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Research Article

A Novel Algorithm Based on 2-Additive Measure and Shapley Value and Its Application in Land Pollution Remediation

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In this article, a new aggregation operator called the Young–Shapley optimal weight (Y-SOW) operator is proposed to aggregate heterogeneous information in group decision-making. The Y-SOW operator combines the Shapley value with the Young inequality. Meanwhile, a series of special cases and main properties of the Y-SOW operator are studied. Furthermore, the dispersion maximization model of the Y-SOW operator is established to obtain the optimal 2-additive measure. In the Shapley value method of the cooperative game, the 2-additive measure not only simplifies the complexity of fuzzy measures but also solves the interaction between attributes. The Shapley value of the 2-additive measure is explored to the weight of the Y-SOW operator. Finally, the Y-SOW operator-based multiattribute group decision (YSMGAD) algorithm is proposed. The application of the YSMGAD algorithm for land pollution remediation is analyzed.

1. Introduction

Group decision-making is rapidly developed to an important branch of modern decision sciences, which helps to gather the wisdom of experts from different fields. The transition from individual to group decision-making is a major step forward to cope with increasingly complex human activities. To this end, group decision-making has been recognized and used in economic, military, agricultural, and other fields [1–6].

In the state-of-art literature, many methods of aggregation operators and their weight determination have been proposed for multiattribute decision-making. Yager [7] introduced an ordered weighted averaging (OWA) operator, where the input arguments were rearranged in the descending order, and the weight vector was only related to its ordered position. Xu and Da [8, 9] developed the ordered weighted geometric averaging (OWGA) operator for multiattribute decision-making. Chen and Liu [10] proposed an extension of the OWA operator called an ordered weighted harmonic mean (OWHA) operator. In 2004, Yager [11] used a generalized mean in the OWA operator and a generalized ordered weighted averaging (GOWA) operator. Based on a minimizing model, Zhou and Chen presented the generalized ordered weighted logarithm averaging (GOWLA) operator [12], generalized ordered weighted harmonic averaging (GOWHA) operator [13], and so on. Merigo et al. expanded the OWA operator and proposed the ordered weighted averaging-weighted average (OWAWA) operator [14], which unified the OWA operator in the same formulation. Other generalizations of the aggregation operators were observed in [12, 15–21]. The operators proposed above only deal with additive or multiplicative information alone. The existing literature is not sufficient to solve the problem when two kinds of information appear simultaneously in group decision-making.

In 1995, Grabisch [22–25] proposed the fuzzy measure as an aggregation operator for multiattribute decision-making. However, the fuzzy measure requires a large number of parameters, which is difficult to implement. In order to reduce the computational complexity, various forms and methods of determining the fuzzy measure have been proposed. The choquet integral [26–28] is a nonlinear integration operator defined on the basis of fuzzy measures, which can effectively deal with the interaction between decision attributes. The premise of the integral is utilized to determine the fuzzy measure, which is complicated. If there are n attributes, $2^n - 2$ parameters are needed. Sugeno [29] proposed a λmeasure that only requires n parameters but cannot fully describe the interaction between attributes. In 1996, Grabisch [30] proposed the k-additive measure. The Shapley value of a single attribute and the interaction among k attributes were determined, the computational complexity was reduced, and the representing ability between the attributes was improved to some extent.

In reality, the attribute value often includes both additive and multiplicative information; however, it is difficult to aggregate precisely in one matrix. Based on the Young inequality and Shapley value, a new optimal operator called the Young–Shapley optimal weight (Y-SOW) operator is proposed. Meanwhile, a series of special cases and the main properties of the Y-SOW operator are studied. Because one certain interaction exists between the attributes, the 2-additive measure is introduced to reduce the complexity of the fuzzy measure. The Shapley value method is the most effective and widely used method in cooperative games. Therefore, the dispersion maximization model based on the 2-additive measure and the Shapley value is established to obtain the optimal 2-additive measure. Some formula and programming models are also provided to effectively determine the 2-additive measure. The Shapley value of the 2-additive measure is used as the weight of the Y-SOW operator. Finally, the Y-SOW operator-based multiattribute group decision (YSMAGD) algorithm is proposed to effectively aggregate the heterogeneous data. However, the Y-SOW operator can only aggregate the real-type data and has certain limitations for the other types of data, such as interval-type data.

The rest of this article is organized as follows. In Section 2, the basic concepts of common aggregation operators, fuzzy measures, and Shapley values are reviewed. In Section 3, the common Y-SOW operator is proposed, and some special cases and ideal characteristics of the Y-SOW operator are also proved. In Section 4, the Y-SOW operator-based multiattribute group decision (YSMAGD) algorithm is established. In Section 5, the YSMAGD algorithm is applied to sequence the remediable location of land pollution. In Section 6, summary is given.

2. Preliminaries

2.1. Several Commonly Used Information Aggregation Operators. The ordered weighted averaging (OWA) operator was proposed by Yager [7] in 1988 and is widely used in a series of decision problems, as defined below.

Definition 1. Let R be the set of real number. AnOWA operator is a mapping, OWA: $R^n \longrightarrow R$ is satisfied:

$$\mathrm{OWA}(a_1, \ldots, a_n) = \sum_{j=1}^{n} w_j b_j, \tag{1}$$

then OWAis called an ordered weighted averaging operator, where b_j is the jth largest of the arguments $a_1, \ldots, a_n$ and the weight vector $W = (w_1, \ldots, w_n)^T$ satisfies $\sum_{j=1}^{n} w_j = 1$, $w_j \in [0,1]$ and $(j = 1, 2, \ldots, n)$.

The OWA operator is an effective aggregation method that rearranges the arguments and then weights it according to the sequence position to weaken the adverse effects of the extreme value. It is characterized by considering only the positional relationship of the arguments in the ordering process. The OWA operator has many desirable properties such as monotonicity, boundedness, idempotency, and permutation invariance. When $b_j = a_j$, holds for all $j = 1, 2, \ldots, n$, then the OWA operator becomes a weighted averaging (WA) operator [31].

Yager also proposed the BUM function Q [32] to calculate the associated weight of the OWA operator, which satisfies $Q(0) = 0$, $Q(1) = 1$, and $Q(x) \leq Q(y)$ for any $0 \leq x \leq y \leq 1$, that is,

$$w_j = Q\left(\frac{j}{n}\right) - Q\left(\frac{j-1}{n}\right), \quad j = 1, 2, \ldots n. \tag{2}$$

The BUM function is called the fuzzy semantic quantization operator, and the expression can be stated as follows:

$$Q(x) = \begin{cases} 0, & x \leq x_1, \\ \dfrac{x - x_1}{x_2 - x_1}, & x_1 \leq x \leq x_2, \\ 1, & x \geq x_2, \end{cases} \tag{3}$$

where x_1, x_2, and x are in the range of $[0,1]$. When we choose the pair $(0, 0.5)$, $(0.3, 0.8)$, and $(0.5, 1)$, the fuzzy linguistic representation is "at least half," "more," and "as many as possible," respectively.

Definition 2. An OWG operator [33] is a mapping, *OWG*: $I^n \longrightarrow I$ and $I = \{x \mid x \geq 0\}$, according to the following formula:

$$\mathrm{OWG}(a_1, \ldots, a_n) = \prod_{j=1}^{n} b_j^{w_j}, \tag{4}$$

then OWG is called an ordered weighted geometric operator, where b_j is the jth largest of the arguments $a_1, \ldots, a_n$, and the weight vector $w = (w_1, \ldots, w_n)^T$ satisfies $\sum_{j=1}^{n} w_j = 1$ and $w_j \in [0,1]$, $j = 1, 2, \ldots, n$.

The generalized weighted averaging (GWA) operator was first presented by Yager [11] based on the generalized mean. Assuming that the fusion result is an n-dimensional function f, we can construct a penalty function $J = \sum_{j=1}^{n} w_j (f^\theta - a_j^\theta)^2$ and the minimization problem:

$$\min J = \sum_{j=1}^{n} w_j \left(f^\theta - a_j^\theta\right)^2. \tag{5}$$

Let $(\partial J/\partial f) = 0$, and the aggregation method for obtaining the GWA operator is shown as follows:

$$\mathrm{GWA}(a_1, \ldots, a_n) = \left(\sum_{j=1}^{n} w_j a_j^{\theta}\right)^{1/\theta}. \quad (6)$$

If the arguments $a_1, \ldots, a_n$ in the GWA operator are arranged in a descending order, the generalized ordered weighted averaging (GOWA) operator can be obtained. The GWA operator also has many desired properties, such as monotonicity, boundedness, idempotency, and permutation invariance. When $\theta = 1$, $\theta = -1$, and $\theta \longrightarrow 0$, then the GWA operator reduces to the weighted averaging (WA) operator, the weighted harmonic averaging (WHA) operator, and the weighted geometric averaging (WGA) operator, respectively.

2.2. Fuzzy Measure and Shapley Value. The aggregation operators used in the traditional multiattribute decision-making are generally based on the premise that the attributes are independent and do not interact with each other, but the actual situations are often interactive or dependent. To deal with the abovementioned phenomenon, the fuzzy measure [12, 20–27] is introduced as follows.

Definition 3 (see [29]). Let N be the attribute set and $P(N)$ be the power set of N. If set function μ: $P(N) \longrightarrow [0,1]$ satisfies the following conditions:

(i) $\mu(\phi) = 0, \mu(N) = 1$

(ii) $\forall A, sB \in P(N), A \subseteq B \Rightarrow \mu(A) \leq \mu(B)$

Then, μ is a fuzzy measure on $P(N)$.

Fuzzy measure is difficult to calculate and requires a large number of parameters. Grabisch proposed the 2-additive fuzzy measure based on pseudo-Boolean function and Mobius transform [29].

Definition 4. Let f: $\{0,1\}^n \longrightarrow R$ be a pseudo-Boolean function [26]. Where $\{0,1\}^n$ denotes the entirety of all n-dimensional Boolean vectors. Let $X = \{x_1, \ldots, x_n\}$, $\forall A \subseteq N$. Any fuzzy measure μ can be seen as a particular case of pseudo-Boolean function denoted by

$$\mu(A) = \sum_{T \subseteq N} \left[a_T \prod_{i \in T} y_i\right], \quad (7)$$

where $a_T \in R$, $y = (y_1, y_2, \ldots, y_n) \in \{0,1\}^n$, and $a_T = \sum_{S \subseteq T} (-1)^{t-s} \mu(S)$ is called the Mobius transform coefficient. Obviously, $y_i = 1$ if and only if $i \in A$. The fuzzy measure μ defined on $(X, P(N))$ is a k-additive fuzzy measure, and the corresponding pseudo-Boolean function is a k-order linear polynomial, that is, $\forall T \in N$, if $|T| > k$, then $a_T = 0$ and $\exists T_0 \in P(N)$, $|T_0| = k \Rightarrow a_{T_0} = 0$.

From Definition 4, we get that 1-additive measures are additive measures and n-additive measures are fuzzy measures. Especially, when $k = 2$, by equation (7) we get a 2-additive measure μ.

Definition 5 (see [30]). For a 2-additive measure μ, $\forall S \subseteq N, s \geq 2$, then

$$\mu(S) = \sum_{i=1}^{n} a_i x_i + \sum_{\{i,j\} \subseteq N} a_{ij} x_i x_j = \sum_{i \in S} a_i + \sum_{\{i,j\} \subseteq S} a_{ij} = \sum_{\{i,j\} \subseteq S} \mu(i,j) - (s-2) \sum_{i \in S} \mu(i), \quad (8)$$

where $\mu(i) = a_i, \mu(i,j) = a_i + a_j + a_{ij}$.

It is well known that a 2-additive measure requires only $n(n+1)/2$ parameters, and it will be simpler to solve a 2-additive measure than to solve a usual fuzzy measure.

Definition 6 (see [30]). Let μbe the fuzzy measure on N,$N = \{1, \ldots, n\}$. $\forall i, j \in N \exists \mu(i)$, $\mu(i,j)$, the 2-additive fuzzy measure satisfies the following conditions:

(i) $\mu(i) \geq 0, \forall i \in N$,

(ii) $\sum_{\{i,j\} \subseteq N} \mu(i,j) - (n-2) \sum_{i \in N} \mu(i) = 1$,

(iii) $\sum_{i \subseteq S \backslash k} (\mu(i,k) - \mu(i)) \geq (s-2)\mu(k), \forall S \in N, s \geq 2$,

where s and n are the cardinalities of $S$and N, respectively.

The Shapley value [34] is determined in a grand coalition N based on the marginal contribution of players to obtain the optimal benefit distribution. In order to avoid the irrationality of the average distribution and show certain rationality and fairness, the Shapley value method is widely used for cooperative games.

Definition 7. Vector $\mathrm{Sh}(N,\mu) = (\mathrm{Sh}_1(N,\mu), \mathrm{Sh}_2(N,\mu) \ldots, \mathrm{Sh}_n(N,\mu))$, and its components are defined by

$$\mathrm{Sh}_i(N,\mu) = \sum_{T:\, i \in T \subseteq N} \frac{(|T|-1)!\,(|N|-|T|)!}{|N|!} [\mu(T) - \mu(T \backslash \{i\})], \quad \forall i \in N, \quad (9)$$

where μ is a fuzzy measure on N and $|N|$ and $|T|$ are the cardinality of set N and T respectively.

Let μ be a superadditive fuzzy measure on N, and it follows that $\mu(T) \geq \mu(T \backslash \{i\})$ for all $\forall i \in T \subseteq N$. Obviously, we have

(i) $\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu) = 1$,

(ii) $\mathrm{Sh}_i(N,\mu) \geq 0$ $(i = 1, \ldots, n)$.

It means that $\mathrm{Sh}(N,\mu)$ can be viewed as a weight vector here, named as the Shapley weight vector.

3. The Young–Shapley Optimal Weight (Y-SOW) Operator

3.1. The Proposed Operator and Its Equivalent Expression

Lemma 1 (Young inequality, see [35]). $\forall a, b \geq 0$, $\lambda \in [0,1]$, it holds that

$$a^{\lambda} b^{1-\lambda} \leq \lambda a + (1-\lambda) b, \tag{10}$$

if and only if $a = b$, and the equation holds.

In the aggregation process, let $a_1, \ldots, a_n$ be the non-negative aggregation arguments and $\mathrm{Sh}_i(N,\mu)$ be a Shapley weighting vector, then function f is strictly monotonically increasing. Construct a penalty function H as follows:

$$H = \sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)\left[\frac{\lambda f(a_i) + (1-\lambda) f(y)}{(f(a_i))^{\lambda}(f(y))^{1-\lambda}} - 1\right], \tag{11}$$

where λ is a parameter that satisfies $\lambda \in [0,1]$. According to the necessary conditions for the existence of extreme values, taking the partial derivative of H with respect to y, there is

$$\frac{\partial H}{\partial y} = \sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)\left[\frac{(1-\lambda) f'(y)(f(a_i))^{\lambda}(f(y))^{1-\lambda} - [\lambda f(a_i) + (1-\lambda) f(y)](f(a_i))^{\lambda}(1-\lambda)(f(y))^{1-\lambda} f'(y)}{(f(a_i))^{2\lambda}(f(y))^{2-2\lambda}}\right]. \tag{12}$$

Setting $(\partial H/\partial y) = 0$, it follows that

$$\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)\left[\frac{(f(a_i))^{\lambda} f(y) f'(y) - [\lambda f(a_i) + (1-\lambda) f(y)](f(a_i))^{\lambda} f'(y)}{(f(a_i))^{2\lambda}}\right] = 0. \tag{13}$$

Equation (13) can be written as follows:

$$\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)\left[\frac{f(y) f'(y) - \lambda f(a_i) f'(y) - (1-\lambda) f(y) f'(y)}{(f(a_i))^{\lambda}}\right] = 0. \tag{14}$$

Equation (14) is transformed into the following formula:

$$f(y) = \frac{\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)(f(a_i))^{1-\lambda}}{\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)(f(a_i))^{-\lambda}}. \tag{15}$$

The function f is strictly monotonically increasing and has a reversible property. Thus, its inverse function exists as follows:

$$y = f^{-1}\left(\frac{\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)(f(a_i))^{1-\lambda}}{\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)(f(a_i))^{-\lambda}}\right), \tag{16}$$

and we called equation (16) as the Young–Shapley optimal weight (Y-SOW) operator.

In order to simplify the structure of the Y-SOW operator, the following equivalent expression is introduced in detail.

Let

$$\widehat{B}_i = \widehat{B}_i(a_1, \ldots, a_n) = \frac{\mathrm{Sh}_i(N,\mu)(f(a_i))^{-\lambda}}{\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)(f(a_i))^{-\lambda}}, \tag{17}$$

then, it holds that $\sum_{i=1}^{n} \widehat{B}_i = 1$. The Y-SOW operator can be equivalently written as

$$\mathrm{Y-SOW}(a_1, \ldots, a_n) = f^{-1}\left(\sum_{i=1}^{n} \widehat{B}_i f(a_i)\right). \tag{18}$$

The following theorem shows the monotonicity of $\widehat{B}_i$ with respect to a_i.

Theorem 1. *Let $\lambda \in [0,1]$, then $\widehat{B}_i$ is monotonically decreasing with respect to a_i.*

Proof. Taking the derivative of $\widehat{B}_i$ with respect to a_i, we have

$$\frac{\partial \widehat{B}_i}{\partial a_i} = \frac{(-\lambda)\mathrm{Sh}_i(N,\mu)(f(a_i))^{-\lambda-1} f'(a_i) \sum_{i\neq j} \mathrm{Sh}_i(N,\mu)(f(a_i))^{-\lambda}}{\left(\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)(f(a_i))^{-\lambda}\right)^2}. \tag{19}$$

Since the function f is monotonically increasing, we get $f'(a_i) \geq 0$. Moreover, function f is nonnegative and $\mathrm{Sh}_i(N,\mu) \geq 0$, $0 \leq \lambda \leq 1$, then we have

$$\frac{\partial \widehat{B}_i}{\partial a_i} \leq 0. \tag{20}$$

Then, $\widehat{B}_i$ is monotonically decreasing. □

Theorem 2. *Let a_i' and a_i $(i = 1, 2, \ldots, n)$ be real numbers, then we have*

(i) If $a_i' \geq a_i$, then $\widehat{B}_i' = \widehat{B}_i'(a_1', \ldots, a_i', \ldots, a_n') \leq \widehat{B}_i(a_1, \ldots, a_i, \ldots, a_n) = \widehat{B}_i$

(ii) If $a_i' \leq a_i$, then $\widehat{B}_i' = \widehat{B}_i'(a_1', \ldots, a_i', \ldots, a_n') \geq \widehat{B}_i(a_1, \ldots, a_i, \ldots, a_n) = \widehat{B}_i$

Proof. It can be known from Theorem 1, so the proof process is omitted. □

3.2. Some Special Cases of the Y-SOW Operator. It is worth noting that the Y-SOW operator includes some special operators when the value of λ is different or the expression of function $f(x)$ changes.

(i) When the weights of players (attributes) are indifferent and independent and $\mathrm{Sh}(N,\mu) = ((1/n), (1/n), \ldots, (1/n))$ is selected, we have

$$\mathrm{Y-SOW}(a_1,\ldots,a_n) = f^{-1}\left(\frac{\sum_{i=1}^{n}(f(a_i))^{1-\lambda}}{\sum_{i=1}^{n}(f(a_i))^{-\lambda}}\right), \tag{21}$$

then equation (21) becomes the Young-optimal weight (Y-OW) operator.

(ii) When $\lambda = 1$, then

$$\mathrm{Y-SOW}(a_1,\ldots,a_n) = f^{-1}\left(\frac{1}{\sum_{i=1}^{n} Sh_i(N,\mu)(1/(f(a_i)))}\right). \tag{22}$$

The Y-SOW operator reduces to the Shapley weighted harmonic averaging (SWHA) operator.

(iii) When $\lambda = 0$, then

$$\mathrm{Y-SOW}(a_1,\ldots,a_n) = f^{-1}\left(\sum_{i=1}^{n}\mathrm{Sh}_i(N,\mu)f(a_i)\right). \tag{23}$$

Equation (23) reduces to quasi-Shapley weighted averaging (QSWA) operator.

(iv) When $f(x) = x$ is selected according to the overall risk attitude, the Y-SOW operator reduces to the following operator with respect to risk neutrality:

$$\mathrm{Y-SOW}(a_1,\ldots,a_n) = \frac{\sum_{i=1}^{n}\mathrm{Sh}_i(N,\mu)(a_i)^{1-\lambda}}{\sum_{i=1}^{n}\mathrm{Sh}_i(N,\mu)(a_i)^{-\lambda}}. \tag{24}$$

(v) When $f(x) = \ln x$ is selected according to the overall risk attitude, the Y-SOW operator is simplified to the following operator in terms of risk aversion:

$$\mathrm{Y-SOW}(a_1,\ldots,a_n) = e^{\left(\sum_{i=1}^{n}\mathrm{Sh}_i(N,\mu)(\ln(a_i))^{1-\lambda}/\sum_{i=1}^{n}\mathrm{Sh}_i(N,\mu)(\ln(a_i))^{-\lambda}\right)}. \tag{25}$$

If $\lambda = 0$, equation (25) is changed to

$$\begin{aligned}\mathrm{Y-SOW}(a_1,\ldots,a_n) &= e^{\left(\sum_{i=1}^{n}\mathrm{Sh}_i(N,\mu)(\ln(a_i))^{1-\lambda}/\sum_{i=1}^{n}\mathrm{Sh}_i(N,\mu)(\ln(a_i))^{-\lambda}\right)}\\ &= e^{\left(\sum_{i=1}^{n}\mathrm{Sh}_i(N,\mu)\ln(a_i)/\sum_{i=1}^{n}\mathrm{Sh}_i(N,\mu)\right)} = e^{\left(\sum_{i=1}^{n}\mathrm{Sh}_i(N,\mu)\ln(a_i)\right)} = e^{\left(\sum_{i=1}^{n}\ln(a_i)^{\mathrm{Sh}_i(N,\mu)\cdot}\right)} = \sum_{i=1}^{n} a_i^{\mathrm{Sh}_i(N,\mu)}.\end{aligned} \tag{26}$$

Equation (26) becomes the Shapley weighted geometric average (SWGA) operator.

(vi) When $f(x) = e^x$ is selected according to the decision makers overall risk attitude, the Y-SOW operator is reduced to the following operator in terms of risk proneness:

$$\mathrm{Y-SOW}(a_1,\ldots,a_n) = \ln\left(\frac{\sum_{i=1}^{n}\mathrm{Sh}_i(N,\mu)\cdot e^{(1-\lambda)\cdot a_i}}{\sum_{i=1}^{n}\mathrm{Sh}_i(N,\mu)\cdot e^{(-\lambda)\cdot a_i}}\right). \tag{27}$$

3.3. Desirable Property of the Y-SOW Operator. The Y-SOW operator has many potential properties, such as monotonicity, idempotency, and boundedness.

Property 1 (monotonicity). $\forall a_i', a_i \in R$, $(i = 1, \ldots, n)$. If $a_i' \geq a_i$, it holds that

$$\mathrm{Y-SOW}(a_1',\ldots,a_n') \geq \mathrm{Y-SOW}(a_1,\ldots,a_n). \tag{28}$$

Proof. Denoting

$$H = f^{-1}\left(\frac{\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)(f(a_i))^{1-\lambda}}{\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)(f(a_i))^{-\lambda}}\right). \quad (29)$$

It follows that

$$\ln f(H) = \ln\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)(f(a_i))^{1-\lambda} - \ln\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)(f(a_i))^{-\lambda}. \quad (30)$$

Taking the derivative of the $\ln f(H)$ with respect to $a_i\,(i = 1, 2, \ldots, n)$, respectively, we get

$$\begin{aligned}\frac{\partial(\ln f(H))}{\partial a_i} &= \frac{\mathrm{Sh}_i(N,\mu)\cdot(1-\lambda)\cdot(f(a_i))^{-\lambda}\cdot f'(a_i)}{\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)(f(a_i))^{1-\lambda}} - \frac{\mathrm{Sh}_i(N,\mu)\cdot(-\lambda)\cdot(f(a_i))^{-\lambda-1}\cdot f'(a_i)}{\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)(f(a_i))^{-\lambda}}\\ &= \mathrm{Sh}_i(N,\mu)\cdot f'(a_i)\cdot\left(\frac{(1-\lambda)\cdot(f(a_i))^{-\lambda}}{\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)(f(a_i))^{1-\lambda}} + \frac{\lambda\cdot(f(a_i))^{-\lambda-1}}{\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)(f(a_i))^{-\lambda}}\right).\end{aligned} \quad (31)$$

Since $f\colon [1,T] \longrightarrow [0,+\infty]$ and f is increasing and nonnegative, we have $f'(a_i)\geq 0$. According to Definition 7, we have $\mathrm{Sh}_i(N,\mu)\geq 0, \lambda\in[0,1]$. Therefore, $(\partial(\ln f(H))/\partial a_i)\geq 0$. This means that $\ln f(H)$ is a monotonically increasing function. Obviously, $f(H)$ is also a monotonically increasing function. Thus, we get

$$\frac{\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)(f(a_i'))^{1-\lambda}}{\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)(f(a_i'))^{-\lambda}} \geq \frac{\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)(f(a_i))^{1-\lambda}}{\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)(f(a_i))^{-\lambda}}, \quad (32)$$

which can be equivalently expressed as

$$\begin{aligned}&f^{-1}\left(\frac{\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)(f(a_i'))^{1-\lambda}}{\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)(f(a_i'))^{-\lambda}}\right)\\ &\geq f^{-1}\left(\frac{\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)(f(a_i))^{1-\lambda}}{\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)(f(a_i))^{-\lambda}}\right).\end{aligned} \quad (33)$$

To sum up, we have $f(a_1',\ldots,a_n')\geq f(a_1,\ldots,a_n)$. Therefore, Property 1 is proved. □

Property 2 (idempotency). $a, a_i\in R$, $(i = 1,\ldots,n)$. If $a_i = a$ for all $i\in\{1,\ldots,n\}$, then

$$\mathrm{Y-SOW}(a_1,\ldots,a_n) = a. \quad (34)$$

Proof:. Denoting

$$\mathrm{Y-SOW}(a_1,\ldots,a_n) = f^{-1}\left(\frac{\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)(f(a_i))^{1-\lambda}}{\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)(f(a_i))^{-\lambda}}\right). \quad (35)$$

Since $a_i = a$, for all $i = 1,\ldots,n$, we have

$$\begin{aligned}\mathrm{Y-SOW}(a_1,\ldots,a_n) &= f^{-1}\left(\frac{\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)(f(a_i))^{1-\lambda}}{\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)(f(a_i))^{-\lambda}}\right)\\ &= f^{-1}\left(\frac{(f(a))^{1-\lambda}\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)}{(f(a))^{-\lambda}\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)}\right)\\ &= f^{-1}(f(a)) = a.\end{aligned} \quad (36)$$

Therefore, Property 2 is proved. □

Property 3 (boundedness). $a_i\in R\,(i = 1,\ldots,n)$and denoting $\max_i a_i = a_{\max}$ and $\min_i a_i = a_{\min}$, then

$$a_{\min}\leq \mathrm{Y-SOW}(a_1,\ldots,a_n)\leq a_{\max}. \quad (37)$$

Proof. Denoting

$$\mathrm{Y-SOW}(a_1,\ldots,a_n) = f^{-1}\left(\frac{\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)(f(a_i))^{1-\lambda}}{\sum_{i=1}^{n} \mathrm{Sh}_i(N,\mu)(f(a_i))^{-\lambda}}\right). \quad (38)$$

By Theorem 1, we have

$$\mathrm{Y-SOW}(a_1,\ldots,a_n) = f^{-1}\left(\sum_{i=1}^{n}\widehat{B}_i f(a_i)\right). \quad (39)$$

Because $\lambda\in[0,1]$, we have

$$\sum_{i=1}^{n}\widehat{B}_i f(a_i)\geq \sum_{i=1}^{n}\widehat{B}_i f(a_{\min}) = f(a_{\min})\sum_{i=1}^{n}\widehat{B}_i = f(a_{\min}), \sum_{i=1}^{n}\widehat{B}_i f(a_i)\leq \sum_{i=1}^{n}\widehat{B}_i f(a_{\max}) = f(a_{\max})\sum_{i=1}^{n}\widehat{B}_i = f(a_{\max}). \quad (40)$$

Thus, we get

$$f(a_{\min}) \le \sum_{i=1}^{n} \widehat{B}_i f(a_i) \le f(a_{\max}). \quad (41)$$

It follows that

$$a_{\min} \le f^{-1}\left(\sum_{i=1}^{n} \widehat{B}_i f(a_i)\right) \le a_{\max}. \quad (42)$$

To sum up, we have

$$a_{\min} \le \mathrm{Y-SOW}(a_1, \ldots, a_n) \le a_{\max}. \quad (43)$$

The proof of Property 3 is completed. □

4. Deriving the Optimal Weight Vector Based on Dispersion Maximization Method

Aggregation operators play a vital role in multiple fields, such as economics, statistics, and management. In case that the attribute associated with weight information is unknown, we establish the optimal weight vector based on the dispersion maximization method.

Suppose $A = \{a_1, \ldots, a_m\}$ is a finite set of alternatives, $C = \{c_1, \ldots, c_n\}$ is a set of attributes and $D = \{d_1, \ldots, d_t\}$ is the set of decision makers and $v = (v_1, \ldots, v_t)^T$ is the weight vector of a decision maker, which satisfies the condition $v_k \in [0,1], \sum_{k=1}^{t} v_k = 1$. Assume that $A^{(k)} = (a_{ij}^{(k)})_{m\times n}$ is the decision matrix given by the decision maker d_k, where the estimated value $a_{ij}^{(k)}$ indicates that the alternative $a_i \in A$ under attributes $c_j \in C$ is given by the decision maker d_k. $\omega = (\omega_1, \omega_2, \cdots, \omega_t)^T$ is the weight of the decision maker, satisfying $\omega_i \in [0,1], \sum_{i=1}^{t} \omega_i = 1$.

In multiattribute decision-making, the attributes cannot be aggregated directly due to their different size. The primitive decision maker matrix has to be normalized. The attributes mainly include three types: benefit attribute, cost attribute, and fixed attribute. The benefit attribute refers to the bigger the better index, the cost attribute refers to the smaller the better index, and the fixed attribute refers to how close is it to a fixed value.

Let $H_i\ (i = 1,2,3)$ be the subscript set of the above three types of attributes in order [10], and the following formula can be used for normalization:

$$r_{ij} = \frac{a_{ij}}{\min_{i\in N} a_{ij}}, \quad j \in H_1, \quad (44)$$

$$r_{ij} = \frac{\max_{i\in N} a_{ij}}{a_{ij}}, \quad j \in H_2, \quad (45)$$

$$r_{ij} = 1 - \frac{|a_{ij} - a_j|}{\max\limits_{i} |a_{ij} - a_j|}, \quad j \in H_3. \quad (46)$$

For multiattribute decision-making, if the attribute u_j can make a bigger difference in the attribute values of all alternatives, it means that u_j plays a bigger role in the ordering of the alternatives and shall be given a bigger weight, vise verse. In particular, if all of the alternatives have no difference in the attribute values under the attribute u_j, u_j will have no effect on the ordering of the alternatives and u_j is zero. Based on that the weight can be determined with the dispersion maximization method.

Because there is a certain interaction between the attributes, in order to simplify the complexity of the fuzzy measure, the Shapley value based on the 2-additive measure is introduced as the weight of the attributes and the traditional Shapley value function has been mentioned in equation (9).

When the fuzzy measure μ is a 2-additive measure, equation (9) is converted to [36–38]

$$\mathrm{Sh}_i(N,\mu) = \frac{3-n}{2}\mu(i) + \sum_{j\in N\setminus i} \frac{1}{2}(\mu(i,j) - \mu(j)), \quad \forall i \in N. \quad (47)$$

In Definition 7, $\mathrm{Sh}_i(N,\mu)$ can be regarded as a weight vector.

In view of Definition 6 and equation (47), the dispersion maximization model based on the 2-additive measure and Shapley value is established as follows:

Model $(M-1)$:

$$\max T = \frac{3-n}{2}\sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{k=1}^{n}\left(r_{ij} - r_{kj}\right)^2 \mu(j) + \frac{1}{2}\sum_{j=1}^{m}\sum_{i=1}^{n}\sum_{k=1}^{n}\left(r_{ij} - r_{kj}\right)^2\left(\sum_{l\in N\setminus j}\mu(j,l) - \mu(l)\right),$$

$$\text{s.t.}\begin{cases} \sum\limits_{l\subseteq S\setminus j} (\mu(j,l)) - \mu(l) \ge (s-2)\mu(c_j), & \forall S\subseteq N, \forall l \in S, s \ge 2, \\ \sum\limits_{\{j,l\}\subseteq N} \mu(j,l) - (n-2)\sum\limits_{j\in N}\mu(j) = 1, \\ \mu(j) \ge 0, \quad j = 1,2,\ldots,n. \end{cases} \quad (48)$$

Finally, the optimal 2-additive measure can be obtained, and the optimal 2-additive measure can be reused to calculate the Shapley value in the Y-SOW operator. Using equation (16) to aggregate the Y-SOW operator, the optimal result is obtained.

Therefore, the Y-SOW operator-based multiattribute group decision (YSMAGD) algorithm is stepped as follows:

Step 1. Assuming that $A^{(k)} = (a_{ij}^{(k)})_{m\times n}$ is the decision matrix given by the decision maker $d_k\ (k = 1,2,\ldots,t)$.

Step 2. By equations (2) and (3), according to the decision makers' preference, the fuzzy linguistic quantifier with the pair (a,b) is selected to compute the

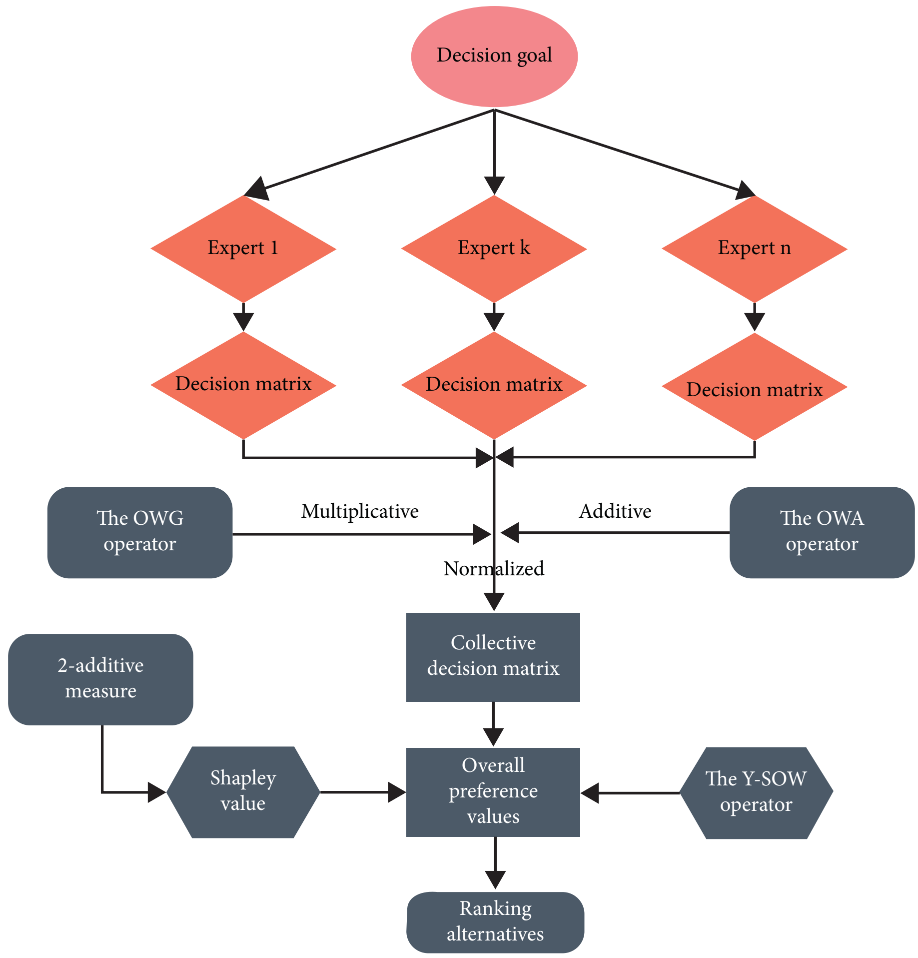

Figure 1: Framework of the YSMAGD algorithm.

weights of the decision makers $w_j = (w_1, w_2, \ldots, w_n)^T$ $(j = 1, 2, \ldots, n)$.

Step 3. Based on equations (1) and (4), the OWA operator and the OWG operator are used to aggregate the additive and multiplicative information of the decision matrix $A^{(k)}$, respectively. Final decision matrix $\overline{A} = (\overline{r}_{ij})_{m\times n}$ is obtained.

Step 4. The final decision matrix $\overline{A}$ is normalized by equations (44)–(46) to obtain a collective decision matrix R.

Step 5. Utilize the dispersion maximization model $(M-1)$ to obtain the optimal 2-additive measure $\mu(i)$ and $\mu(i, j)$.

Step 6. $\mu(i)$ and $\mu(i, j)$ are used to calculate the Shapley value based on equation (47), that is, the weight vector of the Y-SOW operator.

Step 7. Utilize equation (16) to aggregate the normalized matrix R, and the overall preference value $\overline{r_i}(r = 1, \ldots, m)$ of the alternative a_i is obtained.

Step 8. Rank all the alternatives $a_i (i = 1, \ldots, m)$ and select the best one in accordance with the ranking of $\overline{r}_i (r = 1, \ldots, m)$.

Step 9. End.

Based on the above analysis, the framework of the YSMAGD algorithm is illustrated in Figure 1.

5. Numerical Example

The economic development brings us livelihood improvement, but with increasing land pollution. It is investigated that among the total 150 million mu of cultivated land in the country, 32.5 million mu is contaminated by sewage, 2 million mu is occupied by solid waste, and 2 million mu is destroyed, accounting for over 20%. Crops accumulate harmful substances from polluted land, which causes diseases and ultimately endanger human future. However, the prevention and control is still weak. Nowadays, distribution and extent of soil pollution in the country are unclear. As a result, the government lacks specific control measures and capital input, and experts on land science research are also difficult to carry out in depth.

The first task is to select the polluted location for remediation. There are many both objective and subjective factors involved. The evaluation index is usually chaotic and miscellaneous. It is a typical problem of multiattribute aggregation with additive and multiplicative information. The proposed Y-SOW operator above is applied to decision analysis.

Consider site selection for land remediation, that is, the assessment of polluted land. Assume that there are eight land experts $(d_1, d_2, \ldots, d_8)$, evaluating the improvement of rural land from six attributes: c_1 pollution area, $c_2$remediation potential, c_3 realistic feasibility, c_4 fertilizer and pesticide use rate, c_5

Table 1: 1–9 comparison scale.

Scale value	Meaning
1	When F_i is as important as F_j
3	When F_i is slightly more important than F_j
5	When F_i is more important than F_j
7	When F_i is much more important than F_j
9	When F_i is extremely important than F_j
2, 4, 6, 8	A compromise of the above degree

soil pH, and c_6 irrigation guarantee rate. There exist four lands (A, B, C, D), and eight experts score four lands according to the above six attributes.

Step 1. Because the analytic hierarchy process (AHP) [39–41] can easily and flexibly deal with the quantitative problem of decision makers on complex systems, it is a classic weighting method commonly used by decision makers. The core of the method is to compare the degree of importance between the two elements by the 1 to 9 scale method. c_2 and c_3 are multiplicative indicators and cannot be calculated by specific numerical values. Therefore, the AHP scale method is used. Suppose there is a sample land E with a scale of 1 for each attribute. Let land E be F_j and lands A B C D be F_i. The experts compare the four lands A B C D with the sample land E according to the following 1–9 scale (shown in Table 1).

Let us suppose that the decision matrix $A^{(k)} = (a_{ij}^{(k)})_{4\times 6}$ provided by decision makers d_k $(k = 1, 2, \ldots, 8)$ is obtained as follows:

$$A^{(1)} = \begin{pmatrix} 40 & 7 & 6 & 0.6 & 5 & 0.5 \\ 60 & 4 & 5 & 0.7 & 5.5 & 0.3 \\ 55 & 5 & 6 & 0.5 & 7 & 0.7 \\ 70 & 3 & 4 & 0.6 & 8.5 & 0.4 \end{pmatrix},$$
$$A^{(2)} = \begin{pmatrix} 43 & 6 & 7 & 0.4 & 4.8 & 0.4 \\ 58 & 5 & 4 & 0.8 & 5.8 & 0.7 \\ 48 & 7 & 6 & 0.4 & 7.5 & 0.8 \\ 65 & 4 & 3 & 0.7 & 8 & 0.4 \end{pmatrix},$$
$$A^{(3)} = \begin{pmatrix} 48 & 6 & 7 & 0.3 & 5.4 & 0.7 \\ 62 & 5 & 5 & 0.7 & 6 & 0.5 \\ 53 & 7 & 7 & 0.6 & 7.8 & 0.8 \\ 76 & 4 & 4 & 0.7 & 8.7 & 0.4 \end{pmatrix},$$
$$A^{(4)} = \begin{pmatrix} 38 & 8 & 7 & 0.3 & 5.8 & 0.6 \\ 58 & 6 & 6 & 0.6 & 6.6 & 0.7 \\ 50 & 7 & 7 & 0.5 & 7 & 0.7 \\ 80 & 4 & 3 & 0.8 & 8 & 0.2 \end{pmatrix},$$
$$A^{(5)} = \begin{pmatrix} 45 & 6 & 7 & 0.5 & 5.5 & 0.6 \\ 55 & 5 & 5 & 0.6 & 5 & 0.4 \\ 50 & 5 & 7 & 0.5 & 7.4 & 0.7 \\ 73 & 4 & 3 & 0.7 & 8.8 & 0.3 \end{pmatrix},$$
$$A^{(6)} = \begin{pmatrix} 40 & 6 & 7 & 0.4 & 5.3 & 0.7 \\ 58 & 4 & 5 & 0.6 & 5.5 & 0.5 \\ 58 & 5 & 7 & 0.2 & 7.2 & 0.8 \\ 66 & 3 & 3 & 0.8 & 8.2 & 0.4 \end{pmatrix},$$
$$A^{(7)} = \begin{pmatrix} 45 & 6 & 7 & 0.5 & 5.3 & 0.7 \\ 62 & 3 & 5 & 0.7 & 5 & 0.4 \\ 52 & 5 & 7 & 0.4 & 7.5 & 0.7 \\ 71 & 2 & 2 & 0.7 & 9 & 0.3 \end{pmatrix}, \tag{49}$$
$$A^{(8)} = \begin{pmatrix} 47 & 6 & 6 & 0.6 & 5.3 & 0.6 \\ 65 & 3 & 3 & 0.8 & 4.8 & 0.3 \\ 50 & 5 & 7 & 0.5 & 7.2 & 0.7 \\ 75 & 2 & 3 & 0.8 & 8.3 & 0.3 \end{pmatrix}.$$

Step 2. By equations (2) and (3), and according to the decision makers' preference, the fuzzy linguistic quantifier "more," with the pair $(0.3, 0.8)$ is selected to compute all the weights of the decision makers $w_j = (w_1, w_2, \ldots, w_n)^T$ $(j = 1, 2, \ldots, n)$.

$$\begin{aligned} w_1 &= Q\left(\frac{1}{8}\right) - Q(0) = 0, \\ w_2 &= Q\left(\frac{1}{4}\right) - Q\left(\frac{1}{8}\right) = 0, \\ w_3 &= Q\left(\frac{3}{8}\right) - Q\left(\frac{1}{4}\right) = 0.15, \\ w_4 &= Q\left(\frac{1}{2}\right) - Q\left(\frac{3}{8}\right) = 0.25, \\ w_5 &= Q\left(\frac{5}{8}\right) - Q\left(\frac{1}{2}\right) = 0.25, \\ w_6 &= Q\left(\frac{3}{4}\right) - Q\left(\frac{5}{8}\right) = 0.25, \\ w_7 &= Q\left(\frac{7}{8}\right) - Q\left(\frac{3}{4}\right) = 0.1, \\ w_8 &= Q(1) - Q\left(\frac{7}{8}\right) = 0. \end{aligned} \tag{50}$$

Finally, we can derive the weights of the decision makers $w = (0, 0, 0.15, 0.25, 0.25, 0.25, 0.1, 0)^T$.

Step 3. c_2 and c_3 are multiplicative indicators, and the rest are additive indicators, so it cannot be directly aggregated. Based on equations (1) and (4) to aggregate the additive and multiplicative of the decision matrix $A^{(k)}$, obtain $\overline{A}\lim_{x\longrightarrow\infty} = (\overline{r}_{ij})_{m\times n}$:

$$\overline{A} = \begin{pmatrix} 42.4500 & 10.3165 & 10.4335 & 0.4250 & 5.4900 & 0.6500 \\ 58.2500 & 9.8638 & 10.0035 & 0.6250 & 5.6750 & 0.5150 \\ 52.6500 & 10.1308 & 10.4335 & 0.4300 & 7.3200 & 0.7400 \\ 73.2500 & 9.4474 & 9.2511 & 0.7500 & 8.4550 & 0.3150 \end{pmatrix}. \tag{51}$$

Step 4. It is clear that c_2, c_3, and c_6 are benefit attributes, c_1 and c_4 are cost attributes, and c_5 is the fixed attribute.

Equations (44)–(46) are used for normalization to obtain a collective decision matrix $\overline{R}$:

$$\overline{R} = \begin{pmatrix} 1.0000 & 1.0000 & 1.0000 & 1.0000 & 0.0662 & 0.8784 \\ 0.7288 & 0.9561 & 0.9588 & 0.6800 & 0.1225 & 0.6959 \\ 0.8063 & 0.9820 & 0.9820 & 0.9884 & 0.7881 & 1.0000 \\ 0.5795 & 0.9158 & 0.8867 & 0.5667 & 0.0364 & 0.4257 \end{pmatrix}. \quad (52)$$

Step 5. From the above model $(M-1)$, the dispersion maximization model based on the 2-additive measure and Shapley value is established to obtain the optimal 2-additive measure $\mu(i)$ and $\mu(i,j)$.

$$\begin{aligned} &\max(-3.28\mu(1) - 3.55\mu(2) - 4.26\mu(3) - 3.78\mu(4) - 4.54\mu(5) - 3.22\mu(6) + 0.38\mu(1,2) + 0.4\mu(1,3) \\ &\quad + 0.94\mu(1,4) + 1.91\mu(1,5) + 1.12\mu(1,6) + 0.05\mu(2,3) + 0.59\mu(2,4) + 1.56\mu(2,5) + 0.76\mu(2.6) \\ &\quad + 0.61\mu(3,4) + 1.57\mu(3,5) + 0.78\mu(3,6) + 2.12\mu(4,5) + 1.32\mu(4,6) + 2.23\mu(5,6)), \\ &\text{s.t.} \begin{cases} \sum\limits_{l\subseteq S\backslash j} (\mu(j,l)) - \mu(l) \geq (s-2)\mu(j) \forall S \subseteq \{1,2,3,4,5,6\}, \quad \forall l \in S, \quad s \geq 2, \\ \sum\limits_{\{j,l\}\subseteq\{1,2,3,4,5,6\}} \mu(j,l) - 4(\mu(1) + \mu(2) + \mu(3) + \mu(4) + \mu(5) + \mu(6)) = 1, \\ \mu(j) \geq 0, \quad j = 1,2,\ldots,6. \end{cases} \end{aligned} \quad (53)$$

Solving the above linear programming model $(M-1)$, we get

$$\begin{aligned} \mu(1) &= 0.05, \\ \mu(2) &= 0.1, \\ \mu(3) &= 0.1, \\ \mu(4) &= 0.1, \\ \mu(5) &= 0.05, \\ \mu(6) &= 0.05, \\ \mu(1,2) &= 0.0081, \\ \mu(1,3) &= 0.0091, \\ \mu(1,4) &= 0.0013, \\ \mu(1,5) &= 0.2252, \\ \mu(1,6) &= 0.0063, \\ \mu(2,3) &= 0.001, \\ \mu(2,4) &= 0.0028, \\ \mu(2,5) &= 0.2617, \\ \mu(2,6) &= 0.0096, \\ \mu(3,4) &= 0.0096, \\ \mu(3,5) &= 0.2556, \\ \mu(3,6) &= 0.0097, \\ \mu(4,5) &= 0.2701, \\ \mu(4,6) &= 0.0049, \\ \mu(5,6) &= 0.3250. \end{aligned} \quad (54)$$

These are the optimal 2-additive measures.

Step 6. From Equation (47), the Shapley value of the Y-SOW operator is calculated as follows:

$$\begin{aligned}
\mathrm{Sh}_1(N,\mu) &= -\frac{3}{2}\mu(1)+\frac{1}{2}[\mu(1,2)-\mu(2)+\mu(1,3)-\mu(3)+\mu(1,4)-\mu(4)+\mu(1,5)-\mu(5)+\mu(1,6)-\mu(6)] = 0.16,\\
\mathrm{Sh}_2(N,\mu) &= -\frac{3}{2}\mu(2)+\frac{1}{2}[\mu(2,1)-\mu(1)+\mu(2,3)-\mu(3)+\mu(2,4)-\mu(4)+\mu(2,5)-\mu(5)+\mu(2,6)-\mu(6)] = 0.076,\\
\mathrm{Sh}_4(N,\mu) &= -\frac{3}{2}\mu(4)+\frac{1}{2}[\mu(4,1)-\mu(1)+\mu(4,2)-\mu(2)+\mu(4,3)-\mu(4)+\mu(4,5)-\mu(5)+\mu(4,6)-\mu(6)] = 0.229,\\
\mathrm{Sh}_3(N,\mu) &= -\frac{3}{2}\mu(3)+\frac{1}{2}[\mu(3,1)-\mu(1)+\mu(3,2)-\mu(2)+\mu(3,4)-\mu(4)+\mu(3,5)-\mu(5)+\mu(3,6)-\mu(6)] = 0.142,\\
\mathrm{Sh}_5(N,\mu) &= -\frac{3}{2}\mu(5)+\frac{1}{2}[\mu(5,1)-\mu(1)+\mu(5,2)-\mu(2)+\mu(5,3)-\mu(3)+\mu(5,4)-\mu(4)+\mu(5,6)-\mu(6)] = 0.272,\\
\mathrm{Sh}_6(N,\mu) &= -\frac{3}{2}\mu(6)+\frac{1}{2}[\mu(6,1)-\mu(1)+\mu(6,2)-\mu(2)+\mu(6,3)-\mu(3)+\mu(6,4)-\mu(4)+\mu(6,5)-\mu(5)] = 0.121.
\end{aligned} \tag{55}$$

Step 7. Based on equation (16), for convenience, we set $f(x)=x$, $\lambda=1$, and the overall preference value $\overline{r}_i\,(r=1,\ldots,m)$ of the alternative a_i is obtained. Let $\overline{r}_i = Y-SOW(a_1,\ldots,a_n)$. The overall preference values are obtained as

$$\begin{aligned}
\overline{r}_1 &= 4.858,\\
\overline{r}_2 &= 3.247,\\
\overline{r}_3 &= 4.323,\\
\overline{r}_4 &= 8.679.
\end{aligned} \tag{56}$$

Step 8. The ranking is $\overline{r}_4 > \overline{r}_1 > \overline{r}_3 > \overline{r}_2$.

Therefore, the best alternative is a_4. Namely, land D is most needed for remediation.

In this example, we can see that the YSMAGD algorithm can efficiently handle heterogeneous data with the good aggregation. The evaluation index of site selection of polluted land in the above example usually contains objective factors and subjective factors. For example, indicators such as remedial potential and realistic feasibility are multiplicative indicators that cannot be calculated from specific values. Indicators such as pollution area and soil pH are additive indicators that can be obtained through clear numerical measurements. The key to land pollution assessment is to combine both indicators, so decision makers use the Y-SOW operator proposed above to aggregate and use the YSMAGD algorithm for effective land assessment. This is not only beneficial for the government to implement land pollution remediation, but also for experts to conduct land research.

6. Comparison and Conclusion

6.1. Comparison and Analysis

6.1.1. Comparative Analysis of Aggregation Results under Different Values of λ. Because $\lambda \in [0,1]$, there are many options of parameter λ, such as $\lambda=0$, $\lambda=(1/2)$, and $\lambda=1$. Comparing these three situations, the overall preference value is obtained shown as follows:

(i) When $\lambda = 0$,

$$\begin{aligned}
\overline{r}_1 &= 1.368,\\
\overline{r}_2 &= 1.667,\\
\overline{r}_3 &= 1.105,\\
\overline{r}_4 &= 2.222.
\end{aligned} \tag{57}$$

(ii) When $\lambda = (1/2)$,

$$\begin{aligned}
\overline{r}_1 &= 2.269,\\
\overline{r}_2 &= 2.191,\\
\overline{r}_3 &= 1.113,\\
\overline{r}_4 &= 3.730.
\end{aligned} \tag{58}$$

Compare the above three cases and use the obtained overall preference values as a scatter plot, as shown in Figure 2.

Obviously, for $\lambda=0$, $\lambda=(1/2)$, and $\lambda=1$, the results of aggregation are not numerically identical. However, by comparing these results, the same alternative approach can be obtained, namely, land D needs to be repaired the most urgently.

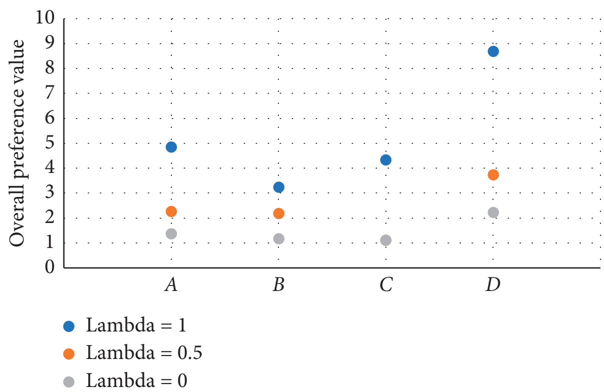

Figure 2: The overall preference values with respect to different λ.

6.1.2. Comparing the Y-SOW Operator with WAA Operator and WGA Operator. Let $a_1 = (1/2)$, $a_2 = 3$, and $a_3 = 8$ be three real numbers. For comparing with the weighted arithmetic averaging (WAA) operator [42] and weighted geometric averaging (WGA) operator [43], $\lambda = 0.5$ and $f(x) = x$ are selected. Assume that decision makers (or attributes) are independent and unrelated, so we set weighted vector $w = \mathrm{Sh}(N, \mu) = ((1/3), (1/3), (1/3))$. Accordingly, the aggregation results calculated by three aggregation operators are shown as follows:

$$\begin{aligned} \mathrm{WAA}(a_1, a_2, a_3) &= 3.833, \\ \mathrm{WGA}(a_1, a_2, a_3) &= 2.289, \\ \mathrm{Y-SOW}(a_1, a_2, a_3) &= 2.246. \end{aligned} \quad (59)$$

We denote $H(\mathrm{WWA})$, $H(\mathrm{WGA})$, and $H(\mathrm{Y-SOW})$ the penalty functions derived by equation (11), respectively. Thus, we have the following results:

$$\begin{aligned} H(\mathrm{WWA}) &= 0.214, \\ H(\mathrm{WGA}) &= 0.172, \\ H(\mathrm{Y-SOW}) &= 0.171. \end{aligned} \quad (60)$$

It is clear that the value of penalty function H with respect to the Y-SOW operator is the smallest. From equation (11), we know that the Y-SOW operator is the optimal aggregation for minimizing the penalty function. This supports the optimality of the Y-SOW operator.

6.2. Brief Conclusion. This paper introduces a new aggregation operator called the Young–Shapley optimal weight (Y-SOW) operator. Some special cases and main properties of the Y-SOW operator are studied. The advantages of this paper are as follows:

(i) The Y-SOW operator solves the problem that the additive and the multiplicative decision information appear simultaneously in the group decision-making.

(ii) In the Shapley value method of cooperative game, the 2-additive measure replaces the original fuzzy measure. This not only can effectively deal with the interaction between decision attributes, but also reduce the computational complexity and improve the representation between attributes.

(iii) To solve the problems of multiattribute group decision-making under attribute interaction, we develop the Y-SOW operator-based multiattribute group decision (YSMAGD) algorithm and establish a linear programming model $(M-1)$.

It is worth noting that we can also apply the Y-SOW operator to the other areas such as deep learning, cluster analysis, and artificial intelligence.

Nevertheless, in this article, we only consider the case where decision information is represented by real numbers. However, in some cases, the decision information may be in other forms, such as interval numbers and fuzzy numbers. This is an issue that needs further study in the near future.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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