Research Article

Dusty Nanofluid Past a Centrifugally Stretching Surface

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This communication reports, the flow of Cu-water dusty nanofluid past a centrifugally stretching surface under the effect of second order slip and convective boundary conditions. The coupled nonlinear ordinary differential equations are get hold of from the partial differential equations which are derived from the conservation of momentum and energy of both nanofluid and dusty phases. Then, using apt resemblance transformation these ordinary differential equations were altered into a dimensionless form and then solved by bvp5c solver in Matlab software. The variation in velocity and temperature profiles of fluid and dusty phases for different parameters are thrash out in depth by figures and tables. The outcomes exhibit that the velocity profile of both fluid and dusty phases boot as the values of the dust particle volume fraction parameter is enlarged. Besides, the magnetic field parameter has similar effect on the velocity profile of both fluid and dusty phases. Also, the results illustrated that temperature profile of both Cu-water nanofluid and dusty particle phases are improved within an enhancement in the values of the temperature relaxation parameter, Cu-particle volume fraction, and Biot number. The results also confirm that for greater values of the magnetic field parameter the values of skin friction coefficient are enlarged for all values of the velocity ratio parameter.

1. Introduction

The investigations on more than one phase flows, in which solid sphere-shaped particles are disseminated in base fluids, are of great interest due to their practical applications in the movement of dust ladder air, fluidization, gas cooling systems to enhance heat transfer process, environmental pollution, powder technology, hydrocyclones, nuclear reactor cooling, dust collection, sedimentation, solid fuel rocket nozzles, petroleum industry, purification of crude oil, etc. Allocated to these appliances, still nowadays several investigators have concerted on examining the momentum and heat relocate characteristics of dusty fluid. For instance, in the year of 1962, Saffman [1] was the earliest person who examined the behavior of laminar flow of the dusty gases. Later on, authors such as Baral [2] and Chakrabarti [3] presented clear description on flow of the dusty gases in their investigations. Furthermore, Venkateshappa et al. [4] gave an analytical solution for flow of dusty gases. Also, Bagewadi and Gireesha [5] conferred the solution for the flow of dusty gases using the Laplace transformation method. The flow of

a dusty viscous fluid through a circular cylinder and pipe has been explored by Damseh [6] and Attia [7]. While, Mishra and Rauta [8] have reported the flow and heat relocate assets of dusty fluid over a stretching upright surface. On the other hand, viscoelastic dusty fluid via a porous medium duct was analyzed by Bhatti et al. [9]. Also, an exact solution of MHD flow of dusty fluid over an expanding surface was inspected by Jalil et al. [10]. Also, Prakash and Makinde [11] and Siddiqua et al. [12] have probed the flow of dusty fluid through an upright canal filled with a saturated porous conduit and wavy surface, respectively. The flow of dusty fluid due to linearly stretching cylinder has been deliberated by Konch and Hazarika [13], and MHD oscillatory flow of dusty fluid in a rotating porous upright canal was studied by Chand et al. [14]. Very recently, Balasubramanian et al. [15] discussed natural nonlinear convection flow of dusty fluid over a stretched cylinder at dissimilar temperatures. Furthermore, Abbas et al. [16] inspected the influences of slip on the flow of a dusty fluid over a porous expanding surface.

Also, in our day technology and industry, nanotechnology is considered to be one of the most momentous

vehicles that donate to the advancement of major industrial rebellion. Thus, several theoretical and experimental studies have been undertaken to advance the performance of industrial production. For instance, Choi and Eastman [17] were the earliest persons endeavor to enhance the thermal performance and conductivity of base fluids by the inclusion of nanoparticles. Later on, an improvement in the thermal conductivity of base fluid by means of nanoparticles were demonstrated by a large number of authors such as Sheikholeslami and Ganji [18], Avramenko et al. [19], Rauf et al. [20], Li et al. [21], Awad et al. [22], Raju et al. [23], Saleem et al. [24], and Animasaun et al. [25]. Besides, other mechanical applications such as in automatic transmission fluids, engine oils, lubricants, and coolants and biomedical applications such as in cancer therapies and nanodrug delivery as well as electronic applications such as cooling microchips in computers have been reported by Buongiorno et al. [26]. Also, several researchers investigated the characteristics of nanofluids and found that the thermal conductivity of base fluids can be improved by the inclusion of nanoparticles. For instance, a comprehensive survey of nanofluids convective heat transport was presented by Buongiorno [27]. An enhancement in the thermal conductivity with carbon nanofluids was experimentally proved by Liu et al. [28], and this outcome was also proved by Yu et al. [29] by considering kerosene-based Fe_3O_4 nanofluids. Moreover, Ibrahim [30] inspected the MHD flow of Maxwell nanofluid over a stretching surface. Also, Ibrahim [31] analyzed the impact of melting heat transport in MHD flow of a nanofluid over an expanding surface.

Furthermore, to be paid to the enhancement of nanotechnology, in addition to the sufficient applications of nanofluid, it is essential to examine the heat relocate assets of dusty nanofluid. Consequently, since both the nanofluid and dusty fluids are helpful in enhancing thermal conductivity of base fluid, numerous investigators have explored the flow and heat transport properties of dusty nanofluid through assorted geometries. Among them, Gireesha et al. [32] scrutinized the impacts of Hall current on two-phase flow of dusty nanofluid. Sandeep et al. [33] examined Nimonic 80A-water and 80A-water-ethylene glycol-based nanofluid embedded with dust particles past a leaky expanding/shrinking cylinder. Also, the flow of nanofluid embedded with dust particles past a expanding surface has been analyzed by Naramgari and Sulochana [34]; furthermore, the MHD flow of dusty nanofluid over a porous extending surface in the presence of a volume fraction of dust and nanoparticles was premeditated by Sandeep et al. [35]. Reddy et al. [36] explored dynamics' possessions of the MHD flow of dusty nanofluid past an upright cone.

To the best of the authors' information, no analyses have been reported on the flow of dusty nanofluid past a centrifugally stretching surface thus far. So, keeping this fact in mind this study aims to examine the flow of Cu-water dusty nanofluid past a centrifugally stretching surface under the effect of second order slip and convective boundary conditions.

FIGURE 1: Flow configuration and coordinate system.

TABLE 1: Thermophysical assets of water and Cu nanoparticles.

Physical properties	ρ (kg/m ³)	c_p (kg ⁻¹ K ⁻¹)	k (Wm ⁻¹ K ⁻¹)
Pure water (H ₂ O)	997.1	4179	0.613
Copper (Cu)	8933	385	401

2. Problem Formulation

Let us consider the steady, laminar two-dimensional flow of an electrically conducting incompressible Cu-water dusty nanofluid over a centrifugally stretching surface coincide with the plane $z = 0$ and the flow cramped to $z > 0$. As shown in Figure 1, r -direction is taken by the side of the centrifugally stretching surface in the direction of motion with velocity $U_w(r) = br$ and the ambient fluid velocity $U_\infty(r) = cr$, where b and c are positive constants and z direction is normal to it by keeping the origin fixed. Also, it is alleged that a heated fluid beneath the surface with temperature T_f is used to alter T_w by the convective heating mode which gives a heat relocate coefficient h_f and the ambient values of temperature of fluid are denoted by T_∞ . The dust particles are presumed to be uniform in size. Sphere-shaped nano and dust particles are considered. A uniform transverse magnetic field of strength B_0 is applied parallel to the z direction. The applied magnetic field is higher when it is compared with the induced magnetic field. Thus, the induced magnetic field and Hall current are negligible for small magnetic Reynolds number. The ordinary fluid- (water-) based nanofluid including copper as a nanoparticle is presumed, and its diameter is presumed to be equal to 100 nm. The physical assets of solid particles and ordinary fluid are described in Table 1.

Bearing in mind the standard boundary layer approximations and the Oberbeck–Boussinesq approximations, the

governing equations of this study under the aforementioned supposition is as follows (Butt et al. [37]):

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \quad (1)$$

$$(1 - \phi_d) \left(u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} \right) = (1 - \phi_p) \left(\frac{\mu_{nf}}{\rho_{nf}} \frac{\partial^2 u}{\partial z^2} \right) + U_\infty \frac{dU_\infty}{dr} - \frac{\rho_p}{\rho_{nf} \varepsilon} (u - u_p) - \frac{\sigma B_0^2}{\rho_{nf}} (u - U_\infty), \quad (2)$$

$$\frac{\partial}{\partial r} (\rho_p u_p) + \frac{\rho_p u_p}{r} + \frac{\partial}{\partial z} (\rho_p w_p) = 0, \quad (3)$$

$$u_p \frac{\partial u_p}{\partial r} + w_p \frac{\partial u_p}{\partial z} = \frac{1}{\varepsilon} (u - u_p), \quad (4)$$

$$u_p \frac{\partial w_p}{\partial r} + w_p \frac{\partial w_p}{\partial z} = \frac{1}{\varepsilon} (w - w_p), \quad (5)$$

$$u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} = \frac{k_{nf}}{(\rho c_p)_{nf}} \frac{\partial^2 T}{\partial z^2} + \frac{\rho_p c_s}{\gamma_t (\rho c_p)_{nf}} (T_p - T), \quad (6)$$

$$u_p \frac{\partial T_p}{\partial r} + w_p \frac{\partial T_p}{\partial z} = \frac{1}{\gamma_t} (T_p - T). \quad (7)$$

With boundary stipulations (Butt et al. [37] and Ibrahim and Gamachu [38]):

$$\begin{aligned} u &= U_w(r) + U_{\text{slip}}, \\ w &= 0, \\ -k \frac{\partial T}{\partial z} &= h_f (T_f - T) \text{ at } z = 0, \\ u &\longrightarrow U_\infty(r) = cr, \\ u_p &\longrightarrow U_\infty(r), \\ w_p &\longrightarrow w, \\ \rho_p &\longrightarrow \rho \omega, \\ T &\longrightarrow T_\infty, \\ T_p &\longrightarrow T_\infty \text{ as } z \longrightarrow \infty, \end{aligned} \quad (8)$$

where (1), (2), and (6) are continuity, momentum, and energy equation of the nanofluid phase, whereas (2)–(4) and (7) are continuity, momentum, and energy equation of the dusty fluid phase, (u, w) and (u_p, w_p) are velocity components of the fluid and dusty phase in the r and z directions, respectively, and U_{slip} is the slip velocity at the surface which is given as follows:

$$U_{\text{slip}} = \Lambda \frac{\partial u}{\partial z} + N \frac{\partial^2 u}{\partial z^2}, \quad (9)$$

where Λ and N are constants. The nanofluid constants are as follows (Sandeep et al. [33]):

$$\begin{cases} \rho_{nf} = (1 - \phi) \rho_f + \phi \rho_s, \\ (\rho c_p)_{nf} = (1 - \phi) (\rho c_p)_f + \phi (\rho c_p)_s, \\ \frac{k_{nf}}{k_f} = \left(\frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)} \right), \\ \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}}. \end{cases} \quad (10)$$

Also, the thermophysical assets of pure water and Cu nanoparticles are discussed as follows:

The following resemblance renovations have been used to make simpler the mathematical formula of the flow fields (Butt et al. [37] and Ibrahim and Gamachu [38]):

$$u(r, z) = ar f'(\xi),$$

$$w(r, z) = -2\sqrt{av} f(\xi),$$

$$\xi = \sqrt{\frac{a}{v}} z,$$

$$u_p = ar F(\xi),$$

$$w_p = 2\sqrt{av} G(\xi), \quad (11)$$

$$\rho_r = H(\xi),$$

$$\theta(\xi) = \frac{T - T_\infty}{T_f - T_\infty},$$

$$\theta_p(\xi) = \frac{T_p - T_\infty}{T_f - T_\infty}.$$

It can be demonstrated that (2) is identically satisfied by equation (1). Inserting (2) into (2)–(7), the overall non-dimensional form of the governing equations is as follows:

$$\begin{aligned} \frac{1 - \phi_d}{(1 - \phi)^{2.5}} f''' + (1 - \phi_d) \left(1 - \phi + \phi \left(\frac{\rho_s}{\rho_f} \right) \right) (2f f'' - f'^2) \\ + C^2 + \beta H(F - f') - M(f' - C) = 0, \end{aligned} \quad (12)$$

$$F^2 + 2GF' + \beta(F - f') = 0, \quad (13)$$

$$2GG' + \beta(f + G) = 0, \quad (14)$$

$$H'G + HG' + HF = 0, \quad (15)$$

$$\left[\frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)} \right] \theta'' + 2Pr \cdot \left(1 - \phi + \phi \left(\frac{(\rho C_p)_s}{(\rho C_p)_f} \right) f \theta' \right) + \frac{2}{3} \beta H (\theta_p - \theta) = 0, \quad (16)$$

$$2G\theta'_p + \beta L_t (\theta_p - \theta) = 0, \quad (17)$$

and the transformed boundary conditions are

$$\begin{aligned} f(\xi) &= 0, \\ f'(\xi) &= 1 + \gamma f'''(\xi) + \delta f''(\xi), \\ \theta(\xi) &= \text{Bt}(\theta(\xi) - 1) \text{ at } \xi = 0, \\ f'(\xi) &= C, \\ F(\xi) &= C, \\ G(\xi) &= -f(\xi), \\ H(\xi) &= \omega, \\ \theta(\xi) &\longrightarrow 0, \\ \theta_p(\xi) &\longrightarrow 0, \text{ as } \xi \longrightarrow \infty, \end{aligned} \quad (18)$$

where μ_{nf} is the dynamic viscosity of the nanofluid, ρ_{nf} is the density of nanofluid, ρ_p is the density of the dusty particles, ν is the kinematic viscosity, $\varepsilon = 1/\widehat{K}$ is the relaxation time of particles, $\widehat{K} = 6\Pi\mu_{nf}d$ is the Stokes' constant, d is the average radius of the dust particles, $M = \sigma B_o^2/a\rho_{nf}$ is the magnetic parameter, $\beta = 1/a\varepsilon$ is the fluid particles interaction parameter, $C = c/b$ is the velocity ratio parameter, $\rho_r = \rho_p/\rho_{nf}$ is the relative density, T and T_p are the temperature of nanofluid and dust particles, respectively, $(\rho C_p)_{nf}$ is the specific heat of nanofluid, $(\rho C_p)_s$ effective heat capacity of the nanoparticle medium, ϕ the volume fraction of nanoparticles, ϕ_d is the volume fraction of the dusty particles, c_s is the specific heat capacity of dust particles ($= 2k_{nf}\gamma_t/3\mu_{nf}\varepsilon$), γ_t is the temperature relaxation time ($= 3\rho_r\gamma_p c_s/2c_p$), γ_p is the velocity relaxation time ($= 1/\widehat{K}$), k_{nf} is the thermal conductivity of nanofluid, and $Pr = \mu_{nf}c_p/k_{nf}$ is the Prandtl number, ω is the density ratio, $L_t = \varepsilon/\gamma_t$ is the temperature relation parameter, $\gamma = \Lambda\sqrt{a\nu}$ is first order slip parameter, and $\delta = Ba/\nu$ is second order slip parameter.

The skin friction coefficient C_f and the Nusselt number Nu_r are, respectively, defined as follows:

$$\begin{aligned} C_f &= \frac{\tau_{wr}}{(1/2)\rho_{nf}(ar)^2}, \\ Nu_r &= \frac{\ell_w}{k(T_f - T_\infty)}, \end{aligned} \quad (19)$$

where τ_{wr} and ℓ_w are wall shear stress and wall heat flux, respectively, and are defined as follows:

$$\begin{aligned} \tau_{wr} &= \mu_{nf} \left(\frac{\partial u}{\partial z} \right) \Big|_{z=0}, \\ \ell_w &= -k \left(\frac{\partial T}{\partial z} \right) \Big|_{z=0}. \end{aligned} \quad (20)$$

Substituting (11) into (20), reduced to:

$$\begin{aligned} \frac{1}{2}C_f(\text{Re}_r)^{1/2} &= \frac{1}{(1-\phi)^{2.5}} f''(0), \\ (\text{Re}_r)^{-1/2} Nu_r &= - \left(\frac{k_{nf}}{k_f} \right) \theta'(0). \end{aligned} \quad (21)$$

3. Numerical Solution

The nondimensional system of equations (12)–(17) with boundary stipulations (12) are extremely joined nonlinear ODEs whose exact solutions are impossible. Hence, the numerical scheme should be employed to get the solution of these equations. In this investigation, the bvp5c solver is utilized to reach the numerical solution for (12)–(17) and these equations are reduced to

$$\begin{aligned} f &= \mathfrak{R}_1, \\ f' &= \mathfrak{R}_2, \\ f'' &= \mathfrak{R}_3, \\ f''' &= \mathfrak{R}'_3, \\ F &= \mathfrak{R}_4, \\ F' &= \mathfrak{R}'_4, \\ G &= \mathfrak{R}_5, \\ G' &= \mathfrak{R}'_5, \\ H &= \mathfrak{R}'_6, \\ H' &= \mathfrak{R}'_6, \\ \theta &= \mathfrak{R}_7, \\ \theta' &= \mathfrak{R}_8, \\ \theta'' &= \mathfrak{R}'_8, \\ \theta_p &= \mathfrak{R}_9, \\ \theta'_p &= \mathfrak{R}'_9. \end{aligned} \quad (22)$$

The compact form of (22) is stated as follows:

$$\begin{pmatrix} \mathfrak{R}'_1 \\ \mathfrak{R}'_2 \\ \mathfrak{R}'_3 \\ \mathfrak{R}'_4 \\ \mathfrak{R}'_5 \\ \mathfrak{R}'_6 \\ \mathfrak{R}'_7 \\ \mathfrak{R}'_8 \\ \mathfrak{R}'_9 \end{pmatrix} = \begin{pmatrix} \mathfrak{R}_2 \\ \mathfrak{R}_3 \\ \frac{(1-\phi)^{2.5}}{(1-\phi_d)} \left((1-\phi_d) \left(1 - \phi + \phi \left(\frac{\rho_s}{\rho_f} \right) \right) (2\mathfrak{R}_1 \mathfrak{R}_3 - \mathfrak{R}_2^2) \right) \\ -\beta \mathfrak{R}_6 (\mathfrak{R}_4 - \mathfrak{R}_2) - C^2 + M (\mathfrak{R}_2 - C) \\ -\frac{1}{2\mathfrak{R}_5} (\mathfrak{R}_4^2 + \beta (\mathfrak{R}_4 - \mathfrak{R}_2)) \\ \frac{\beta}{2\mathfrak{R}_5} (\mathfrak{R}_1 + \mathfrak{R}_5) \\ -\frac{1}{\mathfrak{R}_5} \left(\mathfrak{R}_6 \left(\frac{-\beta}{2\mathfrak{R}_5} (\mathfrak{R}_1 + \mathfrak{R}_5) \right) + \mathfrak{R}_4 \mathfrak{R}_6 \right) \\ \mathfrak{R}_8 \\ \frac{k_f}{k_{nf}} \left(2\text{Pr} \left(1 - \phi + \phi \left(\frac{(\rho c_p)_s}{(\rho c_p)_f} \right) \mathfrak{R}_1 \mathfrak{R}_8 \right) + \frac{2}{3} \beta H (\mathfrak{R}_9 - \mathfrak{R}_7) \right) \\ -\frac{1}{2\mathfrak{R}_2} \beta L_t (\mathfrak{R}_9 - \mathfrak{R}_7) \end{pmatrix}, \quad (23)$$

and the corresponding boundary stipulations are

$$\begin{pmatrix} \mathfrak{R}_0(1) \\ \mathfrak{R}_0(2) - 1 - \gamma \mathfrak{R}_0(3) - \delta \mathfrak{R}'_0(3) \\ \mathfrak{R}_0(8) - \text{Bt} (\mathfrak{R}_0(7) - 1) \\ \mathfrak{R}_{\text{inf}}(2) - C \\ \mathfrak{R}_{\text{inf}}(4) - C \\ \mathfrak{R}_{\text{inf}}(5) + \mathfrak{R}_{\text{inf}}(1) \\ \mathfrak{R}_{\text{inf}}(6) - \omega \\ \mathfrak{R}_{\text{inf}}(7) \\ \mathfrak{R}_{\text{inf}}(9) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \quad (24)$$

4. Outcomes and Discussion

In this investigation, the set of coupled nonlinear ordinary differential equations (12)–(17) with matching boundary stipulations (18) are solved numerically for assorted values of governing parameters using the bvp5c solver in Matlab. In all cases, the bvp5c solver outcomes offered in this toil were obtained using default values of $M = 1$, $\gamma = 1$, $L_t = 1$, $\text{Bt} = 1$, $\delta = -1$, $C = 0.5$, $\omega = 0.5$, $\phi = 0.01$, $\phi_d = 0.1$, $\beta = 0.2$, and

$\text{Pr} = 6.2$. The values of the Prandtl number used are the values assigned to water which is $\text{Pr} = 6.2$.

The effects of the magnetic field parameter M on the fluid velocity, dusty particle velocity, fluid temperature, and dusty particle temperature profiles are illustrated in Figures 2–5, respectively. It is observed from Figures 2 and 3 that the fluid velocity profile and particle velocity profile are drastically declined with an increment in the values of the magnetic field parameter M . Clearly, for greater values of the magnetic field parameter, the Lorentz force increases the resistive forces in the flow region. These forces are resisting both fluid and dusty particle motion and cause the Cu-water dusty nanofluid to sluggish downward. In consequence of this fact, the velocity profiles of both fluid and dusty particle phases are significantly diminished. This result reveals that the magnetic field parameter has similar effect on both fluid and particle velocity profiles. For the reason that the dust particles are engrossed in nanofluid the velocity profiles of the dust phase also pursue the fluid phase. On the other hand, the opposite trend is observed on the fluid temperature and dusty particle temperature with a mount in the values of the magnetic field parameter M , as seen from Figures 4 and 5. That is, for intensification in the values of the magnetic field parameter M , both temperatures of fluid and dusty particle profiles are improved. This is owing to the fact that an

FIGURE 2: Effects of magnetic parameter M on fluid velocity profile. FIGURE 3: Influences of magnetic parameter M on particle velocity profiles. FIGURE 4: Effects of magnetic parameter M on fluid temperature profiles.

FIGURE 5: Influences of magnetic parameter M on particle temperature profile. FIGURE 6: Effects of dusty particle volume fraction ϕ_d on fluid velocity profile. FIGURE 7: Influences of dusty particle volume fraction ϕ_d on particle velocity profile.

FIGURE 8: Effects of dusty particle volume fraction ϕ_d on fluid temperature profile. FIGURE 9: Influences of dusty particle volume fraction ϕ_d on particle temperature profile. FIGURE 10: Influences of nanoparticle volume fraction ϕ on fluid temperature profile.

increase in the values of the magnetic field parameter in the flow region imposes to grow up the Lorentz force in the boundary layer flow which fabricates the Lorentz heating in the energy equation, which can be used as the extra heat source to the flow system. The influences of the dusty particle volume fraction parameter ϕ_d on fluid velocity, particle velocity, and fluid and particle temperature profiles are presented through Figures 6–9, respectively. As observed from the graphs, a boost in the values of the dust particles volume fraction rises the velocity profiles of both fluid and dust phases. Conversely, an augmentation in the values of the dusty particle volume fraction tends to decline temperature profiles of both fluid and dust phases. Figures 10 and 11 reveal the influences of Cu nanoparticle volume fraction on temperature of both fluid and dusty phases,

respectively. As the values of Cu-nanoparticle volume fraction augments in the boundary layer flow, the temperature profile of both fluid and dusty phases are improved. This may take place owing to the fact that an enhancement in the interparticle collision inside the flow region improves the internal heat generation which leads to scale up of the temperature of nanofluid as well as the temperature of dusty particle, as observed from Figures 10 and 11.

The impact of the first order slip parameter γ on velocity profile of fluid is shown in Figure 12. As seen from the figure, fluid velocity profile $f'(\xi)$ is attenuation within the escalating values of γ . This is owing to the fact that an enlargement in the values of first order slip imposes to scale up the slip factor that produces the friction force which permits more fluid to slip past the surface and hence the flow

FIGURE 11: Effects of nanoparticle volume fraction ϕ on dusty particle temperature profile. FIGURE 12: Influences of first order slip parameter γ on fluid velocity profile. FIGURE 13: Influences of second order slip parameter δ on fluid velocity profile.

decelerates. Also, the effect of the second order slip parameter d on the velocity of fluid profile is revealed in Figure 13. It is observed from the graph that the velocity profile of the fluid phase is declined within an enlargement in the values of second order velocity slip. The influence of the velocity ratio parameter C on temperature profile of both fluid and dusty phases are displayed in Figures 14 and 15, respectively. It demonstrates that as the velocity ratio parameter C augments the thermal boundary layer thickness of fluid reduces. Furthermore, the Cu-water-based nanofluid temperature gradient at the surface amplifies (in absolute value) as C swells. As a consequence, Cu-water-based nanofluid temperature profile reduces, as shown in Figure 14. The dusty phase temperature at the surface descends with boost in the velocity ratio parameter, as shown in

Figure 15. Also, the thermal boundary layer thickness of particle drop with the enlarge values of the velocity ratio parameter C .

Figures 16 and 17 display the influences of various values of thermal Biot number Bt on the temperature profile of fluid and dusty phases, respectively. It is observed from the graphs that the fluid and particle temperature at the surface is varying due to convective boundary conditions. The small Biot number gives stronger conduction within the surface and the large Biot number entails stronger convection at the surface. Temperature profile of both fluid and dusty phases via Bt also predicts this behavior. As a result, the temperature of both fluid and particle improved within the boundary layer which makes the values of Bt better. Figure 18 displays the impacts of the temperature relaxation parameter L_t on

FIGURE 14: Influence of velocity ratio parameter C on fluid temperature profile. FIGURE 15: Influences of velocity ratio parameter C on particle temperature profile. FIGURE 16: Influences of the Biot number Bt on fluid temperature profile.

FIGURE 17: Influences of the Biot number Bt on particle temperature profile. FIGURE 18: Influences of temperature relaxation parameter L_t on particle temperature profile.

FIGURE 19: Grid-independence test for velocity profile.

FIGURE 20: Grid-independence test for temperature profile.

TABLE 2: Grid-independence test for velocity and temperature distributions (coarse mesh with 50 points, medium mesh with 100 points, and fine mesh with 500 points).

ξ	Coarse mesh		Medium mesh		Fine mesh	
	$ f''(\xi) $	$ \theta'(\xi) $	$ f''(\xi) $	$ \theta'(\xi) $	$ f''(\xi) $	$ \theta'(\xi) $
1.0	0.170247	0.385450	0.170247	0.385450	0.170247	0.385450
1.5	0.162087	0.399330	0.162087	0.399330	0.162087	0.399330
2.0	0.159982	0.400442	0.159982	0.400442	0.159982	0.400442
2.5	0.159231	0.400413	0.159231	0.400413	0.159231	0.400413
3.0	0.158863	0.400372	0.158863	0.400372	0.158857	0.400347
3.5	0.158647	0.400347	0.158647	0.400347	0.158659	0.400349
4.0	0.158509	0.400333	0.158509	0.400333	0.158509	0.400333

TABLE 3: Validation of the skin friction coefficient $f''(0)$ with those of Butt et al. [37] for variations in C and M when $\beta = \gamma = \delta = \phi_d = \phi = 0$.

C	$M = 0$		$M = 1.0$		$M = 2.0$	
	[37]	Present	[37]	Present	[37]	Present
0.1	-1.1246	-1.124628	-1.4334	-1.433474	-2.1138	-2.113820
0.2	-1.0556	-1.055624	-1.3179	-1.317955	-1.9080	-1.908082
0.5	-0.7534	-0.753446	-0.9002	-0.900182	-1.2456	-1.245600
1.0	0.0000	0.000000	0.0000	0.000000	0.0000	0.000000
1.1	0.1821	0.182072	0.2070	0.207030	0.2691	0.269092
1.2	0.3735	0.373539	0.4224	0.422357	0.5445	0.544557
1.5	1.0009	1.000929	1.1157	1.115733	1.4080	1.408098

TABLE 4: Computation of the skin friction coefficient $f''(0)$ for variations in C and M when $\gamma = 1$, $L_t = 1$, $Bt = 1$, $\delta = -1$, $\omega = 0.5$, $\phi = 0.01$, $\phi_d = 0.1$, $\beta = 0.2$, and $Pr = 6.2$.

C	Skin friction coefficient $f''(0)$					
	$M = 0$	$M = 0.5$	$M = 1.0$	$M = 1.5$	$M = 2.0$	$M = 2.5$
0.2	-0.300327	-0.293325	-0.273886	-0.248239	-0.222174	-0.198719
0.3	-0.265622	-0.258388	-0.239946	-0.216770	-0.193832	-0.173380
0.5	-0.189941	-0.184295	-0.170412	-0.153639	-0.137394	-0.123011
1.0	0.001939	0.001802	0.001326	0.001002	0.000701	0.000476
1.3	0.109053	0.106656	0.099021	0.089916	0.080820	0.072616
1.5	0.177510	0.173137	0.161513	0.147340	0.132966	0.119835

TABLE 5: Computation of the skin friction coefficient $f''(0)$ for variations in C and ϕ_d when $M = 1$, $\gamma = 1$, $L_t = 1$, $Bt = 1$, $\delta = -1$, $\omega = 0.5$, $\beta = 0.2$, $\phi = 0.01$, and $Pr = 6.2$.

C	Skin friction coefficient $f''(0)$					
	$\phi_d = 0.1$	$\phi_d = 0.2$	$\phi_d = 0.3$	$\phi_d = 0.4$	$\phi_d = 0.5$	$\phi_d = 0.6$
0.2	-0.273886	-0.270063	-0.265411	-0.259621	-0.252208	-0.242348
0.3	-0.239946	-0.235566	-0.230336	-0.223970	-0.216035	-0.205827
0.5	-0.170412	-0.164849	-0.158337	-0.150599	-0.141243	-0.129694
1.0	0.001326	0.009360	0.018682	0.029678	0.042904	0.059173
1.3	0.099021	0.107928	0.118274	0.130529	0.145408	0.164043
1.5	0.161513	0.170735	0.181455	0.194189	0.209752	0.229494

TABLE 6: Computation of the skin friction coefficient $f''(0)$ for variations in C and ϕ when $M = 1$, $\gamma = 1$, $L_t = 1$, $Bt = 1$, $\delta = -1$, $\omega = 0.5$, $\beta = 0.2$, $\phi_d = 0.1$, and $Pr = 6.2$.

C	Skin friction coefficient $f''(0)$					
	$\phi = 0.01$	$\phi = 0.05$	$\phi = 0.09$	$\phi = 0.13$	$\phi = 0.17$	$\phi = 0.21$
0.2	-0.273886	-0.278118	-0.281928	-0.285419	-0.288643	-0.291616
0.3	-0.239946	-0.245382	-0.250083	-0.254278	-0.258094	-0.261595
0.5	-0.170412	-0.178756	-0.185462	-0.1911442	-0.196144	-0.200655
1.0	0.001326	-0.015067	-0.026723	-0.035676	-0.042999	-0.049300
1.3	0.099021	0.078212	0.064132	0.053783	0.045628	0.038808
1.5	0.161513	0.138091	0.122680	0.111650	0.103163	0.096201

TABLE 7: Computation of the local Nusselt number $-\theta'(0)$ for variations in C and M when $\gamma = 1$, $L_t = 1$, $Bt = 1$, $\delta = -1$, $\omega = 0.5$, $\beta = 0.2$, $\phi = 0.1$, $\phi_d = 0.1$, and $Pr = 6.2$.

C	Local Nusselt number $-\theta'(0)$					
	$M = 0.0$	$M = 0.5$	$M = 1.0$	$M = 1.5$	$M = 2.0$	$M = 2.5$
0.2	0.748174	0.748275	0.748476	0.748640	0.748741	0.748801
0.3	0.747824	0.747895	0.748039	0.748161	0.748239	0.748287
0.5	0.747010	0.747044	0.747116	0.747181	0.747227	0.747256
1.0	0.744645	0.744647	0.744651	0.744656	0.744660	0.744663
1.3	0.743132	0.743128	0.743121	0.743112	0.743106	0.743101
1.5	0.742107	0.742102	0.742090	0.742077	0.742067	0.742059

TABLE 8: Computation of the local Nusselt number $-\theta'(0)$ for variations in C and ϕ_d when $M = 1$, $\gamma = 1$, $L_t = 1$, $Bt = 1$, $\delta = -1$, $\omega = 0.5$, $\beta = 0.2$, $\phi = 0.1$, and $Pr = 6.2$.

C	Local Nusselt number $-\theta'(0)$					
	$\phi_d = 0.1$	$\phi_d = 0.2$	$\phi_d = 0.3$	$\phi_d = 0.4$	$\phi_d = 0.5$	$\phi_d = 0.6$
0.2	0.748476	0.748489	0.748505	0.748523	0.748545	0.748570
0.3	0.748039	0.748034	0.748028	0.748021	0.748013	0.748004
0.5	0.747116	0.747070	0.747016	0.746952	0.746877	0.746786
1.0	0.744651	0.744491	0.744304	0.744080	0.743807	0.743462
1.3	0.743121	0.742890	0.742619	0.742293	0.741891	0.741378
1.5	0.742090	0.741813	0.741485	0.741090	0.740600	0.739972

TABLE 9: Computation of the local Nusselt number $-\theta'(0)$ for different values of C and Bt when $M = 1$, $\gamma = 1$, $L_t = 1$, $\delta = -1$, $\omega = 0.5$, $\beta = 0.2$, $\phi = 0.1$, $\phi_d = 0.1$, and $Pr = 6.2$.

C	Local Nusselt number $-\theta'(0)$					
	$Bt = 1.0$	$Bt = 2.0$	$Bt = 3.0$	$Bt = 4.0$	$Bt = 5.0$	$Bt = 10.0$
0.2	0.748476	0.856147	0.899268	0.922499	0.937023	0.967488
0.3	0.748039	0.855861	0.899057	0.922333	0.936886	0.967415
0.5	0.747116	0.855256	0.898613	0.921982	0.936596	0.967260
1.0	0.744651	0.853639	0.897421	0.921041	0.935819	0.966846
1.3	0.743121	0.852632	0.896680	0.920455	0.935335	0.966587
1.5	0.742090	0.851954	0.896179	0.920059	0.935008	0.966413

the temperature profile of dusty particle. It is observed that temperature profile of the particle increases within the enhancement in the values of the temperature relaxation parameter L_t in the entire of boundary layer region. Clearly, the temperature relaxation parameter L_t is the ratio of relaxation time of particles to temperature relaxation time. That is, greater the values of temperature relaxation parameter means larger a relaxation time of particles. This improves the thermal conductivity of the flow. As a consequence, the temperature of dusty particle climb within the values of temperature relaxation parameter amplifies. Figures 19 and 20 show the grid-independence test for velocity and temperature distributions. As seen from the figures, enhancing the number of points more than 500, the velocity and temperature distributions are not affected, but only to enhance the compilation time.

Table 2 shows that the grid-independence test is performed to sustain the eight-decimal point accuracy. It is also called the grid invariance test or grid convergence test. We used this test to improve outcomes using successively smaller size and we started by choosing a coarser mesh with 50 number of points. Then, enlarging number of points two times, we acquired a medium mesh with 100 points. Finally, we have a fine mesh with 500 points and get eight-decimal point accuracy in the skin friction coefficient and Nusselt number values. After enlarging the number of points more than 500, the accuracy is not affected but only to increase the compilation time. Table 3 confirms the corroboration of the present upshots with earlier published data in the literature (Butt et al. [37]) for the skin friction coefficient $f''(0)$. The results obtained are in an excellent agreement with the formerly published data available in the literature in limiting conditions for some particular cases of the present study, as seen from Table 3. Thus, the authors are confident that the present results are accurate.

The influences of assorted parameters on the skin friction coefficient $f''(0)$ and local Nusselt number $-\theta'(0)$ are displayed in Tables 4–9. Table 4 reveals the variation of the skin friction coefficient $f''(0)$ for varied values of the velocity ratio parameter C and magnetic field parameter M when other parameters remain constant. It can be observed from the table that as the values of C and M increase, the skin friction coefficient $f''(0)$ enhances. Also, it is appealing to note that there is a change of sign for $C \geq 1$ for all values of M . This is owing to the fact that within a rise in the values of M the resistance forces become more and more, and skin friction force is developed in the boundary layer flow. Consequently, the skin friction coefficient is augmented. Also, it is noted from Table 5 that growing the values of the dusty particle volume fraction parameter ϕ_d near the radially stretching wall, the amount of $f''(0)$ swells. Likewise, augment in the values of the velocity ratio parameter C also causes $f''(0)$ to amplify for all values of ϕ_d . Table 6 illustrates the variation of the skin friction coefficient $f''(0)$ for assorted values of C and Cu-nanoparticle volume fraction ϕ . As seen from the table, boosting in the values of ϕ , outcomes in a decline in the values of $f''(0)$. But, with the raise values of C the values of the skin friction coefficient is declined. That means the effects of C and ϕ on $f''(0)$ is not similar.

Table 7 reveals the impact of C and M on the local Nusselt number. It is perceived that enlargement in the value of the velocity ratio parameter results in drop of $-\theta'(0)$ for all values of M . In addition, it is perceived that for $C \leq 1.0$ the local Nusselt number improves with an increase in the values of M and reverse trend is observed for $C > 1.0$. The variation of $-\theta'(0)$ for dissimilar values of the velocity ratio parameter C and dusty particle volume fraction ϕ_d is illustrated in Table 8. It is clear from the table that augment in the values of ϕ_d results in intensification of $-\theta'(0)$ for all values of C , but the values of $-\theta'(0)$ reduce decreased (fall down) as the values of C increased for all values of ϕ_d . Table 9 presents the impact of C and Bt on the local Nusselt number $\theta'(0)$. As seen from the table, an increment in the values of the Biot number Bt leads to improvement in the rate of heat transfer for all values of C , whereas the rate of heat transfer $\theta'(0)$ is declined within the values of C , becoming better than before.

5. Conclusions

This study examined the boundary layer flow of Cu-water dusty nanofluid past a centrifugally stretching surface under the effect of second order slip and convective boundary conditions. The main observations which have been derived from the study can be concluded as follows:

- (1) Magnetic field parameter has a similar effect on the velocity profile of both fluid and dusty phases
- (2) Dusty particle volume fraction ϕ_d and Cu-nanoparticle volume fraction ϕ have parallel impact on the temperature of nanofluid and dusty particles
- (3) Cu-nanoparticle volume fraction ϕ and velocity ratio parameter C have opposite effect on the temperature of both fluid and dusty phases
- (4) For bigger values of the Biot number Bt and temperature relaxation parameter L_t the temperature profile of the dusty phase is increased
- (5) The effect of C and ϕ on the skin friction coefficient $f''(0)$ is opposite
- (6) Dusty particle volume fraction ϕ_d and Bt have same effect on the local Nusselt number

Nomenclature

b and c :	Rate of the stretching surface
B_o :	Magnetic field strength (T)
Bt:	Thermal Biot number
C_f :	Skin friction coefficient
C :	Velocity ratio parameter
d :	Average radius of the dust particles
h_f :	Heat transfer coefficient
k_{nf} :	Thermal conductivity of nanofluid (W/mK)
k_f :	Thermal conductivity of base fluid (W/mK)
L_t :	Temperature relation parameter
M :	Magnetic field parameter
Nu_r :	Local Nusselt number
Pr :	Prandtl number
ℓ_w :	Wall mass flux

R_{er} : Local Reynolds number
 r and z : Cartesian coordinate (m)
 c_s : Specific heat capacity of dust particles (J/kgK)
 c_p : Specific heat capacity of nanofluid (J/kgK)
 T_p : Temperature of the dust particles (K)
 T_∞ : Temperature of the ambient fluid (K)
 T_f : Convection heat transfer (K)
 U_∞ : Free stream velocity (ms^{-1})
 (u, w) : Velocity components of the fluid (ms^{-1})
 (u_p, w_p) : Velocity components of the dusty phase (ms^{-1})

Greeks

β : Fluid particle interaction parameter
 ξ : Dimensionless similarity variables
 ϕ : Nanoparticle volume fraction (nm)
 ϕ_d : Dusty particle volume fraction (μm or mm)
 θ : Dimensionless temperature of fluid
 θ_p : Dimensionless temperature of dusty particles
 σ : Electric conductivity (m^{-1}S)
 ρ_s : Density of the nanofluid (kgm^{-3})
 ρ_f : Density of the base fluid (kgm^{-3})
 γ : First order slip parameter
 ν : Kinematic viscosity of the fluid (m^2s^{-1})
 μ_{nf} : Dynamic viscosity of the nanofluid (m^2s^{-1})
 τ_{wr} : Shear stress ($\text{kgm}^{-1}\text{s}^{-2}$)

Subscripts

∞ : Condition at the free stream
 w : Condition at the surface.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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