + V_2(j)^2 + n_1(j)^2 + n_2(j)^2 + C_1(j)^2 + C_2(j)^2}}, \quad (6)$$

where $V_i(j)$, $n_i(j)$, and $C_i(j)$ ($i = 1, 2$) are the j th sampling values and N is the number of total samples. The normalized MSE of the Chay twin-neuron network is becoming zero for synchronous state and nonzero for out of synchronization.

Herein, the time sequence interval (600 s and 700 s) and the time step 0.01 s are selected. Thus, the number of total samples is $N = 10000$. The normalized MSEs for different coupling strength k and initial condition φ_0 , as well as different g_1 and $g_{K,V}$ are given in the φ_0 - k plane, as plotted

in Figure 3. The regions padded by red indicate the two connected Chay neurons in sync with $E=0$, whereas the regions padded by other colors denote the two connected Chay neurons loss synchronization with $E>0$. In general, as coupling strength k becoming larger and more negative initial condition φ_0 , the normalized MSEs E drops near zero. Thus, the two connected Chay neurons become sync. Contrarily, the two connected Chay neurons are in asynchronous state with k becoming smaller and small negative or positive φ_0 .

The normalized MSEs is given in Figure 3. The results show that the synchronous behaviors for the memristive synapse-connected Chay twin-neuron network are really associated with the memristor synapse coupling strength and the initial condition. Such an initial condition related synchronous behavior has been rarely reported in previous literature [22].

4. FPGA-Based Hardware Implementation

It is more complex to physically realize the memristive synapse-connected Chay twin-neuron network than a single 3D Chay neuron model by FPGA. Thus, only the memristive synapse-connected Chay twin-neuron model by the digital electronic platform is demonstrated with representativeness. For this aim, fourth-order Runge–Kutta algorithm is utilized to obtain the discrete-time form for model (4) and given as

$$\begin{aligned}
 (V_1)_N &= (V_1)_{N-1} + \frac{l \left[D_1 k_{(V_1)1} + D_2 k_{(V_1)2} + D_3 k_{(V_1)3} + D_4 k_{(V_1)4} \right]}{D_1 + D_2 + D_3 + D_4}, \\
 (n_1)_N &= (n_1)_{N-1} + \frac{l \left[D_1 k_{(n_1)1} + D_2 k_{(n_1)2} + D_3 k_{(n_1)3} + D_4 k_{(n_1)4} \right]}{D_1 + D_2 + D_3 + D_4}, \\
 (C_1)_N &= (C_1)_{N-1} + \frac{l \left[D_1 k_{(C_1)1} + D_2 k_{(C_1)2} + D_3 k_{(C_1)3} + D_4 k_{(C_1)4} \right]}{D_1 + D_2 + D_3 + D_4}, \\
 (V_2)_N &= (V_2)_{N-1} + \frac{l \left[D_1 k_{(V_2)1} + D_2 k_{(V_2)2} + D_3 k_{(V_2)3} + D_4 k_{(V_2)4} \right]}{D_1 + D_2 + D_3 + D_4}, \\
 (n_2)_N &= (n_2)_{N-1} + \frac{l \left[D_1 k_{(n_2)1} + D_2 k_{(n_2)2} + D_3 k_{(n_2)3} + D_4 k_{(n_2)4} \right]}{D_1 + D_2 + D_3 + D_4}, \\
 (C_2)_N &= (C_2)_{N-1} + \frac{l \left[D_1 k_{(C_2)1} + D_2 k_{(C_2)2} + D_3 k_{(C_2)3} + D_4 k_{(C_2)4} \right]}{D_1 + D_2 + D_3 + D_4}, \\
 \varphi_N &= \varphi_{N-1} + \frac{l \left[D_1 k_{\varphi 1} + D_2 k_{\varphi 2} + D_3 k_{\varphi 3} + D_4 k_{\varphi 4} \right]}{D_1 + D_2 + D_3 + D_4},
 \end{aligned} \tag{7}$$

where i is the sampling interval, N is the total number of iterations, and D_q ($q=1, 2, 3, 4$) denotes the coefficient of variation. To achieve the unique expression of intermediate vector $k_{(V_i)q}$, $k_{(n_i)q}$, $k_{(C_i)q}$, and $k_{\varphi q}$ ($i=1, 2$ and $q=1, 2, 3, 4$), the assumption of $k_{(V_i)0} = k_{(n_i)0} = k_{(C_i)0} = k_{\varphi 0} = 0$ is applied. Then, one yields

$$\begin{aligned}
 K_{(V_1)q} &= g_I m_N^3 h_N \left[V_I - \left((V_1)_{N-1} + \frac{lk_{(V_1)q-1}}{D_q} \right) \right] \\
 &+ g_{K,V} n_{N-1}^4 \left[V_K - \left((V_1)_{N-1} + \frac{lk_{(V_1)q-1}}{D_q} \right) \right] \\
 &+ g_{K,C} \frac{(C_1)_{N-1}}{1 + (C_1)_{N-1}} \\
 &\cdot \left[V_K - \left((V_1)_{N-1} + \frac{lk_{(V_1)q-1}}{D_q} \right) \right] \\
 &+ g_L \left[V_L - \left((V_1)_{N-1} + \frac{lk_{(V_1)q-1}}{D_q} \right) \right] \\
 &+ k \tan \varphi_{N-1} \left((V_1)_{N-1} + \frac{lk_{(V_1)q-1}}{D_q} - (V_2)_{N-1} \right), \\
 K_{(n_1)q} &= \frac{d_N - \left((n_1)_{N-1} + \left(lk_{(n_1)q-1} / D_q \right) \right)}{\tau_N}, \\
 K_{(C_1)q} &= \rho \left[m_N^3 h_N (V_C - (V_1)_{N-1}) - K_C \left((C_1)_{N-1} + \frac{lk_{(C_1)q-1}}{D_q} \right) \right], \\
 K_{(V_2)q} &= g_I m_N^3 h_N \left[V_I - \left((V_2)_{N-1} + \frac{lk_{(V_2)q-1}}{D_q} \right) \right] \\
 &+ g_{K,V} n_{N-1}^4 \left[V_K - \left((V_2)_{N-1} + \frac{lk_{(V_2)q-1}}{D_q} \right) \right] \\
 &+ g_{K,C} \frac{(C_2)_{N-1}}{1 + (C_2)_{N-1}} \\
 &\cdot \left[V_K - \left((V_2)_{N-1} + \frac{lk_{(V_2)q-1}}{D_q} \right) \right] \\
 &+ g_L \left[V_L - \left((V_2)_{N-1} + \frac{lk_{(V_2)q-1}}{D_q} \right) \right] \\
 &- k \tan \varphi_{N-1} \left((V_1)_{N-1} - (V_2)_{N-1} - \frac{lk_{(V_2)q-1}}{D_q} \right), \\
 K_{(n_2)q} &= \frac{d_N - \left((n_2)_{N-1} + \left(lk_{(n_2)q-1} / D_q \right) \right)}{\tau_N}, \\
 K_{(C_2)q} &= \rho \left[m_N^3 h_N (V_C - (V_2)_{N-1}) - K_C \left((C_2)_{N-1} + \frac{lk_{(C_2)q-1}}{D_q} \right) \right], \\
 K_{\varphi q} &= (V_1)_{N-1} - (V_2)_{N-1},
 \end{aligned} \tag{8}$$

in which $q=1, 2, 3, 4$, and

FIGURE 3: Normalized MSEs of the memristive synapse-connected Chay twin-neuron network in the $\phi_0 - k$ plane. (a) $g_I = 1250 \text{ mS/cm}^2$ and $g_{K,V} = 1700 \text{ mS/cm}^2$; (b) $g_I = 1850 \text{ mS/cm}^2$ and $g_{K,V} = 1700 \text{ mS/cm}^2$; (c) $g_I = 1925 \text{ mS/cm}^2$ and $g_{K,V} = 1700 \text{ mS/cm}^2$; (d) $g_I = 1800 \text{ mS/cm}^2$ and $g_{K,V} = 1650 \text{ mS/cm}^2$.

FIGURE 4: Hierarchical structure of the Verilog HDL program for the memristive synapse-connected Chay twin-neuron model.

FIGURE 5: RTL schematics of the discrete-time memristive synapse-connected Chay twin-neuron model executed on XC7Z020 FPGA.

FIGURE 6: Hardware measured time-domain waveforms and synchronous transition states with different coupling strength k and initial condition φ_0 . (a) $k = 1$ and $\varphi_0 = -2$ mWb; (b) $k = 3$ and $\varphi_0 = -2$ mWb; (c) $k = 4$ and $\varphi_0 = -2$ mWb; (d) $k = 4$ and $\varphi_0 = 1$ mWb.

$$\begin{aligned}
 (m_i)_N &= \frac{25 + (V_1)_{N-1}}{25 + (V_1)_{N-1} + 40 \left(1 - e^{-0.1(V_i)_{N-1}-2.5} \right) e^{-((V_i)_{N-1}+50)/80}}, \\
 (h_i)_N &= \frac{7 \left(1 + e^{-0.1(V_i)_{N-1}-2} \right) e^{-0.05(V_i)_{N-1}-2.5}}{7 \left(1 + e^{-0.1(V_i)_{N-1}-2} \right) e^{-0.05(V_i)_{N-1}-2.5} + 100}, \\
 (d_i)_N &= \frac{20 + (V_i)_{N-1}}{20 + (V_1)_{N-1} + 12.5 \left(1 - e^{-0.1(V_i)_{N-1}-2} \right) e^{-((V_i)_{N-1}+30)/80}}, \\
 (\tau_i)_N &= \frac{100 \left(1 + e^{-0.1(V_i)_{N-1}-2} \right)}{r_n \left[(20 + (V_i)_{N-1}) + 12.5 e^{-((V_i)_{N-1}+30)/80} \left(1 + e^{-0.1(V_i)_{N-1}-2} \right) \right]}.
 \end{aligned} \tag{9}$$

To achieve the discrete-time memristive synapse-connected Chay twin-neuron model (7), we employ a low-cost yet powerful XC7Z020 FPGA to execute the model for the first time. The FPGA software program using Verilog language is coded, within which the number of iteration $N = 150000$, the sampled interval $i = 0.001$, and the coefficients of variation $D_1 = D_4 = 1$ and $D_2 = D_4 = 2$ are set up. The parameters g_1 , $g_{K,V}$, k , and φ_0 , which impact the firing activities and synchronous behaviors, are changed by the software program to capture the four typical firing activities, time-domain waveforms, and STSs corresponding to the numerical simulations.

The hierarchical structure of the Verilog HDL program is illustrated in Figure 4. A main controller is set to reset the electro-neuron and to start the iteration form of initial conditions for V_p , n_p , C_p , and φ while the power is on. The floating-type operation IPs are contained in the lowest inherent IP layer, which can be instantiated to construct our own customized IP. In the customized IP layer, add-, subtract-, multiply-, divide-, and exponent-operators are instantiated to construct each function of the right side of continuous-time model (4) (expressed as f_{V1} , f_{n1} , f_{C1} , f_{V2} , f_{n2} , f_{C2} , and f_φ). The intermediate variables $m_{1\infty}$,

$h_{1\infty}$, $n_{1\infty}$, $m_{2\infty}$, $h_{2\infty}$, and $n_{2\infty}$ have to be calculated before the calculations of f_V , f_n , and f_C ; thus, a sublayer for intermediate variables is hired in this layer. In the iteration layer, the functions f_{V1} , f_{n1} , f_{C1} , f_{V2} , f_{n2} , f_{C2} , and f_φ are transmitted by time multiplex way to compute the intermediate vectors $k_{(V_i)q}$, $k_{(n_i)q}$, $k_{(C_i)q}$, and $k_{\varphi q}$ ($i = 1, 2$ and $q = 1, 2, 3, 4$) (marked as K_1 , K_2 , K_3 , and K_4) in (8), and then the $(V_1)_N$, $(n_1)_N$, $(C_1)_N$, $(V_2)_N$, $(n_2)_N$, $(C_2)_N$, and φ_N are updated. Finally, an output layer is employed to adjust the output signal for visualizing expediently. The RTL schematic for the digitally electronic twin-neuron network is generated, as shown in Figure 5.

Since the outputs of a FPGA are digital, they are fed to a two-channel 14 bit D/A converter (AD9767) combined with the peripheral circuit to convert the digital outputs into analog ones. Herein, an oscilloscope Agilent DSO-X 3012A is employed to display the output analog signals. Note that, all variables are in the single-precision floating-type during the computation process, so they must be converted into integers and enlarged to the range of $[-8192, 8191]$ to take full use of 14 bits of the DAC digital input ports. The time-domain waveforms of V_1 and V_2 and

FIGURE 7: Experimentally captured trajectories in the C-V phase plane and time-domain waveforms. (a) Periodic bursting behavior for $g_I = 1250 \text{ mS/cm}^2$ and $g_{K,V} = 1700 \text{ mS/cm}^2$; (b) chaotic bursting behavior for $g_I = 1850 \text{ mS/cm}^2$ and $g_{K,V} = 1700 \text{ mS/cm}^2$; (c) chaotic spiking behavior for $g_I = 1925 \text{ mS/cm}^2$ and $g_{K,V} = 1700 \text{ mS/cm}^2$; (d) periodic bursting behavior with burster-5 for $g_I = 1800 \text{ mS/cm}^2$ and $g_{K,V} = 1650 \text{ mS/cm}^2$.

synchronous transition states in the $V_1 - V_2$ plane for different coupling strength k and initial condition ϕ_0 in the Chay bi-neuron network are captured and displayed in Figure 6. The amplitudes of V_1 and V_2 on the oscilloscope are differed from that of MATLAB simulations in Figure 2, but they are proportional with each other due to the same enlargement. It is clearly shown that the proposed FPGA-based digital hardware can verify the synchronous behavior for the memristive synapse-connected Chay twin-neuron network. Note that the initial states in our digital circuit experiment are accurately assigned in the software [44]. This is very different from the randomly sensed way of acquiring initial states by repeatedly switching on and off the power supply in an analog circuit experiment [45]. Hence, the initial states are determined in the digital circuits but undetermined in the analog ones.

Besides, a FPGA-based digital hardware electronic neuron is simply realized to confirm the four representative firing activities in the 3D Chay neuron model. The trajectories in the $V_1 - V_2$ phase plane and time sequences of V_1 and V_2 are captured and displayed in Figure 7. The model parameters are selected the same as these utilized in Figure 1.

5. Conclusion

Four kinds of representative firing activities classified on the dependence of two maximal conductances in a 3D Chay neuron model are briefly reviewed. Then, a memristive synapse-connected Chay twin-neuron network is built, upon which synchronous behaviors are explored by utilizing time-domain waveforms, STSs, and MSEs. The numerical simulations demonstrated the success and effectiveness of employing the memristor synapse to achieve synchronization. It is found that, associating with the large coupling strength and more negative initial condition of the memristor, synchronous behaviors are achieved. An effective approach to implement the electronic neuron and the Chay twin-neuron network via FPGA are employed, from which

the four kinds of representative firing activities of chaotic and periodic bursting/spiking behaviors, as well as the STSs are experimentally captured to confirm the correctness of the numerical ones. Synchronous behavior disclosed in neuronal network can well reveal the benefit for understanding the dynamical intricacy in the biological neurons and reflect the feasibility of diverse neuron-based applications.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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