Research Article

Hesitant Fuzzy Decision-Making Method Based on Correlation Coefficient under Confidence Levels with Application to Multisensor Electronic Reconnaissance

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In this paper, we focus on the hesitant fuzzy decision-making method based on correlation coefficient under confidence levels. Firstly, we propose several correlation coefficients based on confidence levels, and based on the correlation coefficient between attribute-evaluated values and the ideal values, several optimal attribute weights models are constructed. Secondly, we have defined the concepts of module, weight module, projection, and weighted projection in hesitant fuzzy sets on account of the projection theory. Finally, on the strength of our research results above, we construct a hesitant fuzzy multiattribute decision-making method and apply it to the multiattribute decision-making problem in multisensor electronic reconnaissance. Comparative analyses are made via simulation to test the rationality and validity of the proposed method.

1. Introduction

In the field of multiattribute decision-making, the decision-making process is complex and confuses researchers. Due to different knowledge backgrounds and thinking patterns of decision makers, the degree of membership may vary for the same elements and collections. For example, some people give 0.5, other people give 0.6 or 0.7. Especially, all decision makers insist on their own opinions, and they are dead-locked. To deal with this complex situation, the concept of hesitant fuzzy sets was proposed by Torra et al. [1, 2], which allowed the membership of an element in a given set to be different values. Xu and Xia [3, 4] proposed multiple distance measures for hesitant fuzzy sets. Farhadinia [5] studied the relationship between entropy, similarity measure and distance measure of hesitant fuzzy sets (HFSs), and interval-valued hesitant fuzzy sets (IVHFSs). Zhu et al. [6] and Yu et al. [7] proposed dual hesitant fuzzy sets (DHFSs) and researched the basic operation and characteristics of DHFs. The problem of fuzzy multiattribute group decision-making (MAGDM) with different priorities of attribute and expert was studied by Wei [8]. Yu [9] proposed a hesitant fuzzy

multicriteria group decision-making method for performance and development review (PADR) evaluation. Farhadinia [10, 11] presents a series of scoring functions for hesitant fuzzy sets (HFSs) and extended the hesitant fuzzy set (HFS) to its higher order type. Rodriguez et al. [12] present a marvellous overview on hesitant fuzzy sets.

Many classic decision-making methods have been extended to uncertain environments, and some fuzzy multiattribute decision-making methods have been obtained; attribute weights play an important role in summarizing expert assessments in the decision-making process and attracts many researchers' attention. For example, Xu [13] studied some of the group decision-making problems concerning attribute weight information. Park [14] investigated the group decision-making problems, in which all the information provided by the decision makers is presented in the form of interval-valued intuitionistic fuzzy decision matrices. And fuzzy TOPSIS method under incomplete weight information [15–17], fuzzy ELECTRE II methods [18], fuzzy PROMETHEE method [19], fuzzy VIKOR method [20, 21], fuzzy QUALIFLEX method [22], fuzzy TWO-SIDED MATCHING model [23], etc. It can be said

that the research on fuzzy decision-making methods has formed a relatively complete system, which can basically cover decision-making problems in fuzzy environments. However, among these methods, the research on the hesitant fuzzy decision-making method under confidence levels has just started, and the content is less.

In many practical decision-making problems, the evaluation experts are required to provide two types of information such as the performance of the evaluation objects and the familiarity with the evaluation fields (called confidence levels). In this paper, we propose several correlation coefficients based on confidence levels. Then, we have defined the concepts of module, weight module, projection, and weighted projection in hesitant fuzzy sets under confidence levels on account of the projection theory. Finally, we construct a hesitant fuzzy multiattribute decision-making method and apply it to the multiattribute decision-making problem in multi-sensor electronic reconnaissance.

2. Preliminaries

Zadeh [24] first attempted to use membership function for uncertain information. However, in the decision-making process, because there is a lot of uncertain information, experts usually hesitate for one thing or another, and it is difficult to reach a final consensus, while hesitant fuzzy sets can effectively solve such problems.

Definition 1 (see [1, 2]). Let $X = \{x_1, x_2, \dots, x_n\}$ be a fixed set; then, a hesitant fuzzy set (HFS) on X is in terms of a function that when applied to X returns a subset of $[0, 1]$.

For ease of understanding, Xia and Xu [25] represent the HFS with a mathematical symbol:

$$E = \{ \langle x, h_E(x) \rangle \mid x \in X \}, \quad (1)$$

where $h_E(x)$ is a set of some values in $[0, 1]$, denoting the possible membership degrees of the element $x \in X$ to the set E . For convenience, Xia and Xu [25] call $h = h_E(x)$ a hesitant fuzzy element (HFE) and H the set of all the HFEs.

In many practical decision-making problems, most existing hesitant fuzzy aggregation operators do not take into account the confidence level of aggregation parameters provided by information providers. Xia and Xu [26] propose a hesitant fuzzy aggregation operator considering absolute confidence levels.

Definition 2 (see [26]). Let $h_1, h_2, \dots, h_n$ be a collection of HFSs, l_i be the confidence levels of HFS h_i , and $l_i \in [0, 1]$, $w = (w_1, w_2, \dots, w_n)^T$ be the weight vector of them, such that $w_i \in [0, 1]$, $\sum_{i=1}^n w_i = 1$. If

$$\begin{aligned} \text{CIHFWA}(h_1, h_2, \dots, h_n) &= \bigoplus_{i=1}^n w_i (l_i h_i) \\ &= \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2, \dots, \gamma_n \in h_n} \left(1 - \prod_{i=1}^n (1 - l_i \gamma_i)^{w_i} \right), \end{aligned} \quad (2)$$

then CIHFWA is referred to as confidence-induced hesitant fuzzy weighted averaging (CIHFWA) operator. Peculiarly, if $l_1 = l_2 = \dots = l_n = 1$, then the CIHFWA operator reduces to the hesitant fuzzy weighted averaging (HFWA) operator:

$$\begin{aligned} \text{HFWA}(h_1, h_2, \dots, h_n) &= \bigoplus_{j=1}^n w_j h_j \\ &= \cup_{\gamma_1 \in h_1, \gamma_2 \in h_2, \dots, \gamma_n \in h_n} \left(1 - \prod_{j=1}^n (1 - \gamma_j)^{w_j} \right). \end{aligned} \quad (3)$$

Definition 3 (see [25]). Suppose $h_1, h_2, \dots, h_n$ be a collection of hesitant fuzzy sets (HFSs), and let HFOWA be $\Omega_n \rightarrow \Omega$. The hesitant fuzzy ordered weighted averaging (HFOWA) operator:

$$\begin{aligned} \text{HFOWA}(h_1, h_2, \dots, h_n) &= \bigoplus_{j=1}^n w_j h_{\sigma(j)} \\ &= \cup_{\gamma_{\sigma(1)} \in h_{\sigma(1)}, \gamma_{\sigma(2)} \in h_{\sigma(2)}, \dots, \gamma_{\sigma(n)} \in h_{\sigma(n)}} \left(1 - \prod_{j=1}^n (1 - \gamma_{\sigma(j)})^{w_j} \right), \end{aligned} \quad (4)$$

where $h_{\sigma(j)}$ is the j th largest element of h_j ($j = 1, 2, \dots, n$) and $w = (w_1, w_2, \dots, w_n)^T$ is the aggregation-associated vector such that $w_j \in [0, 1]$, $\sum_{j=1}^n w_j = 1$.

3. Models and Method for Determining the Attribute Weights

In a multiattribute decision-making, the expert group is sometimes asked to provide the weights for selected attributes. At all events, the attribute weights are always incomplete because of the short of knowledge and data and the limitation of decision makers' expertise and time. Therefore, how to carry out the alternatives sorting by the known information is an interesting and important question. In this section, we propose a method based on the correlation coefficients of HFEs.

3.1. Correlation Coefficients of HFEs. We first introduce the concept of correlation coefficient, then give several correlation coefficient formulas, and discuss their properties.

Definition 4 (see [25]). For two HFEs h_1 and h_2 , the correlation coefficient of h_1 and h_2 , denoted as $C(h_1, h_2)$, should satisfy the following properties:

$$\begin{aligned} (1) & |C(h_1, h_2)| \leq 1, \\ (2) & C(h_1, h_2) = 1, \\ (3) & C(h_1, h_2) = C(h_2, h_1). \end{aligned} \quad (5)$$

In most cases, $k_{h_1} \neq k_{h_2}$, and for convenience, let $k = \max\{k_{h_1}, k_{h_2}\}$. For proper operation, when comparing them, extend the shorter one to the point where both have the same length. The best way to extend the shorter one is to add a value to it. In fact, we can extend shorter values by adding shorter ones, depending on the DM's risk appetite. The optimists expect the desired outcome and may increase

the maximum, while the pessimists expect the adverse outcome and may increase the minimum.

Based on Definition 4, in [4], the following correlation coefficients for HFEs are given:

$$C_1(h_1, h_2) = \frac{\sum_{j=1}^k h_1^{\sigma(j)} \cdot h_2^{\sigma(j)}}{\max\left\{\sum_{j=1}^k (h_1^{\sigma(j)})^2, \sum_{j=1}^k (h_2^{\sigma(j)})^2\right\}}, \quad (6)$$

$$C_2(h_1, h_2) = \frac{\sum_{j=1}^k h_1^{\sigma(j)} \cdot h_2^{\sigma(j)}}{\left(\sum_{j=1}^k (h_1^{\sigma(j)})^2 \cdot \sum_{j=1}^k (h_2^{\sigma(j)})^2\right)^{1/2}},$$

where $h_1^{\sigma(j)}$ and $h_2^{\sigma(j)}$ are the j th largest values in h_1 and h_2 .

We give another correlation coefficient for HFEs:

$$C_3(h_1, h_2) = \frac{\sum_{j=1}^k h_1^{\sigma(j)} \cdot h_2^{\sigma(j)}}{\left(\sum_{j=1}^k (h_1^{\sigma(j)})^2 + \sum_{j=1}^k (h_2^{\sigma(j)})^2\right)/2}. \quad (7)$$

Although all three formulas satisfy the properties of Definition 4, they each have their own characteristics. We will discuss this issue below.

Theorem 1. Let h_1 and h_2 be two HFEs; then,

$$C_2(h_1, h_2) \leq C_3(h_1, h_2) \leq C_1(h_1, h_2). \quad (8)$$

Proof. Because $\sqrt{ab} \leq (a + b)/2 \leq \max\{a, b\}$, $a, b \geq 0$, we can obtain

$$\frac{\left(\sum_{j=1}^k (h_1^{\sigma(j)})^2 \cdot \sum_{j=1}^k (h_2^{\sigma(j)})^2\right)^{1/2}}{2 \leq \max\left\{\sum_{j=1}^k (h_1^{\sigma(j)})^2, \sum_{j=1}^k (h_2^{\sigma(j)})^2\right\}}, \quad (9)$$

then

$$\begin{aligned} & \frac{\sum_{j=1}^k h_1^{\sigma(j)} \cdot h_2^{\sigma(j)}}{\left(\sum_{j=1}^k (h_1^{\sigma(j)})^2 \cdot \sum_{j=1}^k (h_2^{\sigma(j)})^2\right)^{1/2}} \\ & \leq \frac{\sum_{j=1}^k h_1^{\sigma(j)} \cdot h_2^{\sigma(j)}}{\left(\sum_{j=1}^k (h_1^{\sigma(j)})^2 + \sum_{j=1}^k (h_2^{\sigma(j)})^2\right)/2} \\ & \leq \frac{\sum_{j=1}^k h_1^{\sigma(j)} \cdot h_2^{\sigma(j)}}{\max\left\{\sum_{j=1}^k (h_1^{\sigma(j)})^2, \sum_{j=1}^k (h_2^{\sigma(j)})^2\right\}}. \end{aligned} \quad (10)$$

This completes the proof of Theorem 1.

Considering the importance of the confidence levels, we propose several hesitant fuzzy correlation coefficients based on relative confidence levels and absolute confidence levels and discuss their relationship.

Definition 5. Let h_1 and h_2 be two HFEs, $k = \max(k_{h_1}, k_{h_2})$, and k_{h_1} and k_{h_2} be the length in h_1 and h_2 . Then, we can construct several correlation coefficients under confidence levels for HFEs:

$$\begin{aligned} C_l^1 &= \frac{\sum_{j=1}^k (l_1^{\tau(j)} h_1^{\tau(j)}) (l_2^{\tau(j)} h_2^{\tau(j)})}{\max\left\{\sum_{j=1}^k (l_1^{\tau(j)} h_1^{\tau(j)})^2, \sum_{j=1}^k (l_2^{\tau(j)} h_2^{\tau(j)})^2\right\}}, \\ C_l^2 &= \frac{\sum_{j=1}^k (l_1^{\tau(j)} h_1^{\tau(j)}) (l_2^{\tau(j)} h_2^{\tau(j)})}{\left(\sum_{j=1}^k (l_1^{\tau(j)} h_1^{\tau(j)})^2 \cdot \sum_{j=1}^k (l_2^{\tau(j)} h_2^{\tau(j)})^2\right)^{1/2}}, \\ C_l^3 &= \frac{\sum_{j=1}^k (l_1^{\tau(j)} h_1^{\tau(j)}) (l_2^{\tau(j)} h_2^{\tau(j)})}{\left(\sum_{j=1}^k (l_1^{\tau(j)} h_1^{\tau(j)})^2 + \sum_{j=1}^k (l_2^{\tau(j)} h_2^{\tau(j)})^2\right)/2}, \\ C_l^4 &= \frac{\sum_{j=1}^k (l_1^{\tau(j)} h_1^{\tau(j)}) (l_2^{\tau(j)} h_2^{\tau(j)})}{\max\left\{\sum_{j=1}^k (l_1^{\tau(j)} h_1^{\tau(j)})^2, \sum_{j=1}^k (l_2^{\tau(j)} h_2^{\tau(j)})^2\right\}}, \\ C_l^5 &= \frac{\sum_{j=1}^k (l_1^{\tau(j)} h_1^{\tau(j)}) (l_2^{\tau(j)} h_2^{\tau(j)})}{\left(\sum_{j=1}^k (l_1^{\tau(j)} h_1^{\tau(j)})^2 \cdot \sum_{j=1}^k (l_2^{\tau(j)} h_2^{\tau(j)})^2\right)^{1/2}}, \\ C_l^6 &= \frac{\sum_{j=1}^k (l_1^{\tau(j)} h_1^{\tau(j)}) (l_2^{\tau(j)} h_2^{\tau(j)})}{\left(\sum_{j=1}^k (l_1^{\tau(j)} h_1^{\tau(j)})^2 + \sum_{j=1}^k (l_2^{\tau(j)} h_2^{\tau(j)})^2\right)/2}, \end{aligned} \quad (11)$$

where $h_1^{\sigma(j)}$ and $h_2^{\sigma(j)}$ are the j th largest values in h_1 and h_2 . $l_i^{\tau(j)}$ be the confidence levels of $h_i^{\tau(j)}$ and $l_i^{\tau(j)} \in [0, 1]$, and $l_i^{\tau(j)} = n \times (l_i^{\tau(j)} / \sum_{i=1}^k l_i^{\tau(j)})$ be the relative confidence levels of $h_i^{\tau(j)}$ and $l_i^{\tau(j)} \in [0, 1]$.

Theorem 2. Let h_1 and h_2 be two HFEs, then

$$\begin{aligned} C_l^2(h_1, h_2) &\leq C_l^3(h_1, h_2) \leq C_l^1(h_1, h_2), \\ C_l^5(h_1, h_2) &\leq C_l^6(h_1, h_2) \leq C_l^4(h_1, h_2). \end{aligned} \quad (12)$$

3.2. Models for Determining the Attribute Weights. In real life, there are always some differences between the ideal point of attribute values and the attribute values' vector corresponding to the alternative A_i ($i = 1, 2, \dots, m$). We call $A^* = (a_1^*, a_2^*, \dots, a_n^*)$ is an ideal selection for attribute values, where $a_j^* = \max_i \{a_{ij}\}$, $j = 1, 2, \dots, n$. We define the correlation $C_1(A_i, A^*)$. Obviously, the bigger $C_1(A_i, A^*)$ is, the better the alternative A_i will be. Therefore, a reasonable

weight vector $w^* = (w_1^*, w_2^*, \dots, w_n^*)$ should be determined so that all the correlations are as bigger as possible, which means maximizing the following correlation vector [27–29].

Model 1:

$$\max \left(\sum_{j=1}^n w_j C_1(a_{1j}, a_j^*), \sum_{j=1}^n w_j C_1(a_{2j}, a_j^*), \dots, \sum_{j=1}^n w_j C_1(a_{nj}, a_j^*) \right), \quad (13)$$

under the condition $w \in H$, where H is the set of the known weight information.

To this end, we set up the following multiobjective optimization model.

Model 2:

$$\begin{aligned} \max \quad & \sum_{i=1}^m \sum_{j=1}^n w_j C_1(a_{ij}, a_j^*) \\ \text{s.t.} \quad & w \in H. \end{aligned} \quad (14)$$

Model 2 can be easily solved by the single-objective programming model; then, we can get the optimal solution $w^* = (w_1^*, w_2^*, \dots, w_n^*)$, which can be used as the weight vector of attributes.

If the information about attribute weights is completely unknown, the following multiobjective programming model can be established.

Model 3:

$$\begin{aligned} \max \quad & \sum_{i=1}^m \sum_{j=1}^n w_j C_1(a_{ij}, a_j^*) = \sum_{i=1}^m \sum_{j=1}^n w_j \frac{\sum_{j=1}^l \gamma_1^{\sigma(j)} \cdot \gamma_2^{\sigma(j)}}{\max \left\{ \sum_{j=1}^l (\gamma_1^{\sigma(j)})^2, \sum_{j=1}^l (\gamma_2^{\sigma(j)})^2 \right\}} \\ \text{s.t.} \quad & \sum_{j=1}^n w_j^2 = 1, w_j \geq 0, j = 1, 2, \dots, n. \end{aligned} \quad (15)$$

To solve this model, we construct the Lagrange function:

$$L(w, \lambda) = \sum_{i=1}^m \sum_{j=1}^n w_j C_1(a_{ij}, a_j^*) + \frac{\lambda}{2} \left(\sum_{j=1}^n w_j^2 - 1 \right), \quad (16)$$

where λ is the Lagrange multiplier.

Differentiating equation (16) with respect to w_j ($j = 1, 2, \dots, n$) and λ , set these partial derivatives to zero, which gives the following equation:

$$\begin{cases} \frac{\delta L(w_j, \lambda)}{\delta w_j} = \sum_{i=1}^m C_1(a_{ij}, a_j^*) + \lambda w_j = 0, \\ \frac{\delta L(w_j, \lambda)}{\delta \lambda} = \sum_{j=1}^n w_j^2 - 1 = 0. \end{cases} \quad (17)$$

By solving equation (17), we obtain a simple and precise formula to determine the attribute weights as shown below:

$$w_j^* = \frac{\sum_{i=1}^m C_1(a_{ij}, a_j^*)}{\sum_{j=1}^n \left(\sum_{i=1}^m C_1(a_{ij}, a_j^*) \right)}, \quad (18)$$

which can be used as the weight vector of attributes. Apparently, $w_j^* \geq 0$, for all j .

If $C_1(a_{ij}, a_j^*)$ is replaced with formula (4), then we have

$$w_j^* = \frac{\sum_{i=1}^m \left(\sum_{j=1}^k a_{ij} \cdot a_j^* / \left(\max \left\{ \sum_{j=1}^k (a_{ij})^2, \sum_{j=1}^k (a_j^*)^2 \right\} \right) \right)}{\sum_{j=1}^n \left(\sum_{i=1}^m \left(\left(\sum_{j=1}^k a_{ij} \cdot a_j^* \right) / \left(\max \left\{ \sum_{j=1}^k (a_{ij})^2, \sum_{j=1}^k (a_j^*)^2 \right\} \right) \right) \right)}, \quad j = 1, 2, \dots, n. \quad (19)$$

Through the above analysis, we know that, under the hesitant fuzzy information, both Model 2 and Model 3 can be used to determine attribute weights with incomplete weight information. Model 2 can be used when the weight information is incomplete, and the model can be constructed in various forms. Model 3 can be used when the information about attribute weights is completely unknown, which can be solved easily and obtained as a simple and accurate formula for determining the weight of an attribute [29].

4. Projection Method under Hesitant Fuzzy Environment

Wang [30] proposed a projection method for multiattribute group decision-making, the projection values of evaluation alternatives, and ideal alternatives are used to get the ideal sort. As an excellent measuring tool, the projection method not only reflects the closeness degree between the two objects but also reflects the intrinsic nature of the objects. Now, it is being applied to the decision-making method [31–33]. In this section, we focus on the projection method under hesitant fuzzy environment.

Definition 6 (see [30–33]). Suppose $x = (x_1, x_2, \dots, x_n)$ and $y = (y_1, y_2, \dots, y_n)$ are two vectors, then cosine of included angle between vectors x and y can be defined as follows:

$$\cos(x, y) = \frac{\sum_{i=1}^n x_i y_i}{\sqrt{\sum_{i=1}^n x_i^2} \sqrt{\sum_{i=1}^n y_i^2}}. \quad (20)$$

Evidently, cosine of included angle is in this range: $0 < \cos(x, y) \leq 1$, and the larger the value of $\cos(x, y)$ is, the more the direction of vector x and y is consistent.

Definition 7 (see [30–33]). Suppose $x = (x_1, x_2, \dots, x_n)$, then $|x| = \sqrt{\sum_{i=1}^n x_i^2}$ is a norm of vector x .

A vector consists of two parts, the direction and the norm. The cosine of the angle $\cos(x, y)$ can only measure whether the direction is consistent, but it cannot reflect the magnitude of norm. In order to consider the norm magnitude and cosine of included angle together, the proximity of two vectors can be measured by the projection value, and the definition of projection is given as follows:

Definition 8 (see [30–33]). Suppose $x = (x_1, x_2, \dots, x_n)$ and $y = (y_1, y_2, \dots, y_n)$ are two vectors, and the projection of vector x onto vector y can be defined:

$$\text{Pr } j_y x = \frac{\sum_{i=1}^n x_i y_i}{\sqrt{\sum_{i=1}^n x_i^2} \sqrt{\sum_{i=1}^n y_i^2}} \sqrt{\sum_{i=1}^n x_i^2} = \frac{\sum_{i=1}^n x_i y_i}{\sqrt{\sum_{i=1}^n y_i^2}}. \quad (21)$$

Apparently, the larger the value of $\text{Pr } j_y x$ is, the closer the vector x is to the vector y . For the MADM problem, if the vector y obtains a positive ideal solution and the vector x is used for each alternative, we can obtain the projection of each alternative on the positive ideal solution. The larger the projection value is, the better the alternative is.

Now, we extend the projection theory to the hesitant fuzzy sets and give the concepts of the hesitant fuzzy module, weighted module, projection, and weighted projection.

Definition 9 (see [34]). Suppose $H = (h_1, h_2, \dots, h_n)$ be a vector of HFSs; then,

$$|H| = \sqrt{\sum_{j=1}^n h_j^2}, \quad (22)$$

is called the module of vector H .

In many cases, the weight of h_j should be considered. For example, in the multiple attribute decision-making, sometimes the evaluation experts are asked to provide the weight of every attribute, so we should define the hesitant fuzzy weighted module.

Definition 10 (see [34]). Let $H = (h_1, h_2, \dots, h_n)$ be a vector of HFSs and $w = (w_1, w_2, \dots, w_n)$ be the weight vector of h_j ($j = 1, 2, \dots, n$), such that $w_j \in [0, 1]$, $\sum_{j=1}^n w_j = 1$. Then,

$$|H|_w = \sqrt{\sum_{i=1}^n (w_i |h_i|)^2}. \quad (23)$$

is called the weighted module of vector H .

Definition 11 (see [34]). Suppose $H = (h_1, h_2, \dots, h_n)$ and $E = (e_1, e_2, \dots, e_n)$ be two vectors of HFSs; then, the projection of vector H onto vector E can be defined:

$$\text{Pr}j_E H = \frac{\sum_{i=1}^n h_i e_i}{|E|}. \quad (24)$$

Evidently, the larger the value of $\text{Pr}j_E H$ is, the closer the vector H is to the vector E .

Definition 12. Suppose $H = (h_1, h_2, \dots, h_n)$ and $E = (e_1, e_2, \dots, e_n)$ be two vectors of HFSs. $|E|_w$ is the weighted module of vector E . Let w_i and t_i be the weight of h_i and e_i , such that $w_i \in [0, 1]$, $\sum_{i=1}^n w_i = 1$, $t_i \in [0, 1]$, $\sum_{i=1}^n t_i = 1$. Then, the weighted projection of vector H onto vector E can be defined:

$$\text{Pr}j_{(E)_w}(H)_w = \frac{\sum_{i=1}^n (w_i h_i)(t_i e_i)}{|E|_w}. \quad (25)$$

5. An Approach to Multiple Attribute Decision-Making with Hesitant Fuzzy Information under Confidence Levels

Based on the above models, we develop a practical approach to solve the group decision-making problems, where the information about attribute weights is not completely known or completely unknown, and the attribute values take the form of hesitant fuzzy information. The approach includes the following steps:

Step 1: we construct the decision matrix $A^{(k)} = (a_{ij})_{m \times n}^{(k)}$ with confidence levels, where all the arguments a_{ij} ($i = 1, 2, \dots, m; j = 1, 2, \dots, n$) are HFSs, given by the decision makers, for the alternative $A_i \in A$ with respect to the attribute $G_j \in G$, and the decision makers need to give out the confidence levels l_k ($0 \leq l_k \leq 1$) ($k = 1, 2, \dots, p$).

Step 2: we apply summary operators to aggregate decision makers' assessments into collective evaluation a_{ij} ($i = 1, 2, \dots, m; j = 1, 2, \dots, n$) on each alternative A_i ($i = 1, 2, \dots, m$).

Step 3: we can get the ideal of attribute values $A^* = \{a_1^*, a_2^*, \dots, a_n^*\}$. If the information about the attribute weights is entirely unknown, then we can get the attribute weights $w = (w_1, w_2, \dots, w_n)^T$ by using Model 3. If the information about the attribute weights is partly known, then we solve model 2 to obtain the attribute weights $w = (w_1, w_2, \dots, w_n)^T$.

Step 4: apply formula (25) to calculate the relative weighted projection $\text{Pr}j_{A_w^*} A_i$ of each alternative A_i :

$$\text{Pr}j_{A_w^*} A_i = \frac{\sum_{j=1}^n w_j^2 (a_{ij} \cdot a_j^*)}{|A^*|_w}. \quad (26)$$

Step 5: rank the alternatives A_i ($i = 1, 2, \dots, m$) according to the relative weighted projection

$\text{Pr}j_{A_w^*} A_i$. The larger the $\text{Pr}j_{A_w^*} A_i$ is, the better the alternative A_i is.

6. Numerical Example

In this section, we apply the proposed hesitant fuzzy multi-attribute decision-making method based on signed correlation to multisensor electronic reconnaissance decision problems. In these problems, multiattribute information about the enemy, such as pulse repetition frequency, carrier frequency, power, and pulse width, are detected using various sensors such as Radar, Electronic Support Measures (ESMs), and Infrared, and multiattribute decisions must be made regarding the kind of airborne platform the enemy is using based on these pieces of attribute information. However, in the actual measurement process under a complex electromagnetic environment, the electronic reconnaissance equipment is often interfered by air clutter and enemy signals, which causes a certain degree of information uncertainty that is in line with the characteristics of HFEs. Thus, it is more appropriate to describe these attribute decisions using HFE [35].

Assume that four types of airborne platform (i.e., A_i ($i = 1, 2, 3, 4$)) are present based on the information transmitted by each electronic reconnaissance sensor to the fusion center and that each type of airborne platform has four types of attribute, i.e., pulse repetition frequency (G_1), carrier frequency (G_2), power (G_3), and pulse width (G_4). The fusion center must perform multiattribute fusion judgment on the four types of airborne platform with uncertainty based on the four attribute types. The electronic reconnaissance equipment measures the value of each attribute $G_j \in G$ of each scheme $A_i \in A$, which forms a matrix based on hesitant fuzzy information. ($H = (h_{ij})_{m \times n}$).

To obtain the type of airborne platform, the specific multiattribute decision-making steps are as follows:

Step 1: decision makers assess the alternatives A_i ($i = 1, 2, 3, 4$) with respect to the attributes G_j ($j = 1, 2, 3, 4$), and the decision makers give the confidence levels l_p ($0 \leq l_p \leq 1$). Then, we set up the following hesitant fuzzy decision matrix $A^{(p)} = (a_{ij})_{4 \times 4}^{(p)}$, and the first components in the two types of information are the confidence levels (see Table 1).

Step 2: apply the CIHFHA operator to aggregate all the hesitant fuzzy decision matrix $A^{(p)} = (a_{ij})_{4 \times 4}^{(p)}$ into the aggregate hesitant fuzzy decision matrix $A = a_{ij}$ ($i = 1, 2, 3, 4; j = 1, 2, 3, 4$) (see Table 2).

Step 3: we can obtain the ideal of attribute values $A^* = \{0.36, 0.39, 0.38, 0.39\}$. Because the information about the attribute weights is entirely unknown, we can get the attribute weights $t = (0.24, 0.23, 0.27, 0.26)^T$ by using Model 3.

Step 4: use formula (25) to calculate the relative weighted projection $\text{Pr}j_{A_w^*} A_i$ of each alternative A_i :

TABLE 1: Hesitant fuzzy decision matrix under confidence levels.

	G_1	G_2	G_3	G_4
A_1	$\{(0.7,0.5),(0.3,0.7)\}$	$\{(0.7,0.6),(0.5,0.7)\}$	$\{(0.6,0.5),(0.7,0.6),(0.5,0.8)\}$	$\{(0.8,0.4),(0.7,0.5)\}$
A_2	$\{(0.8,0.3),(0.6,0.4)\}$	$\{(0.6,0.4),(0.4,0.7)\}$	$\{(0.7,0.4),(0.6,0.6)\}$	$\{(0.9,0.4),(0.7,0.5),(0.6,0.6)\}$
A_3	$\{(0.7,0.6),(0.5,0.7),(0.4,0.8)\}$	$\{(0.7,0.4),(0.4,0.6)\}$	$\{(0.5,0.8),(0.3,0.9)\}$	$\{(0.4,0.8),(0.5,0.9)\}$
A_4	$\{(0.7,0.4),(0.5,0.6)\}$	$\{(0.6,0.5),(0.4,0.7),(0.3,0.9)\}$	$\{(0.5,0.4),(0.6,0.7)\}$	$\{(0.6,0.5),(0.4,0.6)\}$

TABLE 2: Comprehensive hesitant fuzzy decision matrix A .

	G_1	G_2	G_3	G_4
A_1	0.28	0.39	0.38	0.34
A_2	0.24	0.26	0.32	0.36
A_3	0.36	0.26	0.34	0.39
A_4	0.29	0.28	0.32	0.27

TABLE 3: Hesitant fuzzy decision matrix $A^{(r)}$.

	G_1	G_2	G_3	G_4
A_1	$\{0.5,0.7\}$	$\{0.6,0.7\}$	$\{0.5,0.6,0.8\}$	$\{0.4,0.5\}$
A_2	$\{0.3,0.4\}$	$\{0.4,0.7\}$	$\{0.4,0.6\}$	$\{0.4,0.5,0.6\}$
A_3	$\{0.6,0.7,0.8\}$	$\{0.4,0.6\}$	$\{0.8,0.9\}$	$\{0.8,0.9\}$
A_4	$\{0.4,0.6\}$	$\{0.5,0.7,0.9\}$	$\{0.4,0.7\}$	$\{0.5,0.6\}$

$$\text{Pr}j_{A_w^*} A_1 = 0.48, \text{Pr}j_{A_w^*} A_2 = 0.41, \text{Pr}j_{A_w^*} A_3 = 0.47, \text{Pr}j_{A_w^*} A_4 = 0.40. \quad (27)$$

Step 5: rank the alternatives A_i ($i = 1, 2, 3, 4$) according to the relative weighted projection $\text{Pr}j_{A_w^*} A_i$:

$$\text{Pr}j_{A_w^*} A_1 > \text{Pr}j_{A_w^*} A_3 > \text{Pr}j_{A_w^*} A_2 > \text{Pr}j_{A_w^*} A_4, \quad (28)$$

so the airborne platform type is A_1 .

If we do with the HFWA operator to aggregate the hesitant fuzzy decision matrix without considering the confidence levels, the main steps are as follows:

Step 1': decision makers evaluate the alternatives A'_i ($i = 1, 2, 3, 4$) with respect to the attributes G'_j ($j = 1, 2, 3, 4$) without considering the confidence levels. Then, we construct the following hesitant fuzzy decision matrix $A^{(r)} = (a_{ij})_{4 \times 4}^{(r)}$ (see Table 3).

Step 2': employ the HFWA operator to aggregate all the hesitant fuzzy decision matrices $A^{(r)} = (a_{ij})_{4 \times 4}^{(r)}$ into the collective hesitant fuzzy decision matrix $A' = a'_{ij}$ ($i = 1, 2, 3, 4; j = 1, 2, 3, 4$) (see Table 4).

Step 3': we can obtain the ideal of attribute values $A'^* = \{0.71, 0.75, 0.86, 0.86\}$. Because the information about the attribute weights is entirely unknown, we can get the attribute weights $t' = (0.25, 0.27, 0.25, 0.23)^T$ by using Model 3.

Step 4': use formula (25) to calculate the relative weighted projection $\text{Pr}j_{A_w^*} A'_i$ of each alternative A'_i :

$$\begin{aligned} \text{Pr}j_{A_w^*} A'_1 &= 0.30, \text{Pr}j_{A_w^*} A'_2 = 0.25, \text{Pr}j_{A_w^*} A'_3 \\ &= 0.36, \text{Pr}j_{A_w^*} A'_4 = 0.30. \end{aligned} \quad (29)$$

Step 5': rank the alternatives A'_i ($i = 1, 2, 3, 4$) according to the relative weighted projection $\text{Pr}j_{A_w^*} A'_i$:

$$\text{Pr}j_{A_w^*} A'_3 > \text{Pr}j_{A_w^*} A'_1 = \text{Pr}j_{A_w^*} A'_4 > \text{Pr}j_{A_w^*} A'_2, \quad (30)$$

so the airborne platform type is A_3 .

If we do with the HFGA operator to aggregate the hesitant fuzzy decision matrix without considering the confidence levels, the main steps are as follows:

Step 1'': decision makers evaluate the alternatives A'_i ($i = 1, 2, 3, 4$) with respect to the attributes G'_j ($j = 1, 2, 3, 4$) without considering the confidence levels. Then, we construct the following hesitant fuzzy decision matrix $A^{(r)} = (a_{ij})_{4 \times 4}^{(r)}$ (see Table 3).

Step 2'': employ the HFGA operator to aggregate all the hesitant fuzzy decision matrices $A^{(r)} = (a_{ij})_{4 \times 4}^{(r)}$ into the collective hesitant fuzzy decision matrix $A'' = a''_{ij}$ ($i = 1, 2, 3, 4; j = 1, 2, 3, 4$) (see Table 5).

Step 3'': we can obtain the ideal of attribute values $A''^* = \{0.70, 0.68, 0.85, 0.85\}$. Because the information about the attribute weights is entirely unknown, we can get the attribute weights $t'' = (0.25, 0.28, 0.24, 0.23)^T$ by using Model 3.

Step 4'': use formula (25) to calculate the relative weighted projection $\text{Pr}j_{A_w^*} A''_i$ of each alternative A''_i :

$$\text{Pr}j_{A_w^*} A''_1 = 0.58, \text{Pr}j_{A_w^*} A''_2 = 0.46, \text{Pr}j_{A_w^*} A''_3 = 0.69, \text{Pr}j_{A_w^*} A''_4 = 0.56. \quad (31)$$

TABLE 4: Comprehensive hesitant fuzzy decision matrix A' .

	G_1	G_2	G_3	G_4
A_1	0.61	0.65	0.66	0.45
A_2	0.35	0.58	0.51	0.51
A_3	0.71	0.51	0.86	0.86
A_4	0.51	0.75	0.58	0.55

TABLE 5: Comprehensive hesitant fuzzy decision matrix A'' .

	G_1	G_2	G_3	G_4
A_1	0.59	0.65	0.62	0.45
A_2	0.35	0.53	0.49	0.49
A_3	0.70	0.49	0.85	0.85
A_4	0.49	0.68	0.53	0.55

Step 5": rank the alternatives A''_i ($i = 1, 2, 3, 4$) according to the relative weighted projection $\text{Pr}j_{A''_w} A''_i$:

$$\text{Pr}j_{A''_w} A''_3 > \text{Pr}j_{A''_w} A''_1 = \text{Pr}j_{A''_w} A''_4 > \text{Pr}j_{A''_w} A''_2, \quad (32)$$

so the airborne platform type is A_3 .

If we do not consider the factor of confidence levels, our proposed operators are simplified to the existing hesitant fuzzy aggregation operators. Nevertheless, the experts' group is completely unaware of the alternatives so that the results are different. In view of these situations, the hesitant fuzzy aggregation operators at the confidence levels proposed in this paper are useful tools. As can be seen from the above analysis, compared with the traditional hesitant fuzzy decision-making methods, the main advantages of our methods are not only that they can adapt to the hesitant fuzzy environment but also that they take into account the confidence levels between decision makers, which makes more feasibility and practicality.

7. Conclusion

In this paper, a new decision analysis method under confidence levels is proposed for decision-making (MADM) problems where attribute values are in the form of hesitant fuzzy numbers and unknown attribute weights. Firstly, several hesitant fuzzy correlation coefficient based on confidence levels are proposed. Secondly, some simple optimization models based on the correlation coefficients under confidence levels for HFEs are used to determine the attribute weight, and the projection method for HFEs is proposed. Finally, a numerical example is used to illustrate the practicability and effectiveness of the method. The results show that the main advantages of the decision analysis methods under confidence levels are not only that they can adapt to the hesitant fuzzy environment but also that they take into account the confidence levels between decision makers, which makes the results more feasibility and practicality. However, expert credibility needs expert's subjective judgment, and there will be some errors in decision-making problems. In the future work, we will try our

best to correct expert credibility and study probabilistic hesitant fuzzy environment under confidence levels [36, 37].

Data Availability

All data included in this study are available from the corresponding author upon request. Most of the data are already in the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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