Research Article

Mathematical and Engineering Aspects of Chemically Reactive Tangent Hyperbolic Nanofluid over a Cone and Plate with Mixed Convection

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Current disquisition is expressed to elucidate the mathematical and engineering aspects of tangent hyperbolic liquid over a cone and plate which has valuable applications in industrial technological instruments such as cone and plate viscometer, spinning column technology, and biomass pyrolysis processing. The role of mixed convection and chemical reaction on physical features of fluid is considered. The attained coupled system is scrutinized by the aid of self-coded shooting method. In order to produce accuracy and refinement in the consequences, the coefficients of Runge–Kutta scheme are obliged. To get complete intellect about the current pagination, the impact of flow controlling variables on pertinent profiles are evaluated in quantitative and qualitative manner. The computations of wall drag force and heat and mass fluxes are presented pictorially and tabularly. From the thorough analysis, it is deduced that surface drag and convective thermal coefficient for cone is more in magnitude compared with those of plate. Similarly, dominance in thermal field is noted for cone compare with that in plate. Increment of velocity profile is revealed with enhancement in mixed convection parameter, whereas contrasting behavior is observed by varying Williamson and magnetic field parameters. Declining aptitude in thermal profile is exhibited against Prandtl number, whereas temperature of fluid mounts by changing the Brownian movement and thermophoresis parameters.

1. Introduction

Transmission of heat is the basic component of engineering which is concerned with the production, consumption, and transformation of energy. Heat transfer procedure has promising applications in thermal-exchanged systems in power houses, cooling and buildings, vehicles conditioner,

and cooling system in processing plants. In all the aforementioned applications, heat is carried with the aid of diversified fluids and efficiency of each working system depends on strong influence of thermal conductivity. For such reasons, researchers are continuously working on the development of techniques to raise the thermal conductivity of utilized fluids. Initially, the idea of dispersing the particles

into host fluid was proposed by Maxwell almost 120 years before. In this regard, Choi and Eastman [1] developed a new categorization of thermalized liquids depending on suspension of metallic nanoparticles with an average size of 100 nm termed as nanofluids. Masuda et al. [2] reconnoitered restricted thermal aptitudes of conventional fluids and added ultrafine elements in the base fluid for enhancement of thermal conductance. Irregular enhancement in thermal conductance of fluid with the addition of nanoconstituents in ethylene glycol-based fluid was bestowed by Eastman et al. [3]. Buongiorno [4] contemplated thermal attribute of nanofluids by examining seven distinctive properties possessed by nanofluids. Zhang et al. [5] explicated nanofluid flow generated by the variable stretched surface with radiation and MHD effects. Hsiao [6] scrutinized thermophysical features for viscoelastic fluid over a stretchable configuration. Sekholeslami et al. [7] accomplished an exploration on CuO-water-based nanofluid by operating control volume-based finite element procedure. Malik et al. [8] scrutinized Casson nanofluid flow generated due to exponential cylinder and heeded the solution by implementing shooting procedure. Haq et al. [9] investigated thermophysical aspects of viscoelastic fluid generated due to the stretched surface. Aboalbashari et al. [10] executed thermophysical aspects in the attendance of slip velocity with Newtonian heating. They used four dissimilar types of nanoparticles to enhance thermophysical characteristics of base fluid. In the aforementioned literature, vast concern related to study of nanofluids with Newtonian and non-Newtonian has been addressed; however, for the sake of interest of readers, few communications are enclosed in [11–14].

Viscosity is the fundamental material property of each flowing object, and conceptualization about viscosity has an important role in the development of multiple fluids. On the basis of this intrinsic feature possessed by fluids, characterizations among them are made. Each unique material has its own behavior when subject to flow due to which they are classified into two main fields, i.e., viscous and in-viscous fluids. Viscous liquids are defined as the liquids in which deformation made by them is linear against appliance of the shear rate. The basic component of life, i.e., air and water are Newtonian fluids, so importance of such fluids cannot be neglected. On the contrary to this, in many cases, non-Newtonian model must be modelled. The joint analysis of tangent hyperbolic fluid over cone and plate has promising real world application such as in cone and plate viscometer which measures the viscosity of coatings, paints, and emulsions. In addition, another most attracting application of the current work is in spinning column technology which is related to the removal of volatile organic compounds from highly viscous polymeric suspensions. Such situation also arises in mining industries where mud and slurries are handled, chemical food industries where different eatable products and other homemade objects are produced, plastic industries where paints emulsion, suspension, and lubrications are prepared, and in biomedical flows. So, the essence of non-Newtonian fluids from rheological and practical sense cannot be neglected. Furthermore, the fluid

dynamists have proposed several fluid models, e.g., Williamson fluid model, tangent hyperbolic model, and Carreau model on the basis of their pseudoplastic nature because they are unable to explicate with Navier Stokes equation. Among these various presented pseudoplastic models tangent hyperbolic fluid have been given less focus. So, present effort is dedicated to predict the intrinsic features of the tangent hyperbolic fluid model. Starting from the pioneering work carried out by Sher et al. [15], probed two-dimensional (2D) magnetized tangent hyperbolic liquids induced by a stretchable sheet. Friedman et al. [16] explained the practical implementation of tangent hyperbolic fluid by describing their utilization in damper coils. Nadeem and Akram [17] scrutinized hyperbolic liquid in asymmetric configuration. Naseer et al. [18] studied thermal aspects of tangent liquid generated by exponential axially stretched cylinder. Jyothi et al. [19] explicated peristaltic movement of tangent liquid through Darcy medium in symmetric channel.

Collaboration of boundary layer theory with electromagnetic forces has attracted promising attendance of researchers. Lorentz was the premier to present the mathematical expression for transversely applied magnetic field by proposing Lorentz force law to measure intensification of magnetic field in fluid flow. As an outcome, Maxwell electromagnetic equation is generated and coupling of it with Navier Stokes equation brings revolution in fluid dynamics. Alfeven [20] followed the concept manipulated by Lorentz and performed mathematical modeling about magnetohydrodynamic (MHD) flows. In recent years, tremendous work commenced on magnetohydrodynamic flow was conducted. Akbar et al. [21] measured reduction in velocity of tangent fluid produced by stretching of surface under the effect of perpendicularly executed MHD. Tain et al. [22] portrayed impact of Lorentz field on viscous liquid along with variable viscosity flow over a stretched surface. Malik et al. [23] evaluated influence of MHD on Eyring-Powel nanofluid caused by the stretched surface. Malik et al. [24] presented investigation about magnetization on viscoelastic fluid over a variably thickened stretching sheet. Bilal et al. [25] reconnoitered features of viscoelastic liquid over nonlinearly stretched surface under the effectiveness of magnetization. Analysis on behavior of hyperbolic liquid in the attendance of magnetic field and internal friction between molecules was manifested by Salahuddin et al. [26]. Some recent studies regarding MHD flow is referred in [27–31].

Inspired from previously acquired literature and reported applications about the pseudoplastic material, present pagination is different and innovative in such a way that it presents the analysis of such type of materials on two configurations jointly, i.e., cone and plate. So, current work excogitates mixed convection tangent hyperbolic flow along with the impact of chemical reaction on proposed joint configuration. According to author's knowledge, such type of work has not been executed so far. Firstly, the nonlinear differential equations concerning with flow fields are written in the form of nonlinear (ordinary differential equations) ODE's by employing permissible transmutations. The attained differential structure is handled by applying

Runge–Kutta (RK) fifth-order method. Fascinating features of influential variables are disclosed tabularly and pictorially. Interpretation regarding the control of multiple engineering factors such as wall friction, heat transfer, and mass flux are also executed. In addition, the assurance of computed data is performed by constructing comparison with the published literature. So, authors have hope that this communication will intend research fraternity towards this direction.

2. Model Development

Consider two-dimensional (2D), time-independent, and magnetized incompressible hyperbolic tangent nanofluid caused by a cone/plate. Variable magnetic field $B(x) = B_0/x(Gr)^{1/2}$ is employed alongside x -direction, as portrayed in Figure 1 (see Figures 2 and 3).

Constitutive conservation equations for tangent hyperbolic fluid are given as

$$\begin{aligned} \text{div } V &= 0, \\ \rho \frac{dV}{dt} &= \text{div } S + \rho b. \end{aligned} \quad (1)$$

The tensorial structure of fluid model is given as

$$\begin{aligned} S &= pI + \tau, \\ \tau &= -[\eta_\infty + (\eta_0 - \eta_\infty) \tanh(\Gamma \dot{\gamma})] \dot{\gamma}. \end{aligned} \quad (2)$$

$\dot{\gamma}$ is defined as follows:

$$\dot{\gamma} = \sqrt{\frac{1}{2} \Pi}, \quad (3)$$

$$\Pi = \frac{1}{2} \text{trac} [\text{grad } V + (\text{grad } V)^t]^2.$$

By using $\eta_\infty = 0$ and $\Gamma \dot{\gamma} < 1$, the component of extra-stress tensor is expressed as

$$\begin{aligned} \tau &= -\eta_0 (\Gamma \dot{\gamma})^n \dot{\gamma}, \\ \tau &= -\eta_0 (1 + \Gamma \dot{\gamma} - 1)^n \dot{\gamma}, \\ \tau &= -\eta_0 (1 + n[\Gamma \dot{\gamma} - 1]) \dot{\gamma}, \end{aligned}$$

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0,$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} = \nu(1-n) \frac{\partial^2 u}{\partial y^2} + \nu n \Gamma \sqrt{2} \frac{\partial u}{\partial y} \frac{\partial^2 u}{\partial y^2}$$

$$+ g(\rho\beta)_T (T - T_\infty) \cos \gamma - u \frac{v}{k} - \sigma B_0^2 u,$$

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \alpha \frac{\partial^2 T}{\partial y^2} + \tau \left(\frac{D_T}{T_\infty} \left(\frac{\partial T}{\partial y} \right)^2 + D_B \frac{\partial T}{\partial y} \frac{\partial C}{\partial y} \right),$$

$$u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} = D_B \frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty} \left(\frac{\partial^2 T}{\partial y^2} \right) - k_r (C - C_\infty), \quad (4)$$

subjected to the boundary conditions (BC's)

FIGURE 1: Physical Configuration.

$$u = 0, v = 0, \quad \text{at } y = 0, \text{ as } u \rightarrow 0,$$

$$T = T_w, C = C_w, \quad T \rightarrow T_\infty, C \rightarrow C_\infty, \text{ as } y \rightarrow \infty. \quad (5)$$

By using similarity transformations,

$$\eta = \frac{y}{x} (Gr)^{1/4},$$

$$T = T_\infty + (T_w - T_\infty) \theta(\eta),$$

$$Gr = \frac{\rho g \beta_\tau (T_w - T_\infty)}{\nu^2},$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad (6)$$

$$\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty},$$

$$u = \frac{\nu}{x} (Gr)^{1/2} f'(\eta),$$

$$v = -\frac{\nu}{x} (Gr)^{1/4} \left(\frac{\eta}{4} f'(\eta) - \frac{1}{4} f'(\eta) \right),$$

where η is the using variable similarity.

The transformed expression are defined as follows and referred to [24] and [26]:

$$(1-n)f''' + \frac{f f''}{4} - \frac{f'^2}{2} + nWe f'' f''' - (M+K)f' + \lambda\theta = 0, \quad (7)$$

$$\theta'' + \frac{Pr}{4} \theta' f + Pr \theta' \phi' N_b + Pr N_t \theta'^2 = 0, \quad (8)$$

$$\phi'' + \frac{Le}{4} f \phi' + \frac{N_t}{N_b} \theta'' + Le \gamma \phi = 0. \quad (9)$$

The related transformed boundary conditions are

$$f(0) = 0, f'(0) = 0, \theta(0) = 1, \phi(0) = 1, \quad \text{at } \eta = 0,$$

$$f'(\infty) = 0, \theta(\infty) = 0, \phi(\infty) = 0, \quad \text{at } \eta \longrightarrow \infty,$$

$$\text{Pr} = \frac{\nu}{\alpha},$$

$$\text{We} = \frac{\nu}{x^2} 2\Gamma(Gr)^{3/4},$$

$$M = \frac{\sigma B_0^2 x^2}{(Gr)^{1/2} \rho},$$

$$K = \frac{x^2}{k(Gr)^{1/2}},$$

$$\lambda = \frac{g\beta_T(T_w - T_\infty)\cos\gamma}{\nu^2(Gr)},$$

$$N_b = \frac{\tau D_B(C_w - C_\infty)}{\nu},$$

$$N_t = \frac{\tau D_t(T_w - T_\infty)}{\nu T_\infty},$$

$$\text{Le} = \frac{\nu}{D_B},$$

$$\gamma = \frac{k_r}{\rho g \beta_\tau}.$$

The wall shear factor and is given by

$$\tau_w = \mu \left((1-n) \frac{\partial u}{\partial r} + \frac{n\Gamma}{\sqrt{2}} \left(\frac{\partial u}{\partial r} \right)^2 \right),$$

$$q_w = -k \left(\frac{\partial T}{\partial y} \right),$$

$$q_m = -k \left(\frac{\partial C}{\partial y} \right),$$

at $y = 0$.

$$\frac{C_f \text{Re}_x^{1/2}}{2} = \left[(1-n) f''(0) + n \text{We} f''(0) \right],$$

where $\text{Re}_x^{1/2} = (\nu u/x^2)(Gr)^{3/4}$.

Local Nusselt (Nu) and Sherwood numbers (Sh) are given by

$$\begin{aligned} Nu &= -\theta'(0), \\ Sh &= -\phi'(0), \end{aligned} \quad (12)$$

where Third Affiliated Hospital of Sun Yat-Sen University.

3. Implemented Methodology

From the overview of the literature, an extensive number of methodologies are proposed to handle fluid dynamic problems. Like in other current analysis is modelled in form of differential equations, which is easier to handle by way of shooting method in collaboration with the RK-Fehlberg scheme. The schematic diagram is shown in Figure 2. The schematic diagram is shown in Figure 3. Since it applies on initial value problem (IVP), so equations (7)–(9) are transmuted into 1st order ODE's by letting $y_1 = f$, $y_2 = f'$, $y_3 = f''$, $y_4 = \theta$, $y_5 = \theta'$, $y_6 = \phi$, and $y_7 = \phi'$; then, equations (7)–(9) are reduced to

$$y_1' = y_2,$$

$$y_2' = y_3,$$

$$y_3' = \frac{1}{(1 + \text{We}^2 y_3)} \left[-\left(\frac{y_3 y_1}{4} - \frac{(y_2')^2}{2} \right) + (M + K)y_2 - \lambda y_4 \right],$$

$$y_4' = y_5,$$

$$y_5' = -\left[\frac{\text{Pr}}{4} y_1 y_5 + \text{Pr} y_5 y_7 N_b + \text{Pr} N_t y_5^2 \right],$$

$$y_6' = y_7,$$

$$y_7' = -\left[+\frac{\text{Le}}{4} y_1 y_7 + \frac{N_t}{N_b} y_5' + \text{Le} \gamma y_6 \right]. \quad (13)$$

The boundary conditions become

$$\begin{aligned} y_1(0) &= 0, y_2(0) = 0, y_4(0) = 1, y_6(0) = 1, \quad \text{at } \eta = 0, \\ y_2(\infty) &\longrightarrow 0, y_4(\infty) \longrightarrow 0, y_6(\infty) \longrightarrow 0, \quad \text{at } \eta \longrightarrow \infty. \end{aligned} \quad (14)$$

The involved central processing unit time is 16.7891 seconds. The computed solution will converge if boundary residuals ($R_1(u_1, u_2, u_3), R_2(u_1, u_2, u_3), R_3(u_1, u_2, u_3)$) are less than tolerance error, i.e., 10^{-6} . Otherwise, initial approximations are modified with Newton method and the process is repeated unless it meets the desired convergence criterion. Boundary residuals are defined such as

$$\begin{aligned} R_1(u_1, u_2, u_3) &= y_2(\infty) - \hat{y}_2(\infty), \\ R_2(u_1, u_2, u_3) &= y_4(\infty) - \hat{y}_4(\infty), \\ R_3(u_1, u_2, u_3) &= y_6(\infty) - \hat{y}_6(\infty), \end{aligned} \quad (15)$$

FIGURE 2: Schematic diagram of the shooting method.

where $\hat{y}_2(\infty)$, $\hat{y}_4(\infty)$, and $\hat{y}_6(\infty)$ are computed boundary values.

4. Graphs

4.1. Interpretation of Results. This segment is dedicated to explicate the impact of primitive parameters on concerning profiles such as momentum, thermal, mass, wall shear stress, heat flux, and mass flux distributions. Firstly, the equations are modelled by using rheological and conservation laws for the flow problem. Similarity transformations are obliged to convert the attained system of differential equations into ordinary differential form to execute solution. RK-Fehlberg method in collaboration with shooting scheme is implemented to manipulate result from system of equations. Table 1 examines compatibility of present findings by establishing comparison with Anil and Roy [25]. Total exactness about consequences of local Nusselt number $-\theta'(0)$ with the existing literature is manifested. From the physical point of view, the credibility of current work is also assured by the finding increasing aptitude in Nusselt number against uplifting magnitude of Pr . Tables 2–4 divulge numerical data about surface drag coefficient, heat flux, and mass flux coefficients against different parameters. Table 1 measures the variation in wall shear stress against magnetic parameter M and porosity parameter K . It is found that, by increasing M and K , the magnitude of skin friction coefficient for plate is more than in case of cone. It is justified physically that, for plate, the connection of fluid particles with the surface is more in comparison to cone. So, more resistance is offered to fluid particles in case of plate than cone. Table 2 expresses the extensive enhancement in convective heat transfer coefficient for plate in comparison to cone. Physically, it is because of the

reason that more contact is made by fluid particles with plate surface and hence more heat is transferred. Table 3 also addresses similar increasing manner in mass flux coefficient for plate than in cone against different flow controlling parameters. Figure 3 shows the gomety of the problem. Figure 4 explains delineation in velocity with respect to magnetic parameter M for fluid flow over a cone/plate. Physically, magnetic field parameter M induces resistive forces in the flow and is always applied in a perpendicular direction to decrease the movement of fluid particles at high velocity. Figure 5 represents deceasing impact of Weissenberg number We on momentum profile for cone/plate cases. Physical justification behind this effect is that Weissenberg number We is the ratio of relaxation time to retardation time thus by increasing We the fluid particles takes more time to retain their original position and thus the velocity down surges. Figure 6 represents enhancing attribute in velocity against mixed convection parameter λ . Physically, due to increased mixed convection parameter, buoyancy forces are generated in the flow which creates temperature difference in flow domain. Thus, as an outcome, the velocity of fluid enriches. Figure 7 reveals diminishing impact on thermal distribution for intensification in magnitude to Pr . This is because of the fact that Pr expresses ratio of viscous to thermal diffusions and with the uplift of Pr thermal diffusion decrements and the temperature decreases. Figure 8 depicts the impact of N_t on thermal profile. It is found that increases in thermophoresis parameter N_t fluid moves from hot to cold region hence thermal profile of the fluid decreases. Figure 9 demonstrates the influence of Brownian movement parameter N_b on thermal profile. It is evidenced that by enhancing the Brownian motion parameter N_b the temperature profile enhances and boundary layer thickness reduced. Figure 10 depicts

FIGURE 3: Computation of residual error for convergence.

TABLE 1: Comparison of the numerical results $-\theta'(0)$ for magnitude of λ and Pr when $\gamma = 0, N_t = 0, N_b \rightarrow 0, Le = 0, We = 0$, and $M = 0$.

Pr	λ	Anil and Roy [25]	Present findings $[-\theta'(0)]$
0.7	0	0.4305	0.4586
	1	0.6127	0.6324
	10	1.0175	0.99843
1	0	0.557294	0.55829
	1	0.721982	0.72318
	10	1.170983	1.17197
10	0	1.4042	1.4342
	1	1.5885	1.5795
	10	2.3528	2.3988

TABLE 2: Computation showing different values of skin friction coefficient M and k .

$\frac{C_f Re_x^{1/2}}{2}$			
M	K	Cone	Plate
0.1	0.1	0.1001	0.9708
	0.2	0.0956	0.0957
	0.3	0.0917	0.0918
0.5	0.1	0.0850	0.0852
	0.2	0.0822	0.0824
	0.3	0.0796	0.0798
0.9	0.1	0.0751	0.0753
	0.2	0.0731	0.0733
	0.3	0.0713	0.0715

decreasing change in concentration distribution and associated boundary thickness against N_b . It is due to the fact that, by increasing Brownian motion parameter, the

velocity of fluid particles increases and concentration decreases. Figure 11 discloses the reducing behavior of concentration profile against thermophoresis parameter N_t . By

TABLE 3: Numerical data of $-\theta'(0)$ with variations of Pr and N_b for cone and plate.

Pr	N_b	Nusselt number	
		Cone	Plate
1	0.1	0.9044	0.5323
	0.2	0.7588	0.5074
	0.3	0.2618	0.4921
1.2	0.1	0.2825	0.5442
	0.2	0.6546	0.5259
	0.3	0.2492	0.5079
1.4	0.1	0.2747	0.5618
	0.2	0.3538	0.5408
	0.3	0.4713	0.5715

TABLE 4: Computation showing $-\phi'(0)N_b = N_t = We = M = \lambda = 0.1$ and $Pr = 1$ with cone and plate.

γ	Le	$\phi'(0)$	
		Cone	Plate
0.1	0.1	0.7993	1.0046
	0.2	0.7873	0.9533
	0.3	0.7551	0.8977
0.3	0.1	0.7865	0.9923
	0.2	0.7475	0.9109
	0.3	0.7062	0.8154
0.5	0.1	0.7735	0.9798
	0.2	0.7053	0.8652
	0.3	0.6296	0.7183

FIGURE 4: Bearing of M on $f'(\eta)$.

increasing thermophoresis parameter, the fluid particles move from hot to cold region and as an outcome the concentration profile decreases. Figure 12 describes the features of Le on mass distribution. It is clear that mass transfer increases with Lewis number Le due to improvement in nanoparticles volume

FIGURE 5: Bearing of We on $f'(\eta)$. FIGURE 6: Bearing of λ on $f'(\eta)$.

fraction. Figure 13 explicates the impression of chemical reaction parameter γ on the mass transfer rate. It is clear that, for large values of chemical reactions parameter γ , $\phi(\eta)$ depreciates. The concentration profile reduces with increment in chemical reactions for $\gamma > 0$ while chemical reaction has opposite behavior for $\gamma < 0$. Figure 14 shows the role of magnetization M and Weissenberg number We on wall drag coefficients. It is depicted that the skin friction coefficient grid as both parameters increases. Figure 15 depicts the effects of wall drag coefficients for various values of parameters M and λ . This phenomenon causes the increase of skin friction with increase in M

FIGURE 7: Bearing of Pr on $\theta(\eta)$. FIGURE 9: Bearing of N_b on $\theta(\eta)$. FIGURE 8: Bearing of N_t on $\theta(\eta)$. FIGURE 10: Bearing of N_b on $\phi(\eta)$.

and λ . Figure 16 illustrated the variation of thermophoresis and Brownian moment parameter on the thermal transfer rate. It is observed that when N_t is increased, the local thermal transfer rate reduced.

5. Concluding Remarks

This work is presented to disclose the features of mixed convection flow of tangent hyperbolic nanofluid over a cone/plate in the presence of variable magnetic field strength, porosity aspects, and chemical reactions. Mathematical modeling of concerning problem is manifested in the form of partial differential expressions. The attained partial

differential expressions are transmuted into ordinary differential equation by implementing similarity approach. Afterwards, the solution of attained equations is solved by implementing the shooting method along with the Runge–Kutta–Fehlberg method.

The following key findings have been derived from the analysis:

Magnitude of drag coefficient for plate is more than in the case of cone

Momentum distribution and associated boundary layer increase in case of cone than plate

FIGURE 11: Bearing of N_t on $\phi(\eta)$. FIGURE 13: Bearing of γ on $\phi(\eta)$. FIGURE 12: Bearing of Le on $\phi(\eta)$. FIGURE 14: Influence We of skin friction.

Intensification in trends of thermal profile is manifested in cone compared with that of flat plate

Incrementing trend in temperature against inciting values of N_b and N_t is depicted

Concentration profile shows decrement pattern against upsurging values of Brownian movement parameter N_b and Lewis number Le

Nomenclature

$B(x)$: Magnetic field along x -direction
 M : Magnetic parameter
 B_0 : Strength of magnetic field

Pr : Prandtl number
 Gr : Grashof number
 BCs : Boundary conditions
 ρ : Density of fluid
 RK : Runge-Kutta
 V : Velocity vector
 MHD : Magnetohydrodynamic
 S : Cauchy stress tensor
 $2D$: Two dimensional
 b : Body force
 n : Power law index
 d/dt : Material time derivative
 We : Weissenberg number
 τ : Extrastress tensor

FIGURE 15: Influence of λ on skin friction. FIGURE 16: Bearing of N_t on $-\theta'(0)$.

λ :	Mixed convection parameter
η_0 :	Viscosity at zero shear stress
N_t :	Thermophoresis parameter
η_∞ :	Viscosity at infinite shear stress
N_b :	Brownian motion parameter
$\dot{\gamma}$:	Time rate constant
Le :	Lewis number
A_1 :	First rivlin-ericksoe tensor
γ :	Chemical reaction parameter
θ :	Dimensionless temperature
$C_f Re_x^{1/2}$:	Skin friction coefficient
ϕ :	Dimensionless concentration
$Nu Re_x^{-1/2}$:	Nusselt number

T_w :	Wall temperature
$Sh Re_x^{-1/2}$:	Sherwood number
T_∞ :	Ambient temperature
τ_w :	Wall shear stress
C_w :	Wall concentration
q_w :	Wall heat flux.

Data Availability

Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All authors equally contributed to this manuscript and approved the final version.

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