Research Article

Self-Operated Store or Franchised Store? Optimal Decisions for Online-to-Offline Supply Chain with a Demand Shift

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Received 9 September 2020; Revised 7 November 2020; Accepted 27 November 2020; Published 15 December 2020

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Mutual shifts in offline and online demand have become the norm in supply chain operations. The online-to-offline (O2O) supply chain system consists of a platform vendor, a physical store, and a product. The platform vendor sells the product directly online and governs either the centralized decision-making of a self-operated store or the decentralized decision-making of a franchised store offline. In this study, supply chain decision models with and without demand shifts are constructed to obtain optimal wholesale and selling prices and to maximize profit. The coordination mechanism under decentralized decision-making is designed to optimize the O2O supply chain, and the validity and applicability of the model are verified by numerical simulation. Results show that, regardless of whether a store is self-operated or franchised, the total profit of the system increases, and online and offline prices depend on a range of demand shifts. With an increased proportion of online demand shifts, the offline selling price and total profit of the system increase, whereas the online selling price and profit of the platform vendor decrease under decentralized decision-making. When the fixed transfer payment fee is within a certain range, a two-part-tariff contract can effectively coordinate the supply chain. This study not only contributes to the theoretical literature on O2O supply chain systems but also provides practical decision-making support for managers.

1. Introduction

Recent developments in e-commerce have made it very convenient for customers to buy commodities and services. Therefore, online-to-offline (O2O) sales are a widespread phenomenon. In the e-commerce era, many traditional enterprises have invested in the Internet, and substantial enterprises engage in both online and offline sales to achieve sustainable development [1]. In 2015, the merger of Meituan and Dianping became the hottest topic in discussions of industry on the Internet; this merger also pushed O2O to the forefront. Freshhema, Dianping, Ganji, and Koubei have all embraced the O2O business model. In this connection, researchers have designed a sustainable supply chain that considers life cycle assessment [2] and analyzed social welfare and price competition in a mixed duopoly [3]. At the same time, customers have shown an increasing preference for the diversification and individualization of products, causing companies to pursue a competitive advantage through adopting differentiated strategies.

The traditional dual-channel model mainly involves the traditional e-commerce model of an offline physical channel and an online network channel. Users passively accept the system, and the relationship between partners is a game relationship. The O2O model pays more attention to the integration of online and offline channels, integrates online and offline resources for collaborative development, and enables users to participate in the whole e-commerce process. Offline experiences feed online resources, and reciprocally, online resources invigorate offline experiences. The relationship between the partners creates an ecosystem of win-win cooperation. The goal of supply chain management is hence no longer static coordination, but rather dynamic continuous optimization and coordination.
As O2O business modes and logistics technologies continue to develop, retail firms have become deeply integrated with the Internet in terms of internal operations, services, and product sales. An increasing number of firms are combining resources to achieve optimized resource allocation. For example, firms such as Suning, GOME, Jetsetter, Uniqlo, JD.com, Didi Taxi, Tencent, and Airbnb, having adopted dual-channel practices, now coordinate decision-making issues that mainly involve pricing systems, channel conflicts, inventory strategies, and consumer preferences. Zhang et al. [4] examined pricing decisions and services in a dual-channel supply chain. Madani and Rasti-Barzoki [5] discovered that subsidy rates benefit the sustainability of supply chains. Optimal distribution strategies were studied by Li et al. [6]. Research aiming to improve understanding of the O2O process was also evaluated by Reefke and Sundaram [7].

The mutual shift in offline and online demand has become normalized, affecting the decision-making of O2O players. The Internet has served as a new marketing mode, and more and more customers buy goods and services through the Internet as it grows in popularity. Major businesses have used a combination of online shopping malls and offline experience stores or self-built APPs (mobile client applications) to promote the integration of “online + offline” resources. Uniqlo provides a two-dimensional code scanning service and offline store query system for its physical stores through its APP and allows consumers to pay for products online and pick them up from offline stores. Uniqlo thus diverts consumers from online to offline resources and promotes the development of the O2O mode with its strong store network and high-quality offline retail services. As an online shopping platform trusted by consumers, JD.com has maintained deep cooperation with the Yonghui Supermarket in realizing O2O operations in China, successfully bringing online consumers to offline physical stores for consumption. The company has further enhanced its customers’ experience by better serving offline customers through both direct and franchised stores. Mettersbonwe, a casual apparel brand, has established offline experience stores, effectively operated direct and franchised stores, and shared online and offline resources. The two-dimensional code and computer terminal area available through the Suning.com store also supports free online and offline orders, while adopting the same price strategy for online and offline sales to achieve two-way drainage. In 2019, the total transaction volume of Tianmao’s shopping carnival “Double Eleven” reached approximately $38.3 billion, reflecting annual growth of 26%.

In analyzing O2O supply chain systems, three issues must be addressed:

1. Under the O2O business model, ways of developing an optimized game decision model to maximize profit for platform vendors remain unexplored. Further research is needed to understand the effects of consumer demand shift behaviors.

2. The dominant strategy of a platform vendor (i.e., self-operated versus franchised stores) should be based on the demand shift proportion. Coordinating online and offline prices and business strategies to cope with market demand shifts is critical to improving overall performance.

3. In the real market, game players compete for a market share for their own benefit. It is thus essential to determine which channels should be selected to improve economic efficiency.

The present study makes three contributions to research on selling as a part of business operations. First, it examines the influence of self-operated versus franchised stores on optimal performance, further enhancing understanding of online and offline selling strategies. Second, we explore how consumer demand shifts affect merchants’ sales strategies. Configurations of products lead to a differentiation between online and offline prices, making it more difficult to draw product price comparisons and to keep shift effects aligned with the intentions of the business. Finally, a coordination mechanism is designed to optimize the O2O supply chain. The present work can thus help companies make operational decisions as they practice emerging forms of O2O commerce, improving economic performance through marketing functions.

The general framework of this article is as follows. A literature review is given in Section 2, and model assumptions are provided in Section 3. In Sections 4 and 5, O2O supply chain decision models involving a shift in demand versus no shift in demand are constructed. In Section 6, a two-part-tariff contract is designed, and numerical simulation analyses are given in Section 7. Finally, Section 8 concludes.

2. Literature Review

In recent decades, many scholars and practitioners have been concerned with sustainable operations management, with researchers focusing on this domain as a means to expand knowledge about supply chains. A smaller body of research has been devoted to issues of supply chain management for self-operated versus franchised stores. In particular, in connection with O2O supply chain collaboration, the issues of consumer demand, channel selection, and the profit of game players have been discussed. Two main streams of research on this subject can be described as follows.

In one of the research streams, the literature has focused on pricing, coordination, operations management, and channel selection in supply chains. Matsumura and Ogawa [8] focused on quantity contracts and prices in a mixed duopoly. He et al. [9] looked at pricing and inventory decisions of perishable products in a supply chain. The products’ deterioration rate has significant impacts on pricing and inventory decisions. Cheema and Papatla [10] examined the importance of Internet experiences and product categories and compared offline to online information. Chintagunta et al. [11], providing several metrics, investigated the quantification of costs when consumers move from online to offline to purchase groceries. Hua et al.
[12] analyzed lead time decisions made in an online direct channel versus traditional channels. A two-stage optimization technique is used for centralized and decentralized decision-making. Lu and Liu [13] proposed that young people who lack shopping experience via traditional channels are more likely to make purchases online. Panda et al. [14] analyzed coordination and distribution strategies that consider corporate social responsibility. Zhang et al. [15] looked at how channel structure affects pricing decisions and how to choose the optimal channel structure accordingly. A purely offline retailer might open an online channel with little demand, just to get a wholesale price reduction.

Meanwhile, Pei and Yan [16] showed that mechanisms can be used effectively to motivate information sharing and to ensure the impact of an e-tailer’s return policy. Zhang and Wang [17] studied how a firm’s fairness concerns influence three-party supply chain coordination, showing through comparisons how online and traditional channel equilibrium prices, issues of demand, and the common retailer’s profit are managed. Ji et al. [18] discussed initial carbon allowance allocation rules resulting from the O2O retail supply chain. Their results show that government decision-making plays a key role in sustainable development. Luo et al. [19] explored retailers’ product choice decisions in the face of heterogeneous customer demands in the context of retail supply chain management. A revenue sharing contract is designed to achieve Pareto improvement. He et al. [20] focused on government subsidies and channel structure in a closed-loop supply chain. Governments can encourage manufacturers to adopt the expected channel structure by setting different levels of subsidies. Wang et al. [21] established different decision models in a fuzzy supply chain environment. He et al. [22] looked at corporate social responsibility (CSR) in an O2O tourism supply chain (TSC). TSCs should pay more attention to CSR and increase the fit between consumer channels and resale channels so as to create more total utility.

In the other research stream, scholars have considered demand-related decision problems encountered in supply chains. Liu et al. [23] developed a newsvendor model to explore channel coordination and returns handling decisions with demand uncertainty. When the consumer’s return handling cost is increased, the expected market demand will be reduced. Cao et al. [24] focused on distribution channel choice under conditions of demand uncertainty. They found that manufacturers can distribute more goods through their own retail stores and that independent retailers are optimally used when manufacturers sell highly substitutable products. While considering the cost of product returns, Ofek et al. [25] analyzed and compared prices and the amount of profit generated when dealing with monopolies versus duopolies. The demands of consumers differ with respect to online channels and physical store channels. If there is a difference in the likelihood of a return for different products, the retailer may only offer products with a lower return probability online and restrict the sale of risky products to traditional stores. Hu and Lu [26] developed a model for e-tailing and retailing distribution channels, analyzing the influence of demand uncertainty on expected profit and retail services using a stochastic comparison method. Balakrishnan et al. [27] looked at the effects of browse-and-switch behaviors on online pricing strategies and established an economic model in light of these effects. In this case, customers first visit a brick-and-mortar retail store to examine a product, and then decide whether to purchase it online or in store. Gao and Su [28] discussed the effects of buying products online and picking them up in store; they found that, although this channel-switching behavior can increase customer flow, the shift from online to offline store fulfillment may reduce profit margins. Cao et al. [29] analyzed the effects of an “online-to-store” channel on retailers’ profitability and demand allocations. Such channel-switching behavior may generate additional demand while increasing retailers’ operating costs. Thus, the retailer must select products for this channel carefully.

For their part, Wang et al. [30] discussed the effect of demand uncertainty and price competition on sustainable supply chains. Long and Shi [31] focused on pricing strategies used by an online travel agency and a tour operator in an O2O model. The corresponding offline and online demand functions are described, and cooperative and non-cooperative pricing equilibria are obtained from a Stackelberg and Bertrand game. Jing [32] examined the competition between online and traditional retailers in the context of showrooming. The demand for traditional retailers increases when an online store is opened. A return policy set by an online retailer relaxes competition but slightly reduces its demand. In 2019, Zhang et al. [33] discussed the “preorder-online, pickup-in-store” (POPU) strategy in a competition versus a monopoly case. The online and offline shoppers’ demand is variable in different cases.

To facilitate comparison of this prior research with the present study and highlight what is innovative about this paper, Table 1 shows how the current study relates to recently published scholarship on demand-related decision problems.

In sum, the related literature has focused on channel selection and pricing decisions in O2O supply chains. In the context of upstream suppliers, the nature of shifts in consumer demand among game players has not been quantitatively researched. To address this gap, the present paper explores e-commerce both with shifts and without shifts in demand as a context, establishes an O2O supply chain decision-making model for offline experience stores (including the platform vendor’s self-operated store as well as franchised stores), and designs a two-part-tariff contract. The study’s conclusions contribute to theoretical discussions of O2O supply chain systems, while providing decision-making support for managers and suggesting how sustainable performance can be enhanced through marketing functions.

3. Model Assumptions

This paper explores a supply chain model in which a product is sold by a platform vendor and also in an offline experience store. The offline experience store can take two forms: a self-operated store (centralized decision-making) and a
franchised store (decentralized decision-making). The platform vendor sells products to the offline franchised store. The wholesale price is \( w \) and the unit production cost is \( c \). The vendor sells products to consumers through online direct sales and offline franchising (self-employed) channels. See Table 2 for further notation and explanations.

Online costs include distribution costs, platform operating costs, advertising and promotion costs, and others. Offline costs include facility costs, store rent, labor costs, decoration costs, and others. In general, the online selling price is lower than the offline selling price. Referring to the previous literature, we assume that the retail prices of different channels are different, with such price differentiation being a common business behavior. Consumers learn about the functions, prices, materials, and fabrication processes of products from online platforms. Then, in order to avoid returns caused by mismatched products, they often go to offline physical stores to experience and purchase the matching products. This behavioral pattern constitutes the phenomenon of consumer demand shift from online to offline shopping. The demand shift proportion \( \theta \) is determined by the market. The platform vendor will keep statistical records about consumer choices, formulate marketing strategies according to their desires, and guide consumers through channel-switching practices as necessary.

As assumed in previous studies (see Wang et al. [30]; Wei and Chen [34]), for cases involving no shift in consumer demand, the consumer’s demand function via the online channel is given by \( q_o = \mu a - \beta p_o + \gamma p_i \), while the consumer’s demand function via the offline channel is given by \( q_s = (1 - \mu) a - \beta p_s + \gamma p_o \), where \( \mu \in [0, 1] \). Following previous research (see Wei and Chen [34]), the authors assume that \( 0 < \gamma < \beta \).

In the case of a shift in consumer demand, the demand function via the online channel is given by \( q_{o1} = \mu a - \beta p_o + \gamma p_i - \theta \beta \). The demand function via the offline channel is then \( q_{s1} = (1 - \mu) a - \beta p_s + \gamma p_o + \theta \beta \), where \( \theta \in [\mu - 1, \mu] \) is the demand shift proportion and \( \theta \neq 0, \theta > 0 \) indicates that product demand shifts from online to offline, and \( \theta < 0 \) indicates that product demand shifts from offline to online. To make the model more intuitive and consistent with reality, in the O2O supply chain model, the demand shift proportion is taken as an endogenous variable; this allows the analyst to explore the impact of the demand shift proportion on price, demand, and profit. In addition, a numerical simulation analysis is carried out to verify the applicability and effectiveness of the model.

In keeping with real market conditions, the following necessary assumptions are made: \( c > 0, q_o > 0, q_s > 0, \) and \( c < w < p_i \), where \( i = o, s \). These assumptions imply that the unit production cost and selling prices are non-negative. However, the unit production cost must be lower than the wholesale and sales prices. Figure 1 describes the model; 1(a) shows the scenario for a self-operated store, while 1(b) shows the scenario for a franchised store.

### 4. An O2O Supply Chain Decision Model without a Demand Shift

#### 4.1. The Offline Store as the Self-Operated Store of the Platform Vendor

This scenario is regarded as the centralized decision-making model for the O2O supply chain (Zhang et al. [4]; Pei and Yan [16]). The goal is to set appropriate prices that benefit the entire system.

The profit of the O2O supply chain is given by

\[
\pi_r = (p_o - c)q_o + (p_s - c)q_s.
\]

The platform vendor sets the product price. The first group is designated as the profit made by the platform vendor through online direct sales, and the second group is defined as the platform vendor’s profit from offline sales. That is,

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### Table 1: Recently published studies on demand-related decision problems.

<table>
<thead>
<tr>
<th>Authors (year)</th>
<th>Demand model</th>
<th>Decision variables</th>
<th>Situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cao et al. [24]</td>
<td>Linear demand function</td>
<td>Market demand; unit retailing cost</td>
<td>Channel stability analysis; the equilibrium channel structures</td>
</tr>
<tr>
<td>Ofek et al. [25]</td>
<td>Hotelling model</td>
<td>Pricing and service</td>
<td>Channel strategy of retailers; the impact of product returns</td>
</tr>
<tr>
<td>Gao and Su [28]</td>
<td>Random demand</td>
<td>Market demand; the hassle cost</td>
<td>Channel switching behavior; customers strategically choose channels</td>
</tr>
<tr>
<td>Zhang et al. [33]</td>
<td>POPU model based on consumer surplus</td>
<td>Shoppers’ demand; unit travel cost</td>
<td>“Preorder-online, pickup-in-store” (POPU) strategy</td>
</tr>
<tr>
<td>Liu et al. [23]</td>
<td>Stochastic model</td>
<td>Ordering</td>
<td>Returns handling strategy, and coordination</td>
</tr>
<tr>
<td>This study</td>
<td>Stackelberg game model</td>
<td>Demand shift proportion; pricing</td>
<td>The impact of demand shift; coordination strategy</td>
</tr>
</tbody>
</table>

#### Table 2: Detailed notation and explanations.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w )</td>
<td>Franchisee’s unit wholesale price</td>
</tr>
<tr>
<td>( \beta )</td>
<td>Self-price sensitivity coefficient</td>
</tr>
<tr>
<td>( p_i )</td>
<td>Unit sale price through offline channel</td>
</tr>
<tr>
<td>( p_o )</td>
<td>Unit sale price through online channel</td>
</tr>
<tr>
<td>( c )</td>
<td>Platform vendor’s unit production cost</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Consumers’ online preference coefficient</td>
</tr>
<tr>
<td>( a )</td>
<td>Maximum market size</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Cross-price sensitivity coefficient</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Demand shift proportion</td>
</tr>
</tbody>
</table>
\[ \pi_c = (p_o - c) (\mu a - \beta p_o + \gamma p_s) + (p_s - c) [(1 - \mu) a - \beta p_s + \gamma p_o]. \]  

Proposition 1. Under the sales strategy of self-operated stores maintained by the platform vendor, the optimal offline price \( p_{s}^{*} \), online price \( p_{o}^{*} \), and O2O supply chain profit \( \pi_c \) are, respectively,

\[ p_{s}^{*} = \frac{c \beta + c \gamma^2 - a \beta \mu + a \gamma \mu}{2(\beta^2 - \gamma^2)}, \]
\[ p_{o}^{*} = \frac{c \beta^2 + a \gamma - c \gamma^2 + a \beta \mu - a \gamma \mu}{2(\beta^2 - \gamma^2)}, \]
\[ \pi_c = \frac{1}{4(\beta^2 - \gamma^2)} \left\{ a^2 \left[ -2 \gamma (-1 + \mu) + \beta (1 - 2 \mu + 2 \mu^2) \right] + 2 a c (\gamma^2 + \beta \gamma) \right\}. \]

Proposition 1 shows that there are optimal, unique offline and online prices for the platform vendor. The managerial implication of this proposition is as follows: for the platform vendor, under the sales strategy of self-operated stores, setting the optimal selling price not only benefits the vendor, but also maximizes the profit of the O2O supply chain as a whole. This result leads us to Lemma 1.

Lemma 1. Under the sales strategy of a self-operated store, the optimal online price \( p_{s}^{*} \) monotonically increases around consumers' online preference coefficient \( \mu \). The optimal offline price \( p_{o}^{*} \) monotonically decreases around consumers' online preference coefficient \( \mu \).

At the same time, prices in offline physical stores will be reduced, guiding consumers to buy the product in physical stores. Business managers should reasonably control online and offline demand to maintain a balance of demand in different channels. In addition, the rates of online and offline price variation are equal but trend in opposing directions, in line with actual market operations. Thus, business managers should adopt appropriate marketing strategies to promote market demand over different channels.

Lemma 2. The O2O supply chain’s profit \( \pi_c \) is a strictly differentiable concave function at \( (p_s, p_o) \).
4.2. The Offline Store as the Franchised Store of the Platform Vendor. This scenario is regarded as a decentralized decision-making model of the O2O supply chain. A Stackelberg game model treats this scenario as a type of non-cooperative game. The offline store is the franchisee's store where the franchisee and platform vendor make independent decisions to pursue profit maximization.

The profit of the platform vendor is given by

\[ \pi_o = (p_o - c)(\mu a - p_x + \gamma p_o) + (w - c)(1 - \mu) a - \beta p_x + \gamma p_o. \]  

That is,

\[ \pi_o = (p_o - c)(\mu a - \beta p_x + \gamma p_o) + (w - c)(1 - \mu) a - \beta p_x + \gamma p_o. \]  

Proposition 2. Under the sales strategy of a franchised store, the optimal offline price \( p^d_o \), online price \( p^d \), unit wholesale price \( w^d \), platform vendor profit \( \pi_o \), and franchisee's profit \( \pi_s \) are, respectively,

\[
\begin{align*}
  p^d_o &= \frac{c(\beta - \gamma)(\beta + \gamma)^2 + a(\gamma^2 - 3\beta^2)(\mu - 1) + 2\beta\gamma\mu}{4(\beta^2 - \beta\gamma^2)}, \\
  p^d &= \frac{c\beta^2 + ay - cy^2 + a\beta\mu - a\gamma\mu}{2(\beta^2 - \gamma^2)}, \\
  w^d &= \frac{a\beta + c\beta^2 - cy^2 - a\beta\mu + a\gamma\mu}{2(\beta^2 - \gamma^2)}, \\
  \pi_o &= \frac{1}{8(\beta^2 - \beta\gamma^2)} \left\{ \left[ c^2 (\beta - \gamma)^2 (3\beta^2 + 4\beta y + \gamma^2) - 2ac(\beta^2 - \gamma^2) (\beta + y + \beta\mu - \gamma\mu) 
  + a^2 (\gamma^2 (\mu - 1)^2 - 4\beta\gamma\mu (\mu - 1) + \beta^2 (1 - 2\mu + 3\mu^2)) \right] \right\} \\
  \pi_s &= \frac{(c\beta - cy + a\mu - a)^2}{16\beta}.
\end{align*}
\]

This proposition shows that there are unique optimal prices and unit wholesale prices. Under the sales strategy of a franchised store, setting the optimal selling price is beneficial not only to the platform vendor, but also to the franchisee.

Proposition 3. For the scenario involving no shift in demand, comparing online and offline prices for centralized and decentralized decision-making yields the following relation:

\[
\begin{align*}
p^d_o &= p^d, & \text{if } \gamma < \beta < \gamma + \frac{a(1 - \mu)}{c}, \\
p^d_o &= p^d, & \text{if } \gamma > \beta + \frac{a(1 - \mu)}{c}.
\end{align*}
\]

Proposition 3 indicates that consumers do not need to consider whether the platform vendor adopts the self-operation mode or franchising mode, as the online sales price remains unchanged. However, for products sold through the offline store, when the price sensitivity coefficient of the products is low, the prices of products sold by the franchisee are higher than the prices of products sold through the platform vendor’s self-operated store. Business managers should therefore consider the influence of the self-price sensitivity coefficient.

For the self-operated and franchised stores of the platform vendor, comparing the profit margins of the O2O supply chain gives \( \Delta \pi = \pi_c - \pi_d = [c(\beta - \gamma) + a(-1 + \mu)]^2 / 16\beta > 0 \). Thus, more profit can be obtained under the centralized decision-making mode. In other words, the self-operated store of the platform vendor is more competitive. Under the decentralized decision-making mode, due to the existence of a double marginal effect, and given that each player is pursuing profit maximization, the O2O supply chain cannot be coordinated, and overall operational efficiency is reduced.
5. The O2O Supply Chain Decision Model with a Demand Shift

5.1. The Offline Store as the Self-Operated Store of the Platform Vendor. This scenario is regarded as the centralized decision-making model of the O2O supply chain. The goal is to set an appropriate price to maximize the benefits of the entire system.

The overall profit of the O2O supply chain is

\[ \pi_1 = (p_o - c)q_o + (p_s - c)q_s. \]  

That is,

\[ \pi_1 = (p_o - c)[\mu a - \beta p_o + \gamma p_s - a\theta] + (p_s - c)[(1 - \mu)a - \beta p_s + \gamma p_o + a\theta]. \]  

Proposition 4. For the scenario involving a shift in demand, under the sales strategy of the platform vendor’s self-operated store, the optimal demand shift proportion \( \theta^* \), offline price \( p_{o1}^* \), online price \( p_{s1}^* \), and O2O supply chain profit \( \pi_{c1} \) are, respectively,

\[ \theta^* = \mu - \frac{1}{2} \]

\[ p_{o1}^* = \frac{a\beta + c\beta^2 - cy^2 + a\beta\theta - a\gamma\theta - a\mu + ay\mu}{2(\beta^2 - y^2)}. \]

\[ p_{s1}^* = \frac{c\beta^2 + a\gamma - cy^2 + a\beta\theta + a\gamma\theta + a\mu - ay\mu}{2(\beta^2 - y^2)}. \]

\[ \pi_{c1} = \frac{1}{4(\beta^2 - y^2)} \left[ -2\alpha^2 \gamma (\theta^2 - 2\theta\mu - \mu^2) + 2ac(-\beta^2 + \gamma^2) - 2c^2(\beta^2 - y^2)(-\beta + \gamma) + a^2 \beta \left[ 1 + 2\beta^2 + \theta(2 - 4\mu) - 2\mu + 2\mu^2 \right] \right]. \]

By substituting the optimal solution \( \theta^* \) of \( \theta \) into \( p_{o1}^* \), \( p_{s1}^* \), \( \pi_{c1} \), a simplified expression can be obtained. To facilitate the following discussion, this simplified expression of the proposition is adopted. Although the proof of Proposition 4 is omitted for brevity, this proposition shows that there is an optimal demand shift proportion, which maximizes revenue for the platform vendor. The platform vendor guides consumers to engage in channel-switching behavior by adopting appropriate online-to-offline marketing strategies, such as “online order, offline pickup,” “online learning, offline purchase,” and so on.

5.2. The Offline Store as the Franchised Store of the Platform Vendor. This scenario is regarded as the decentralized decision-making model of the O2O supply chain. A Stackelberg game model considers this scenario as a type of non-cooperative game. The offline store is the franchisee’s store, and the franchisee and platform vendor independently make decisions in pursuit of profit maximization (Balakrishnan et al. [27]).

The profit of the platform vendor is given by

\[ \pi_{o1} = (p_o - c)q_o + (w - c)q_s. \]

That is,

\[ \pi_{o1} = (p_o - c)[\mu a - \beta p_o + \gamma p_s - a\theta] + (w - c)[(1 - \mu)a - \beta p_s + \gamma p_o + a\theta]. \]  

The profit of the franchisee is \( \pi_{s1} = (p_s - w)q_s. \)

That is,

\[ \pi_{s1} = (p_s - w)[(1 - \mu)a - \beta p_s + \gamma p_o + a\theta]. \]  

The overall profit of the O2O supply chain is then

\[ \pi_{d1} = \pi_{o1} + \pi_{s1} = (p_o - c)q_o + (p_s - c)q_s. \]  

Proposition 5. In the scenario involving a shift in demand, under the sales strategy of the platform vendor’s franchised store, the optimal demand shift proportion \( \theta^d \), online price \( p_{o1}^d \), optimal offline price \( p_{s1}^d \), unit wholesale price \( w_{d1} \), platform vendor profit \( \pi_{o1}^d \), and franchisee’s profit \( \pi_{s1}^d \) are, respectively,
Proposition 6. In the scenario involving a shift in demand, comparing online and offline prices for centralized and decentralized decision-making gives the following relation:

\[
p^d_{oi} = p^c_{oi};
\]

\[
p^d_{si} > p^c_{si}, \quad \text{if } \gamma < \beta < \gamma + \frac{a(1 + \theta - \mu)}{c},
\]

\[
p^d_{si} < p^c_{si}, \quad \text{if } \beta > \gamma + \frac{a(1 + \theta - \mu)}{c}.
\] (15)

As observed from Proposition 6, regardless of whether the platform vendor adopts a self-operation or a franchising mode, the online sales price remains unchanged. When the price sensitivity coefficient of goods is low, the prices of goods sold by the franchisee are higher than those of goods sold in the platform vendor’s self-operated store. For either the self-operated or the franchised store, the platform vendor does not need to adjust online selling prices; rather, the vendor only has to consider the sales strategy for the offline channel to cope with the influence of consumer channel-switching behaviors on market share and operating efficiency.

Proposition 7. A comparison of the wholesale price and online and offline sales prices before and after a shift in consumer demand shows the following:

The centralized decision-making scenario:

\[
\begin{align*}
p^c_0 &< p^c_{d1}, \quad \text{if } \mu - 1 < \theta < 0, \\
p^c_0 &> p^c_{d1}, \quad \text{if } 0 < \theta \leq \mu
\end{align*}
\]

The decentralized decision-making scenario:

\[
\begin{align*}
p^d_0 &< p^d_{d1}, \quad \text{if } \mu - 1 < \theta < 0, \\
p^d_0 &> p^d_{d1}, \quad \text{if } 0 < \theta \leq \mu
\end{align*}
\]

From Proposition 7, regardless of what sales strategy the platform vendor adopts, when demand shifts from online to offline (0 < \theta \leq \mu), the online selling price becomes lower than it was before the shift. However, the selling price of the offline store becomes higher than it was before the shift, and
this price change can be used to manage the channel-conversion behavior of consumers. When demand shifts from offline to online (μ < 0), the online selling price becomes lower than it was before the shift, but the selling price of the offline store becomes lower than it was before the shift. In addition, under a decentralized decision mode, when demand shifts from offline to online, the wholesale price is increased to ensure that the platform vendor’s profit is maintained. “Online purchase, offline experience” channel-switching behaviors provide customers a satisfying shopping experience and reduce the probability that they will return products due to mismatched products purchased online. Business managers should therefore focus on the impact of the demand shift proportion when it comes to consumers’ channel-switching behaviors.

For the scenario involving a shift in demand, comparing the profit margin of the O2O supply chain gives Δπ = π₁ − π₁₄ = [c (−β + γ) + a (1 + θ − μ)]²/16β > 0. Not surprisingly, the platform vendor can make more profit from their self-operated store than from a franchised store. Under the decentralized decision mode, the operational efficiency of the O2O supply chain is reduced, meaning that the supply chain requires further coordination and improvement.

6. Coordination Mechanism of the O2O Supply Chain

Since the participants in the supply chain follow the rules of economic maximization, the online platform vendor and offline franchisee pursue their own profit maximization schemes; their doing so has a “double marginal effect” and cannot achieve Pareto optimization under decentralized decision-making, reducing O2O supply chain system efficiency. Referring to previous literature [9, 35, 36], we apply a two-part-tariff contract to achieve a Pareto improvement under the O2O business model. In an actual commercial operation, the franchisee pays a service fee to coordinate game players, and with shifts in consumer demand, eliminating or weakening the “double marginal effect” can be facilitated by a two-part-tariff contract of the form (wT,F). Here, wT represents the wholesale price of the product sold by the platform vendor to the franchisee, and F(F > 0) is the fixed transfer fee paid by the franchisee to the platform vendor.

The online platform vendor’s profit under a two-part-tariff contract is

\[ \pi_{o1}^T = (p_o - c)q_{o1} + (w - c)q_{s1} + F. \]  

The offline franchisee’s profit is

\[ \pi_{s1}^T = (p_s - w)q_{s1} + F. \]

Proposition 8. Under decentralized decision-making, supply chain members can realize coordination through a two-part-tariff contract when contract parameters (wT,F) satisfy

\[ w^T = \frac{2cβ^2 + ay - 2cβγ}{2(β - γ)(β + γ)}, \quad F \in [F_1, F_2] \]

\[ \begin{cases} F_1 = \frac{(y^2 - β^2 + 2βγ^2)(c(−β + γ) + a (1 + θ - μ))^2}{4(−2β^2 + γ^2)^2} \\ F_2 = \frac{β(β + γ)(c(−β + γ) + a(1 + θ - μ))^2}{4(β - γ)(2β^2 - γ^2)} \end{cases} \]

(18)

This proposition means that, under decentralized decision-making, the O2O supply chain can be effectively coordinated by designing an appropriate coordination mechanism. With the help of this mechanism, the gaining party compensates the losing party with part of the proceeds through transfer payments, so that both parties can benefit. When contract parameters meet certain conditions, participants can make optimal decisions to achieve the same benefits brought by centralized decision-making, and improve overall performance. This coordination mechanism is conducive to decision-makers and consumers jointly creating a win-win business ecosystem.

7. Numerical Simulation Analysis

To verify the validity and applicability of the model, this section discusses a shift in consumer demand from online to offline, considering the influence of the magnitude of this shift on the sales strategies of the platform vendor and the franchisee. To correlate relevant assumptions, and based on the existing literature, to simplify the calculation, parameters are set as a = 100, β = 0.6, γ = 0.3, μ = 0.4, c = 50. If the relevant assumptions are satisfied, other values are also available and do not affect the results of the study. Based on these considerations, we analyze the influence of changes in the demand shift proportion on prices, demand, and profit, in order to provide a theoretical basis for the decision-making of business managers (Lu and Liu [13]; Ji et al. [18]). To present the research conclusions more intuitively, graphs and tables are presented. The results are shown in Figures 2–4 and Tables 3–5.

Figure 2 shows changes in online and offline prices that occur with variations in the demand shift proportion. Under the two sales strategies, the online (offline) price decreases (increases), with an increase in the demand proportion. In addition, Table 3 shows that when the demand shift proportion (θ) remains at the same level, the offline price under centralized decision-making (p^T_{o1}) is lower than the offline price under decentralized decision-making (p^T_{s1}). For example, when θ = 0.1, then p^T_{o1} = 119.44 and p^T_{s1} = 142.36, and we observe that p^T_{o1} < p^T_{s1}. Regardless of the strategy adopted, the online prices of the platform vendor are equal.
while the online price of the platform vendor ($p_{o1}$) is lower than that of the offline store ($p_{s1}$)—such that the price is significantly affected by the shift proportion $\theta$. For example, under centralized decision-making, when $\theta$ changes between (0.1, 0.2), $p_{o1}$ changes between (97.22, 91.67), and $p_{s1}$ changes between (119.44, 125.00), it is evident that $\theta$ has a significant impact on online and offline selling prices. As further observed, $p_{o1} < p_{s1}$. The platform vendor can lower the price of the online channel and promote products through marketing strategies, causing demand to shift from offline to online; the vendor can also dig deeper into potential demand and improve online sales performance.

The curves shown in Figures 3 and 4 illustrate fluctuations in demand and profit. The overall system’s profit ($\pi_d$) shows an increasing trend. When the demand shift proportion ($\theta$) is maintained at the same level as shown in Table 4, online demand ($q_{o1}$) under centralized decision-making is lower than online demand under the decentralized scenario, and offline demand ($q_{s1}$) is higher than offline demand under the decentralized scenario. Table 5 shows changes in the profit of the game players. The overall system profit ($\pi_d$) achieved under decentralized decision-making is lower than that achieved under the centralized scenario, due to the existence of a “double marginal effect,” resulting in a reduction of the system’s profit. According to both Tables 4 and 5, under the decentralized scenario, when online demand ($q_{o1}$) changes from 18.13 to 10.63, the vendor platform’s online channel profit ($\pi_{o1}$) drops from 1675.35 to 1661.46, but the offline franchisee’s profit ($\pi_{s1}$) is not as high. This indicates that $\theta$ has a more significant impact on the profit of the platform vendor than on the profit of the franchisee. However, according to the O2O supply chain, the overall system profit ($\pi_d$) shows an increasing trend.
Combined with the analyses in Figures 2–4 and Tables 3–5, we can deduce the following managerial insights. (1) Regardless of whether one is managing a self-operated or a franchised store, the demand shift proportion has a significant impact on the prices, demand, and profit of all parties. Business managers should thus evaluate the channel-switching behaviors of consumers to maintain a balance of demand across channels. (2) The platform vendor guides consumers to engage in channel-switching behavior by adopting appropriate online-to-offline marketing strategies, such as “online order, offline pickup,” “online learning, offline purchase,” and so forth. Physical stores provide customers a satisfying shopping experience and reduce the probability that they will return products due to mismatched products purchased online. (3) The platform vendor improves operational efficiency and reduces costs in self-operated stores as well as franchised stores. The integration of the online virtual economy and the offline real economy is thus conducive to expanding market scale and enabling firms to adapt to dynamic market environments.

8. Conclusions

Consumer channel-switching behaviors create opportunities for e-commerce development, and merchants have innovated on their business models in response to these opportunities. We studied pricing and operational decisions made under mutual shifts of offline and online demand. Reasonable operational decisions make merchants more competitive, helping them achieve Pareto improvements and optimize social resources. Our three main conclusions can be summarized as follows.

First, we constructed O2O supply chain decision models both with and without shifts in demand. The platform vendor sells directly online, while the offline approach involves the two strategies of self-operated or franchised stores obtaining the optimal prices for the players involved and maximizing their respective profits. Regardless of which strategy is adopted, the overall system’s profit increases. With the shift in demand from online to offline, the price offered on the online shopping platform is lower than it was before the shift, while the price offered in the offline store is higher than it was before the shift. When demand shifts from offline to online, the price offered on the online shopping platform is higher than it was before the shift, while the price offered in the offline store is lower than it was before the shift.

Second, a shift in consumer demand affects the platform vendor’s sales strategies. The online demand shift proportion is directly proportional to the offline price and overall system’s profit, and it is inversely proportional to the online price and platform vendor’s profit under decentralized decision-making.

Finally, a coordination mechanism can be used to improve supply chain performance. Since supply chain participants are influenced by economic considerations, a “double marginal effect” is observed under decentralized decision-making, preventing the achievement of Pareto optimization. When the fixed transfer payment fee is within a certain range, the design of a two-part-tariff contract can coordinate game players. This coordination mechanism can help different stakeholders achieve improved decision-making efficiency and build a win-win business ecosystem.

This paper thus offers practical managerial insights, but there are still limitations. First, our model assumes that consumer demand changes linearly with price sensitivity to achieve fusion symbiosis and a seamless connection between the online virtual economy and offline real economy. This method is adopted as a practical approach to studying random demand and its effects on the development of sustainable marketing strategies. In addition, while we assume that a platform vendor operates under a two-line system, an offline brick-and-mortar store can also be operated through two-line operations, whereby users participate in a holistic shopping and purchasing experience. Outcomes of the latter scenario are worth studying in the future (Appendixes A and B).

Appendix

A. Proofs of Propositions

Proof of Proposition 1. With (1), from the derivative, we obtain

\[
\frac{\partial \pi_c}{\partial p_s} = a - 2p_s\beta + c(\beta - \gamma) + 2p_c\gamma - a\mu, \\
\frac{\partial \pi_c}{\partial p_o} = -2p_s\beta + c(\beta - \gamma) + 2p_c\gamma + a\mu, \\
\frac{\partial^2 \pi_c}{\partial p_o \partial p_s} = \frac{\partial^2 \pi_c}{\partial p_s \partial p_o} = 2\gamma, \\
\frac{\partial^2 \pi_c}{\partial p_o^2} = \frac{\partial^2 \pi_c}{\partial p_s^2} = -2\beta < 0.
\]  
(A.1)
From known conditions, $\beta > \gamma > 0$, and we in turn obtain the value of the Hessian matrix:

$$
\begin{vmatrix}
\frac{\partial^2 \pi_c}{\partial p_o^2} & \frac{\partial^2 \pi_c}{\partial p_o \partial p_s} \\
\frac{\partial^2 \pi_c}{\partial p_s \partial p_o} & \frac{\partial^2 \pi_c}{\partial p_s^2}
\end{vmatrix} = 4\beta^2 - 4\gamma^2 > 0. \quad (A.2)
$$

Thus, $\pi_c$ is a strictly differentiable concave function at $(p_o, p_s)$. Via backward induction, make the first derivative equal to zero to obtain the optimal solution of online and offline product prices:

$$
\begin{align*}
p_s &= \frac{a + c\beta - cy + 2p_s\gamma - a\mu}{2\beta} \\
p_o &= \frac{c\beta - cy + 2p_s\gamma + a\mu}{2\beta}.
\end{align*} \quad (A.3)
$$

From simultaneous equations, obtain the optimal solution of products’ retail prices:

$$
\begin{align*}
p_d^d &= \frac{a\beta + c\beta^2 - cy^2 - a\beta\mu + ay\mu}{2(\beta^2 - \gamma^2)} \\
p_o^d &= \frac{c\beta^2 + ay - cy^2 + a\beta\mu - ay\mu}{2(\beta^2 - \gamma^2)}.
\end{align*} \quad (A.4)
$$

Include (1) to obtain the profit of the O2O supply chain:

$$
\pi_c = \frac{1}{4(\beta^2 - \gamma^2)} \left\{ \beta^4 \left[ -2\gamma (-1 + \mu) \mu + \beta (1 - 2\mu + 2\mu^2) \right] + 2ac(-\beta^2 + \gamma^2) + 2c^2(\beta + \gamma) (\beta - \gamma)^2 \right\}. \quad (A.5)
$$

This completes the proof.

**Proof of Proposition 2.** The first and second partial derivatives of $p_s$ are obtained from the franchisee’s profit function (4): $d\pi_c/dp_s = a - a\mu - 2p_s\beta + w\beta + p_o\gamma$ and $d^2\pi_c/dp_s^2 = -2\beta < 0$. As a result, $\pi_c(p_s)$ is a strictly concave function at $p_s$. When $(d\pi_c/dp_s) = 0$, then

$$
p_s = \frac{a + w\beta + p_o\gamma - a\mu}{2\beta}. \quad (A.6)
$$

Substitute it to (3), take the derivative of $w$ and $p_o$, and set the first derivative as equal to zero to obtain the optimal solution:

$$
\begin{align*}
p_{o1}^d &= \frac{c\beta^2 + ay - cy^2 + a\beta\theta + ay\theta + a\beta\mu - ay\mu}{2(\beta^2 - \gamma^2)} \\
w_{o1}^d &= \frac{a\beta + c\beta^2 - cy^2 + a\beta\theta - ay\theta - a\beta\mu + ay\mu}{2(\beta^2 - \gamma^2)}.
\end{align*} \quad (A.10)
$$

Replace these in $p_s$ to obtain the optimal solution:

$$
p_s^* = \frac{c(\beta - \gamma)(\beta + \gamma)^2 + a(\gamma^2 - 3\beta^2) (\mu - 1) + 2\beta y\mu}{4(\beta^2 - \beta\gamma^2)}. \quad (A.8)
$$

For the decentralized decision-making strategy, replace $p_o^d$, $p_s^*$, and $w_d$ with (3) and (4) to obtain the profit of the platform vendor and franchisee. This completes the proof.

**Proof of Proposition 3.** For the scenario involving no shift in demand, since the expression of the online price of decentralized decision-making $(p_o^d)$ is the same as that of the centralized scenario $(p_o^c)$, we know that $p_o^d = p_o^c$. Compare the offline price of decentralized decision-making $(p_s^d)$ with the offline price of centralized decision-making $(p_s^c)$ to obtain $p_s^c > p_s^d$, when $a - c\beta + cy - a\mu < 0$, i.e., $\gamma < \beta < \gamma + a(1 - \mu)/c$, then $p_s^c > p_s^d$; when $a - c\beta + cy - a\mu < 0$, i.e., $\beta > \gamma + a(1 - \mu)/c$, then $p_s^c > p_s^d$. This completes the proof.

**Proof of Proposition 5.** For the scenario involving a shift in demand, the first and second partial derivatives of $p_s$ are obtained from the franchisee’s profit function (11): $d\pi_c/dp_s = -2p_s\beta + w\beta + p_o\gamma + a(1 + \theta - \mu)$ and $d^2\pi_c/dp_s^2 = -2\beta < 0$. As a result, $\pi_c(p_s)$ is a strictly concave function at $p_s$. When $(d\pi_c/dp_s) = 0$, then

$$
p_s = \frac{a + w\beta + p_o\gamma + a\theta - a\mu}{2\beta}. \quad (A.9)
$$

Substitute it to (9), take the derivative of $w$ and $p_o$, and set the first derivative as equal to zero to obtain the optimal solution:

$$
\begin{align*}
p_{o1}^d &= \frac{c\beta^2 + ay - cy^2 + a\beta\theta + ay\theta + a\beta\mu - ay\mu}{2(\beta^2 - \gamma^2)} \\
w_{o1}^d &= \frac{a\beta + c\beta^2 - cy^2 + a\beta\theta - ay\theta - a\beta\mu + ay\mu}{2(\beta^2 - \gamma^2)}.
\end{align*} \quad (A.10)
$$

Replace these in $p_s$ to obtain the optimal solution:
\[ p^d_{s1} = \frac{1}{4(\beta^2 - \beta y^2)} \left\{ (c(\beta - y)(\beta + y))^2 + a\left[ 3\beta^2 (1 + \theta - \mu) \right. \right. \\
\left. \left. + y^2(-1 - \theta + \mu) + 2\beta y(-\theta + \mu) \right] \right\} \]

(A.11)

For decentralized decision-making, replace \( p^d_{c}, p^d_{s_1}, \) and \( w_{d1} \) with (9) and (11) to obtain the profit of platform vendor \((\pi_o)\) and franchisee \((\pi_{s_1})\). This completes the proof.

**Proof of Proposition 7.** For the scenario involving a shift in demand, since the expression of the online price of decentralized decision-making \((p^d_{c1})\) is the same as that of the centralized scenario \((p^c_{c1})\), we know that \( p^d_{c1} = p^c_{c1} \). By comparing the offline price of decentralized decision-making \((p^d_{s_1})\) with the offline price of centralized decision-making \((p^c_{s_1})\), we obtain \( p^d_{s_1} > p^c_{s_1} \). For the numerator, when \( a - c(\beta - y)(\beta + y) + 2\beta a\theta + a\gamma(1 + \theta - \mu) > 0 \), \( \gamma < \beta < y + a(1 + \theta - \mu)/c \), then \( p^d_{s_1} > p^c_{s_1} \); when \( a - c(\beta - y)(\beta + y) + 2\beta a\theta + a\gamma(1 + \theta - \mu) > 0 \), \( \beta > y + a(1 + \theta - \mu)/c \), then \( p^d_{s_1} < p^c_{s_1} \). This completes the proof.

**Proof of Proposition 6.** For the scenario involving a shift in demand, since the expression of the online price of decentralized decision-making \((p^d_{o})\) is the same as that of the centralized scenario \((p^c_{o})\), we know that \( p^d_{o} = p^c_{o} \). By comparing the offline price of decentralized decision-making \((p^d_{o})\) with the offline price of centralized decision-making \((p^c_{o})\), we obtain \( p^d_{o} > p^c_{o} \). For the numerator, when \( a - c(\beta - y)(\beta + y) + 2\beta a\theta + a\gamma(1 + \theta - \mu) > 0 \), \( \gamma < \beta < y + a(1 + \theta - \mu)/c \), then \( p^d_{o} > p^c_{o} \); when \( a - c(\beta - y)(\beta + y) + 2\beta a\theta + a\gamma(1 + \theta - \mu) > 0 \), \( \beta > y + a(1 + \theta - \mu)/c \), then \( p^d_{o} < p^c_{o} \). This completes the proof.

**Proof of Proposition 8.** From the franchisee’s profit function (16), take the first partial derivative of \( p^c \) and set it at zero to obtain the expression of \( p^c \). Replace it with (13), take the first derivative of \( p^c \), and set it at zero. Prices after coordination are then, respectively,

\[ p^c_T = \frac{1}{2(2\beta^2 - \gamma^2)} \left[ 2c\beta^2 + ay - c\beta y + 2\omega\beta y - cy^2 - 2a\beta\theta + a\gamma \theta + 2a\beta\mu - a\mu \right], \]

\[ p^c_{s_1} = \frac{1}{2\beta - \gamma} \left[ 4\omega\beta\gamma - cy(-2\beta^2 + \beta y + y^2) + 4a\beta^2 (1 + \theta - \mu) + a\gamma^2(-1 - \theta + \mu) + 2a\beta y(-\theta + \mu) \right]. \]

(A.12)

To ensure that the profit of the O2O supply chain under centralized decision-making is equal to that after coordination, \( \{ p^c_{o} = p^c_{o}, \ p^c_{s_1} = p^c_{s_1} \} \) must be satisfied by solving \( p^c_{o} = p^c_{o} \)

\[ w^c = 2c\beta^2 + ay - c\beta y/2(\beta - y)(\beta + y) \]

We then have \( w^c_1 = 2c\beta^2 + ay - c\beta y/2(\beta - y)(\beta + y) \). Since participants of the supply chain are economically driven, the benefit of the whole supply chain will only be considered when their own interests are maximized. Under the coordination mechanism, the profit of each game player is no lower than the profit achieved under decentralized decision-making, and both sides will achieve Pareto improvement. From simultaneous equations \( \{ \pi^c_{o} \geq \pi^c_{o}, \ \pi^c_{s_1} \geq \pi^c_{s_1} \} \), the decision range of fixed transfer fee \( F \) is obtained as \( F \in [F_1, F_2] \), where

\[ F_1 = \frac{\left( y^3 - \beta^3 + 2\beta y^2 \right) [c(-\beta + y) + a(1 + \theta - \mu)]^2}{4(2\beta^2 + \gamma^2)^3}, \]

\[ F_2 = \frac{\beta(\beta + y)(c(-\beta + y) + a(1 + \theta - \mu))^2}{4(\beta - y)(2\beta^2 - y^2)}. \]

(A.13)

This completes the proof.

**B. Proofs of Lemmas**

**Proof of Lemma 1.** Consider the first derivative of prices \( p^c_{o} \) and \( p^c_{s_1} \) with respect to \( \mu \), which show that \( d^2 p^c_{o}/d\mu = -a/2(\beta + y) < 0 \), \( d^2 p^c_{s_1}/d\mu = a/2(\beta + y) > 0 \), and \( d^2 p^c_{o}/d\mu = -d^2 p^c_{s_1}/d\mu = a/2(\beta + y) > 0 \). This completes the proof.
Proof of Lemma 2. The second partial derivatives of $p_o$ and $p_s$ are obtained from (1), from which we obtain Hessian matrix:

$$
H = \begin{bmatrix}
\frac{\partial^2 \pi_c}{\partial p_o \partial p_o} & \frac{\partial^2 \pi_c}{\partial p_o \partial p_s} \\
\frac{\partial^2 \pi_c}{\partial p_s \partial p_o} & \frac{\partial^2 \pi_c}{\partial p_s \partial p_s}
\end{bmatrix}
= \begin{bmatrix}
-2\beta & 2y \\
2y & -2\beta
\end{bmatrix}.
$$

(B.1)

The sequential principal minor is in turn $-2\beta < 0$, $-2\beta < 0$, $4\beta^2 - 4y^2 > 0$, and the Hessian matrix is strictly negative. As a result, $\pi_c$ is a strictly concave function at $(p_o, p_s)$. This completes the proof.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This research was funded by the National Natural Science Foundation of China (No. 71472026). The authors thank a number of instructors of management and economics for their helpful comments and insights. The authors also thank the participants of a number of group discussions for their helpful suggestions. The authors especially thank Institute of Internet Industry, Tsinghua University, for providing the article’s framework.

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