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Research Article

Delivery Service Optimization When the Information on the Consumers' Time Values Is Asymmetric

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When consumers' time values are discrete and asymmetric, three delivery service schemes, which are single ordinary delivery, single expedited delivery, and a mixture of ordinary and expedited delivery, were analyzed and compared. From the perspective of the firm, the optimal lead time and price in each delivery service scheme were determined based on the principal-agent theory. The results show that when the proportion of higher time value consumers is not above a specific threshold, providing a mixture of delivery services outperforms providing single delivery services. Otherwise, providing a single expedited delivery service becomes the best choice. As the consumers' time value increases, the optimal lead time of the delivery service that is targeted to them should be decreased and the optimal price should be increased. If the firm provides a mixture of delivery services, then the optimal lead time of expedited delivery should be equal to that of the single expedited delivery scheme, while the optimal lead time of ordinary delivery should be longer than that of the single ordinary delivery scheme. Moreover, the optimal lead time of ordinary delivery in the mixed case should be increased with the proportion of higher time value consumers until it reaches the maximum length of time that all consumers could bear.

1. Introduction

As the network economy is booming, providing the right delivery service for online shopping consumers is becoming increasingly more important. The purchase decision-making behavior of consumers is directly affected by how the companies' delivery services are designed, for which the delivery lead time and price are usually the two critical and most influential factors. As such, many e-tailers, express companies, and take-out businesses have offered diversified delivery services based on the lead time and price. For example, JD.com, the biggest online retailer in China, charges consumers different prices for delivery services such as "211 Guaranteed Delivery" and "Instant delivery," whose difference essentially lies in the delivery lead time or delivery speed (see https://help.jd.com/user/issue/list-81.html).

According to microeconomic theory, rational consumers always choose the commodities or services that maximize their utility. However, consumers with different time values may perceive different levels of utility, even for the same delivery service. In addition, consumers' time value is usually unknown to the companies. Under such an asymmetric environment, how a business can determine which service scheme to offer and then set the lead time and price in the scheme to attract consumers and maximize its interests is both challenging and crucial.

By constructing principal-agent models, this paper aims to help a firm optimally design its delivery service for its online shoppers, whose time values are discrete and asymmetric. Profit maximization is the objective. Three service schemes, which are a single ordinary delivery service, a single expedited delivery service, and a mixture of ordinary and expedited delivery services, are the potential choices, and the lead time and the price in each service scheme are the decision variables.

The contributions of this paper are the following. First, most research on delivery service optimization is conducted in an information symmetric context (Huang et al. [1], Pekgün et al. [2], and Modak and Kelle [3]). In this paper, we assume that consumers' time values are asymmetric and find the effect of consumers' time values on the delivery service optimization. Second, we comparatively investigate three delivery schemes and specify a threshold of the proportion of higher time value consumers as the delivery scheme choice index. We show that providing mixed services is not always the best choice, which challenges the notion that providing mixed and differentiated services is generally believed to be the optimal service scheme (Hsu and Li [4]).

The rest of this paper is organized as follows. Section 2 gives the literature review, Section 3 provides the notations and assumptions, and Section 4 develops the mathematical formulations and specifies the optimal decisions. Section 5 discusses the effects of the key parameters on the optimal decisions and compares the proposed delivery schemes. Section 6 numerically illustrates the models. Finally, Section 7 provides the conclusions and future research suggestions.

2. Literature Review

There are three bodies of existing literature which are relevant to our research. The first body of literature deals with the delivery lead time. Hsu and Li [4] determined the optimal number and duration of service cycles for Internet shopping by adopting a discriminating strategy based on time-dependent consumer demand. Zhu [5] established a multinomial logit model to determine the best allocation time for online shopping customers' orders.

The second body of literature consists of papers dealing with pricing issues for delivery services. The studies along this line are mature and have considered a variety of factors, including the interaction of commodity pricing and delivery service pricing (Leng et al. [6] and Gümüs et al. [7]), single and multiple product transactions (Jiang et al. [8]), different service modes such as door-to-door delivery and pick-up in stores (Wang and Liu [9]), and threshold pricing in contingent free shipping (Qing and Dong [10] and Song et al. [11]). In particular, Yao et al. [12], Chen et al. [13], and Lu et al. [14] studied pricing decisions based on the delivery lead time when differentiated delivery services were provided.

Last, there are several papers pertaining to the joint optimization of the delivery lead time and price, which is most relevant to our research. Hua et al. [15] determined the optimal decisions regarding the delivery time and prices in a dual-channel supply chain. Xu et al. [16] investigated the manufacturer's pricing and delivery time decisions and the e-tailer's percentage fee decisions when consumers have private information. Huang et al. (2013) and Pekgün et al. [2] explored the price and lead time competition of two firms when consumers are sensitive to price and delivery lead time. Modak and Kelle [3] extended the works of Huang et al. [1] and Pekgün et al. [2] to study random demand. Note that, except for Xu et al. [16], all the other researches were conducted by assuming a symmetric information background, which is different from our asymmetric information background. However, our study is also different from Xu et al.'s [16] in that we focus on comparing three different delivery schemes, while the other study mainly dealt with the manufacturer's single delivery scheme and the retailer's fee decision. In addition, we consider the maximum length of time that all consumers could bear, which exists in practice and cannot be negligible.

3. Context and Assumptions

We consider a model with a firm needing to design its delivery service menu for a fixed population of N consumers. The consumers' time values are private information and are not known to the firm. However, the firm knows that consumers can be divided into two types. One type is characterized by a higher time value, which means that these consumers are willing to pay more for one unit of time saved; and the other is characterized by a lower time value. The proportion of higher time value consumers is λ and $\lambda \in [0, 1]$.

The firm attempts to match the two types of consumers with two kinds of delivery services, which are expedited delivery and ordinary delivery. The delivery lead times and prices of the two delivery services are noted as $t = \{t_o, t_e\}$ and $p = \{p_o, p_e\}$, where $t_e < t_o$ and $p_o \le p_e$, respectively. The subscripts $i = o, e$ refer to ordinary delivery and expedited delivery, respectively.

Since the consumers with a higher time value prefer expedited delivery and the consumers with a lower time value prefer ordinary delivery, we also use the subscripts $i = o, e$ to refer to higher time value consumers and lower time value consumers, respectively, to avoid complexity and reflect the association. For consumer type i, his willingness to pay the firm is assumed to be $\mu + \theta_i (T - t)$, where μ is the willingness to pay for the firm's other attributes like service attitude or commodity quality; θ_i represents his willingness to pay for one unit of time saved, which is not detectable to the firm, as mentioned earlier; and T is the maximum length of time that all consumers could bear. When $t = T$, all the consumers' willingness to pay drops to μ, and so μ can also be seen as the lowest willingness to pay. We assume all the consumers are risk-neutral, and thus the net utility of a consumer with time value θ_i equals $\mu + \theta_i (T - t) - p$.

We assume that the cost of providing delivery service is $c(t) = \nu + (k/t)$ (Xu et al., [16]), where ν is the fixed cost and (k/t) is the variable cost inversely proportional to the length of delivery lead time t. It can be verified that $c'(t) < 0$ and $c''(t) > 0$, which conforms to the mechanism of a marginal cost increase.

To ensure that the firm would not provide a delivery service whose lead time reaches or is longer than the maximum length of time T, it is specified here that $c(t)|_{t=T} = \nu + (k/t) \ge \mu$ and $c'(t)|_{t=T} = (k/T^2) < \theta_0 < \theta_e$, which means that the firm will experience a loss if the delivery lead time exceeds T and the firm will lose more if the time continues to be extended.

4. The Delivery Lead Time and Pricing Decisions in Different Service Schemes

In the following, the optimal delivery lead time and pricing decisions will be, respectively, solved to maximize the firm's profit ($\prod$) in two big or three small classifications of delivery schemes, which are providing two single delivery services (expedited delivery or ordinary delivery) and providing a mixture of delivery services (expedited delivery and ordinary delivery), based on the analysis of consumers' behavior. The superscript $j = 1$ refers to the single delivery service cases and $j = 2$ refers to the mixed delivery service case.

4.1. Providing Single Ordinary Delivery. When the firm provides single ordinary delivery, it will capture all the lower time value consumers' surplus by pricing the service at precisely these consumers' marginal willingness to pay. Therefore, $p_o = \theta_o (T - t_o) + \mu$, implying that the consumers with a lower time value obtain zero net utility.

Since $\theta_o \leq \theta_e$, $\mu + \theta_e (T - t_o^{1*}) - p_o^{1*} > 0$. This implies that the consumers with a higher time value would get positive utility, and thus they would accept the service scheme.

Therefore, the firm's problem can be formulated as follows:

$$\begin{aligned} \prod\nolimits_o^1 &= \max_{\{t_o, p_o\}} \{(1 - \lambda) N [p_o - c(t_o)] + \lambda N [p_o - c(t_o)]\} \\ &= N [p_o - c(t_o)], \end{aligned} \tag{1}$$

such that

$$p_o = \theta_o (T - t_o) + \mu, \tag{2}$$

$$t_o \leq T. \tag{3}$$

By applying equation (2) and $c(t_o) = \nu + (k/t_o)$ to equation (1), we can get $t_o^{1*} = \sqrt{(k/\theta_o)}$. Since $c'(t)|_{t=T} = (k/T^2) < \theta_0 < \theta_e$, $t_o^{1*} = \sqrt{(k/\theta_o)} < T$ in equation (3) holds. Therefore, $t_o^{1*} = \sqrt{(k/\theta_o)}$ is the optimal solution for $\prod_o^1$. Then, we insert t_o^{1*} back into equation (2), and we have $p_o^{1*} = \mu + \theta_o T - \sqrt{k\theta_o}$. From these results, we have the following proposition.

Proposition 1. *When the firm only provides ordinary delivery, the optimal delivery lead time is* $t_o^{1*} = \sqrt{(k/\theta_o)}$ *and the optimal price is* $p_o^{1*} = \mu + \theta_o T - \sqrt{k\theta_o}$.

4.2. Providing Single Expedited Delivery. When the firm only provides a single expedited delivery service, it will capture all the higher time value consumers' surplus by pricing the service precisely according to these consumers' marginal willingness to pay. Therefore, $p_e = \theta_e (T - t_e) + \mu$, implying that consumers with a higher time value obtain zero net utility.

Since $\theta_o \leq \theta_e$, $\mu + \theta_o (T - t_e) - p_e < 0$, which implies that the consumers with a lower time value obtain negative utility. Thus, they would not accept the service scheme.

Therefore, in this case, the firm's problem can be formulated as follows:

$$\begin{aligned} \prod\nolimits_e^1 &= \max_{\{t_e, p_e\}} \lambda N [p_e - c(t_e)] \\ \text{s.t} \quad & p_e = \theta_e (T - t_e) + \mu \\ & t_e \leq T. \end{aligned} \tag{4}$$

Using the same approach as described in the single ordinary delivery scenario, the optimal lead time and pricing decisions can be shown in the following proposition.

Proposition 2. *When the firm only provides expedited delivery, the optimal delivery lead time is* $t_e^{1*} = \sqrt{(k/\theta_e)}$ *and the optimal price is* $p_e^{1*} = \mu + \theta_e T - \sqrt{k\theta_e}$.

4.3. Providing Mixed Ordinary and Expedited Delivery. When the firm simultaneously provides ordinary and expedited delivery services, its primary goal is to earn more money by making the higher time value consumers choose expedited delivery services and the lower time value consumers choose the ordinary delivery services, which are, respectively, targeted to them. However, when consumers' classes are unobservable, consumers are free to selfishly choose between the different services offered. If the firm simply offers a mixture of ordinary and expedited delivery with no other strategy, the higher time value consumers could hide their type and act as a lower time value consumer to choose the delivery that is intended for the lower time value consumers and pay less money to earn more utility. Therefore, the firm should elaborately determine (t_i, p_i) in delivery service i to give consumers an incentive to reveal their real types. Mathematically, the firm's problem can be formulated as follows:

$$\Pi^2 = \max_{\{t_o, p_o\}, \{t_e, p_e\}} \lambda N [p_e - c(t_e)] + (1 - \lambda) N [p_o - c(t_o)], \tag{5}$$

such that

$$\mu + \theta_o (T - t_o) \geq p_o, \tag{6}$$

$$\mu + \theta_e (T - t_e) \geq p_e, \tag{7}$$

$$\mu + \theta_o (T - t_o) - p_o \geq \mu + \theta_o (T - t_e) - p_e, \tag{8}$$

$$\mu + \theta_e (T - t_e) - p_e \geq \mu + \theta_e (T - t_o) - p_o, \tag{9}$$

$$t_e < t_o \leq T, \tag{10}$$

where equations (6) and (7) represent the individual rationality constraint. This means that customers choose the delivery service only if they receive at least a zero level of utility; otherwise, the consumers will choose to go to another firm. Equations (8) and (9) are the incentive compatibility constraints that guarantee that each consumer reveals his time value by choosing his favorite delivery that is targeted for him. Equation (10) is the delivery time constraint requiring the lead time in both deliveries to be shorter than the maximum length of time and the expedited delivery lead time to be shorter than the ordinary one.

Since $\theta_e \geq \theta_o$, we have $\theta_e(T-t_e)-p_e \geq \theta_e(T-t_o)-p_o \geq \theta_o(T-t_o)-p_o \geq 0$ from equations (6) and (9). Hence, equation (7) is redundant. In addition, equation (6) needs to be tight to maximize profits. Then, equation (8) also becomes redundant as a result of equation (6) being tight and $\theta_e(T-t_e)-p_e \geq \theta_e(T-t_o)-p_o \geq \theta_o(T-t_o)-p_o \geq 0$.

Now, we can rewrite the firm's problem as

$$\Pi^2 = \max_{\{t_o,p_o\},\{t_e,p_e\}} \lambda N\left[p_e - c(t_e)\right] + (1-\lambda)N\left[p_o - c(t_o)\right], \tag{11}$$

such that

$$\mu + \theta_o(T-t_o) = p_o, \tag{12}$$

$$\theta_e(T-t_e) - p_e = \theta_e(T-t_o) - p_o, \tag{13}$$

$$t_e < t_o \leq T. \tag{14}$$

By inserting equations (12) and (13) into equation (11), we can obtain $t_e^2 = \sqrt{k/\theta_e}$ and $t_o^2 = \sqrt{(1-\lambda)k/(\theta_o-\lambda\theta_e)}$.

Now, we check the satisfaction of equation (14), which we ignored so far. It is apparent that $t_e^2 = \sqrt{k/\theta_e} < T$ regarding $c'(t)|_{t=T} = (k/T^2) < \theta_0 < \theta_e$; thus, $t_e^2 = \sqrt{(1-\lambda)k/(\theta_e-\lambda\theta_e)} < t_o^2 = \sqrt{(1-\lambda)k/(\theta_o-\lambda\theta_e)}$ holds because of $\theta_o < \theta_e$. However, whether t_o^2 is larger than T depends on the value of λ. When $0 \leq \lambda < (T^2\theta_o - k/T^2\theta_e - k)$, $\theta_o - \lambda\theta_e > 0$ and $\sqrt{(1-\lambda)k/(\theta_o-\lambda\theta_e)} < T$; hence, $t_o^{2*} = t_o^2 = \sqrt{(1-\lambda)k/(\theta_o-\lambda\theta_e)}$, that is, t_o^2 is the optimal solution. However, when $(T^2\theta_o - k/T^2\theta_e - k) \leq \lambda \leq 1$, $t_o^2 = \sqrt{(1-\lambda)k/(\theta_o-\lambda\theta_e)} > T$, implying that t_o^2 here is not the optimal solution. According to the constraint in equation (14), we have that $t_o^{2*} = T$ is the optimum in this case.

Substituting t_e^{2*} and t_o^{2*} into equations (12) and (13), p_o^{2*} and p_e^{2*} can be obtained, as shown in the following proposition.

Proposition 3. *When the firm provides mixed ordinary and expedited delivery services, we have the following:*

(1) If $0 \leq \lambda < (T^2\theta_o - k/T^2\theta_e - k)$, the optimal lead time is $t_e^{2*} = \sqrt{k/\theta_e}$ and the optimal price is $p_e^{2*} = \mu + \theta_o T - \sqrt{k\theta_e} + (\theta_e - \theta_o)\sqrt{(1-\lambda)k/(\theta_o-\lambda\theta_e)}$ for expedited delivery, and the optimal lead time is $t_o^{2*} = \sqrt{(1-\lambda)k/(\theta_o-\lambda\theta_e)}$ and the optimal price is $p_o^{2*} = \mu + \theta_o T - \theta_o\sqrt{(1-\lambda)k/(\theta_o-\lambda\theta_e)}$ for ordinary delivery.

(2) If $(T^2\theta_o - k/T^2\theta_e - k) \leq \lambda \leq 1$, the optimal lead time is $t_e^{2*} = \sqrt{k/\theta_e}$ and the optimal price is $p_e^{2*} = \mu + \theta_e T - \sqrt{k\theta_e}$ for expedited delivery, and the optimal lead time is $t_o^{2*} = T$ and the optimal price is $p_o^{2*} = \mu$ for ordinary delivery.

From the above proposition, it can be seen that, in the mixed service scheme, the net utility of lower time value consumers always equals zero. However, for consumers with a higher time value, their net utility is positive when $\lambda < (T^2\theta_o - k/T^2\theta_e - k)$ since $\mu + \theta_e(T - t_e^{2a*}) - p_e^{2a*} = (\theta_e - \theta_o)(T - \sqrt{(1-\lambda)k/(\theta_o-\lambda\theta_e)}) > 0$ and equals zero when $\lambda \geq (T^2\theta_o - k/T^2\theta_e - k)$ since $\mu + \theta_e(T - t_e^{2*}) - p_e^{2*} = 0$.

5. Analysis and Comparison

To understand the impacts of the parameters on the optimal decisions and the corresponding total profit in the different service schemes, sensitivity analysis and comparative research are conducted here to help companies to choose the optimal schemes (see Corollary 2) and to make the optimal decisions (see Corollary 1).

Corollary 1. *The optimal values of the lead time and price in the three schemes satisfy the following properties:*

(a) For $j=1$ and $j=2$ $(\partial t_o^{j*}/\partial\theta_o) \leq 0$, $(\partial t_e^{j*}/\partial\theta_e) \leq 0$, $(\partial p_o^{j*}/\partial\theta_o) \geq 0$, and $(\partial p_e^{j*}/\partial\theta_e) \geq 0$; and for $j=2$, $(\partial t_o^{j*}/\partial\theta_e) \geq 0$, $(\partial t_e^{j*}/\partial\theta_o) = 0$, $(\partial p_e^{j*}/\partial\theta_o) \geq 0$, and $(\partial p_o^{j*}/\partial\theta_e) \leq 0$.

(b) For $j=1$, $(\partial t_o^{j*}/\partial\lambda) = (\partial t_e^{j*}/\partial\lambda) = (\partial p_o^{j*}/\partial\lambda) = (\partial p_e^{j*}/\partial\lambda) = 0$; and for $j=2$, $(\partial t_o^{j*}/\partial\lambda) \geq 0$, $(\partial p_o^{j*}/\partial\lambda) \leq 0$, $(\partial t_e^{j*}/\partial\lambda) = 0$, $(\partial p_e^{j*}/\partial\lambda) \geq 0$.

(c) $t_e^{2*} = t_e^{1*}$, $t_o^{2*} > t_o^{1*}$, $p_e^{1*} \geq p_e^{2*}$, and $p_o^{1*} > p_o^{2*}$.

Property (a) in Corollary 1 expresses that the higher the time value of one type of consumers, the shorter the optimal lead time and the higher the optimal price of the delivery service that is intended for them (ordinary delivery for lower time value consumers and expedited delivery for higher time value service). However, in a mixed delivery service scheme, the consumers' time value also affects the other type of delivery service besides their matching type. Property (a) is easy to prove by solving the first derivative of the optimal lead time and price versus time value θ_i; therefore, the proof is omitted here.

Property (b) in Corollary 1 indicates that when the firm only provides a single delivery service, its optimal lead time and price are irrelevant to the distribution of the consumers. However, when the firm simultaneously provides two delivery services, the optimal lead time for ordinary delivery should increase with the proportion of higher time value consumers, while its optimal price should be decreased, and the optimal price for expedited delivery should increase with the proportion of the higher time value consumers, while its optimal lead time should remain unchanged.

Proof for Property (b). When $j=1$, obviously the expressions of t_o^1 and p_o^1 in Proposition 1 and t_e^1 and p_e^1 in Proposition 2 do not contain λ; thus, the first half is proved. We move on to the latter half when $j=2$.

Let $y = ((1-\lambda)k/\theta_o - \lambda\theta_e)$. Then, $(\partial y/\partial\lambda) = (\theta_e(1-\lambda)k/(\theta_o-\lambda\theta_e)^2) - (k/\theta_o - \lambda\theta_e) = (k(\theta_e-\theta_o)/(\theta_o - \lambda\theta_e)^2) > 0$. In addition, there exists $(\partial t_o^{2*}/\partial y) = (1/2)y^{-(1/2)} > 0$ for $0 \leq \lambda < (T^2\theta_o - k/T^2\theta_e - k)$ and $(\partial t_o^{2*}/\partial y) = 0$ for $(T^2\theta_o - k/T^2\theta_e - k) \leq \lambda \leq 1$; thus, $(\partial t_o^{2*}/\partial\lambda) = (\partial t_o^{2*}/\partial y) * (\partial y/\partial\lambda) \geq 0$. This means that the optimal lead time of ordinary delivery service t_o^{2*} is increasing with the proportion of the higher time value consumers λ until t_o^{2*} reaches the maximum time T. Similarly, we can prove that $(\partial\lambda/\partial t_e^{j*}) = 0$, $(\partial p_o^j/\partial\lambda) \le 0$, and $(\partial p_e^j/\partial\lambda) \ge 0$. This completes the proof.

In Property (c), $t_e^{2*} = t_e^{1*}$ and $t_o^{2*} > t_o^{1*}$ imply that when the firm simultaneously provides two differentiated delivery services, the consumers with a higher time value obtain the same speed for expedited delivery, but the consumers with a lower time value get a lower speed for ordinary delivery, implying a worsened service level for them. In addition, $p_e^{1*} \ge p_e^{2*}$ and $p_o^{1*} > p_o^{2*}$ indicate that although the lead time is unchanged, the price for expedited delivery is decreased, and thus the net utility of the consumers with a higher time value is no less than that in the single expedited delivery case; therefore, they obtain nonnegative gains from information asymmetry. In contrast, the net utility of the consumers with a lower time value is the same as that of the single ordinary delivery case, which equals zero since their negative utility from the lead time extension and their positive utility from the price reduction cancel each other out. Since Property (c) can be easily proved by comparing and contrasting Proposition 1, Proposition 2, and Proposition 3, the proof is omitted here.

The above has investigated the characters of the equilibrium decision in different service schemes. Now we compare their profits.

Corollary 2. *The profits in different delivery service schemes satisfy the following:*

(d) $(\partial\Pi_o^{1*}/\partial\lambda) = 0$, $(\partial\Pi_e^{1*}/\partial\lambda) > 0$, $(\partial\Pi^{2*}/\partial\lambda) > 0$.

(e) There exists $\lambda_0 \in (0, (T^2\theta_o - k/T^2\theta_e - k)]$ *such that, for* $\lambda \le \lambda_0$, $\Pi_e^{2*} \le \Pi^{2*}$ *and* $\Pi_o^{1*} \le \Pi^{2*}$*; and, for* $\lambda > \lambda_0$, $\Pi_e^{1*} > \Pi^{2*} > \Pi_o^{1*}$.

Proof for Property (d) in Corollary 2. We know from Proposition 1 and Proposition 2 that $\Pi_o^{1*} = N(\mu - \nu + \theta_o T - 2\sqrt{k\theta_o})$ and $\Pi_e^{1*} = \lambda N(\mu - \nu + \theta_e T - 2\sqrt{k\theta_e})$. It is clear that $(\partial\Pi_o^{1*}/\partial\lambda) = 0$ and $(\partial\Pi_e^{1*}/\partial\lambda) > 0$. Now we emphasize proving $(\partial\Pi^{2*}/\partial\lambda) > 0$. We consider it in two cases: where $0 \le \lambda < (T^2\theta_o - k/T^2\theta_e - k)$ and where $(T^2\theta_0 - k/T^2\theta_e - k) \le \lambda \le 1$. For $0 \le \lambda < (T^2\theta_o - k/T^2\theta_e - k)$, $(1/N) * (\partial\Pi^{2*}/\partial\lambda) = [p_e^{2*} - c(t_e^{2*})] - [p_o^{2*} - c(t_o^{2*})] + \{\lambda(\partial[p_e^{2*} - c(t_e^{2*})]/\partial t_o^{2*}) + (1-\lambda)(\partial[p_o^{2*} - c(t_o^{2*})]/\partial t_o^{2*})\}(\partial t_o^{2*}/\partial\lambda)$. When $[p_o^{2*} - c(t_o^{2*})] = \mu - \nu + \theta_o(T - t_o^{2*}) - k/t_o^{2*}$ and $[p_e^{2*} - c(t_e^{2*})] = \mu - \nu + \theta_o(T - t_o^{2*}) + \theta_e(t_o^{2*} - t_e^{2*}) - k/t_e^{2*}$, we have $[p_e^{2*} - c(t_e^{2*})] - [p_o^{2*} - c(t_o^{2*})] = \theta_e(t_o^{2*} - t_e^{2*}) + ((k/t_o^{2*}) - (k/t_e^{2*})) = (k(\sqrt{((1-\lambda)\theta_e/\theta_o - \lambda\theta_e)} - 1)(t_o^{2*} - t_e^{2*})/t_o^{2*}t_e^{2*}) > 0$ from $t_o^{2*} = \sqrt{(1-\lambda)k/(\theta_o - \lambda\theta_e)}$ and $\theta_e > \theta_o$. In addition, we have $\lambda(\partial[p_e^{2*} - c(t_e^{2*})]/\partial t_o^{2*}) + (1-\lambda)(\partial[p_o^{2*} - c(t_o^{2*})]/\partial t_o^{2*}) = 0$ because of $(\partial[p_o^{2*} - c(t_o^{2*})]/\partial t_o^{2*}) = -\theta_o + (k/(t_o^{2*})^2)$ and $(\partial[p_e^{2*} - c(t_e^{2*})]/\partial t_o^{2*}) = -\theta_o + \theta_e$. Therefore, $(1/N) * (\partial\Pi^{2*}/\partial\lambda) > 0$ and subsequently $(\partial\Pi^{2*}/\partial\lambda) > 0$ for $0 \le \lambda < (T^2\theta_o - k/T^2\theta_e - k)$.

For $(T^2\theta_o - k/T^2\theta_e - k) \le \lambda \le 1$, $\Pi^{2*} = \lambda N(\mu - \nu + \theta_e T - 2\sqrt{k\theta_e}) + (1-\lambda)N(\mu - \nu - (k/t))$. Since $c'(t)|_{t=T} = (k/T^2) < \theta_0 < \theta_e$, $(1/N) * (\partial\Pi^{2*}/\partial\lambda) = (\mu - \nu + \theta_e T - 2\sqrt{k\theta_e}) - (\mu - \nu - (k/t)) = (\sqrt{\theta_e T} - \sqrt{k/T})^2 > 0$. Therefore, $(\partial\Pi^{2*}/\partial\lambda) > 0$ for $(T^2\theta_0 - k/T^2\theta_e - k) \le \lambda \le 1$. This completes the proof.

Proof for Property (e) in Corollary 2. If $\lambda = 0$, then $t_o^{2*} = \sqrt{(1-\lambda)k/(\theta_o - \lambda\theta_e)} = \sqrt{k/\theta_o} = t_o^{1*}$, and $p_o^{2*} = \mu + \theta_o T - \theta_o\sqrt{(1-\lambda)k/(\theta_o - \lambda\theta_e)} = p_o^{1*}$; as a result, $\Pi^{2*} = N[p_o^{2*} - c(t_o^{2*})] = \Pi_o^{1*}$. Note that $(\partial\Pi^{2*}/\partial\lambda) > 0$ and $(\partial\Pi^{1*}/\partial\lambda) = 0$. Then, $\Pi_o^{1*} \le \Pi^{2*}$ holds. Next, we turn to compare Π^{2*} and Π_e^{1*}. If $\lambda = 0$, $\Pi^{2*} = \Pi_o^{1*} = N(\mu - \nu + \theta_o T - 2\sqrt{k\theta_o}) > \Pi_e^{1*} = 0$ holds; and if $\lambda = (T^2\theta_o - k/T^2\theta_e - k)$, $\Pi^{2*} = \lambda N(\mu - \nu + \theta_e T - 2\sqrt{k\theta_e}) + (1-\lambda)N(\mu - \nu - (k/t)) \le \Pi_e^{1*} = \lambda N(\mu - \nu + \theta_e T - 2\sqrt{k\theta_e})$ with $\mu - \nu \le (k/t)$. Moreover, we have proved that $(\partial\Pi_e^{1*}/\partial\lambda) > 0$ and $(\partial\Pi^{2*}/\partial\lambda) > 0$ in Property (d); therefore, a threshold value λ_0 must exist in $(0, (T^2\theta_o - k/T^2\theta_e - k)]$ to make $\Pi^{2*} = \Pi_e^{1*}$ hold. Further, $\Pi_e^{1*} < \Pi^{2*}$ for $\lambda < \lambda_0$ and $\Pi_e^{1*} \ge \Pi^{2*}$ for $\lambda \ge \lambda_0$.

With the combination of $\Pi_o^{1*} < \Pi^{2*}$, we have $\Pi_e^{1*} \le \Pi^{2*}$ and $\Pi_o^{1*} \le \Pi^{2*}$ for $\lambda \le \lambda_0$ and $\Pi_e^{1*} > \Pi^{2*} > \Pi_o^{1*}$ for $\lambda > \lambda_0$. This completes the proof.

From Property (d) in Corollary 2, it can be seen that, except for the single ordinary delivery case, the single expedited delivery case and the mixed delivery case are both positively affected by the distribution of consumers regarding profits. In addition, we note from Property (e) that when the proportion of consumers with a higher time value is not above a specific threshold value, providing mixed delivery services outperforms providing a single delivery service; but when it is above the value, providing a single expedited delivery service performs the best and providing a single ordinary delivery service performs the worst. As is evident from Corollary 2, providing mixed services is not always the best choice for companies with consumers having an acceptable maximum length for the lead time and asymmetric time values.

6. Numerical Illustration

In this section, we provide numerical simulations to qualitatively illustrate the characteristics of the equilibrium obtained with reasonable values of the parameters in the Chinese market.

We use the following values: μ = Y3, k = Y2, ν = Y2.5, θ_e = Y8/D, θ_o = Y4/D, $T = 2\,D$, $N = 1 * 10^6$, and $\lambda = (1/3)$.

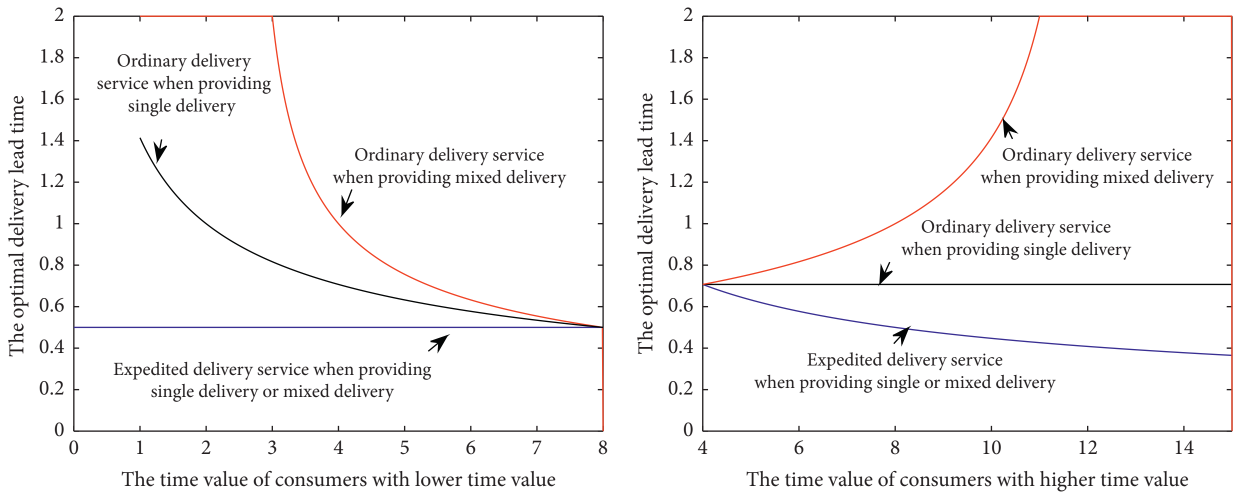
According to Proposition 1-Propostion 3, the optimal values of the decision variables and the consequent profits under the different service schemes are presented in Table 1.

From Table 1, note that providing mixed delivery services results in a profit of Y $3.167 * 10^6$, which is more than that in the single ordinary or expedited cases. Thus, providing mixed delivery services is the optimal service scheme in our numerical setting.

Then, we use the following Figures 1(a) and 1(b) to illustrate Corollary 1. As is shown in Figure 1(a), the optimal lead time of ordinary delivery decreases as the time value of lower time value consumers increases. The intuition is that when the consumers value time more, the delivery speed should be accelerated to cater to their needs. The same is true with Figure 1(b), which shows that the optimal lead time of expedited delivery is also decreasing as the time value of

Table 1: The optimal decisions and the resulting profits under different delivery service schemes.

	Providing single ordinary delivery	Providing single ordinary delivery	Providing mixed ordinary and expedited delivery
Lead time for ordinary delivery (*D*)	0.707	—	1
Price for ordinary delivery (Y)	8.172	—	7
Lead time for expedited delivery (*D*)	—	0.5	0.5
Price for expedited delivery (Y)	—	15	11
Firm's profit (Y)	$2.843 * 10^6$	$2.833 * 10^6$	$3.167 * 10^6$



(a)

(b)

Figure 1: (a) Effect of θ_o on the optimal delivery lead time. (b) Effect of θ_e on the optimal delivery lead time.

higher time value consumers increases. From both Figures 1(a) and 1(b), we can also find that the optimal lead time of ordinary delivery service in a mixed delivery scheme is always longer than that in a single ordinary delivery scheme, while the lead time of the expedited delivery service in a mixed delivery scheme is always the same as that in a single expedited delivery scheme. The result is similar to that of many previous studies involving adverse selection (Hui et al. [17]). In the delivery service context, this suggests that if a firm plans to add expedited delivery to its service menu as a value-added service, then the previous ordinary delivery service should be slowed down to prevent the higher value consumers from choosing the ordinary delivery.

Next, to show the effect of the proportion of higher time value consumers λ on the choice of delivery schemes, we use the following Figure 2 to present how changes in λ influence the optimal value of the maximum expected total profit in different schemes.

It is obvious in Figure 2 that the increase of λ has significant positive effects on the profits of providing mixed delivery services and providing single expedited delivery service but has no effect on the profit of providing single ordinary delivery service. Noe that, for $\lambda \leq \lambda_0 = 0.389$, the optimal service scheme is providing mixed delivery services, since it brings the most profit, whereas, for $\lambda > \lambda_0 = 0.389$, the optimal service scheme changes to providing a single expedited delivery service.

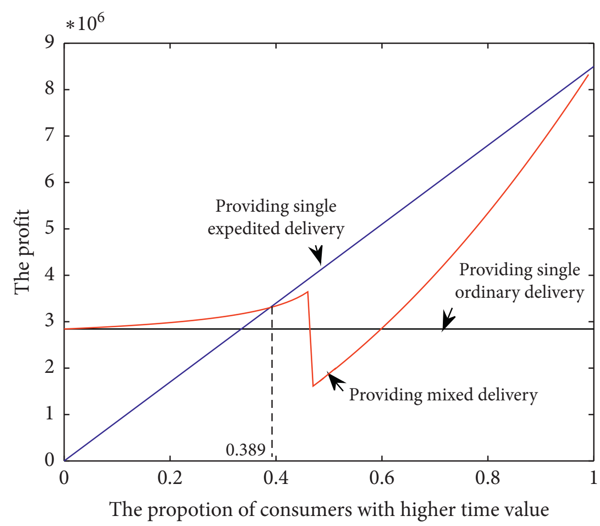


Figure 2: Effect of the proportion of consumers with higher time value (λ) on profits.

Figure 2 illustrates the findings of Corollary 2. The important managerial insight behind this is that the firm should exploit the distribution of consumers and craft its optimal delivery service scheme strategy according to the critical threshold given in Corollary 2. Evidently, the proportion of consumers with higher time value usually varies across products and markets (regions). When a product is innovative or fashionable or when there is a large high-income group in the market, the proportion is relatively larger than that in other conditions. In this case, providing a single expedited delivery service may be wiser.

7. Conclusions and Future Research

As is known, delivering products with the promised lead time and the optimal price, which are important preferences of customers, is widely desired by companies in fiercely competitive markets.

In this paper, we, respectively, investigated the optimal delivery lead time and pricing decisions in three delivery schemes when consumers' time values are asymmetric and discrete, and then we analyzed the firm's strategic choice regarding which delivery scheme to adopt. The problems we have solved and the conclusions we have obtained are as follows.

First, should the firm provide single ordinary delivery, single expedited delivery, or mixed ordinary and expedited delivery? It is found that when the proportion of higher time value consumers is relatively low and not larger than a threshold value (a function of the variable costs, the consumer's time value, and the acceptable maximum length of time parameters), the firm should choose to provide mixed ordinary and expedited delivery. Otherwise, it is advised to provide single expedited delivery.

Second, how should the firm determine its lead time and price? The results suggest that, for a specific kind of delivery service, the optimal lead time should decrease and the optimal price should increase with its target consumers' time value (ordinary delivery for lower time value consumers and expedited delivery for higher time value consumers). In addition, if the firm provides a mixture of delivery services, then the optimal price of expedited delivery should not be higher than that of a single expedited delivery scheme, and the optimal lead time should remain the same. However, the optimal price of ordinary delivery should be lower than that of the single ordinary delivery scheme, its optimal lead time should be increased, and it is advised to go up with the proportion of higher time value consumers until the maximum length of time that all consumers could bear is reached.

In summary, our research sheds light on delivery service design and optimization when consumers have private time value information and have an acceptable maximum length of time. There are many possible extensions to this study. The service schemes could be extended to include more services, the model could be extended to consider the influence of competitors, and the interaction of commodity price with the delivery lead time and price can also be considered as a research topic.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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