

# Research Article **Multiattribute Group Decision-Making Method Using a Genetic K-Means Clustering Algorithm**

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Multiattribute group decision-making (MAGDM) problems are characterized by the large number, uneven levels, and bounded rationality of decision-makers; multiple attributes and fuzziness of decision problems; and complex group behaviours. Considering these characteristics, we propose a MAGDM method using a genetic *K*-means clustering algorithm. First, we briefly review the traditional multiattribute decision-making method based on prospect theory (PT) and trapezoidal intuitionistic fuzzy numbers (TrIFNs) under the premise of human bounded rationality and uncertain decision environment. Then, the aggregation model of decision information given by decision-makers is established using the genetic *K*-means algorithm in order to determine optimal clustering results. Each clustering center represents decision information given by decision-makers in each cluster, and the weight of each clustering center is determined by considering the tightness of decision information within a cluster and the count of decision-makers in each cluster. Finally, the ranking of schemes is obtained according to the comparison rules of TrIFNs. We design comparison simulation experiments to test the proposed method and the simulation results demonstrate that the proposed method is apprehensible and feasible to solve MAGDM problems.

# 1. Introduction

Multiattribute group decision-making (MAGDM) is a problem in which multiple decision-makers make decisions on multiple schemes under the premise of multiple attributes [1–3]. The basis of the MAGDM problem is multiattribute decision-making (MADM). MADM is characterized by bounded rationality of decision-makers as well as multiple attributes and fuzziness of decision problems. In addition to the characteristics of the MADM problem, the characteristics of the MAGDM problem include a large number and uneven levels of decision-makers and complex group behaviours.

Prospect theory (PT) can accurately describe and explain decision-makers' judgment and choice behaviours and thus can solve MADM problems under the condition of human bounded rationality [4, 5]. The problems of uncertainty of the decision-making conditions and fuzziness of decision information exist in many MADM problems, which are effectively solved by intuitionistic fuzzy decision-making methods. The trapezoidal intuitionistic fuzzy number (TrIFN) [6, 7] is usually used to solve MADM problems with uncertain decision conditions and fuzzy decision information. To solve MADM problems, a method combining PT with TrIFNs and various derived methods have been widely used. Reference [8] developed a prospect value determination method based on multiple reference points under a trapezoidal intuitionistic fuzzy environment. Reference [9] solved the MADM problem of wind energy by combining PT and TrIFNs. Reference [10] proposed an MAGDM method using the ITFN weighted geometric operator and hybrid geometric operator to obtain the collective overall values.

MAGDM problems mainly include the determination of decision-maker's weights, consensus analysis, aggregation of decision information, and ranking of schemes. Decision-makers' weights can be determined by methods such as subjective, objective, and subjective-objective weighting [11–13]. Pang et al. [14] proposed an adaptive consensus method for MAGDM based on adaptive experts' weights and explicit guidance rules. Sun and Ma [15] proposed an approach to consensus measurement of linguistic preference relations. The aggregation of decision information and ranking of schemes are the two aspects that can best embody the characteristics of MAGDM. Chen et al. [16] proposed aggregation operators based on interval-valued intuitionistic fuzzy numbers. Andrej [17] proposed a hybrid Delphi and aggregation-disaggregation procedure of MAGDM. Shakeel [18] proposed a cubic averaging aggregation operator. Under the premise of human bounded rationality and uncertain decision-making environment, the representative papers that systematically study MAGDM methods are as follows: Li and Chen [19] proposed a novel TOPSIS based on PT and TrIFNs, and the aggregation operator and ranking strategy can effectively obtain the final decision scheme. Yuan and Li [20] proposed an aggregation operator based on the Choquet integral and PT. In addition to the above studies, MAGDM problems can also be solved by methods using data mining and cluster analysis. Shankar and Kumar [21] used a K-means clustering algorithm to solve the MAGDM problem in the field of healthcare systems. Liu and Chen [22] proposed an improved K-means algorithm to solve the large group decision-making problem. Zhao [23] proposed an improved K-means clustering algorithm based on interval similarity measurement to solve the MAGDM problem. However, existing research has the following shortcomings: (a) decision information is still expressed in real numbers, interval numbers, and language values in some studies, thus leading to poor adaptability in processing uncertain and fuzzy information; (b) the preference of decision-makers is based on expected utility theory in some studies, which is inconsistent with the nature of human preference and judgment based on comparison; (c) some decisionmaking methods require decision-makers whose decision preference largely deviates from the group preference to change their decision preference, which incurs great cost; (d) the aggregation methods of decision information are too subjective, which cannot objectively reflect the decision information given by decisionmakers; (e) the existing MAGDM methods based on the K-means clustering algorithm [24] are easily trapped in local optimum.

Considering the above shortcomings of the MAGDM methods based on clustering algorithms, we propose a MAGDM method using a genetic K-means clustering algorithm on the basis of PT and TrIFNs. This paper is organized as follows. First, we briefly review the framework of the PT and TrIFN and their operational rules in Section 2. Then, in Section 3, we propose the method to solve the MAGDM problem using the genetic *K*-means clustering algorithm and propose its details. Finally, in Section 4, we perform simulations to verify whether the proposed method is apprehensible and feasible to solve MAGDM problems. The proposed MAGDM method fully reflects the objectivity of decision-making, realizes the learning of clustering numbers automatically, and overcomes the shortcomings of MAGDM methods above.

## 2. Brief Review of the Prospect Theory and Intuitionistic Trapezoidal Fuzzy Number

The traditional method to solve MADM problems is based on PT and TrIFNs, which lays the foundation of our proposed MADM method. On the basis of the PT framework, the decision information is expressed in the TrIFN form. In this section, the PT and the TrIFN are briefly reviewed, respectively.

2.1. Framework of PT. The basic unit of PT is the prospect, which is denoted as  $f = (x_1, p_1; x_2, p_2; ...; x_n, p_n)$ . The reference point, which is used to describe the attributes of MADM, is given according to the subjective feelings of the decision-maker.  $x_i$  is the gain or loss value of the  $i^{\text{th}}$  attribute value compared to the  $i^{\text{th}}$  attribute value of the reference point.  $p_i$  is the probability of  $x_i$ . The most important feature of PT is that it reflects the value of change rather than the absolute value, which is also consistent with the fact that decision-makers are more sensitive to change. The prospect is evaluated by the prospect value pv(f) as follows:

$$pv(f) = \sum_{i=1}^{n} w(p_i)v(x_i), \qquad (1)$$

where  $w(p_i)$  and  $v(x_i)$  are the weight function and the value function of the *i*<sup>th</sup> attribute, respectively.  $w(p_i)$  and  $v(x_i)$  are given by

$$\nu(x_{i}) = \begin{cases} x_{i}^{\alpha}, & x_{i} \ge 0, \\ \\ -\delta(-x_{i})^{\beta}, & x_{i} < 0, \end{cases}$$
(2)

$$w(p_i) = \frac{p_i^{\gamma}}{[p_i^{\gamma} + (1 - p_i)^{\gamma}]^{(1/\gamma)}},$$
(3)

where  $\alpha$  and  $\beta$  are concave and convex degrees of  $x_i$ , respectively, with  $\alpha < 1$  and  $\beta < 1$ ;  $\delta$  indicates that the loss curve is steeper than the gain curve, and  $\delta > 1$  indicates that decision-makers do not accept losses;  $\gamma$  is the fitting parameter.

2.2. TrIFN and Its Operational Rules. Let  $\tilde{a} = \langle (a_1, a_2, a_3, a_4), (b_1, b_2, b_3, b_4); \mu_{\tilde{a}}, \nu_{\tilde{a}} \rangle$  be a TrIFN. Parameters  $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ , and  $b_4$  are real numbers that satisfy the condition  $b_1 \le a_1 \le b_2 \le a_2 \le b_3 \le a_3 \le b_4 \le a_4$ . The membership function and nonmembership function are, respectively,

$$\mu_{\widetilde{a}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & \mu_{\widetilde{a}}, a_1 \le x < a_2, \\ \\ \mu_{\widetilde{a}}, & a_2 \le x \le a_3, \\ \\ \frac{a_4-x}{a_4-a_3}, & \mu_{\widetilde{a}}, a_3 \le x \le a_4, \\ \\ 0, & x \text{ equals any other value,} \end{cases}$$

(4)

$$v_{\widetilde{a}}(x) = \begin{cases} \frac{b_2 - x + v_{\widetilde{a}}(x - b_1)}{b_2 - b_1}, & b_1 \le x < b_2, \\ v_{\widetilde{a}}, & b_2 \le x \le b_3, \\ \frac{x - b_3 + v_{\widetilde{a}}(b_4 - x)}{b_4 - b_3}, & b_3 \le x \le b_4, \\ 0, & x \text{ equals any other value}, \end{cases}$$
(5)

where  $\mu_{\widetilde{a}} \in [0, 1]$ ,  $\nu_{\widetilde{a}} \in [0, 1]$ , and  $0 \le \mu_{\widetilde{a}} + \nu_{\widetilde{a}} \le 1$ .

Before reviewing operational rules, we set  $\tilde{a}_1 = \langle (a_{11}, a_{12}, a_{13}, a_{14}), (b_{11}, b_{12}, b_{13}, b_{14}); \mu_{\tilde{a}_1}, \nu_{\tilde{a}_1} \rangle$  and  $\tilde{a}_2 = \langle (a_{21}, a_{22}, a_{23}, a_{24}), (b_{21}, b_{22}, b_{23}, b_{24}); \mu_{\tilde{a}_2}, \nu_{\tilde{a}_2} \rangle$  as two TrIFNs. The operational rules of TrIFNs are defined as follows:

$$\tilde{a}_{1} \oplus \tilde{a}_{2} = \langle (a_{11} + a_{21}, a_{12} + a_{22}, a_{13} + a_{23}, a_{14} + a_{24}), (b_{11} + b_{21}, b_{12} + b_{22}, b_{13} + b_{23}, b_{14} + b_{24}); \mu_{\tilde{a}_{1}} + \mu_{\tilde{a}_{2}} - \mu_{\tilde{a}_{1}} \mu_{\tilde{a}_{2}}, \nu_{\tilde{a}_{1}} \nu_{\tilde{a}_{2}} \rangle,$$

$$(6)$$

$$\lambda \tilde{a}_{1} = \begin{cases} \langle (\lambda a_{11}, \lambda a_{12}, \lambda a_{13}, \lambda a_{14}), (\lambda b_{11}, \lambda b_{12}, \lambda b_{13}, \lambda b_{14}); 1 - (1 - \mu_{\tilde{a}_{1}})^{\lambda}, \nu_{\tilde{a}_{1}}^{\lambda} \rangle, & \lambda \ge 0, \\ \langle (\lambda a_{14}, \lambda a_{13}, \lambda a_{12}, \lambda a_{11}), (\lambda b_{14}, \lambda b_{13}, \lambda b_{12}, \lambda b_{11}); 1 - (1 - \mu_{\tilde{a}_{1}})^{-\lambda}, \nu_{\tilde{a}_{1}}^{-\lambda} \rangle, & \lambda < 0, \end{cases}$$
(7)

$$\tilde{a}_{1}^{\ \lambda} = \langle \left(a_{11}^{\lambda}, a_{12}^{\lambda}, a_{13}^{\lambda}, a_{14}^{\lambda}\right), \left(b_{11}^{\lambda}, b_{12}^{\lambda}, b_{13}^{\lambda}, b_{14}^{\lambda}\right); \mu_{\tilde{a}_{1}}^{\lambda}, 1 - \left(1 - \nu_{\tilde{a}_{1}}\right)^{\lambda} \rangle, \quad \lambda \ge 0, a_{ij}^{\lambda} > 0, b_{ij}^{\lambda} > 0,$$

$$\tag{8}$$

 $\tilde{a}_{1} \otimes \tilde{a}_{2} = \langle (a_{11}a_{21}, a_{12}a_{22}, a_{13}a_{23}, a_{14}a_{24}), (b_{11}b_{21}, b_{12}b_{22}, b_{13}b_{23}, b_{14}b_{24}); \mu_{\tilde{a}_{1}}\mu_{\tilde{a}_{2}}, \nu_{\tilde{a}_{1}} + \nu_{\tilde{a}_{2}} - \nu_{\tilde{a}_{1}}\nu_{\tilde{a}_{2}} \rangle.$   $\tag{9}$ 

The expected value of  $\tilde{a}_1$  is as follows:

$$EV(\tilde{a}_{1}) = \frac{1}{8} \Big[ \mu_{\tilde{a}_{1}} (a_{11} + a_{12} + a_{13} + a_{14}) + (1 - \nu_{\tilde{a}_{1}}) (b_{11} + b_{12} + b_{13} + b_{14}) \Big].$$
(10)

The score function and accuracy function of  $\tilde{a}_1$  are

$$S(\tilde{a}_1) = \mathrm{EV}(\tilde{a}_1) \cdot \left(\mu_{\tilde{a}_1} - \nu_{\tilde{a}_1}\right), \tag{11}$$

$$A(\tilde{a}_1) = \mathrm{EV}(\tilde{a}_1) \cdot \left(\mu_{\tilde{a}_1} + \nu_{\tilde{a}_1}\right), \tag{12}$$

where  $EV(\tilde{a}_1)$  is the expected value of  $\tilde{a}_1$ .

The rules for comparing  $\tilde{a}_1$  and  $\tilde{a}_2$  are as follows: If  $S(\tilde{a}_1) > S(\tilde{a}_2)$ ,  $\tilde{a}_1 > \tilde{a}_2$ . If  $S(\tilde{a}_1) = S(\tilde{a}_2)$  and  $A(\tilde{a}_1) > A(\tilde{a}_2)$ ,  $\tilde{a}_1 > \tilde{a}_2$ . If  $S(\tilde{a}_1) = S(\tilde{a}_2)$  and  $A(\tilde{a}_1) = A(\tilde{a}_2)$ ,  $\tilde{a}_1 = \tilde{a}_2$ .

Hamming distance is used as the distance measured between  $\tilde{a}_1$  and  $\tilde{a}_2$  as follows:

$$d(\tilde{a}_{1},\tilde{a}_{2}) = \frac{1}{8} \sum_{j=1}^{4} \left( \left| \mu_{\tilde{a}_{1}}a_{1j} - \mu_{\tilde{a}_{2}}a_{2j} \right| + \left( 1 - \nu_{\tilde{a}_{1}} \right) b_{1j} - \left( 1 - \nu_{\tilde{a}_{2}} \right) b_{2j} \right| \right).$$
(13)

The values of attribute and weight are given by decisionmakers in the form of TrIFN.

#### **3. Proposed MAGDM Method**

Aggregation of decision information of decision-makers in one group by using the genetic *K*-means clustering algorithm is the key to the proposed MAGDM method. The decision information set  $\tilde{F}^l$ , which is formed by decision information (provided by decision-makers) of the *l*<sup>th</sup> decision scheme, is partitioned into different clusters and each cluster center represents the decision information of each cluster. The weight of each clustering center is determined by considering the tightness of decision information within a cluster and the count of decision-makers in each cluster. Then the aggregation results are taken as the basis for ranking. In the following section, the aggregation of decision information by using the genetic *K*-means clustering algorithm and the calculation steps of the proposed method are introduced.

3.1. Aggregation of Decision Information by Using K-Means Clustering Algorithm. The K-means clustering algorithm is sensitive to the initial cluster centers, cannot determine the optimal number of clusters, and is easily trapped in local optima. Genetic algorithms can overcome the shortcomings of K-means clustering algorithms and realize the automatic learning of the cluster count. The decision information expressed by TrIFNs is clustered by a genetic K-means clustering algorithm as follows. k pieces of decision information in  $\vec{F}$  are randomly selected as the initial cluster centers. Then, each piece of the remaining decision information is assigned to the cluster center of the nearest distance. The cluster centers are recalculated to minimize the squared error criterion E converges:

$$E^{l} = \sum_{j=1}^{K} \sum_{\widetilde{f}_{i}^{l} \in C_{j}^{l}} \left\{ d\left[ pv\left(\widetilde{f}_{i}^{l}\right), \widetilde{m}_{j}^{l} \right] \right\},$$
(14)

where k is the number of clusters,  $C_j^l$  is the  $j^{\text{th}}$  cluster of the  $l^{\text{th}}$  decision scheme,  $\tilde{m}_j^l$  is the cluster center of cluster  $C_j^l$ ,  $\tilde{f}_i^l$  is the decision information of the  $l^{\text{th}}$  decision scheme given by the  $i^{\text{th}}$  decision-maker, and  $pv(\tilde{f}_i^l)$  is the prospect value of  $(\tilde{f}_i^l)$ :

$$pv\left(\tilde{f}_{i}^{l}\right) = \sum_{j=1}^{n} \left[w\left(\tilde{p}_{ij}^{l}\right)v\left(\tilde{x}_{ij}^{l}\right)\right],\tag{15}$$

where  $w(\tilde{p}_{ij}^l)$  and  $v(\tilde{x}_{ij}^l)$  are the weight function and the value function of the  $j^{\text{th}}$  attribute in  $\tilde{f}_i^l$ , respectively.  $\tilde{x}_{ij}^l$  is a TrIFN and the value function  $v(\tilde{x}_{ij}^l)$  is

$$\nu(\tilde{x}_{ij}^{l}) = \begin{cases} d(\tilde{x}_{ij}^{l}, \tilde{r}_{ij}), & \tilde{x}_{ij}^{l} \ge \tilde{r}_{ij}, \\ -\delta [d(\tilde{x}_{ij}^{l}, \tilde{r}_{ij})]^{\beta}, & \tilde{x}_{ij}^{l} < \tilde{r}_{ij}, \end{cases}$$
(16)

where  $\tilde{r}_{ij}$  is the  $j^{\rm th}$  reference point given by the  $i^{\rm th}$  decision-maker.

Before clustering the group decision information, the outliers should be eliminated. The threshold value of Hamming distance is denoted as  $d_{max}$  and the threshold value of exceed  $d_{max}$  is denoted as  $\delta$ . If the count of  $d[pv(\tilde{f}_i), pv(\tilde{f}_j)]; j = 1, 2, ..., n$  more than  $d_{max}$  exceeds  $\delta$ , the *i*<sup>th</sup> decision information should be eliminated. The process of elimination is process of consensus analysis.

The genetic variant of the *K*-means clustering algorithm involves the evolution of chromosomes, and the final result is obtained by genetic operations such as selection, crossover, and mutation. The steps involved in the genetic *K*-means clustering algorithm are as follows:

$G_1$	G <sub>2</sub>	 $G_k$

FIGURE 1: Encoding of cluster centers as genes of a chromosome.

- (a) Real-number encoding is adopted to transform the cluster centers into genes  $G_1, G_2, \ldots, G_k$  on a chromosome, as shown in Figure 1. The encoding values are integers from 1 to *n*, where *n* is the number of decision-makers. The size of a chromosome varies with the number of clusters.
- (b) The clustering results should satisfy tightness and separability requirements. Tightness means that the schemes within one cluster are as similar as possible, while separability means that the schemes in different clusters are as different as possible. Therefore, we define the fitness function fit  $D^l$  of the genetic *K*-means clustering algorithm as

$$\operatorname{fit} D^{l} = \frac{G_{b}^{l}}{b + a E^{l}},$$
(17)

where *a* and *b* are positive constant coefficients and  $G_b^l$  is the sum of the distances between different clusters when dealing with the *l*<sup>th</sup> decision scheme, given as

$$G_{b}^{l} = \frac{2}{k(k-1)} \sum_{i=1}^{k} \sum_{j=i+1}^{k} \left\| \tilde{m}_{i} - \tilde{m}_{j} \right\|^{2},$$
(18)

which is used to quantify separability. The fitness function fit  $D^l$  indicates that the clustering is better the pieces of decision information in the same cluster are closer to each other other ( $E^l$  has a smaller value) and the cluster centers of different clusters are farther from each other ( $G_h^l$  has a larger value).

Using genetic operations including roulette-based selection and single-point crossover (denoted as  $P_c$ ), we propose a new mutation operation that leads to the automatic learning of the optimal number of clusters k. The population chromosome with the largest fitness value is selected as the model chromosome of the optimal cluster number. Other chromosomes in the same population should learn from this model to achieve better fitness by decreasing or increasing the number of genes of chromosomes. The chromosomes in the initial population have the same length, a small number of initial clusters are set, and an increasing trend is assumed when the first mutation occurs. With the reoccurrence of mutation operation, the offspring chromosomes decrease or increase based on whether their lengths are longer or shorter than the model chromosome, respectively. On the one hand, decreasing the number of genes is achieved by eliminating the nearest genes to cluster centers of the model chromosome. On the other hand, the number of genes is increased by adding the farthest schemes to the cluster centers of the model chromosome.

The decision information is partitioned into k clusters. The weight of decision information in each cluster is determined by considering two factors, namely, the tightness of decision information within a cluster and the count of decision-makers in each group. Let  $c_t$  and  $E_t^l$  be the count of decision-makers and the squared error criterion of decision information in the  $t^{\text{th}}$  cluster. Then, the weight  $\mu_t^l$  of the  $t^{\text{th}}$  cluster of the  $l^{\text{th}}$  decision scheme is

$$\mu_t^l = \frac{\overline{c}_t + \overline{E}_t^l}{2},\tag{19}$$

where  $\overline{c}_t$  and  $\overline{E}_t^l$  are the normalized values of  $c_t$  and  $E_t^l$ , respectively.

$$\overline{c}_t = \frac{c_t}{n},\tag{20}$$

$$\overline{E}_t^l = \frac{E_t^l}{\sum_{t=1}^k E_t^l},\tag{21}$$

where n is the count of decision-makers.

The aggregation result of the  $l^{\text{th}}$  decision scheme can be written as follows.

$$a^{l} = \sum_{t=1}^{t=T} \mu_{t}^{l} C_{t}^{l}, \qquad (22)$$

where T is the count of clusters.

This aggregation method considers both intercluster information aggregation and intracluster information aggregation, which can fully integrate decision information. Compared with other MAGDM based on clustering methods, the proposed MAGDM method can realize the learning of clustering numbers automatically.

*3.2. Calculation Steps of the Proposed MAGDM Method.* The calculation steps of the proposed MAGDM method are as follows.

3.2.1. Construction of Decision Information Matrix. Let  $\tilde{X}^l$  and  $\tilde{P}^l$  be the attributes and weights of decision information matrices of the  $l^{\text{th}}$  decision scheme:

$$\tilde{X}^{l} = \begin{bmatrix} \tilde{x}_{11}^{l} & \tilde{x}_{12}^{l} & \cdots & \tilde{x}_{1m}^{l} \\ \tilde{x}_{21}^{l} & \tilde{x}_{22}^{l} & \cdots & \tilde{x}_{2m}^{l} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{x}_{n1}^{l} & \tilde{x}_{n2}^{l} & \cdots & \tilde{x}_{nm}^{l} \end{bmatrix},$$
(23)

$$\tilde{P}^{l} = \begin{bmatrix} \tilde{P}_{11}^{l} & \tilde{P}_{12}^{l} & \cdots & \tilde{P}_{1m}^{l} \\ \tilde{P}_{21}^{l} & \tilde{P}_{22}^{l} & \cdots & \tilde{P}_{2m}^{l} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{p}_{n1}^{l} & \tilde{p}_{n2}^{l} & \cdots & \tilde{p}_{nm}^{l} \end{bmatrix},$$
(24)

where  $\tilde{x}_{ij}^l$  and  $\tilde{p}_{ij}^l$  are the  $j^{\text{th}}$  attribute and the weight of the  $l^{\text{th}}$  decision scheme given by the  $i^{\text{th}}$  decision-maker, n is the count of decision-makers, and m is the count of attributes.  $\tilde{x}_i^l = [\tilde{x}_{i1}^l, \tilde{x}_{i2}^l, \dots, \tilde{x}_{im}^l]$  is called the  $l^{\text{th}}$  decision scheme given by the  $i^{\text{th}}$  decision-maker.

3.2.2. Aggregation of Reference Points. The reference points matrix  $\hat{R}$  is given according to the subjective feelings of the decision-makers as

$$\widetilde{R} = \begin{bmatrix} \widetilde{r}_{11} & \widetilde{r}_{12} & \cdots & \widetilde{r}_{1m} \\ \widetilde{r}_{21} & \widetilde{r}_{22} & \cdots & \widetilde{r}_{2m} \\ \vdots & \vdots & \vdots & \vdots \\ \widetilde{r}_{n1} & \widetilde{r}_{n2} & \cdots & \widetilde{r}_{nm} \end{bmatrix},$$
(25)

where  $\tilde{r}_{ij}$  is the  $j^{\text{th}}$  reference point given by the  $i^{\text{th}}$  decisionmaker. The ordered weighted average (OWA) operator is applied to aggregate the reference point  $\tilde{r}$  given by decisionmakers. TrIFNs of each column in the matrix  $\tilde{R}$  are sorted by comparing rules of TrIFNs (Section 2). The sorted  $\tilde{R}$  is written as  $\tilde{R}^s$  as follows:

$$\widetilde{R}^{s} = \begin{bmatrix} \widetilde{r}_{11}^{s} & \widetilde{r}_{12}^{s} & \cdots & \widetilde{r}_{1m}^{s} \\ \widetilde{r}_{21}^{s} & \widetilde{r}_{22}^{s} & \cdots & \widetilde{r}_{2m}^{s} \\ \vdots & \vdots & \vdots & \vdots \\ \widetilde{r}_{n1}^{s} & \widetilde{r}_{n2}^{s} & \cdots & \widetilde{r}_{nm}^{s} \end{bmatrix}.$$
(26)

Then, the reference point  $\tilde{r}$  is obtained as

$$\widetilde{r} = \left[ \bigoplus_{i=1}^{n} w_1 \widetilde{r}_{i1}^s, \bigoplus_{i=1}^{n} w_2 \widetilde{r}_{i2}^s, \dots, \bigoplus_{i=1}^{n} w_m \widetilde{r}_{im}^s \right],$$
(27)

where  $w = [w_1, w_2, ..., w_m]$  is the weighted vector relevant to OWA.

3.2.3. Determination of the Gain or Loss Values. The gain or loss values  $d(\tilde{x}_{ij}^l, \tilde{r}_j)$  of the  $j^{\text{th}}$  attribute of the  $l^{\text{th}}$  decision scheme given by the  $i^{\text{th}}$  decision-maker are obtained by formula (13). Then, the decision information set  $\tilde{F}^l = [\tilde{f}_1^l, \ldots, \tilde{f}_i^l, \ldots, \tilde{f}_n^l]$  is obtained, and  $\tilde{f}_i^l = [d(\tilde{x}_{i1}^l, \tilde{r}_1), \tilde{p}_{i1}^l; d(\tilde{x}_{i2}^l, \tilde{r}_2), \tilde{p}_{i2}^l; \ldots; d(\tilde{x}_{im}^l, \tilde{r}_m), \tilde{p}_{im}^l]$  is the prospect used to express decision information of the  $l^{\text{th}}$  decision scheme given by the  $i^{\text{th}}$  decision-maker.

3.2.4. Computation and Normalization of Prospect Values. The prospect value  $pv^{l}(\tilde{f}_{i})$  of  $\tilde{f}_{i}^{l}$  is obtained by formula (15). Let  $pv^{l}(\tilde{f}_{i})$  be written as follows:

$$pv^{l}(\tilde{f}_{i}) = \left\langle \left( pv^{l}_{11}(\tilde{f}_{i}), pv^{l}_{12}(\tilde{f}_{i}), pv^{l}_{13}(\tilde{f}_{i}), pv^{l}_{14}(\tilde{f}_{i}) \right), \left( pv^{l}_{21}(\tilde{f}_{i}), pv^{l}_{22}(\tilde{f}_{i}), pv^{l}_{23}(\tilde{f}_{i}), pv^{l}_{24}(\tilde{f}_{i}) \right); \mu_{pv^{l}}(\tilde{f}_{i}), v_{pv^{l}}(\tilde{f}_{i}) \right\rangle.$$
(28)

The prospect value  $pv^l(\tilde{f}_i)$  is normalized.

$$\overline{p}v^{l}(\widetilde{f}_{i}) = \left\langle \left( \frac{pv_{11}^{l}(\widetilde{f}_{i}) - \gamma_{11}^{l_{-}}}{\gamma_{11}^{l_{+}} - \gamma_{11}^{l_{-}}}, \frac{pv_{12}^{l}(\widetilde{f}_{i}) - \gamma_{12}^{l_{-}}}{\gamma_{13}^{l_{+}} - \gamma_{13}^{l_{-}}}, \frac{pv_{14}^{l}(\widetilde{f}_{i}) - \gamma_{14}^{l_{-}}}{\gamma_{14}^{l_{+}} - \gamma_{14}^{l_{-}}} \right), \\ \left( \frac{pv_{21}^{l}(\widetilde{f}_{i}) - \zeta_{21}^{l_{-}}}{\zeta_{21}^{l_{-}} - \zeta_{21}^{l_{-}}}, \frac{pv_{22}^{l}(\widetilde{f}_{i}) - \zeta_{22}^{l_{-}}}{\zeta_{21}^{l_{+}} - \zeta_{22}^{l_{-}}}, \frac{pv_{23}^{l}(\widetilde{f}_{i}) - \zeta_{23}^{l_{-}}}{\zeta_{23}^{l_{+}} - \zeta_{23}^{l_{-}}}, \frac{pv_{24}^{l}(\widetilde{f}_{i}) - \zeta_{24}^{l_{-}}}{\zeta_{24}^{l_{+}} - \zeta_{24}^{l_{-}}} \right); \mu_{pv^{l}}(\widetilde{f}_{i}), \nu_{pv^{l}}(\widetilde{f}_{i}) \right\rangle, \tag{29}$$

where  $\overline{p}v^{l}(\tilde{f}_{i})$  is the normalized value of  $pv^{l}(\tilde{f}_{i})$ ;  $\gamma_{1s}^{l}$ , (s = 1, 2, 3, 4) and  $\gamma_{2s}^{l}$  are  $\min_{i=1}^{n} [pv^{l}_{1s}(\tilde{f}_{i})]$  and  $\min_{i=1}^{n} [pv^{l}_{2s}(\tilde{f}_{i})]$ , respectively; and  $\gamma_{1s}^{l+}$  and  $\gamma_{2s}^{l+}$  are  $\min_{i=1}^{n} [pv^{l}_{1s}(\tilde{f}_{i})]$  and  $\max_{i=1}^{n} [pv^{l}_{2s}(\tilde{f}_{i})]$ , respectively.

3.2.5. Consensus Analysis and Aggregation of Decision Information Given by all Decision-Makers. The population and the maximum number of iterations are denoted by P and TM, respectively. The consensus analysis and aggregation procedure are the same as those described in Section 3.1.

3.2.6. Ranking of Decision Schemes. The count of decision schemes is *L*. All decision schemes perform Steps 1–5. All expected values  $EV(a^l)$  of  $a^l (l = 1, 2, ..., L)$  are computed according to formula (10). Then, score function  $S(a^l)$  and accuracy function  $A(a^l)$  values are computed. Ranking of all decision schemes is done according to the comparing rules of TrIFNs (Section 2).

#### 4. Simulations and Analyses of Results

4.1. Simulations. We assume that stock investors decide whether to buy a stock based on gain or loss of stock. The stock investors select one of the three stocks  $A = \{A_1, A_2, A_3\}$ . The attributes of stocks are denoted as  $\tilde{X} = {\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{x}_4, \tilde{x}_5}, \text{ where } \tilde{x}_1 \text{ means big gain, } \tilde{x}_2 \text{ means}$ small gain,  $\tilde{x}_3$  means basically unchanged,  $\tilde{x}_4$  means small loss, and  $\tilde{x}_5$  means big loss. Ten decision-makers  $D = \{D_1, D_2, \dots, D_{10}\}$  are invited to make decisions. The values of attributes S evaluated by decision-makers D are listed in Table 1. The values of weights the  $P = \{\tilde{p}_1, \tilde{p}_2, \tilde{p}_3, \tilde{p}_4, \tilde{p}_5\} \text{ and }$ reference points  $\overline{R} = \{\overline{r}_1, \overline{r}_2, \dots, \overline{r}_{10}\}$  provided by decision-makers D are listed in Tables 2 and 3, respectively.  $w_1, w_2, \ldots, w_{10} = 0.1$ .  $d_{max}$  is 0.85 and  $\delta$  is 2 (values of  $d_{max}$  and  $\delta$  should be given according to a number of comparative experiments based on other MAGDM methods). The parameters of the genetic clustering algorithm are as follows: the population *P* is 5, the maximum number of iterations TM is 8, the genetic crossover  $P_c$  is 0.8, the initial value of k is 2, and the positive coefficients a and b are 2 and 1.2, respectively. The genetic K-means algorithm is executed 20 times independently (Tables 1–3). We compare our proposed method with the methods proposed by [20, 22, 23] and [10]. The parameters of the other three methods are taken from literature [10, 20, 22, 23].

4.2. Analyses of Results. Figures 2–4 present the curves of varying k values (in the experiment with the best clustering result  $G_b^l$ ) in aggregation procedures of schemes  $A_1$ ,  $A_2$ , and  $A_3$ , respectively, and they indicate that the values of k in the genetic K-means clustering algorithm realize automatic learning. Table 4 presents the ranking and computational time of four methods.

Zhao [23] uses interval-valued intuitionistic fuzzy numbers to express the decision information, so we adopted the parameters and operational details of Zhao's *K*-means clustering algorithm to conduct the experiments in the present study. Although the ranking of schemes obtained by this method is the same as our proposed method, there are two shortcomings of Zhao's method: first, the decision information is expressed in the form of interval-valued intuitionistic fuzzy numbers, and, second, the computational time is longer than that in our proposed method because the *k* values are determined by using the traversing method.

Liu and Chen [22] modify the clustering results on the basis of the gradient descent method. After computation, the ranking of schemes is different from that of our proposed method. The values of  $G_b^1$ ,  $G_b^2$ , and  $G_b^3$  obtained by Liu and Chen's method are 0.155, 0.163, and 0.146, respectively, whereas those obtained by our proposed method are 0.326, 0.320, and 0.402, respectively, clearly indicating that our proposed method yields higher values. The results indicate that, in Liu and Chen's method, the *k* value is determined before the simulation, which leads to the lower performance of this clustering method compared with our proposed method.

Yuan and Li [20] use the framework of the PT and TrIFN to solve the MAGDM problem. The preferred scheme is  $A_2$ , which is the same as our proposed method, but the alternatives are different from those obtained by our proposed method. The difference between our proposed method and Yuan and Li's method is the aggregation method used. As a representative MAGDM method, Yuan and Li's method comprehensively considers the factors in the MAGDM problem. The decision results of all MAGDM methods are neither absolutely good nor absolutely poor and the quality of MAGDM methods depends on whether the factors in the decision-making process are fully considered. Therefore, we cannot just compare whether the results of Yuan and Li's method and our proposed method are absolutely good or poor. The simulation results of these two methods can be used for mutual reference.

Wu and Cao [10] use the TrIFN weighted geometric operator and hybrid geometric operator to obtain the

TABLE 1: Attributes evaluated by decision-makers.

-		~	~	~	~	~
D	Α	$\widetilde{x}_1$	$\widetilde{x}_2$	$\widetilde{x}_3$	$\widetilde{x}_4$	$\widetilde{x}_5$
	4	<(0.1, 0.2, 0.3, 0.4), (0.1,	<(0.1, 0.3, 0.5, 0.7), (0.1,	<(0.2, 0.3, 0.4, 0.5), (0.2,	<(0.1, 0.2, 0.3, 0.4), (0.1,	<(0.1, 0.3, 0.5, 0.7), (0.3,
	$A_1$	0.2, 0.3, 0.4; $0.7, 0.2>$	0.3, 0.5, 0.7); 0.6, 0.3>	0.3, 0.4, 0.5; $0.5, 0.4$ >	0.2, 0.3, 0.4; $0.7, 0.2>$	0.4, 0.5, 0.6); 0.7, 0.2>
D	4	<(0.2, 0.3, 0.4, 0.5), (0.2,	<(0.4, 0.5, 0.7, 0.8), (0.4,	<(0.5, 0.6, 0.8, 0.9), (0.5,	<(0.5, 0.6, 0.7, 0.8), (0.5,	<(0.4, 0.5, 0.7, 0.9), (0.4,
$D_1$	$A_2$	0.3, 0.4, 0.5); 0.5, 0.4>	0.5, 0.7, 0.8); 0.5, 0.4>	0.6, 0.8, 0.9); 0.4, 0.3>	0.6, 0.7, 0.8); 0.4, 0.3>	0.5, 0.7, 0.9); 0.5, 0.4>
	4	<(0.1, 0.3, 0.5, 0.7), (0.1,	<(0.3, 0.4, 0.7, 0.8), (0.3,	<(0.2, 0.3, 0.4, 0.5), (0.2,	<(0.4, 0.5, 0.7, 0.8), (0.4,	<(0.3, 0.4, 0.6, 0.7), (0.3,
	$A_3$	0.3, 0.5, 0.7); 0.6, 0.3>	0.4, 0.7, 0.8); 0.7, 0.2>	0.3, 0.4, 0.5); 0.5, 0.4>	0.5, 0.7, 0.8); 0.5, 0.4>	0.4, 0.6, 0.7); 0.7, 0.2>
		<(0.2, 0.3, 0.5, 0.6), (0.2,		<(0.3, 0.4, 0.6, 0.7), (0.3,		
	$A_1$	(0.2, 0.5, 0.6); 0.5, 0.4>	0.7, 0.8, 0.9); 0.4, 0.3>	0.4, 0.6, 0.7); 0.7, 0.2>	0.3, 0.6, 0.8); 0.6, 0.3>	0.6, 0.8, 0.9); 0.4, 0.3>
		<(01 02 03 04) (01		<(0.4, 0.5, 0.7, 0.8), (0.4,		<(0.3, 0.5, 0.7, 0.9), (0.3,
$D_2$	$A_2$	0.2, 0.3, 0.4); 0.7, 0.2>	0.4, 0.7, 0.8); 0.7, 0.2>	0.5, 0.7, 0.8); 0.5, 0.4>	0.3, 0.6, 0.8); 0.6, 0.3>	0.5, 0.7, 0.9); 0.6, 0.3>
		<(05.06.08.09)(05.06.08.09)	<(0.2, 0.4, 0.6, 0.8), (0.2,	<(0.6, 0.7, 0.8, 0.9), (0.6,		
	$A_3$	0.6, 0.8, 0.9); 0.4, 0.3>	0.4, 0.6, 0.8); 0.4, 0.3>	0.7, 0.8, 0.9); 0.4, 0.3>	0.6, 0.7, 0.8); 0.4, 0.3>	0.4, 0.6, 0.7); 0.7, 0.2>
				<(0.2, 0.4, 0.7, 0.9), (0.2,	<(0.2, 0.4, 0.6, 0.8), (0.2,	<(0.4, 0.5, 0.7, 0.8), (0.4,
	$A_1$	<(0.6, 0.7, 0.8, 0.9), (0.6, 0.7, 0.8, 0.9); 0.4, 0.3>	<(0.4, 0.5, 0.7, 0.9), (0.4, 0.5, 0.7, 0.9); 0.5, 0.4>	(0.2, 0.4, 0.7, 0.9); (0.2, 0.4, 0.7, 0.9); 0.4, 0.3>	(0.2, 0.4, 0.6, 0.8); (0.2, 0.4, 0.6, 0.8); (0.2, 0.4, 0.6, 0.8); 0.4, 0.3>	(0.4, 0.5, 0.7, 0.8); (0.4, 0.5, 0.7, 0.8); 0.5, 0.4>
$D_3$	$A_2$	<(0.4, 0.5, 0.7, 0.8), (0.4, 0.5, 0.7, 0.8); 0.5, 0.4>		<(0.3, 0.5, 0.7, 0.9), (0.3,		
			0.3, 0.5, 0.7; $0.6, 0.3 >$	0.5, 0.7, 0.9; $0.6, 0.3 >$	0.6, 0.8, 0.9; $0.4, 0.3 >$	0.4, 0.7, 0.9; $0.4, 0.3 >$
	$A_3$	<(0.5, 0.6, 0.8, 0.9), (0.5,		<(0.2, 0.3, 0.5, 0.6), (0.2, 0.3, 0.5, 0.6); 0.5, 0.4>	<(0.4, 0.5, 0.7, 0.8), (0.4, 0.5, 0.7, 0.8); 0.5, 0.4>	
	-	0.0, 0.0, 0.0), 0.4, 0.5	0.4, 0.5, 0.6); 0.7, 0.2>			0.4, 0.7, 0.8); 0.7, 0.2>
	$A_1$	<(0.4, 0.5, 0.6, 0.7), (0.4,		<(0.3, 0.4, 0.6, 0.7), (0.3,		
	1	0.5, 0.6, 0.7); 0.5, 0.4>	0.4, 0.7, 0.9); 0.4, 0.3>	0.4, 0.6, 0.7); 0.7, 0.2>	0.7, 0.8, 0.9); 0.4, 0.3>	0.3, 0.6, 0.7); 0.5, 0.4>
$D_4$	$A_2$	<(0.2, 0.3, 0.5, 0.6), (0.2,		<(0.5, 0.6, 0.7, 0.8), (0.5,	<(0.3, 0.4, 0.6, 0.7), (0.3,	
4	2	0.3, 0.5, 0.6); 0.5, 0.4>	0.5, 0.7, 0.8); 0.5, 0.4>	0.6, 0.7, 0.8); 0.4, 0.3>	0.4, 0.6, 0.7); 0.7, 0.2>	0.3, 0.5, 0.6); 0.5, 0.4>
	$A_3$	<(0.4, 0.5, 0.7, 0.8), (0.4,		<(0.4, 0.5, 0.6, 0.7), (0.4,		
	5	0.5, 0.7, 0.8); 0.5, 0.4>	0.3, 0.5, 0.7); 0.6, 0.3>	0.5, 0.6, 0.7); 0.5, 0.4>	0.3, 0.6, 0.8); 0.6, 0.3>	0.3, 0.5, 0.7); 0.6, 0.3>
	$A_1$	<(0.2, 0.4, 0.7, 0.9), (0.2,		<(0.3, 0.4, 0.7, 0.8), (0.3,		
	11	0.4, 0.7, 0.7, 0.7, 0.4, 0.57	0.7, 0.8, 0.9); 0.4, 0.3>	0.4, 0.7, 0.8); 0.7, 0.2>	0.4, 0.7, 0.9); 0.4, 0.3>	0.3, 0.6, 0.8); 0.6, 0.3>
$D_5$	$A_2$	<(0.2, 0.4, 0.6, 0.8), (0.2,		<(0.4, 0.5, 0.7, 0.8), (0.4,		
125	112	0.4, 0.6, 0.8); 0.4, 0.3>	0.3, 0.6, 0.8); 0.6, 0.3>	0.5, 0.7, 0.8); 0.5, 0.4>	0.4, 0.6, 0.7); 0.7, 0.2>	0.4, 0.6, 0.8); 0.4, 0.3>
	$A_3$	<(0.3, 0.4, 0.6, 0.7), (0.3,		<(0.4, 0.5, 0.7, 0.9), (0.4,		
	113	0.4, 0.6, 0.7); 0.7, 0.2>	0.4, 0.5, 0.6); 0.7, 0.2>	0.5, 0.7, 0.9); 0.5, 0.4>	0.5, 0.7, 0.8); 0.5, 0.4>	0.4, 0.7, 0.9); 0.4, 0.3>
	Δ	<(0.5, 0.6, 0.8, 0.9), (0.5,	<(0.4, 0.5, 0.7, 0.9), (0.4,	<(0.6, 0.7, 0.8, 0.9), (0.6,	<(0.4, 0.5, 0.7, 0.8), (0.4,	<(0.6, 0.7, 0.8, 0.9), (0.6,
	$A_1$	0.6, 0.8, 0.9); 0.4, 0.3>	0.5, 0.7, 0.9); 0.5, 0.4>	0.7, 0.8, 0.9); 0.4, 0.3>	0.5, 0.7, 0.8); 0.5, 0.4>	0.7, 0.8, 0.9); 0.4, 0.3>
$D_6$	Δ	<(0.4, 0.5, 0.7, 0.8), (0.4,	<(0.1, 0.2, 0.3, 0.4), (0.1,	<(0.2, 0.3, 0.5, 0.6), (0.2,	<(0.3, 0.5, 0.7, 0.9), (0.3,	<(0.1, 0.3, 0.5, 0.7), (0.1,
$\nu_6$	$A_2$	0.5, 0.7, 0.8); 0.5, 0.4>	0.2, 0.3, 0.4); 0.7, 0.2>	0.3, 0.5, 0.6); 0.5, 0.4>	0.5, 0.7, 0.9); 0.6, 0.3>	0.3, 0.5, 0.7); 0.6, 0.3>
	Δ	<(0.3, 0.4, 0.7, 0.8), (0.3,	<(0.2, 0.3, 0.5, 0.6), (0.2,	<(0.1, 0.3, 0.5, 0.7), (0.1,	<(0.5, 0.6, 0.8, 0.9), (0.5,	<(0.4, 0.5, 0.6, 0.7), (0.4,
	$A_3$	0.4, 0.7, 0.8); 0.7, 0.2>	0.3, 0.5, 0.6); 0.5, 0.4>	0.3, 0.5, 0.7); 0.6, 0.3>	0.6, 0.8, 0.9); 0.4, 0.3>	0.5, 0.6, 0.7); 0.5, 0.4>
	Δ	<(0.2, 0.3, 0.6, 0.7), (0.2,	<(0.6, 0.7, 0.8, 0.9), (0.6,	<(0.2, 0.3, 0.6, 0.7), (0.2,	<(0.5, 0.6, 0.8, 0.9), (0.5,	<(0.1, 0.2, 0.4, 0.5), (0.1,
	$A_1$	0.3, 0.6, 0.7); 0.5, 0.4>	0.7, 0.8, 0.9); 0.4, 0.3>	0.3, 0.6, 0.7); 0.5, 0.4>	0.6, 0.8, 0.9); 0.4, 0.3>	0.2, 0.4, 0.5); 0.7, 0.2>
D	Δ	<(0.2, 0.4, 0.6, 0.8), (0.2,	<(0.1, 0.3, 0.6, 0.8), (0.1,	<(0.4, 0.5, 0.7, 0.8), (0.4,	<(0.6, 0.7, 0.8, 0.9), (0.6,	<(0.2, 0.3, 0.5, 0.6), (0.2,
$D_7$	$A_2$	0.4, 0.6, 0.8); 0.4, 0.3>	0.3, 0.6, 0.8); 0.6, 0.3>	0.5, 0.7, 0.8); 0.5, 0.4>	0.7, 0.8, 0.9); 0.4, 0.3>	0.3, 0.5, 0.6); 0.5, 0.4>
	Δ	<(0.2, 0.3, 0.4, 0.5), (0.2,	<(0.2, 0.4, 0.7, 0.9), (0.2,	<(0.1, 0.2, 0.4, 0.5), (0.1,	<(0.1, 0.2, 0.5, 0.6), (0.1,	<(0.3, 0.4, 0.6, 0.7), (0.3,
	$A_3$	0.3, 0.4, 0.5); 0.5, 0.4>	0.4, 0.7, 0.9); 0.4, 0.3>	0.2, 0.4, 0.5); 0.7, 0.2>	0.2, 0.5, 0.6); 0.7, 0.2>	0.4, 0.6, 0.7); 0.7, 0.2>
	4	<(0.3, 0.5, 0.7, 0.9), (0.3,	<(0.5, 0.6, 0.8, 0.9), (0.5,	<(0.2, 0.4, 07, 0.9), (0.2,	<(0.2, 0.3, 0.4, 0.5), (0.2,	<(0.3, 0.4, 0.5, 0.6), (0.3,
	$A_1$	0.5, 0.7, 0.9); 0.6, 0.3>	0.6, 0.8, 0.9); 0.4, 0.3>	0.4, 0.7, 0.9); 0.4, 0.3>	0.3, 0.4, 0.5); 0.5, 0.4>	0.4, 0.5, 0.6); 0.7, 0.2>
P	A	<(0.5, 0.6, 0.8, 0.9), (0.5,	<(0.4, 0.5, 0.7, 0.8), (0.4,	<(0.4, 0.5, 0.6, 0.7), (0.4,	<(0.4, 0.5, 0.7, 0.9), (0.4,	
$D_8$	$A_2$	0.6, 0.8, 0.9); 0.4, 0.3>	0.5, 0.7, 0.8); 0.5, 0.4>	0.5, 0.6, 0.7); 0.5, 0.4>	0.5, 0.7, 0.9); 0.5, 0.4>	0.4, 0.6, 0.7); 0.7, 0.2>
	A	<(0.6, 0.7, 0.8, 0.9), (0.6,	<(0.3, 0.4, 0.5, 0.6), (0.3,	<(0.6, 0.7, 0.8, 0.9), (0.6,		<(0.4, 0.5, 0.7, 0.8), (0.4,
	$A_3$	0.7, 0.8, 0.9); 0.4, 0.3>	0.4, 0.5, 0.6); 0.7, 0.2>	0.7, 0.8, 0.9); 0.4, 0.3>	0.3, 0.6, 0.7); 0.5, 0.4>	0.5, 0.7, 0.8); 0.5, 0.4>
		<(0.1, 0.2, 0.4, 0.5), (0.1,		<(0.1, 0.2, 0.5, 0.6), (0.1,		
	$A_1$	0.2, 0.4, 0.5); 0.7, 0.2>	0.3, 0.5, 0.6); 0.5, 0.4>	0.2, 0.5, 0.6); 0.7, 0.2>	0.7, 0.8, 0.9); 0.4, 0.3>	0.4, 0.7, 0.8); 0.7, 0.2>
_		<(0.2, 0.4, 0.6, 0.8), (0.2,		<(0.3, 0.5, 0.7, 0.9), (0.3,		
$D_9$	$A_2$	0.4, 0.6, 0.8); 0.4, 0.3>	0.2, 0.3, 0.4); 0.7, 0.2>	0.5, 0.7, 0.9); 0.6, 0.3>	0.2, 0.4, 0.5); 0.7, 0.2>	0.3, 0.5, 0.6); 0.5, 0.4>
		<(0.4, 0.5, 0.7, 0.9), (0.4,		<(0.5, 0.6, 0.7, 0.8), (0.5,		
	$A_3$	0.5, 0.7, 0.9); 0.5, 0.4>	0.3, 0.5, 0.7); 0.6, 0.3>	0.6, 0.7, 0.8); 0.4, 0.3>	0.3, 0.6, 0.7); 0.5, 0.4>	0.2, 0.3, 0.4); 0.7, 0.2>
		<(0.3, 0.4, 0.6, 0.7), (0.3,		<(0.2, 0.4, 0.7, 0.9), (0.2,	<(0.4, 0.5, 0.7, 0.8), (0.4,	<(0.5, 0.6, 0.8, 0.9), (0.5,
	$A_1$	(0.3, 0.4, 0.0, 0.7), (0.3, 0.4, 0.6, 0.7); 0.7, 0.2>	(0.2, 0.3, 0.4, 0.5); (0.2, 0.3, 0.4, 0.5); 0.5, 0.4>	(0.2, 0.4, 0.7, 0.9); (0.2, 0.4, 0.7, 0.9); 0.4, 0.3>	(0.4, 0.5, 0.7, 0.8); (0.4, 0.5, 0.7, 0.8); 0.5, 0.4>	(0.5, 0.6, 0.8, 0.9); (0.5, 0.6, 0.8, 0.9); (0.4, 0.3>
		<(0.4, 0.5, 0.7, 0.8), (0.4,		<(0.2, 0.4, 07, 0.9), (0.2,	<(0.1, 0.3, 0.5, 0.7), (0.1,	
$D_{10}$	$A_2$	(0.4, 0.5, 0.7, 0.8); (0.4, 0.5, 0.7, 0.8); 0.5, 0.4>	(0.5, 0.6, 0.8, 0.9); (0.5, 0.6, 0.8, 0.9); (0.5, 0.6, 0.8, 0.9); 0.4, 0.3>	(0.2, 0.4, 07, 0.9); (0.2, 0.4, 0.7, 0.9); 0.4, 0.3>	(0.1, 0.3, 0.3, 0.3, 0.7), (0.1, 0.3, 0.5, 0.7); 0.6, 0.3>	(0.4, 0.3, 0.7, 0.9); (0.4, 0.5, 0.7, 0.9); 0.5, 0.4>
		<(0.4, 0.5, 0.6, 0.7), (0.4,		<(0.1, 0.3, 0.5, 0.7), (0.1,		
	$A_3$	(0.4, 0.5, 0.6, 0.7); 0.5, 0.4>	(0.2, 0.5, 0.5, 0.0), (0.2, 0.3, 0.5, 0.6); 0.5, 0.4>	0.3, 0.5, 0.7); 0.6, 0.3>	0.3, 0.6, 0.8); 0.6, 0.3>	(0.1, 0.2, 0.4, 0.5); (0.1, 0.2)
		0.0, 0.0, 0.7, 0.0, 0.4/	0.0, 0.0, 0.0, 0.0, 0.4/	0.0, 0.0, 0.7, 0.0, 0.07	0.0, 0.0, 0.0/, 0.0, 0.0/	0.2, 0.1, 0.0), 0.7, 0.2/

TABLE 2: Weights evaluated by decision-makers.

			8	7		
D	Α	$\tilde{x}_1$	$\tilde{x}_2$	$\widetilde{x}_3$	$\widetilde{x}_4$	$\widetilde{x}_5$
		<(0.2, 0.3, 0.6, 0.7), (0.2,	<(0.6, 0.7, 0.8, 0.9), (0.6,	<(0.1, 0.2, 0.5, 0.6), (0.1,	<(0.1, 0.2, 0.4, 0.5), (0.1,	
	$A_1$	0.3, 0.6, 0.7); 0.5, 0.4>	0.7, 0.8, 0.9); 0.4, 0.3>	0.2, 0.5, 0.6); 0.7, 0.2>	0.2, 0.4, 0.5); 0.7, 0.2>	0.6, 0.8, 0.9); 0.4, 0.3>
P		<(01 03 05 07) (01		<(0.6, 0.7, 0.8, 0.9), (0.6,		
$D_1$	$A_2$	0.3, 0.5, 0.7); 0.6, 0.3>	0.3, 0.6, 0.8); 0.6, 0.3>	0.7, 0.8, 0.9); 0.4, 0.3>	0.5, 0.7, 0.8); 0.5, 0.4>	0.3, 0.5, 0.6); 0.5, 0.4>
		<(01 03 06 08) (01		<(0.2, 0.3, 0.4, 0.5), (0.2,	<(0.2, 0.3, 0.5, 0.6), (0.2,	<(0.5, 0.6, 0.8, 0.9), (0.5,
	$A_3$	0.3, 0.6, 0.8); 0.6, 0.3>	0.6, 0.7, 0.8); 0.4, 0.3>	0.3, 0.4, 0.5); 0.5, 0.4>	0.3, 0.5, 0.6); 0.5, 0.4>	0.6, 0.8, 0.9); 0.4, 0.3>
		<(0.4, 0.5, 0.6, 0.7), (0.4,	<(0.4, 0.5, 0.7, 0.9), (0.4,	<(0.2, 0.3, 0.6, 0.7), (0.2,	<(0.5, 0.6, 0.8, 0.9), (0.5,	<(0.5, 0.6, 0.8, 0.9), (0.5,
	$A_1$	0.5, 0.6, 0.7); 0.5, 0.4>	0.5, 0.7, 0.9); 0.5, 0.4>	0.3, 0.6, 0.7); 0.5, 0.4>	0.6, 0.8, 0.9); 0.4, 0.3>	0.6, 0.8, 0.9); 0.4, 0.3>
D	Δ	<(0.4, 0.5, 0.7, 0.8), (0.4,	<(0.1, 0.2, 0.3, 0.4), (0.1,	<(0.2, 0.3, 0.5, 0.6), (0.2,	<(0.3, 0.5, 0.7, 0.9), (0.3,	<(0.3, 0.4, 0.6, 0.7), (0.3,
$D_2$	$A_2$	0.5, 0.7, 0.8); 0.5, 0.4>	0.2, 0.3, 0.4); 0.7, 0.2>	0.3, 0.5, 0.6); 0.5, 0.4>	0.5, 0.7, 0.9); 0.6, 0.3>	0.4, 0.6, 0.7); 0.7, 0.2>
	Δ	<(0.6, 0.7, 0.8, 0.9), (0.6,	<(0.2, 0.3, 0.4, 0.5), (0.2,	<(0.2, 0.4, 07, 0.9), (0.2,	<(0.2, 0.3, 0.5, 0.6), (0.2,	<(0.2, 0.3, 0.6, 0.7), (0.2,
	$A_3$	0.7, 0.8, 0.9); 0.4, 0.3>	0.3, 0.4, 0.5); 0.5, 0.4>	0.4, 0.7, 0.9); 0.4, 0.3>	0.3, 0.5, 0.6); 0.5, 0.4>	0.3, 0.6, 0.7); 0.5, 0.4>
	4	<(0.1, 0.2, 0.5, 0.6), (0.1,	<(0.6, 0.7, 0.8, 0.9), (0.6,	<(0.2, 0.4, 0.7, 0.9), (0.2,	<(0.4, 0.5, 0.7, 0.8), (0.4,	<(0.1, 0.3, 0.6, 0.8), (0.1,
	$A_1$	0.2, 0.5, 0.6); 0.7, 0.2>	0.7, 0.8, 0.9); 0.4, 0.3>	0.4, 0.7, 0.9); 0.4, 0.3>	0.5, 0.7, 0.8); 0.5, 0.4>	0.3, 0.6, 0.8); 0.6, 0.3>
D	Δ	<(0.2, 0.4, 0.6, 0.8), (0.2,	<(0.3, 0.4, 0.7, 0.8), (0.3,	<(0.1, 0.2, 0.3, 0.4), (0.1,	<(0.2, 0.3, 0.6, 0.7), (0.2,	<(0.4, 0.5, 0.7, 0.8), (0.4,
$D_3$	$A_2$	0.4, 0.6, 0.8); 0.4, 0.3>	0.4, 0.7, 0.8); 0.7, 0.2>	0.2, 0.3, 0.4); 0.7, 0.2>	0.3, 0.6, 0.7); 0.5, 0.4>	0.5, 0.7, 0.8); 0.5, 0.4>
	Δ	<(0.4, 0.5, 0.7, 0.8), (0.4,		<(0.1, 0.2, 0.5, 0.6), (0.1,	<(0.1, 0.2, 0.4, 0.5), (0.1,	<(0.6, 0.7, 0.8, 0.9), (0.6,
	$A_3$	0.5, 0.7, 0.8); 0.5, 0.4>	0.7, 0.8, 0.9); 0.4, 0.3>	0.2, 0.5, 0.6); 0.7, 0.2>	0.2, 0.4, 0.5); 0.7, 0.2>	0.7, 0.8, 0.9); 0.4, 0.3>
	Δ	<(0.3, 0.4, 0.6, 0.7), (0.3,	<(0.5, 0.6, 0.8, 0.9), (0.5,	<(0.6, 0.7, 0.8, 0.9), (0.6,	<(0.2, 0.3, 0.5, 0.6), (0.2,	<(0.3, 0.4, 0.6, 0.7), (0.3,
	$A_1$	0.4, 0.6, 0.7); 0.7, 0.2>	0.6, 0.8, 0.9); 0.4, 0.3>	0.7, 0.8, 0.9); 0.4, 0.3>	0.3, 0.5, 0.6); 0.5, 0.4>	0.4, 0.6, 0.7); 0.7, 0.2>
$D_4$	Δ	<(0.4, 0.5, 0.6, 0.7), (0.4,	<(0.4, 0.5, 0.7, 0.8), (0.4,	<(0.6, 0.7, 0.8, 0.9), (0.6,	<(0.2, 0.4, 0.6, 0.8), (0.2,	<(0.2, 0.3, 0.5, 0.6), (0.2,
$\nu_4$	$A_2$	0.5, 0.6, 0.7); 0.5, 0.4>	0.5, 0.7, 0.8); 0.5, 0.4>	0.7, 0.8, 0.9); 0.4, 0.3>	0.4, 0.6, 0.8); 0.4, 0.3>	0.3, 0.5, 0.6); 0.5, 0.4>
	$A_3$	<(0.3, 0.4, 0.7, 0.8), (0.3,		<(0.2, 0.3, 0.5, 0.6), (0.2,	<(0.1, 0.3, 0.6, 0.8), (0.1,	
	113	0.4, 0.7, 0.8); 0.7, 0.2>	0.4, 0.6, 0.8); 0.4, 0.3>	0.3, 0.5, 0.6); 0.5, 0.4>	0.3, 0.6, 0.8); 0.6, 0.3>	0.3, 0.5, 0.6); 0.5, 0.4>
	Δ	<(0.4, 0.5, 0.7, 0.8), (0.4,	<(0.1, 0.3, 0.6, 0.8), (0.1,	<(0.6, 0.7, 0.8, 0.9), (0.6,	<(0.6, 0.7, 0.8, 0.9), (0.6,	<(0.2, 0.4, 07, 0.9), (0.2,
	$A_1$	0.5, 0.7, 0.8); 0.5, 0.4>	0.3, 0.6, 0.8); 0.6, 0.3>	0.7, 0.8, 0.9); 0.4, 0.3>	0.7, 0.8, 0.9); 0.4, 0.3>	0.4, 0.7, 0.9); 0.4, 0.3>
$D_5$	$A_2$	<(0.1, 0.3, 0.5, 0.7), (0.1,		<(0.3, 0.5, 0.7, 0.9), (0.3,	<(0.1, 0.3, 0.6, 0.8), (0.1,	
25	112	0.5, 0.5, 0.7, 0.0, 0.5	0.5, 0.7, 0.9); 0.5, 0.4>	0.5, 0.7, 0.9); 0.6, 0.3>	0.3, 0.6, 0.8); 0.6, 0.3>	0.3, 0.4, 0.5); 0.5, 0.4>
	$A_3$	<(0.2, 0.3, 0.6, 0.7), (0.2,		<(0.1, 0.2, 0.4, 0.5), (0.1,	<(0.3, 0.4, 0.6, 0.7), (0.3,	
	5	0.3, 0.6, 0.7); 0.5, 0.4>	0.2, 0.3, 0.4); 0.7, 0.2>	0.2, 0.4, 0.5); 0.7, 0.2>	0.4, 0.6, 0.7); 0.7, 0.2>	0.4, 0.7, 0.8); 0.7, 0.2>
	$A_1$	<(0.1, 0.2, 0.5, 0.6), (0.1,		<(0.2, 0.3, 0.6, 0.7), (0.2,		
	1	0.2, 0.5, 0.6); 0.7, 0.2>	0.4, 0.6, 0.7); 0.7, 0.2>	0.3, 0.6, 0.7); 0.5, 0.4>	0.4, 0.7, 0.9); 0.4, 0.3>	0.5, 0.7, 0.8); 0.5, 0.4>
$D_6$	$A_2$	<(0.3, 0.4, 0.6, 0.7), (0.3,		<(0.3, 0.4, 0.6, 0.7), (0.3,		
		0.4, 0.6, 0.7); 0.7, 0.2><(0.4, 0.5, 0.7, 0.9), (0.4,	0.5, 0.7, 0.8); 0.5, 0.4><(0.2, 0.3, 0.5, 0.6), (0.2,	0.4, 0.6, 0.7); 0.7, 0.2><(0.3, 0.5, 0.7, 0.9), (0.3,	0.4, 0.6, 0.8); 0.4, 0.3><(0.5, 0.6, 0.8, 0.9), (0.5,	0.3, 0.5, 0.6); 0.5, 0.4><(0.4, 0.5, 0.6, 0.7), (0.4,
	$A_3$	(0.4, 0.5, 0.7, 0.9); (0.4, 0.5, 0.7, 0.9); 0.5, 0.4>	(0.2, 0.3, 0.5, 0.0), (0.2, 0.3, 0.5, 0.6); 0.5, 0.4>	(0.5, 0.7, 0.9); 0.6, 0.3>	(0.5, 0.6, 0.8, 0.9); (0.5, 0.6, 0.8, 0.9); (0.5, 0.6, 0.8, 0.9); 0.4, 0.3>	(0.4, 0.5, 0.6, 0.7); (0.4, 0.5, 0.6, 0.7); 0.5, 0.4>
				<(0.2, 0.3, 0.5, 0.6), (0.2,		
	$A_1$	<(0.1, 0.3, 0.5, 0.7), (0.1, 0.3, 0.5, 0.7); 0.6, 0.3>	<(0.1, 0.2, 0.4, 0.5), (0.1, 0.2, 0.4, 0.5); 0.7, 0.2>	<(0.2, 0.3, 0.5, 0.6); (0.2, 0.3, 0.5, 0.6); (0.2, 0.3, 0.5, 0.6); 0.5, 0.4>	<(0.1, 0.3, 0.6, 0.8); (0.1, 0.3, 0.6, 0.8); 0.6, 0.3>	<(0.1, 0.2, 0.5, 0.6), (0.1, 0.2, 0.5, 0.6); 0.7, 0.2>
				<(0.5, 0.6, 0.8, 0.9), (0.5, 0.4)		<(0.2, 0.3, 0.5, 0.6), (0.2, <(0.2, 0.3, 0.5, 0.6), (0.2,
$D_7$	$A_2$	0.4, 0.7, 0.9); 0.4, 0.3>	(0.2, 0.5, 0.4, 0.5); (0.2, 0.3, 0.4, 0.5); 0.5, 0.4>	0.6, 0.8, 0.9); 0.4, 0.3>	0.5, 0.7, 0.8); 0.5, 0.4>	(0.2, 0.5, 0.5, 0.0), (0.2, 0.3, 0.5, 0.6); 0.5, 0.4>
		<(0.1, 0.2, 0.5, 0.6), (0.1,	<(0.1, 0.2, 0.5, 0.6), (0.1,	<(0.1, 0.2, 0.3, 0.4), (0.1,	<(0.6, 0.7, 0.8, 0.9), (0.6,	<(0.1, 0.3, 0.6, 0.8), (0.1,
	$A_3$	0.2, 0.5, 0.6); 0.7, 0.2>	0.2, 0.5, 0.6); 0.7, 0.2>	0.2, 0.3, 0.4); 0.7, 0.2>	0.7, 0.8, 0.9); 0.4, 0.3>	0.3, 0.6, 0.8); 0.6, 0.3>
		<(0.2, 0.4, 0.6, 0.8), (0.2,		<(0.2, 0.4, 0.7, 0.9), (0.2,	<(0.2, 0.3, 0.4, 0.5), (0.2,	
	$A_1$	(0.2, 0.1, 0.0, 0.0), (0.2, 0.4, 0.6, 0.8); 0.4, 0.3>	0.4, 0.6, 0.7); 0.7, 0.2>	(0.2, 0.1, 0.7, 0.9); (0.2, 0.4, 0.3>	(0.2, 0.3, 0.1, 0.5); (0.2, 0.3, 0.4, 0.5); 0.5, 0.4>	0.6, 0.8, 0.9); 0.4, 0.3>
_		<(0.1, 0.2, 0.5, 0.6), (0.1,		<(0.2, 0.3, 0.5, 0.6), (0.2,	<(0.4, 0.5, 0.7, 0.9), (0.4,	<(0.3, 0.4, 0.6, 0.7), (0.3,
$D_8$	$A_2$	0.2, 0.5, 0.6); 0.7, 0.2>	0.5, 0.7, 0.8); 0.5, 0.4>	0.3, 0.5, 0.6); 0.5, 0.4>	0.5, 0.7, 0.9); 0.5, 0.4>	0.4, 0.6, 0.7); 0.7, 0.2>
		<(05060708)(05	<(0.6, 0.7, 0.8, 0.9), (0.6,	<(0.3, 0.5, 0.7, 0.9), (0.3,	<(0.4, 0.5, 0.7, 0.8), (0.4,	<(0.4, 0.5, 0.6, 0.7), (0.4,
	$A_3$	0.6, 0.7, 0.8); 0.4, 0.3>	0.7, 0.8, 0.9); 0.4, 0.3>	0.5, 0.7, 0.9); 0.6, 0.3>	0.5, 0.7, 0.8); 0.5, 0.4>	0.5, 0.6, 0.7); 0.5, 0.4>
		<(0.1, 0.3, 0.6, 0.8), (0.1,		<(0.5, 0.6, 0.8, 0.9), (0.5,	<(0.2, 0.3, 0.5, 0.6), (0.2,	
	$A_1$	0.3, 0.6, 0.8); 0.6, 0.3>	0.7, 0.8, 0.9); 0.4, 0.3>	0.6, 0.8, 0.9); 0.4, 0.3>	0.3, 0.5, 0.6); 0.5, 0.4>	0.4, 0.7, 0.9); 0.4, 0.3>
P		<(01 02 04 05) (01		<(0.2, 0.3, 0.4, 0.5), (0.2,	<(0.2, 0.4, 07, 0.9), (0.2,	
$D_9$	$A_2$	0.2, 0.4, 0.5); 0.7, 0.2>	0.5, 0.6, 0.7); 0.5, 0.4>	0.3, 0.4, 0.5); 0.5, 0.4>	0.4, 0.7, 0.9); 0.4, 0.3>	0.4, 0.7, 0.8); 0.7, 0.2>
	Α	<(0.2, 0.3, 0.5, 0.6), (0.2,		<(0.3, 0.4, 0.7, 0.8), (0.3,	<(0.5, 0.6, 0.7, 0.8), (0.5,	
	$A_3$	0.3, 0.5, 0.6); 0.5, 0.4>	0.4, 0.5, 0.6); 0.7, 0.2>	0.4, 0.7, 0.8); 0.7, 0.2>	0.6, 0.7, 0.8); 0.4, 0.3>	0.4, 0.6, 0.8); 0.4, 0.3>
	4	<(0.1, 0.3, 0.5, 0.7), (0.1,		<(0.2, 0.3, 0.5, 0.6), (0.2,	<(0.3, 0.4, 0.6, 0.7), (0.3,	
	$A_1$	0.3, 0.5, 0.7); 0.6, 0.3>	0.3, 0.6, 0.7); 0.5, 0.4>	0.3, 0.5, 0.6); 0.5, 0.4>	0.4, 0.6, 0.7); 0.7, 0.2>	0.7, 0.8, 0.9); 0.4, 0.3>
D	Α	<(0.4, 0.5, 0.7, 0.9), (0.4,		<(0.3, 0.5, 0.7, 0.9), (0.3,	<(0.4, 0.5, 0.7, 0.8), (0.4,	<(0.1, 0.2, 0.5, 0.6), (0.1,
$\nu_{10}$	$A_2$	0.5, 0.7, 0.9); 0.5, 0.4>	0.2, 0.3, 0.4); 0.7, 0.2>	0.5, 0.7, 0.9); 0.6, 0.3>	0.5, 0.7, 0.8); 0.5, 0.4>	0.2, 0.5, 0.6); 0.7, 0.2>
	Δ	<(0.2, 0.4, 0.7, 0.9), (0.2,		<(0.6, 0.7, 0.8, 0.9), (0.6,	<(0.3, 0.4, 0.5, 0.6), (0.3,	
	$A_3$	0.4, 0.7, 0.9); 0.4, 0.3>	0.3, 0.4, 0.5); 0.5, 0.4>	0.7, 0.8, 0.9); 0.4, 0.3>	0.4, 0.5, 0.6); 0.7, 0.2>	0.5, 0.7, 0.8); 0.5, 0.4>

TABLE 3: Reference points given by decision-makers.

D	Α	~	ž	ž	~	ĩ
D	A	$\tilde{x}_1$	$\widetilde{x}_2$	$\widetilde{x}_3$	$\widetilde{x}_4$	$\widetilde{x}_5$
	$A_1$	<(0.2, 0.3, 0.4, 0.5), (0.2,			<(0.1, 0.3, 0.5, 0.7), (0.1,	
	11	0.3, 0.4, 0.5); 0.5, 0.4>	0.7, 0.8, 0.9); 0.4, 0.3>	0.3, 0.5, 0.6); 0.5, 0.4>	0.3, 0.5, 0.7); 0.6, 0.3>	0.3, 0.6, 0.8); 0.6, 0.3>
$D_1$	A.	<(0.4, 0.5, 0.7, 0.8), (0.4,		<(0.5, 0.6, 0.8, 0.9), (0.5,		
$\nu_1$	$A_2$	0.5, 0.7, 0.8); 0.5, 0.4>	0.7, 0.8, 0.9); 0.4, 0.3>	0.6, 0.8, 0.9); 0.4, 0.3>	0.5, 0.7, 0.9; $0.5, 0.4$ >	0.5, 0.7, 0.8); 0.5, 0.4>
	Δ	<(0.4, 0.5, 0.6, 0.7), (0.4,	<(0.2, 0.4, 0.7, 0.9), (0.2,	<(0.1, 0.3, 0.5, 0.7), (0.1,	<(0.3, 0.4, 0.7, 0.8), (0.3,	<(0.6, 0.7, 0.8, 0.9), (0.6,
	$A_3$	0.5, 0.6, 0.7); 0.5, 0.4>	0.4, 0.7, 0.9); 0.4, 0.3>	0.3, 0.5, 0.7); 0.6, 0.3>	0.4, 0.7, 0.8); 0.7, 0.2>	0.7, 0.8, 0.9); 0.4, 0.3>
		<(0.3, 0.4, 0.7, 0.8), (0.3,	<(0.3, 0.4, 0.6, 0.7), (0.3,	<(0.2, 0.3, 0.5, 0.6), (0.2,	<(0.4, 0.5, 0.7, 0.8), (0.4,	<(0.1, 0.2, 0.4, 0.5), (0.1,
	$A_1$	0.4, 0.7, 0.8); 0.7, 0.2>	0.4, 0.6, 0.7); 0.7, 0.2>	0.3, 0.5, 0.6); 0.5, 0.4>	0.5, 0.7, 0.8); 0.5, 0.4>	0.2, 0.4, 0.5); 0.7, 0.2>
P		<(0.5, 0.6, 0.7, 0.8), (0.5,	<(0.4, 0.5, 0.6, 0.7), (0.4,	<(0.6, 0.7, 0.8, 0.9), (0.6,		
$D_2$	$A_2$	0.6, 0.7, 0.8); 0.4, 0.3>	0.5, 0.6, 0.7); 0.5, 0.4>	0.7, 0.8, 0.9); 0.4, 0.3>	0.7, 0.8, 0.9); 0.4, 0.3>	0.3, 0.6, 0.8); 0.6, 0.3>
		<(0.1, 0.2, 0.4, 0.5), (0.1,		<(0.2, 0.4, 0.6, 0.8), (0.2,	<(0.1, 0.3, 0.6, 0.8), (0.1,	
	$A_3$	0.2, 0.4, 0.5); 0.7, 0.2>	0.3, 0.5, 0.6); 0.5, 0.4>	0.4, 0.6, 0.8); 0.4, 0.3>	0.3, 0.6, 0.8); 0.6, 0.3>	0.3, 0.6, 0.7); 0.5, 0.4>
		<(0.3, 0.5, 0.7, 0.9), (0.3,		<(0.4, 0.5, 0.6, 0.7), (0.4,		
	$A_1$	(0.5, 0.7, 0.9); 0.6, 0.3>	(0.4, 0.5, 0.7, 0.8); (0.4, 0.5, 0.7, 0.8); 0.5, 0.4>	(0.4, 0.5, 0.6, 0.7); (0.4, 0.5, 0.6, 0.7); 0.5, 0.4>	(0.5, 0.4, 0.0, 0.7); (0.5, 0.4, 0.6, 0.7); 0.7, 0.2>	(0.2, 0.3, 0.5, 0.0); (0.2, 0.3, 0.5, 0.6); 0.5, 0.4>
		<(0.2, 0.3, 0.5, 0.6), (0.2,		<(0.6, 0.7, 0.8, 0.9), (0.6,		
$D_3$	$A_2$	(0.2, 0.3, 0.5, 0.0), (0.2, 0.3, 0.5, 0.6); 0.5, 0.4>				
			0.5, 0.7, 0.8); 0.5, 0.4><(0.4, 0.5, 0.7, 0.9), (0.4,	0.7, 0.8, 0.9); 0.4, 0.3><(0.3, 0.5, 0.7, 0.9), (0.3,	0.3, 0.6, 0.8; $0.6, 0.3$	0.7, 0.8, 0.9; $0.4, 0.3 >$
	$A_3$	<(0.2, 0.4, 0.6, 0.8), (0.2,				
	-	0.4, 0.6, 0.8); 0.4, 0.3>	0.5, 0.7, 0.9); 0.5, 0.4>	0.5, 0.7, 0.9); 0.6, 0.3>	0.3, 0.6, 0.8); 0.6, 0.3>	0.6, 0.7, 0.8); 0.4, 0.3>
	$A_1$	<(0.3, 0.4, 0.7, 0.8), (0.3,		<(0.1, 0.3, 0.6, 0.8), (0.1,		
	1	0.4, 0.7, 0.8); 0.7, 0.2>	0.4, 0.6, 0.7); 0.7, 0.2>	0.3, 0.6, 0.8); 0.6, 0.3>	0.5, 0.7, 0.9); 0.5, 0.4>	0.4, 0.7, 0.9); 0.4, 0.3>
$D_4$	$A_2$	<(0.2, 0.3, 0.4, 0.5), (0.2,				
4	2	0.3, 0.4, 0.5); 0.5, 0.4>	0.2, 0.3, 0.4); 0.7, 0.2>	0.4, 0.7, 0.9); 0.4, 0.3>	0.7, 0.8, 0.9); 0.4, 0.3>	0.5, 0.7, 0.8); 0.5, 0.4>
	$A_3$	<(0.5, 0.6, 0.8, 0.9), (0.5,	<(0.2, 0.3, 0.6, 0.7), (0.2,	<(0.2, 0.4, 0.6, 0.8), (0.2,		<(0.2, 0.3, 0.5, 0.6), (0.2,
	3	0.6, 0.8, 0.9); 0.4, 0.3>	0.3, 0.6, 0.7); 0.5, 0.4>	0.4, 0.6, 0.8); 0.4, 0.3>	0.7, 0.8, 0.9); 0.4, 0.3>	0.3, 0.5, 0.6); 0.5, 0.4>
	Δ	<(0.1, 0.3, 0.6, 0.8), (0.1,	<(0.2, 0.3, 0.5, 0.6), (0.2,	<(0.4, 0.5, 0.7, 0.9), (0.4,	<(0.3, 0.4, 0.6, 0.7), (0.3,	<(0.4, 0.5, 0.6, 0.7), (0.4,
	$A_1$	0.3, 0.6, 0.8); 0.6, 0.3>	0.3, 0.5, 0.6); 0.5, 0.4>	0.5, 0.7, 0.9); 0.5, 0.4>	0.4, 0.6, 0.7); 0.7, 0.2>	0.5, 0.6, 0.7); 0.5, 0.4>
$D_5$	Δ	<(0.1, 0.3, 0.5, 0.7), (0.1,	<(0.3, 0.4, 0.5, 0.6), (0.3,	<(0.4, 0.5, 0.7, 0.8), (0.4,	<(0.1, 0.2, 0.4, 0.5), (0.1,	<(0.5, 0.6, 0.8, 0.9), (0.5,
$D_5$	$A_2$	0.3, 0.5, 0.7); 0.6, 0.3>	0.4, 0.5, 0.6); 0.7, 0.2>	0.5, 0.7, 0.8); 0.5, 0.4>	0.2, 0.4, 0.5); 0.7, 0.2>	0.6, 0.8, 0.9); 0.4, 0.3>
	Δ	<(0.4, 0.5, 0.6, 0.7), (0.4,	<(0.2, 0.4, 0.7, 0.9), (0.2,	<(0.5, 0.6, 0.8, 0.9), (0.5,	<(0.2, 0.3, 0.6, 0.7), (0.2,	<(0.2, 0.4, 0.6, 0.8), (0.2,
	$A_3$	0.5, 0.6, 0.7); 0.5, 0.4>	0.4, 0.7, 0.9); 0.4, 0.3>	0.6, 0.8, 0.9); 0.4, 0.3>	0.3, 0.6, 0.7); 0.5, 0.4>	0.4, 0.6, 0.8); 0.4, 0.3>
		<(0.2, 0.4, 07, 0.9), (0.2,	<(0.4, 0.5, 0.7, 0.9), (0.4,	<(0.2, 0.3, 0.4, 0.5), (0.2,	<(0.2, 0.3, 0.5, 0.6), (0.2,	<(0.3, 0.4, 0.6, 0.7), (0.3,
	$A_1$	0.4, 0.7, 0.9); 0.4, 0.3>	0.5, 0.7, 0.9); 0.5, 0.4>	0.3, 0.4, 0.5); 0.5, 0.4>	0.3, 0.5, 0.6); 0.5, 0.4>	0.4, 0.6, 0.7); 0.7, 0.2>
D	4	<(0.6, 0.7, 0.8, 0.9), (0.6,	<(0.6, 0.7, 0.8, 0.9), (0.6,	<(0.1, 0.3, 0.6, 0.8), (0.1,	<(0.3, 0.4, 0.7, 0.8), (0.3,	<(0.4, 0.5, 0.7, 0.8), (0.4,
$D_6$	$A_2$	0.7, 0.8, 0.9); 0.4, 0.3>	0.7, 0.8, 0.9); 0.4, 0.3>	0.3, 0.6, 0.8); 0.6, 0.3>	0.4, 0.7, 0.8); 0.7, 0.2>	0.5, 0.7, 0.8); 0.5, 0.4>
	4	<(0.6, 0.7, 0.8, 0.9), (0.6,	<(0.1, 0.2, 0.5, 0.6), (0.1,	<(0.3, 0.5, 0.7, 0.9), (0.3,	<(0.1, 0.3, 0.6, 0.8), (0.1,	<(0.2, 0.3, 0.4, 0.5), (0.2,
	$A_3$	0.7, 0.8, 0.9); 0.4, 0.3>	0.2, 0.5, 0.6); 0.7, 0.2>	0.5, 0.7, 0.9); 0.6, 0.3>	0.3, 0.6, 0.8); 0.6, 0.3>	0.3, 0.4, 0.5); 0.5, 0.4>
		<(0.3, 0.4, 0.5, 0.6), (0.3,	<(0.2, 0.3, 0.6, 0.7), (0.2,	<(0.2, 0.3, 0.5, 0.6), (0.2,	<(0.2, 0.3, 0.5, 0.6), (0.2,	<(0.1, 0.3, 0.6, 0.8), (0.1,
	$A_1$	0.4, 0.5, 0.6); 0.7, 0.2>	0.3, 0.6, 0.7); 0.5, 0.4>	0.3, 0.5, 0.6); 0.5, 0.4>	0.3, 0.5, 0.6); 0.5, 0.4>	0.3, 0.6, 0.8); 0.6, 0.3>
_					<(0.1, 0.2, 0.3, 0.4), (0.1,	
$D_7$	$A_2$	0.4, 0.6, 0.7); 0.7, 0.2>	0.5, 0.7, 0.8); 0.5, 0.4>	0.3, 0.5, 0.7); 0.6, 0.3>	0.2, 0.3, 0.4); 0.7, 0.2>	0.3, 0.6, 0.8); 0.6, 0.3>
		<(0.3, 0.4, 0.6, 0.7), (0.3,			<(0.5, 0.6, 0.8, 0.9), (0.5,	<(0.2, 0.3, 0.5, 0.6), (0.2,
	$A_3$	0.4, 0.6, 0.7); 0.7, 0.2>	0.3, 0.6, 0.8); 0.6, 0.3>	0.4, 0.7, 0.8); 0.7, 0.2>	0.6, 0.8, 0.9); 0.4, 0.3>	0.3, 0.5, 0.6); 0.5, 0.4>
		<(0.3, 0.5, 0.7, 0.9), (0.3,	<(0.2, 0.4, 0.7, 0.9), (0.2,	<(0.2, 0.4, 0.6, 0.8), (0.2,		<(0.5, 0.6, 0.7, 0.8), (0.5,
	$A_1$	(0.5, 0.7, 0.9); (0.6, 0.3)	(0.2, 0.4, 0.7, 0.9); (0.2, 0.4, 0.7, 0.9); 0.4, 0.3>	(0.2, 0.4, 0.0, 0.0), (0.2, 0.4, 0.6, 0.8); 0.4, 0.3>	(0.2, 0.4, 07, 0.9); (0.2, 0.4, 0.7, 0.9); 0.4, 0.3>	(0.5, 0.6, 0.7, 0.8); (0.5, 0.6, 0.7, 0.8); (0.4, 0.3>
		$<(04\ 05\ 07\ 08)\ (04$		<(0.2, 0.3, 0.4, 0.5), (0.2,		
$D_8$	$A_2$	0.5, 0.7, 0.8); 0.5, 0.4>	0.5, 0.7, 0.9); 0.5, 0.4>	0.3, 0.4, 0.5); 0.5, 0.4>	0.3, 0.5, 0.6); 0.5, 0.4>	0.4, 0.7, 0.8); 0.7, 0.2>
		<(0.6, 0.7, 0.8, 0.9), (0.6,		<(0.3, 0.4, 0.7, 0.8), (0.3,		
	$A_3$	0.7, 0.8, 0.9); 0.4, 0.3>	(0.1, 0.5, 0.0, 0.7); (0.1, 0.5, 0.6, 0.7); 0.5, 0.4>	0.4, 0.7, 0.8); 0.7, 0.2>	0.4, 0.6, 0.7); 0.7, 0.2>	0.2, 0.4, 0.5); 0.7, 0.2>
	$A_1$	<(0.5, 0.6, 0.8, 0.9), (0.5,		<(0.3, 0.4, 0.5, 0.6), (0.3,		
	1	0.6, 0.8, 0.9); 0.4, 0.3>	0.3, 0.6, 0.7); 0.5, 0.4>	0.4, 0.5, 0.6); 0.7, 0.2>	0.5, 0.7, 0.8); 0.5, 0.4>	0.2, 0.5, 0.6); 0.7, 0.2>
$D_9$	$A_2$	<(0.1, 0.2, 0.3, 0.4), (0.1,		<(0.6, 0.7, 0.8, 0.9), (0.6,	<(0.4, 0.5, 0.7, 0.9), (0.4,	<(0.4, 0.5, 0.7, 0.8), (0.4,
, i	-	0.2, 0.3, 0.4); 0.7, 0.2>	0.2, 0.4, 0.5); 0.7, 0.2>	0.7, 0.8, 0.9); 0.4, 0.3>	0.5, 0.7, 0.9); 0.5, 0.4>	0.5, 0.7, 0.8); 0.5, 0.4>
	$A_3$	<(0.5, 0.6, 0.7, 0.8), (0.5,		<(0.6, 0.7, 0.8, 0.9), (0.6,		
	3	0.6, 0.7, 0.8); 0.4, 0.3>	0.6, 0.8, 0.9); 0.4, 0.3>	0.7, 0.8, 0.9); 0.4, 0.3>	0.4, 0.5, 0.6); 0.7, 0.2>	0.3, 0.6, 0.8); 0.6, 0.3>
	Α	<(0.2, 0.3, 0.6, 0.7), (0.2,		<(0.2, 0.4, 0.6, 0.8), (0.2,		<(0.2, 0.3, 0.5, 0.6), (0.2,
	$A_1$	0.3, 0.6, 0.7); 0.5, 0.4>	0.4, 0.7, 0.9); 0.4, 0.3>	0.4, 0.6, 0.8); 0.4, 0.3>	0.4, 0.7, 0.8); 0.7, 0.2>	0.3, 0.5, 0.6); 0.5, 0.4>
D	$A_2$	<(0.4, 0.5, 0.6, 0.7), (0.4,		<(0.1, 0.2, 0.5, 0.6), (0.1,		<(0.3, 0.5, 0.7, 0.9), (0.3,
L 10	- 12	0.5, 0.6, 0.7); 0.5, 0.4>	0.3, 0.5, 0.7); 0.6, 0.3>	0.2, 0.5, 0.6); 0.7, 0.2>	0.5, 0.7, 0.8); 0.5, 0.4>	0.5, 0.7, 0.9); 0.6, 0.3>
	$A_3$	<(0.6, 0.7, 0.8, 0.9), (0.6,		<(0.1, 0.3, 0.5, 0.7), (0.1,		
	3	0.7, 0.8, 0.9); 0.4, 0.3>	0.5, 0.7, 0.8); 0.5, 0.4>	0.3, 0.5, 0.7); 0.6, 0.3>	0.3, 0.6, 0.8); 0.6, 0.3>	0.3, 0.4, 0.5); 0.5, 0.4>

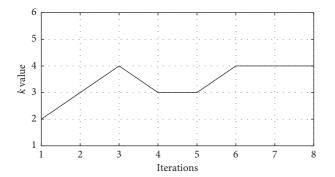


FIGURE 2: Curve of varying k value in the aggregation procedure of scheme  $A_1$ .

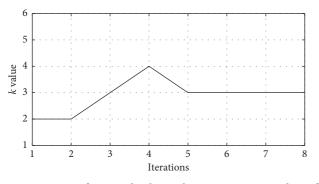


FIGURE 3: Curve of varying k value in the aggregation procedure of scheme  $A_2$ .

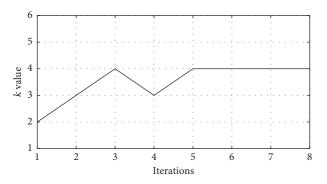


FIGURE 4: Curve of varying k value in the aggregation procedure of scheme  $A_3$ .

TABLE 4: Ranking of schemes and computational time.

Method	Ranking of schemes	Computational time (s)
Our proposed method	$A_2 \succ A_1 \succ A_3$	18.029
Zhao [23]	$A_2 \succ A_1 \succ A_3$	74.332
Liu & Chen [22]	$A_1 \succ A_3 \succ A_2$	5.956
Yuan & Li [20]	$A_2 \succ A_3 \succ A_1$	19.353
[10]	$A_2 \succ A_3 \succ A_1$	33.251

collective overall values. The characteristic of the aggregation is on the basis of attribute independence and decisionmakers' risk preference. The preferred scheme is scheme  $A_2$ , which is the same as our proposed method, but the alternatives are different from those obtained by our proposed method. The computational time of this method is medium among the five algorithms.

The performance of the proposed method is better than the other methods analysed in this study on the basis of the clustering concept. In addition, the preferred scheme by our method is the same as that by Yuan and Li's method. Furthermore, the proposed method is apprehensible and feasible to solve the MAGDM problems. It enriches the theory of solving MAGDM problems based on clustering algorithms.

## 5. Conclusion and Further Research

In this study, we proposed a MAGDM method using a genetic K-means clustering algorithm. The simulation results demonstrate that our proposed method is apprehensible and feasible to solve MAGDM problems. In the future, we will use the proposed MAGDM method to deal with other types of decision information, such as (a) trapezoidal fuzzy numbers in consensus models for MAGDM under trapezoidal fuzzy numbers environment; (b) hesitant fuzzy linguistic information in additiveconsensus of hesitant fuzzy linguistic preference relation with a new expansion principle for hesitant fuzzy linguistic term sets local, and hesitant fuzzy linguistic information in feedback mechanism based on consensus-derived for consensus building in MAGDM with hesitant fuzzy linguistic preference relations; and (c) linguistic fuzzy information in algorithms for improving additive consensus of linguistic preference relations with an integer optimization model.

## **Data Availability**

The simulation data used to support the findings of this study are included within the article.

## **Conflicts of Interest**

The authors declare that they have no conflicts of interest.

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