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Research Article

$\mathscr{H}_2$ Control of Markovian Jump Systems with Input Saturation and Incomplete Knowledge of Transition Probabilities

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This paper proposes an $\mathscr{H}_2$ state-feedback controller for Markovian jump systems with input saturation and incomplete knowledge of transition probabilities. The proposed controller is developed using second-order matrix polynomials of an incomplete transition rate to derive less conservative stabilization conditions. The proposed controller not only guarantees $\mathscr{H}_2$ performance but also rejects matched disturbances. The effectiveness of the proposed method is demonstrated using three numerical examples.

1. Introduction

Over the last few decades, Markovian jump systems (MJSs) have been recognized as one of the most effective models for the representation of dynamic systems subjected to random and abrupt variations. Thus, numerous studies have been conducted to analyze and synthesize MJSs [1–7]. The findings of these studies have been applied in various practical systems, such as networked control systems [8], manufacturing systems [8], economic systems [9], power systems [10], and actuator saturation [11]. In particular, studies have focused on the analysis and synthesis of ideal MJSs having exact values of transition probabilities [12].

However, such MJSs with exactly known transition probabilities have limited scope for application in practical systems because it is difficult to obtain complete knowledge of transition probabilities. Thus, recent studies on controller synthesis have focused on MJSs with incomplete knowledge of transition probabilities. Such studies have employed the free-connection weighting method and linear matrix inequalities (LMIs) [13–16].

However, several practical systems suffer from input saturation because of the physical limitations of the control system [17–19]. It is well known that input saturation generally degrades control system performance and system stability [20].

Thus, the control synthesis problem should be considered with input saturation in practical systems. In particular, the stochastic stabilization problem for MJSs subjected to actuator saturation was studied based on exactly known transition probabilities [21, 22]. Furthermore, the stabilization of saturated MJSs with incomplete knowledge of transition probabilities was studied using the free-connection weighting matrix approach [11]. In addition, the stabilization condition for MJSs in the presence of both partially unknown transition rates and input saturation was proposed [23].

To the best of the author's knowledge, intensive studies on the $\mathscr{H}_2$ control of MJSs with input saturation and incomplete knowledge of transition probabilities have not been conducted thus far. A previous study stabilized nonhomogeneous MJSs with input saturation [24]; however, the findings are not applicable to practical systems because disturbances were not considered.

Thus, an $\mathscr{H}_2$ stabilization condition for MJSs with input saturation and incomplete knowledge of transition probabilities is proposed herein. The main contributions of this study are as follows:

This is the first proposal to propose a stabilization condition to accomplish stochastic stability and guarantee $\mathscr{H}_2$ performance for MJSs with input saturation and incomplete knowledge of transition probabilities.

The proposed controller consists of two parts: a linear control part to guarantee $\mathscr{H}_2$ performance and a nonlinear control part to reject the matched disturbances.

Based on the proposed relaxation method using the second-order matrix polynomials of the incomplete transition rate, this paper presents less conservative stabilization conditions for estimating the domain of attraction.

The effectiveness of the proposed controller is demonstrated using two numerical examples and a practical example.

The remainder of this paper is organized as follows. Section 2 provides a description of the system and some preliminary results. Section 3 introduces the proposed $\mathscr{H}_2$ controller for MJSs with input saturation and incomplete knowledge of transition probabilities. Section 4 presents the simulations of three examples for verifying the proposed controller. Section 5 concludes the paper.

Notation. The notations $X \geq Y$ and $X > Y$ indicate that $X - Y$ is positive semidefinite and positive definite, respectively. In symmetric block matrices, $(*)$ is used as an ellipsis for terms that are induced by symmetry. Furthermore, $\mathbf{He}(X) = X + X^T$ stands for any matrix X, and $\mathbf{E}[\cdot]$ denotes the mathematical expectation. For any matrices S_i and S_{ij},

$$[S_i]_{i\in\{1,2,\ldots,N\}} = [S_{11}, S_{12}, \ldots, S_{1N}],$$
$$[S_{ij}]_{i,j\in\{1,2,\ldots,N\}} = \begin{bmatrix} S_{11} & S_{12} & \cdots & S_{1N} \\ S_{21} & S_{22} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \vdots \\ S_{N1} & \cdots & \cdots & S_{NN} \end{bmatrix}. \quad (1)$$

We also use $\|x\|_p$ to indicate the p-norm of x, i.e., $\|x\|_p \triangleq (|x_1|^p + \cdots + |x_n|^p)^{(1/p)}$, $p \geq 1$. $\lambda_{\min}(X)$ and $\lambda_{\max}(X)$ denote a minimum eigenvalue and a maximum eigenvalue of X, respectively. The notation e_k indicates a unit vector with a single nonzero entry at the kth position, i.e., $e_k \triangleq [0 \ldots \underbrace{1}_{k^{\text{th}}} \ldots 0]^T$.

2. System Description and Preliminaries

Consider the following continuous-time MJS with input saturation and a matched disturbance:

$$\dot{x}(t) = A(r_t)x(t) + B(r_t)\{\text{sat}(u(t)) + d(t)\}, \quad (2)$$

$$z(t) = C(r_t)x(t), \quad (3)$$

where $x(t) \in \mathfrak{R}^n$ is the state, $u(t) \in \mathfrak{R}^m$ is the control input, $d(t) \in \mathfrak{R}^m$ is the matched disturbance, and $z(t) \in \mathfrak{R}^q$ is the controlled output. The matched disturbance $d(t)$ is assumed to be $|e_k^T d(t)| < \varepsilon$. Here, $\{r_t, t \geq 0\}$ is a continuous-time Markov jumping process in a finite set $D = \{1, 2, 3, \ldots, N\}$ with mode transition probabilities:

$$P(r_{t+\delta t} = j \mid r_t = i) = \begin{cases} \pi_{ij}\delta t + o(\delta t), & \text{if } i \neq j, \\ 1 + \pi_{ij}\delta t + o(\delta t), & \text{if } i = j, \end{cases} \quad (4)$$

where $\delta t > 0$, $\lim_{\delta t \longrightarrow 0}(o(\delta t)/\delta t) = 0$, and π_{ij} is the transition rate from mode i to j at time $t + \delta t$. For $r_t = i \in D$, to simplify the notation, $A(r_t) = A_i$, $B(r_t) = B_i$, $D(r_t) = D_i$, and $C(r_t) = C_i$. Further, $\text{sat}(\cdot)$ denotes a saturation operator, which is defined as

$$[\text{sat}(u)]_i \triangleq \begin{cases} [u]_i, & |[u]_i| < \mu, \\ \mu, & |[u]_i| \geq \mu, \\ -\mu, & |[u]_i| \leq \mu, \end{cases} \quad (5)$$

where $\mu(>\varepsilon)$ is the saturation level. Furthermore, the transition rate matrix Π belongs to

$$S_\Pi \triangleq \left\{ [\pi_{ij}]_{i,j\in D} | 0 \leq \pi_{ij}, \quad \text{for } i \neq j, \pi_{ii} = -\sum_{j=1, i\neq j}^{N} \pi_{ij} \right\}. \quad (6)$$

In view of the aforementioned relations, π_{ij} accords with the following relationships, for all $i, j \in D$:

$$\nu_{ij}\pi_{ij} \geq 0, \sum_{j=1}^{N} \pi_{ij} = 0, -\nu_{ij}\pi_{ij}(\pi_{ij} + \pi_{ii}) \geq 0, \quad (7)$$

where

$$\nu_{ij} = \begin{cases} 1, & i \neq j, \\ -1, & i = j. \end{cases} \quad (8)$$

For future convenience, two sets are defined with respect to the measurability of the transition rate for $i, j \in D$:

$$D_i^+ \triangleq \{ j | \pi_{ij} \text{ is known for } i \},$$
$$D_i^- \triangleq \{ j | \pi_{ij} \text{ is unknown for } i \}. \quad (9)$$

The following lemma and definitions are introduced as preliminaries required to prove the theorems presented in the subsequent sections.

Lemma 1 (see [25]). *Let $u, v \in \mathfrak{R}^m$,*

$$u = [\, u_1 \;\; u_2 \;\; \ldots \;\; u_m \,]^T,$$
$$v = [\, v_1 \;\; v_2 \;\; \ldots \;\; v_m \,]^T. \quad (10)$$

Assume that $|e_k^T v| \leq \mu$ for all $k \in [1, m]$, and then

$$\text{sat}(u) \in \mathbf{Co}\{E_s u + E_s^- v |, s \in [1, 2^m]\}, \quad (11)$$

where E_s denotes a diagonal matrix with all possible combinations of 1 and 0 diagonal entries, $E_s^- \triangleq I - E_s$, and $\mathbf{Co}$ is the convex hull.

Definition 1 (see [21]). A set $\mathcal{S} \in \mathfrak{R}^n$ is called the domain of attraction in the mean square sense of (2), if for any initial mode $r_0 \in D$ and initial state $x(0) \in \mathcal{S}$, the state $x(t)$ of (2) satisfies

$$\lim_{T\longrightarrow\infty}\left(\int_0^T \mathbf{E}\left[\| x(\tau)\|^2\right]\mathrm{d}\tau | x(0), r_0\right) < x^T(0)\Psi x(0), \quad (12)$$

where $\Psi > 0$.

3. Main Results

This section considers the design problem of the $\mathscr{H}_2$ state-feedback controller.

A controller is proposed for system (2) as follows:

$$u(t) = K(r_t)x(t) + \overline{u}(r_t, x(t)), \quad (13)$$

where $K(r_t)$ is the linear controller part to guarantee the $\mathscr{H}_2$ performance and $\overline{u}(r_t, x(t))$ is the nonlinear controller part to reject the matched disturbance $B(r_t)d(t)$. Thus, the proposed controller is designed to stochastically stabilize and minimize the upper bound of the following linear quadratic cost:

$$J(t) = \mathbf{E}\left[\int_t^\infty x^T(\tau)Q(r_t)x(\tau)\mathrm{d}\tau\right], \quad (14)$$

where $Q(r_t) = Q_i \geq 0$. Here, for $r_t = i \in D$, $K(r_t) = K_i$ and $\overline{u}(r_t, x(t)) = \overline{u}_i(x(t))$. Using system (2) and the proposed controller (13), the resultant closed-loop system is expressed as follows:

$$\dot{x}(t) = A_i x(t) + B_i\{\mathrm{sat}(K_i x(t) + \overline{u}_i(x(t))) + d(t)\}. \quad (15)$$

Theorem 1. *Consider system (15) with input saturation and incomplete knowledge of the transition rate. For $i, j \in D$, $s \in [1, 2^m]$, and $k \in [1, m]$, suppose that there exist symmetric matrices $\overline{P}_i$ and R_{ij}, matrices $\overline{K}_i$, $\overline{H}_i$, Λ_{ij}, Y_{ij}, S_{i0}, and S_{ij}, and a scalar γ such that*

$$\overline{P}_i > 0, \quad (16)$$

$$\Lambda_{ij} + \Lambda_{ij}^T > 0, Y_{ij} + Y_{ij}^T > 0, \quad (17)$$

$$\begin{bmatrix} R_{ij} & \overline{P}_i \\ (*) & \overline{P}_j \end{bmatrix} > 0, \quad i \neq j, \quad (18)$$

$$\begin{bmatrix} \overline{\Gamma}_s^i & \left[\Gamma_j^i\right]_{j\in D_i^-} \\ (*) & \left[\Gamma_{jl}^i\right]_{j,l\in D_i^-} \end{bmatrix} < 0, \quad (19)$$

$$\begin{bmatrix} \gamma & x^T(0) \\ x(0) & \overline{P}_0 \end{bmatrix} > 0, \quad (20)$$

$$\begin{bmatrix} \overline{P}_i & \overline{H}_i^T e_k \\ e_k^T\overline{H}_i & (\mu - \varepsilon)^2 \end{bmatrix} > 0, \quad (21)$$

$$\begin{bmatrix} 1 & x^T(0) \\ x(0) & \overline{P}_i \end{bmatrix} > 0, \quad (22)$$

where

$i \in D_i^+$

$$\begin{aligned}
\overline{\Gamma}_s^i &= \overline{\Omega}_s^i + \Pi_i^+ E^T\mathbf{He}(S_{i0})E + \sum_{j\in D_i^+}\nu_{ij}\pi_{ij}E^T\mathbf{He}(\Lambda_{ij})E \\
&\quad - \sum_{j\in D_i^+}\nu_{ij}\pi_{ij}(\pi_{ij} + \pi_{ii})E^T\mathbf{He}(Y_{ij})E, \\
\Gamma_j^i &= \frac{1}{2}E^T G_{ij} + E^T(S_{i0} + \Pi_i^+ S_{ij}) + \nu_{ij}E^T\Lambda_{ij} - \nu_{ij}\pi_{ii}E^T Y_{ij}, \\
\Gamma_{jj}^i &= \mathbf{He}(S_{ij}) - \nu_{ij}\mathbf{He}(Y_{ij}), \\
\Gamma_{jl}^i &= S_{il} + S_{ij},
\end{aligned} \quad (23)$$

$i \in D_i^-$

$$\begin{aligned}
\overline{\Gamma}_s^i &= \overline{\Omega}_s^i + \Pi_i^+ E^T\mathbf{He}(S_{i0})E + \sum_{j\in D_i^+}\nu_{ij}\pi_{ij}E^T\mathbf{He}(\Lambda_{ij})E \\
&\quad - \sum_{j\in D_i^+}\nu_{ij}\pi_{ij}^2 E^T\mathbf{He}(Y_{ij})E, \\
\Gamma_j^i &= \frac{1}{2}E^T G_{ij} + E^T S_{i0} + \Pi_i^+ E^T S_{ij} + \nu_{ij}E^T\Lambda_{ij} \\
&\quad - \sum_{j\in D_i^+} c_{ij}\nu_{ij}\pi_{ij}E^T Y_{ij}, \\
\Gamma_{jj}^i &= \mathbf{He}(S_{ij}) - 2\nu_{ij}\mathbf{He}(Y_{ij}), \\
\Gamma_{jl}^i &= \begin{cases} S_{il} + S_{ij}, & i \neq l, \\ S_{il} + S_{ij} - \nu_{ij}Y_{ij}, & i = l, \end{cases} \\
\Pi_i^+ &= \sum_{j\in D^+}\pi_{ij}, \\
\overline{\Omega}_s^i &\triangleq \begin{bmatrix} \mathbf{He}(A_i\overline{P}_i + B_i E_s\overline{K}_i + B_i E_s^-\overline{H}_i) + \sum_{j\in D_i^+}\pi_{ij}G_{ij} & \overline{P}_i \\ \overline{P}_i & -Q_i^{-1} \end{bmatrix}, \\
G_{ij} &\triangleq \kappa_{ij}R_{ij} + (1 - \kappa_{ij})\overline{P}_i, \\
\kappa_{ij} &= \begin{cases} 1, & i \neq j, \\ 0, & i = j, \end{cases} \\
E &= [\,I \quad 0\,] \in \mathfrak{R}^{n\times 2n}.
\end{aligned} \quad (24)$$

Then, the set $\cap_{i=1}^N\Omega(P_i)$ is contained in the domain of attraction, and the proposed system (15) is stochastically stable with the $\mathscr{H}_2$ cost in (14) guaranteed by γ. Furthermore, the proposed controller is constructed as $u(t) = K_i x(t) + \overline{u}_i(x(t))$ for mode i, where $K_i = \overline{K}_i\overline{P}_i^{-1}$ and each component of $\overline{u}_i(x(t))$ is defined as

$$[\overline{u}_i(x(t))]_k = -\varepsilon\mathrm{sgn}(e_k^T B_i^T P_i x(t)). \quad (25)$$

Proof. Consider the following mode-dependent control input $u(t)$ and the auxiliary input $v(t)$:

$$\begin{aligned} u(t) &= K(r_t)x(t) + \overline{u}_i(x(t)), \\ v(t) &= H(r_t)x(t) + \overline{u}_i(x(t)), \end{aligned} \tag{26}$$

where $v(t)$ is used to handle the input saturation in Lemma 1. For the representation method (10) in Lemma 1, the following condition should be satisfied:

$$\left| e_k^T H(r_t)x(t) + e_k^T \overline{u}_i(x(t)) \right| \le \mu. \tag{27}$$

From the definition of $\overline{u}_i(x(t))$ in (25), the left side of (27) can be derived as follows:

$$\begin{aligned} &\left| e_k^T H(r_t)x(t) + e_k^T \overline{u}_i(x(t)) \right| \\ &\quad \le \left| e_k^T H(r_t)x(t) \right| + \left| e_k^T \overline{u}_i(x(t)) \right| \\ &\quad = \left| e_k^T H(r_t)x(t) \right| + \varepsilon. \end{aligned} \tag{28}$$

Then, the sufficient condition for (27) is given as follows:

$$\left| e_k^T H(r_t)x(t) \right| \le \mu - \varepsilon. \tag{29}$$

Therefore, the representation method in Lemma 1 can be used if $x(t) \in L(H(r_t))$ for $k \in [1, m]$, where

$$L(H(r_t)) = \left\{ x(t) \in \mathfrak{R}^n \middle| \left| e_k^T H(r_t)x(t) \right| \le \mu - \varepsilon \right\}. \tag{30}$$

To establish a set invariance condition [25], the ellipsoid $\Omega(P(r_t)) \triangleq \left\{ x(t) \in \mathfrak{R}^n | xt(t)^T nPq(r_t)hx_(t)x \le 71 \right\}$ is in the linear region $L(H(r_t))$, that is, for $k \in [1, m]$,

$$x^T(t)P_i x(t) > x^T(t)H_i^T e_k \frac{1}{(\mu - \varepsilon)^2} e_k^T H_i x(t), \tag{31}$$

or equivalently,

$$\begin{bmatrix} P_i & H_i^T e_k \\ e_k^T H_i & (\mu - \varepsilon)^2 \end{bmatrix} > 0. \tag{32}$$

Then, multiplying both sides of the above equation by $\text{diag}\{P_i^{-1}, I\}$ yields (20), where $\overline{P}_i = P_i^{-1}$ and $\overline{H}_i = H_i \overline{P}_i$.

Let us choose $V(x(t)) = x^T(t)P(r_t)x(t)$ as a Lyapunov function, where $P(r_t)$ is a positive definite matrix. Then, from the weak infinitesimal operator ∇ of the Markov process, $\nabla V(x(t))$ is given by

$$\begin{aligned} \nabla V &= \lim_{\delta t \longrightarrow 0} \frac{1}{\delta t} \left(\mathbf{E}\left[V\left(x(t+\delta t), r_{t+\delta t} = j\right) | x(t), r_t = i \right] \right. \\ &\quad \left. - V\left(x(t), r_t = i\right) \right) \\ &= \frac{\mathrm{d}V(x(t), i)}{\mathrm{d}t} + \sum_{j=1}^{N} \pi_{ij} V(x(t), j) \\ &= 2x^T(t)P_i \dot{x}(t) + x^T(t) \sum_{j=1}^{N} \pi_{ij} P_j x(t). \end{aligned} \tag{33}$$

According to the convex property and condition (21), there exist variables η_s such that

$$\begin{aligned} \text{sat}(u(t)) &= \sum_{s=1}^{2^m} \eta_s \{E_s u(t) + E_s^- v(t)\} \\ &= \sum_{s=1}^{2^m} \eta_s \{E_s K(r_t)x(t) + E_s^- H(r_t)x(t) + \overline{u}_i(x(t))\} \\ &= \sum_{s=1}^{2^m} \eta_s \{E_s K(r_t)x(t) + E_s^- H(r_t)x(t)\} + \overline{u}_i(x(t)), \end{aligned} \tag{34}$$

where $\sum_{s=1}^{2^m} \eta_s = 1$.

Then, $\nabla V(x(t))$ can be rewritten as

$$\begin{aligned} \nabla V(x(t)) &= 2x^T(t) \left\{ P_i A_i + \sum_{s=1}^{2^m} \eta_s P_i B_i \left(E_s K_i + E_s^- H_i\right) \right\} x(t) \\ &\quad + 2x^T(t)P_i B_i \left(\sum_{s=1}^{2^m} \eta_s \overline{u}_i(x(t)) + d(t) \right) + x^T(t) \sum_{j=1}^{N} \pi_{ij} P_j x(t) \\ &= \sum_{s=1}^{2^m} \eta_s \left[2x^T(t)\{P_i A_i + P_i B_i (E_s K_i + E_s^- H_i)\} + 2x^T(t)P_i B_i (\overline{u}_i(x(t)) + d(t)) + x^T(t) \sum_{j=1}^{N} \pi_{ij} P_j x(t) \right]. \end{aligned} \tag{35}$$

Furthermore, from (25) and $|e_k^T d(t)| < \varepsilon$, we have

$$2x^T(t)P_i B_i \left(\overline{u}_i(x(t)) + d(t)\right) \le 0. \tag{36}$$

Thus, if the following condition holds, for $i \in D$ and $s \in [1, 2^m]$,

$$\mathbf{He}\left(P_i A_i + P_i B_i E_s K_i + P_i B_i E_s^- H_i\right) + Q_i + \sum_{j=1}^{N} \pi_{ij} P_j < 0, \tag{37}$$

then, from (36) and (35), $\nabla V(x(t))$ can be expressed as the following relation:

$$\nabla V(x(t)) \le 2\sum_{s=1}^{2^m} \eta_s x^T(t)\{P_iA_i + P_iB_i(E_sK_i + E_s^- H_i)\} + x^T(t)\sum_{j=1}^{N}\pi_{ij}P_jx(t) < -x^T(t)Q_ix(t). \tag{38}$$

Using the generalized Dynkin's formula [26], the above relation allows

$$\begin{aligned} \mathbf{E}[V(t)] - V(0) &= \mathbf{E}\left[\int_0^t \nabla V(\tau)d\tau | x(0), r_0\right] \\ &< -\mathbf{E}\left[\int_0^t x^T(\tau)Q(r_\tau)x(\tau)d\tau | x(0), r_0\right] \\ &\le -\min_{i\in D}(\lambda_{\min}(Q_i))\mathbf{E}\left[\int_0^t \|x(\tau)\|^2 d\tau | x(0), r_0\right], \end{aligned} \tag{39}$$

which leads to

$$\begin{aligned} \min_{i\in D}(\lambda_{\min}(Q_i))\mathbf{E}\left[\int_0^t \|x(\tau)\|^2 d\tau | x(0), r_0\right] &< V(0) - \mathbf{E}[V(t)] \\ &\le V(0), \end{aligned} \tag{40}$$

because the following equation is valid:

$$\mathbf{E}\left[\int_0^t \|x(\tau)\|^2 d\tau | x(0), r_0\right] < \frac{V(0)}{\min_{i\in D}(\lambda_{\min}(Q_i))}. \tag{41}$$

From (41), it is allowed that

$$\lim_{T\longrightarrow\infty} \mathbf{E}\left[\int_0^t \|x(\tau)\|^2 d\tau | x(0), r_0\right] \le x^T(0)\Psi x(0), \tag{42}$$

where

$$\Psi = \frac{\max_{r_0\in D}(\lambda_{\max}(P_{r_0}))}{\min_{i\in D}(\lambda_{\min}(Q_i))} > 0. \tag{43}$$

Furthermore, from (39), we have

$$J(0) < V(0) = x^T(0)P(r_0)x(0), \tag{44}$$

which guarantees the $\mathscr{H}_2$ cost through (20), indicating that $x^T(0)P(r_0)x(0) < \gamma$ using the Schur complement.

Subsequently, by pre- and postmultiplying (37) with P_i^{-1}, we have

$$\mathbf{He}(\mathscr{A}_s^i) + \overline{P}_iQ_i\overline{P}_i + \sum_{j=1}^{N}\pi_{ij}\overline{P}_iP_j\overline{P}_i < 0, \tag{45}$$

where $\mathscr{A}_s^i = A_i\overline{P}_i + B_iE_s\overline{K}_i + B_iE_s^-\overline{H}_i$, $\overline{P}_i = P_i^{-1}$, and $\overline{K}_i = K_i\overline{P}_i$.

Note that for $i = j$, $\overline{P}_iP_j\overline{P}_i = \overline{P}_i$, and for $i \ne j$, (18) leads to $\overline{P}_iP_j\overline{P}_i \le R_{ij}$. Equation (45) holds because of the following condition:

$$\mathbf{He}(\mathscr{A}_s^i) + \overline{P}_iQ_i\overline{P}_i + \sum_{j=1}^{N}\pi_{ij}G_{ij} < 0, \tag{46}$$

where $G_{ij} \triangleq \kappa_{ij}R_{ij} + (1-\kappa_{ij})\overline{P}_i$.

Applying the Schur complement to (46) yields

$$\begin{bmatrix} \mathbf{He}(\mathscr{A}_s^i) + \sum_{j=1}^{N}\pi_{ij}G_{ij} & \overline{P}_i \\ \overline{P}_i & -Q_i^{-1} \end{bmatrix} < 0. \tag{47}$$

To derive the LMI conditions, (47) can be written as follows:

$$\Omega_s^i \triangleq \overline{\Omega}_s^i + \sum_{j\in D_i^-}\pi_{ij}E^TG_{ij}E < 0. \tag{48}$$

In addition, according to condition (7), the following equations can be derived from (19):

$$C_i^1 \triangleq \mathbf{He}\left(\left(\Pi_i^+ + \sum_{j\in D_i^-}\pi_{ij}\right)E^T\left(S_{i0} + \sum_{j\in D_i^-}\pi_{ij}S_{ij}\right)E\right) = 0, \tag{49}$$

$$C_i^2 \triangleq \sum_{j=1}^{N}\nu_{ij}\pi_{ij}E^T\mathbf{He}(\Lambda_{ij})E \ge 0, \tag{50}$$

$$C_i^3 \triangleq -\sum_{j=1}^{N}\nu_{ij}\pi_{ij}(\pi_{ij} + \pi_{ii})E^T\mathbf{He}(Y_{ij})E \ge 0. \tag{51}$$

Then, the positive semidefinite matrix L^i is constructed using (49)–(51) in the following form:

$$\begin{aligned} L^i &\triangleq C_i^1 + C_i^2 + C_i^3 \\ &= \overline{L}^i + \sum_{j\in D_i^-}\pi_{ij}\mathbf{He}(L_j^iE) \\ &\quad + \sum_{j\in D_i^-}\sum_{\substack{l\in D_i^- \\ l>j}}\pi_{ij}\pi_{il}E^T\mathbf{He}(L_{jl}^i)E \\ &\quad + \sum_{\substack{l\in D_i^- \\ l=j}}\pi_{ij}^2E^T\mathbf{He}(L_{jj}^i)E \ge 0, \end{aligned} \tag{52}$$

where

$i \in D_i^+$

$$\overline{L}^i = \Pi_i^+ E^T \mathbf{He}(S_{i0})E + \sum_{j\in D_i^+} \pi_{ij} E^T \mathbf{He}(\nu_{ij}\Lambda_{ij})E - \sum_{j\in D_i^+} \pi_{ij}^2 E^T \mathbf{He}(\nu_{ij}Y_{ij})E - \sum_{j\in D_i^+} \pi_{ij}\pi_{ii} E^T \mathbf{He}(\nu_{ij}Y_{ij})E, \tag{53}$$

$$L_j^i = E^T(S_{i0} + \Pi_i^+ S_{ij}) + E^T \nu_{ij}\Lambda_{ij} - E^T \nu_{ij}\pi_{ii}Y_{ij},$$

$$L_{jl}^i = S_{il} + S_{ij},$$

$$L_{jj}^i = S_{il} - \nu_{ij}Y_{ij},$$

$$i \in D_i^-$$

$$\begin{aligned}
\overline{L}^i &= \Pi_i^+ E^T \mathbf{He}(S_{i0})E + \sum_{j\in D_i^+} \pi_{ij} E^T \mathbf{He}(\nu_{ij}\Lambda_{ij})E - \sum_{j\in D_i^+} \pi_{ij}^2 E^T \mathbf{He}(\nu_{ij}Y_{ij})E, \\
L_j^i &= E^T(S_{i0} + \Pi_i^+ S_{ij}) + E^T(\nu_{ij}\Lambda_{ij}) - \sum_{j\in D_i^+} c_{ij}\pi_{ij}E^T(\nu_{ij}Y_{ij}), \\
L_{jl}^i &= \begin{cases} S_{il} + S_{ij}, & i \neq l, \\ S_{il} + S_{ij} - \nu_{ij}Y_{ij}, & i = l, \end{cases} \\
L_{jj}^i &= S_{ij} - 2\nu_{ij}Y_{ij},
\end{aligned} \tag{54}$$

where

$$c_{ij} = \begin{cases} 1, & i = j, \\ 0, & i \neq j. \end{cases} \tag{55}$$

Based on the S-procedure, if $\Omega_s^i < 0$ whenever $L^i \geq 0$, the following sufficient condition is formulated:

$$L^i + \Omega_s^i < 0, \tag{56}$$

which can be converted to the following LMI condition:

$$\begin{bmatrix} I \\ [\pi_{ij}E]_{j\in D_i^-} \end{bmatrix}^T \begin{bmatrix} \overline{\Gamma}_s^i & [\Gamma_j^i]_{j\in D_i^-} \\ (*) & [\Gamma_{jl}]_{j,l\in D_i^-} \end{bmatrix} \begin{bmatrix} I \\ [\pi_{ij}E]_{j\in D_i^-} \end{bmatrix} < 0, \tag{57}$$

where

$$\overline{\Gamma}_s^i = \overline{\Omega}_s^i + \overline{L}^i,$$
$$\Gamma_j^i = \frac{1}{2}E^T G_{ij} + L_j^i, \tag{58}$$
$$\Gamma_{jl}^i = L_{jl}^i.$$

Then, (57) holds because of the LMI conditions (16)–(19). □

4. Numerical Examples

In this section, the $\mathscr{H}_2$ performance is investigated through numerical examples to verify the effectiveness of the proposed method.

4.1. Example 1. Consider an MJS with four modes ($N = 4$), whose system matrices are

$$\begin{aligned}
A_1 &= \begin{bmatrix} 0.35 & -7.30 \\ 1.48 & 0.81 \end{bmatrix}, \\
A_2 &= \begin{bmatrix} 0.89 & -3.11 \\ 1.48 & 0.21 \end{bmatrix}, \\
A_3 &= \begin{bmatrix} -0.11 & -0.85 \\ 2.31 & -0.10 \end{bmatrix}, \\
A_4 &= \begin{bmatrix} -0.17 & -1.48 \\ 1.59 & -0.27 \end{bmatrix}, \\
B_1 &= \begin{bmatrix} 0.57 \\ 1.23 \end{bmatrix}, \\
B_2 &= \begin{bmatrix} 0.78 \\ -0.49 \end{bmatrix}, \\
B_3 &= \begin{bmatrix} 1.34 \\ 0.39 \end{bmatrix}, \\
B_4 &= \begin{bmatrix} -0.38 \\ 1.07 \end{bmatrix}, \\
C_1 &= \begin{bmatrix} 0.0 & -0.1 \end{bmatrix}, \\
C_2 &= \begin{bmatrix} 0.1 & 0.0 \end{bmatrix}, \\
C_3 &= \begin{bmatrix} 0.0 & 0.1 \end{bmatrix}, \\
C_4 &= \begin{bmatrix} 0.1 & 0.0 \end{bmatrix}, \\
\Pi &= \begin{bmatrix} -1.3 & 0.2 & \pi_{13} & \pi_{14} \\ \pi_{21} & \pi_{22} & 0.3 & 0.3 \\ 0.6 & \pi_{32} & -1.5 & \pi_{34} \\ 0.4 & \pi_{42} & \pi_{43} & \pi_{44} \end{bmatrix}, \\
Q_1 &= Q_2 = Q_3 = Q_4 = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}, \\
\varepsilon &= 0.1, \mu = 1,
\end{aligned} \tag{59}$$

where π_{13}, π_{14}, π_{21}, π_{22}, π_{32}, π_{34}, π_{42}, π_{43}, and π_{44} are the unknown transition rates. The following sets can be obtained using the transition rate matrix Π:

$$\begin{aligned}
&D_1^+ = \{1,2\}, D_2^+ = \{3,4\}, D_3^+ = \{1,3\}, D_4^+ = \{1\}, \\
&D_1^- = \{3,4\}, D_2^- = \{1,2\}, D_3^- = \{2,4\}, D_4^- = \{2,3,4\}.
\end{aligned} \tag{60}$$

Considering the initial condition $x(0) = [\,0.2 \;\; -0.15\,]^T$, the state trajectories of the closed-loop system shown in Figure 1 are stochastically stable with incomplete knowledge of transition rates under the input saturation and matched

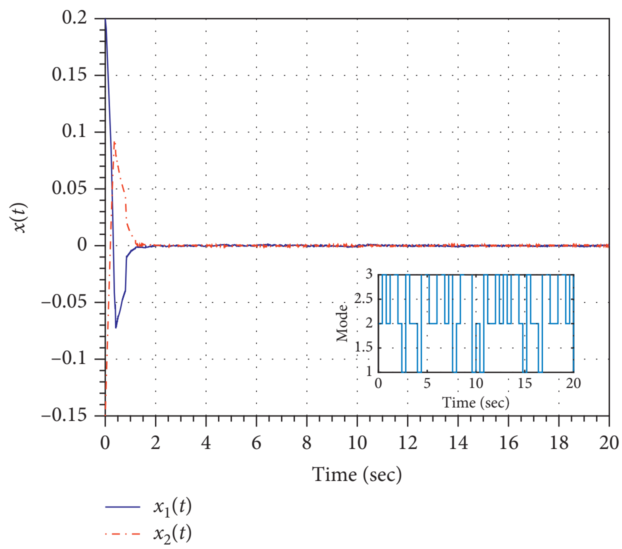

FIGURE 1: State trajectories for Example 1.

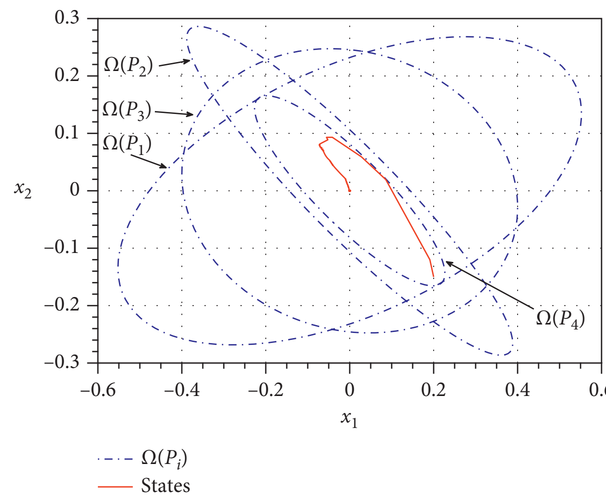

FIGURE 2: Domain of attraction for Example 1.

disturbances. Here, we set $r_0 = 2$ and $d(t) = 0.1\sin(2t^2 - 0.7)$.

According to Theorem 1, the $\mathscr{H}_2$ performance $\gamma = 0.2803$, and the proposed controller gains are obtained as follows:

$$\begin{aligned} K_1 &= \left[3.2384\times 10^5 \quad -2.1342^6 \right], \\ K_2 &= \left[-1.5913\times 10^6 \quad -7.8766\times 10^5 \right], \\ K_3 &= \left[-8.9047\times 10^5 \quad -8.0083\times 10^5 \right], \\ K_4 &= \left[-9.0151\times 10^6 \quad -1.5631\times 10^7 \right], \\ P_1 &= \begin{bmatrix} 4.3829 & -4.5290 \\ -4.5290 & 1.8561 \end{bmatrix}, \\ P_2 &= \begin{bmatrix} 47.968 & 60.404 \\ 60.404 & 88.256 \end{bmatrix}, \\ P_3 &= \begin{bmatrix} 6.3342 & 1.1945 \\ 1.1945 & 16.543 \end{bmatrix}, \\ P_4 &= \begin{bmatrix} 480.521 & 96.073 \\ 96.073 & 151.11 \end{bmatrix}. \end{aligned} \tag{61}$$

Figure 2 presents the domain of attraction for the proposed controller. As shown in the figure, the state trajectory of the closed-loop system (15) converges to the origin as time progresses, as long as the initial state is in $\cap_{i=1}^{4}\Omega(P_i)$.

4.2. Example 2. Consider the following multiinput system with three different modes ($N = 3$) [24]:

$$\begin{aligned} A_1 &= \begin{bmatrix} -2.5 & 0.3 & 0.8 \\ 1 & -3 & 0.2 \\ 0 & 0.5 & -2 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} -2.5 & 1.2 & 0.3 \\ -0.5 & 5 & -1 \\ 0.25 & 1.2 & 5 \end{bmatrix}, \\ A_3 &= \begin{bmatrix} 2 & 1.5 & -0.4 \\ 2.2 & 3 & 0.7 \\ 1.1 & 0.9 & -2 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} 0.707 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\ B_2 &= \begin{bmatrix} 0.707 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.707 \end{bmatrix}, \\ B_3 &= \begin{bmatrix} 0.707 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \\ \Pi &= \begin{bmatrix} -3 & \pi_{12} & \pi_{13} \\ \pi_{21} & \pi_{22} & 1 \\ \pi_{31} & 0.3 & \pi_{33} \end{bmatrix}, \\ Q_1 &= Q_2 \\ \varepsilon &= 0.01, \mu \end{aligned} \tag{62}$$

where π_{12}, π_{13}, π_{21}, π_{22}, π_{31}, and π_{33} are the unknown transition rates. The following sets can be obtained using the transition rate matrix Π:

$$\begin{aligned} &D_1^+ = \{1\}, D_2^+ = \{3\}, D_3^+ = \{2\}, \\ &D_1^- = \{2,3\}, D_2^- = \{1,2\}, D_3^- = \{1,3\}. \end{aligned} \tag{63}$$

According to Theorem 1, the $\mathscr{H}_2$ performance $\gamma = 0.0803$, and the proposed controller gains are obtained as follows:

$$K_1 = \begin{bmatrix} -94.941 & -12.248 & -2.4019 \\ -8.6349 & -61.833 & -6.8275 \\ -1.7987 & -6.8828 & -76.698 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -24.110 & -3.7947 & -0.69917 \\ -2.7040 & -13.943 & -2.0031 \\ -0.69626 & -2.8494 & -26.380 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} -93.208 & -25.607 & 18.331 \\ -18.085 & -67.048 & -14.084 \\ 12.952 & -14.079 & -76.181 \end{bmatrix},$$

$$P_1 = \begin{bmatrix} 1.8198 \times 10^{-1} & 2.3490 \times 10^{-2} & 3.0898 \times 10^{-3} \\ 2.3490 \times 10^{-2} & 1.6739 \times 10^{-1} & 1.8759 \times 10^{-2} \\ 3.0898 \times 10^{-3} & 1.8759 \times 10^{-2} & 2.0758 \times 10^{-1} \end{bmatrix},$$

$$P_2 = \begin{bmatrix} 3.3589 \times 10^{-1} & 5.3394 \times 10^{-2} & 9.2638 \times 10^{-3} \\ 5.3394 \times 10^{-2} & 2.7157 \times 10^{-1} & 4.0095 \times 10^{-2} \\ 9.2638 \times 10^{-3} & 4.0095 \times 10^{-2} & 3.6452 \times 10^{-1} \end{bmatrix},$$

$$P_3 = \begin{bmatrix} 2.5519 \times 10^{-1} & 7.0131 \times 10^{-2} & -5.1063 \times 10^{-2} \\ 7.0131 \times 10^{-2} & 2.5930 \times 10^{-1} & 5.4768 \times 10^{-2} \\ -5.1063 \times 10^{-2} & 5.4768 \times 10^{-2} & 2.9602 \times 10^{-1} \end{bmatrix}. \quad (64)$$

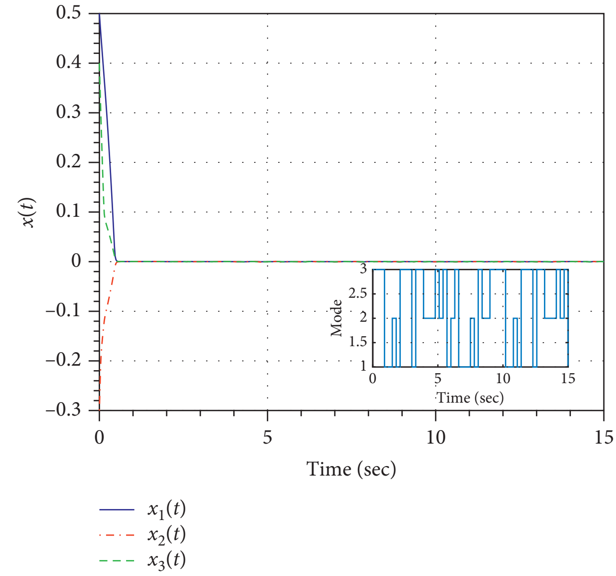

Figure 3: State trajectories for Example 2.

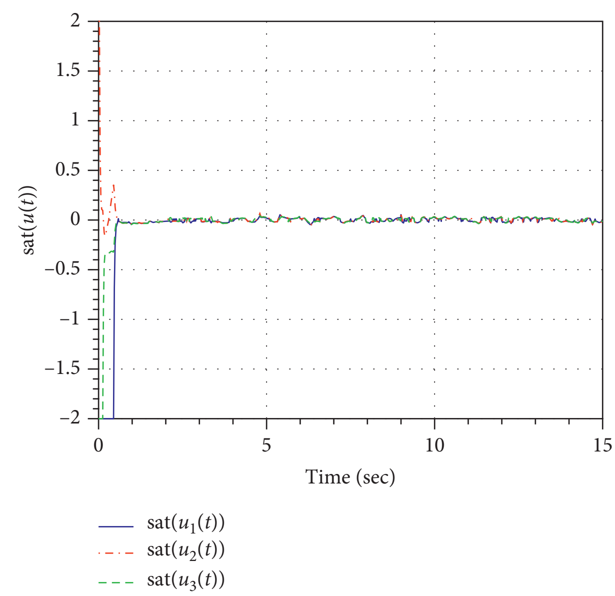

Figure 4: Control input for Example 2.

Figure 3 shows the state trajectories and the mode evolution obtained using the aforementioned controller gains. Figure 4 shows the saturated control input, where $x(0) = [0.5 \;\; -0.3 \;\; -0.4]^T$ and $r_0 = 3$. Here, we set $d(t) = 0.01 \sin(t^2 + 0.1)$. Figures 5 and 6 show the domains of attraction on the $x_1(t) - x_2(t)$ and $x_2(t) - x_3(t)$ planes, respectively. As shown in the figures, the state trajectory of the closed-loop system (15) converges to the origin as time progresses, as long as the initial state is in $\cap_{i=1}^{3} \Omega(P_i)$. These figures show that the proposed controller stabilizes the MJS with input saturation and incomplete knowledge of the transition rates under the matched disturbance.

4.3. Example 3. Consider the following inverted pendulum system controlled using a DC motor [27]:

$$\begin{aligned} \dot{x}_1(t) &= x_2(t), \\ \dot{x}_2(t) &= \frac{g}{l} \sin x_1(t) + \frac{NK_m}{ml^2} x_3(t), \\ L_a \dot{x}_3(t) &= K_b N x_2(t) - R(r_t) x_3(t) + \operatorname{sat}(u(t)), \end{aligned} \quad (65)$$

where $x_1(t)$ is the angle of the inverted pendulum, $x_2(t)$ is the angular velocity, $x_3(t)$ is the input current, $u(t)$ is the control input voltage, g is the acceleration of gravity, m and l are the mass and length of the inverted pendulum, respectively, K_b is the back-EMF constant, K_m is the motor torque constant, and N is the gear ratio. Here, $R(r_t)$ is the resistance in the DC motor, which is defined as

$$R(r_t) = \begin{cases} R_a, & \text{if } r_t = 1, \\ R_b, & \text{if } r_t = 2. \end{cases} \quad (66)$$

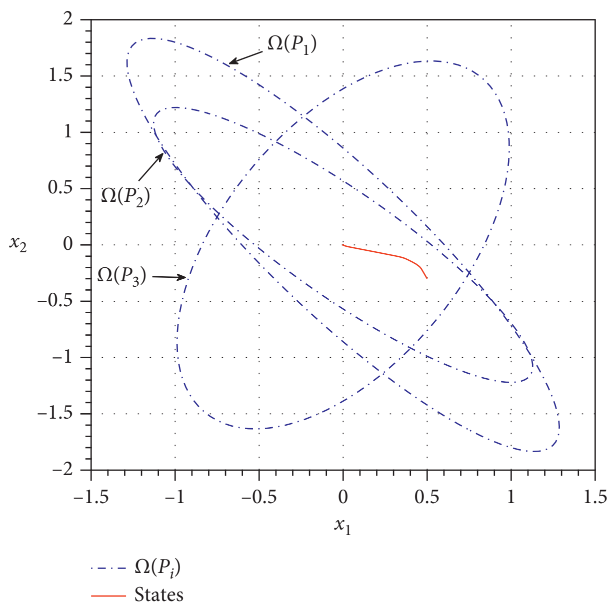

Figure 5: Domain of attraction ($x_1(t) - x_2(t)$ planes) for Example 2.

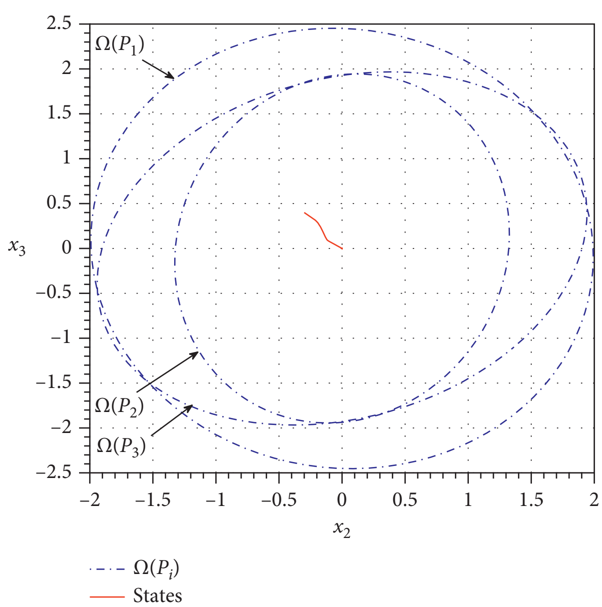

Figure 6: Domain of attraction ($x_2(t) - x_3(t)$ planes) for Example 2.

Let $L_a = 1$, $g = 9.8\,(\mathrm{m/s^2})$, $l = 1\,\mathrm{m}$, $m = 1\,\mathrm{kg}$, $N = 10$, $K_m = 0.1\,(\mathrm{Nm/A})$, $K_b = 0.1\,(\mathrm{Vs/rad})$, $R_a = 1\Omega$, and $R_b = 0.5\Omega$.

Using the aforementioned parameters, system (65) can be *linearized* as the following MJS with two modes:

$$\dot{x}(t) = A(r_t)x(t) + B(r_t)\{\mathrm{sat}(u(t)) + d(t)\},$$
$$z(t) = C(r_t)x(t), \tag{67}$$

where

$$x(t) = [\,x_1(t)\;\; x_2(t)\;\; x_3(t)\,]^T,$$
$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 9.8 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix},$$
$$A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 9.8 & 0 & 1 \\ 0 & 1 & -0.5 \end{bmatrix},$$
$$B_1 = B_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \tag{68}$$
$$C_1 = [\,0.1\;\; 0\;\; 0\,],$$
$$C_2 = [\,0.2\;\; 0\;\; 0\,],$$
$$\Pi = \begin{bmatrix} -0.6127 & 0.6127 \\ \pi_{21} & \pi_{22} \end{bmatrix},$$
$$d(t) = 0.01e^{-0.5t}\sin 10t^2,$$
$$\varepsilon = 0.01,\; \mu = 12,$$

where π_{21} and π_{22} are the unknown transition rates. Here, it is assumed that the matched disturbance $d(t)$ exists. Based on Theorem 1, the $\mathscr{H}_2$ performance $\gamma = 0.1399$, and the proposed controller gains are obtained as follows:

$$K_1 = [\,-2.6199\times 10^7\;\; -8.6656\times 10^6\;\; -1.9736\times 10^6\,],$$
$$K_2 = [\,-1.25\times 10^7\;\; -4.1531\times 10^7\;\; -1.0244\times 10^7\,],$$
$$P_1 = \begin{bmatrix} 1.0289\times 10^2 & 3.2962\times 10^1 & 2.9323 \\ 3.2962\times 10^1 & 1.0739\times 10^1 & 9.6989\times 10^{-1} \\ 2.9323 & 9.6989\times 10^{-1} & 2.2090\times 10^{-1} \end{bmatrix},$$
$$P_2 = \begin{bmatrix} 0.0698\times 10^4 & 2.1904\times 10^2 & 1.3991\times 10^1 \\ 2.1904\times 10^2 & 6.9178\times 10^1 & 4.6483 \\ 1.3991\times 10^1 & 4.6483 & 1.1465 \end{bmatrix}. \tag{69}$$

Based on the aforementioned control gains, Figure 7 shows the state trajectories for $x(0) = [\,-0.1\;\; 0.2\, 0\,]^T$ and the mode evolution r_t. As shown in the figure, the state trajectories of the closed-loop systems with the proposed controller converge to zero as time progresses.

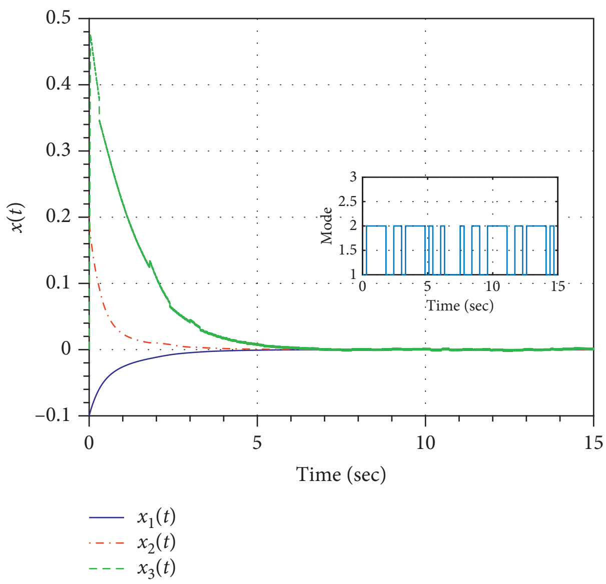

FIGURE 7: State trajectories for Example 3.

5. Conclusion

This paper proposed an $\mathscr{H}_2$ mode-dependent state-feedback controller for MJSs with input saturation and an incomplete knowledge of transition probabilities. Specifically, an invaluable relaxation method was developed into the second-order matrix polynomials of the unknown transition rate using all possible slack variables for the incomplete transition rates to obtain less conservative stabilization conditions. Consequently, the proposed controller guaranteed $\mathscr{H}_2$ performance and removed the matched disturbances. The effectiveness of the proposed controller was demonstrated using three examples.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

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