

Research Article

Productivity Evaluation of Vertical Wells Incorporating Fracture Closure and Reservoir Pressure Drop in Fractured Reservoirs

Lun Zhao,¹ Zifei Fan,¹ Mingxian Wang ^{1,2}, Guoqiang Xing ¹, Wenqi Zhao,¹ Chengqian Tan,² and Youyou Cheng³

¹Research Institute of Petroleum Exploration and Development, China National Petroleum Corporation, Beijing 100083, China

²School of Earth Science and Engineering, Xi'an Shiyou University, Xi'an 710065, China

³School of Petroleum Engineering, Xi'an Shiyou University, Xi'an 710065, China

Correspondence should be addressed to Mingxian Wang; wangmingxian89@126.com

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In most oilfields, many wells produce in pseudo-steady-state period for a long time. Because of large reservoir pressure drop in this period, fractured reservoirs always show strong stress sensitivity and fracture closure is likely to occur near wellbores. The primary goal of this study is to evaluate productivity of vertical wells incorporating fracture closure and reservoir pressure drop. Firstly, a new composite model was developed to deal with stress sensitivity and fracture closure existed in fractured reservoirs. Secondly, considering reservoir saturation condition, new pseudo-steady productivity equations for vertical wells were derived by using the proposed composite system. Thirdly, related inflow performance characteristics and influence of some factors on them were also discussed in detail. Results show that fracture closure has a great effect on vertical well inflow performance and fracture closure radius is negatively correlated with well productivity. In this composite model, the effects of stress sensitivity of the inner and outer zone on well productivity are rather different. The inner zone's stress sensitivity affects well productivity significantly, but the outer zone's stress sensitivity just has a weak effect on the productivity. Strong stress sensitivity in the inner zone leads to low well productivity, and both inflow performance and productivity index curves bend closer to the bottom-hole pressure axis with stress sensitivity intensifying. Meanwhile, both maximum productivity and optimal bottom-hole pressure can be achieved from inflow performance curves. In addition, reservoir pressure is positively correlated with vertical well productivity. These new productivity equations and inflow performance curves can directly provide quantitative reference for optimizing production system in fractured reservoirs.

1. Introduction

In fossil reservoirs, the investigation of well inflow performance is an important method to evaluate and predict well productivity, which belongs to the category of well test and is widely used in oilfields for its simplicity and practicality [1, 2].

Through a series of numerical simulation, Vogel first presented a dimensionless inflow performance curve suitable for dissolved gas drive reservoirs and laid a solid foundation for the study of inflow performance [3]. Based on Vogel's work, Standing defined a flow efficiency to reflect the perfection of oil wells and established the inflow

performance relationship of imperfect wells [4], which was the extension of Vogel equation. Based on the empirical equation of gas well productivity, Fetkovich proposed a new inflow performance relationship, namely, Fetkovich equation [5]. Similar to Vogel's approach deriving vertical wells' inflow relationship, Bendakhli and Aziz developed a dimensionless horizontal well inflow equation by combining the Vogel equation and Fetkovich equation [6]. Later, Cheng used numerical simulation to study the productivity of inclined and horizontal wells in dissolved gas drive reservoirs and obtained the inflow relationships of different wellbores with slant angles by regression [7]. The above inflow relationships greatly enrich the theory of inflow

performance analysis and provide quantitative basis for the optimization of production system, especially for different well types. However, all these equations have some implicit assumptions, such as that reservoir is homogeneous and reservoir pressure remains constant, which means these equations do not take stress sensitivity and reservoir pressure drop into account. Thus, it is less likely to incorporate fracture closure caused by stress sensitivity into these equations. Therefore, the above equations have some limitations and deficiencies, which may lead to large error in field applications, especially in fractured reservoirs having severe stress sensitivity and fracture closure in late-time production [8].

Some researchers have put forward some effective methods to consider stress-sensitivity effect in the simulation and characterization of fractured reservoirs [9–12]. Nur and Yilmaz first defined a stress-sensitive coefficient to characterize and quantify permeability variation [9]. By using this newly defined parameters, Kikani and Pedrosa derived a permeability formula in the form of exponential function, which takes pressure variable into account [10]. The exponential expression is widely applied to describe stress sensitivity [13–17]. Considering actual reservoir situation, some scholars extended these inflow relationships to reservoirs with stress sensitivity by using the previous exponential relationship. Song considered the influence of stress sensitivity and starting pressure gradient on inflow performance and obtained the inflow equation suitable for low-permeability reservoirs [18]. Wang et al. applied pseudo-pressure to deformed medium reservoirs and derived the inflow equation considering permeability change [19]. Tian et al. established a steady-state productivity equation for reservoirs with starting pressure gradient, stress sensitivity, and fluid viscosity variability [20]. They considered stress sensitivity in reservoirs, but ignored fracture closure near wellbores caused by stress sensitivity. In addition, the previous exponential relationship implies that reservoir pressure remains unchanged during production.

Obviously, all the above inflow equations have some limitations to some extent. For example, they may not consider stress sensitivity and fracture closure in fractured reservoirs, or not consider oil-gas two-phase flow, or assume steady-state flow happened in reservoirs, or assume reservoir pressure remains constant during production. At present, there is a lack of inflow equations that take all the above factors into account, including stress sensitivity, fracture closure, two-phase flow, and reservoir pressure variation, which is exactly the problem expected to be solved in this paper.

The main purpose of this paper is to investigate the productivity of vertical wells incorporating fracture closure and reservoir pressure drop in fractured reservoirs. This paper is organized as follows. Firstly, by dividing a fractured reservoir into two zones, i.e., fracture closure zone and unclosed fracture zone, a composite model was developed to deal with the issue of stress sensitivity and fracture closure. Secondly, for different reservoir saturation, new pseudo-steady productivity equations for vertical

wells were derived by using the proposed composite system. Thirdly, inflow performance curves and productivity index curves under different parameter values were presented and effects of some key factors on vertical well productivity and optimal bottom-hole pressure were also investigated in detail.

2. Physical Model of Vertical Well in Fractured Reservoir with Fracture Closure

For a fractured reservoir (made up of natural fracture and matrix), stress sensitivity of the natural fracture system is stronger than that of the matrix system. When reservoir pressure drops significantly during production, fracture closure first happens near the wellbore. Under this circumstance, the reservoir near the wellbore behaves as a single medium. Then, a radial discontinuity of physical properties occurs around the wellbore, and the reservoir can be divided into two zones: fracture closure zone (inner zone) and unclosed fracture zone (outer zone). Thus, the idea of composite reservoir can be used to deal with stress sensitivity and fracture closure appeared in fractured reservoirs [21–23].

Figure 1 is a schematic illustration of a vertical well with fracture closure near the wellbore in a fractured reservoir. This composite model has two concentric zones. In the inner zone, the fracture around the wellbore closes at a certain radius and only the matrix system provides flow ability for the hydrocarbon fluid. In the outer zone, the fracture system does not close and provides main flow ability together with the matrix system. To simplify this physical model, some necessary assumptions are demonstrated as follows:

- (1) For this fractured reservoir, the permeability of the matrix and natural fracture system are k_m and k_f , respectively.
- (2) The reservoir is circular in shape and closed in the outer boundary. Its seepage radius and thickness are r_e and h , respectively. The radius of the fracture closure area is r_f and just corresponds to the radius of the inner zone.
- (3) The outer zone's initial equivalent permeability is k_{10} (equal to the sum of k_f and k_m), and its stress sensitivity coefficient is α_1 . Also, the inner zone's initial equivalent permeability is k_{20} (equal to k_m), and the corresponding stress sensitivity coefficient is α_2 [24].
- (4) The reservoir's initial pressure and bubble-point pressure are p_0 and p_b , respectively. This reservoir may be saturated ($p_0 < p_b$) or unsaturated ($p_0 > p_b$). The average reservoir pressure during production is p_{avg} , and the pressure at the closed radius is p_f .
- (5) The hydrocarbon fluid in this reservoir is slightly compressible and has constant viscosity μ , volume factor B , and total compressibility C_t , and flow in this model obeys Darcy's law.
- (6) A vertical well is located at the center of the circular reservoir. Its wellbore radius is r_w and bottom-hole pressure is p_{wf} .

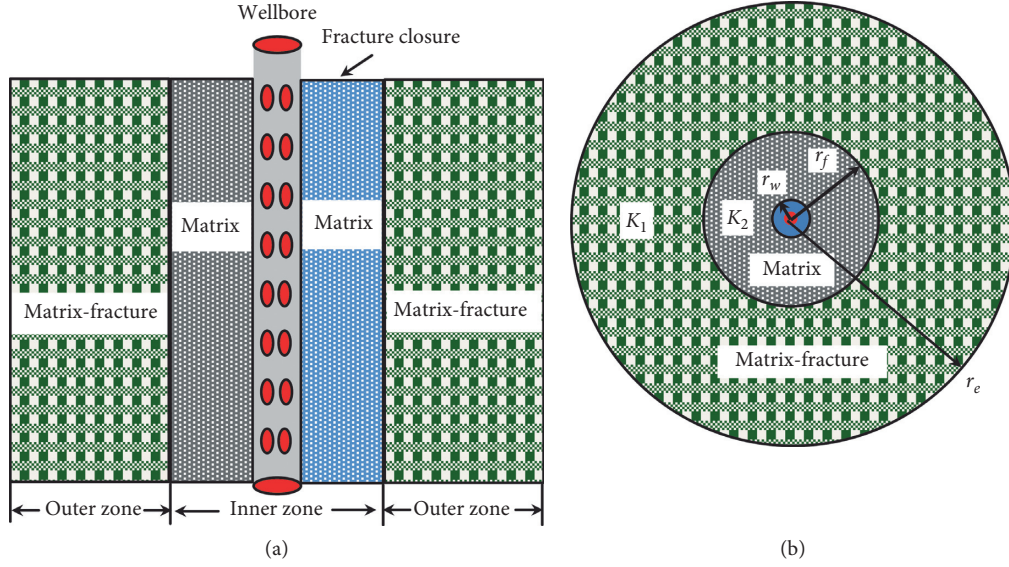


FIGURE 1: Schematic of the composite model of a vertical well in a fractured reservoir with fracture closure. (a) Cutaway view of the composite model. (b) Top view of the composite model.

3. Productivity Equation considering Reservoir Pressure Drop but without Fracture Closure in Fractured Reservoirs

In this section, we focus on the derivation of the productivity equation considering reservoir pressure drop and stress sensitivity, but without fracture closure. This is the basis for deriving the productivity equation considering fracture closure in the next section. When there is no fracture closure in the reservoir, it is not necessary to divide the reservoir into two zones. The initial equivalent permeability and stress sensitivity coefficient throughout the fractured reservoir are k_{a0} (equal to k_{10}) and α_k (equal to α_1), respectively.

For closed reservoirs, pseudo-steady flow occurs when pressure disturbance reaches the reservoir boundary. At this point, the total production comes from the fluid and rock's expansion caused by reservoir pressure drop [25].

According to the physical definition of total compressibility C_t , the total volume of fluid derived out by the reservoir's elastic energy is

$$V = C_t V_f (p_0 - p_{\text{avg}}) = C_t [\pi(r_e^2 - r_w^2)h](p_0 - p_{\text{avg}}). \quad (1)$$

Then, the production rate of the vertical well can be deduced by taking the derivative with respect to time:

$$QB = \frac{dV}{dt} = -C_t \pi (r_e^2 - r_w^2) h \frac{dp_{\text{avg}}}{dt}. \quad (2)$$

At any reservoir radius, the flow rate through the cylindrical section can be described as

$$q_r = -C_t \pi (r_e^2 - r^2) h \frac{dp_{\text{avg}}}{dt}. \quad (3)$$

In combination with equations (2) and (3), we can get the following relationship:

$$\frac{q_r}{QB} = \frac{r_e^2 - r^2}{r_e^2 - r_w^2}. \quad (4)$$

Because the wellbore radius is far less than the reservoir radius, we can have the approximation $r_e^2 - r_w^2 \approx r_e^2$; thus, the above equation can be simplified to

$$q_r = \left(1 - \frac{r^2}{r_e^2}\right) QB. \quad (5)$$

The flow velocity at any cylindrical section in the circular reservoir is equal to

$$v_r = \frac{q_r}{86400 \times 2\pi r h} = \frac{QB}{5.4287 \times 10^5 r_e h} \left(\frac{r_e}{r} - \frac{r}{r_e}\right). \quad (6)$$

According to the unit of each parameter, defining in the nomenclature part, there is a conversion coefficient of 86400 in equation (6).

Considering fractured reservoir's stress sensitivity, seepage velocity in the form of Darcy's law can be obtained [9]. Furthermore, equation (6) can be written as

$$v_r = \frac{k_{a0} \exp[-\alpha_K (p_0 - p)]}{10^3 \mu} \frac{dp}{dr} = \frac{QB}{5.4287 \times 10^5 r_e h} \left(\frac{r_e}{r} - \frac{r}{r_e}\right). \quad (7)$$

By separating variable and integrating in equation (7) and considering the wellbore condition, the formation pressure distribution can be given by

$$p(r) = p_{wf} + \frac{1}{\alpha_K} \ln \left[1 + \frac{\alpha_K \mu B Q}{542.87 k_{a0} e^{-\alpha_K (p_0 - p_{wf})} h} \cdot \left(\ln \frac{r}{r_w} - \frac{r^2 - r_w^2}{2r_e^2} \right) \right]. \quad (8)$$

Considering $r_w^2 \leq r_e^2$, equation (8) can be simplified as

$$p(r) = p_{wf} + \frac{1}{\alpha_K} \ln \left[1 + \frac{\alpha_K \mu B Q}{542.87 k_{a0} e^{-\alpha_K (p_0 - p_{wf})} h} \cdot \left(\ln \frac{r}{r_w} - \frac{r^2}{2r_e^2} \right) \right]. \quad (9)$$

The pressure at the outer boundary is assumed to be p_e . By separating variable and integrating in equation (7) and considering the outer boundary condition, another form of formation pressure distribution can be obtained:

$$p(r) = p_e + \frac{1}{\alpha_K} \ln \left[1 - \frac{\alpha_K \mu B Q}{542.87 k_{a0} e^{-\alpha_K (p_0 - p_e)} h} \cdot \left(\ln \frac{r_e}{r} - \frac{r_e^2 - r^2}{2r_e^2} \right) \right]. \quad (10)$$

Define the following pseudo-pressure function:

$$U(r) = \frac{1}{\alpha_K} \exp[-\alpha_K (p_0 - p)]. \quad (11)$$

By combining equations (10) and (11), we can get

$$U(r) = U(r_e) - \frac{\mu B Q}{542.87 k_{a0} h} \left(\ln \frac{r_e}{r} + \frac{r^2}{2r_e^2} - \frac{1}{2} \right). \quad (12)$$

Then, the average pseudo-pressure function for the formation within the seepage area can be written as follows:

$$\begin{aligned} \bar{U}(r) &= \frac{\int_{r_w}^{r_e} U(r) \cdot 2\pi r dr}{\pi(r_e^2 - r_w^2)} = U(r_e) - \frac{\mu B Q}{542.87 k_{a0} h} \\ &\cdot \left(\frac{r_e^2 + r_w^2}{4r_e^2} - \frac{r_w^2}{r_e^2 - r_w^2} \ln \frac{r_e}{r_w} \right) \approx U(r_e) - \frac{\mu B Q}{2171.47 k_{a0} h}. \end{aligned} \quad (13)$$

If the formation pressure corresponding to $\bar{U}(r)$ is p_{avg} , recalling the definition of pseudo-pressure function and equation (13), the following equation can be derived:

$$p_{avg} = p_0 + \frac{1}{\alpha_K} \ln \left\{ \exp[-\alpha_K (p_0 - p_e)] - \frac{\alpha_K \mu B Q}{2171.47 k_{a0} h} \right\}. \quad (14)$$

Using equation (9) to represent the boundary pressure p_e , we have

$$p_e = p_{wf} + \frac{1}{\alpha_K} \ln \left[1 + \frac{\alpha_K \mu B Q}{542.87 k_{a0} e^{-\alpha_K (p_0 - p_{wf})} h} \left(\ln \frac{r_e}{r_w} - \frac{1}{2} \right) \right]. \quad (15)$$

Substituting equation (15) into equation (14), we can eliminate the term of boundary pressure p_e and get the productivity relationship without boundary pressure:

$$p_{avg} = p_0 + \frac{1}{\alpha_K} \ln \left[e^{-\alpha_K (p_0 - p_{wf})} + \frac{\alpha_K \mu B Q}{542.87 k_{a0} h} \left(\ln \frac{r_e}{r_w} - \frac{3}{4} \right) \right]. \quad (16)$$

Equation (16) can be easily converted into the following productivity equation, which is similar to the Fetkovich equation and includes the average reservoir pressure:

$$Q = \frac{542.87 k_{a0} h}{\mu B (\ln(r_e/r_w) - (3/4))} \frac{e^{-\alpha_K (p_0 - p_{avg})} - e^{-\alpha_K (p_0 - p_{wf})}}{\alpha_K}. \quad (17)$$

Equation (17) is the pseudo-steady productivity equation of vertical wells considering stress sensitivity and reservoir pressure drop but without fracture closure in fractured reservoirs. Using this equation, we can easily calculate the productivity index (J) by introducing bottom-hole pressure difference ($p_{avg} - p_{wf}$) into equation (17).

4. Productivity Equation Incorporating Reservoir Pressure Drop and Fracture Closure in Fractured Reservoirs

Based on the derivation in the last section, we further derived the productivity equation of a vertical well incorporating reservoir pressure drop and fracture closure by using the previous composite model (Figure 1).

Here, we applied the exponential stress sensitivity relation to describe stress sensitivity in this fractured reservoir. The permeability expression of the outer and inner zones can be presented as follows [26]:

$$K_1 = K_{10} e^{-\alpha_1 (p_0 - p)}, \quad (18)$$

$$K_2 = K_{20} e^{-\alpha_2 (p_0 - p)} = K_{20} e^{-\alpha_2 (p_0 - p_w)} e^{-\alpha_2 (p_w - p)}. \quad (19)$$

For saturated and unsaturated reservoirs, single-phase flow (oil phase) or two-phase flow (oil phase and gas phase) may occur near the wellbore. The flow characteristics of two situations are significantly different, and it is necessary to discuss them separately.

4.1. Unsaturated Reservoirs. When $p_{wf} \geq p_b$, flow in the reservoir is single-phase flow; when $p_{wf} < p_b$, two-phase flow occurs near the wellbore.

4.1.1. Single-Phase Flow (Oil Phase). In equation (17), using the closure radius and the pressure at the closure radius to replace the wellbore radius and bottom-hole pressure, respectively, the pseudo-steady productivity equation for the outer zone (unclosed fracture zone) can be written as

$$Q = \frac{542.87 k_{10} h}{\mu B (\ln(r_e/r_f) - (3/4))} \frac{e^{-\alpha_1 (p_0 - p_{avg})} - e^{-\alpha_1 (p_0 - p_f)}}{\alpha_1}. \quad (20)$$

The above equation can be converted into the following form:

$$\bar{p}_r - p_f = -\frac{1}{\alpha_1} \ln \left[1 - \frac{\alpha_1 \mu B Q}{542.87 k_{10} h e^{-\alpha_1 (p_0 - p_{avg})}} \left(\ln \frac{r_e}{r_f} - \frac{3}{4} \right) \right]. \quad (21)$$

Using the derivation in the last section, the inner zone can be regarded as a closed reservoir with a radius equal to the closed radius and a boundary pressure equal to p_f . Thus, applying seepage velocity (equation (7)) to this zone, we can obtain the seepage velocity in the inner zone:

$$v_r = \frac{k_{20} \exp[-\alpha_2 (p_0 - p)]}{10^3 \mu} \frac{dp}{dr} = \frac{QB}{5.4287 \times 10^5 r_f h} \cdot \left(\frac{r_f}{r} - \frac{r}{r_f} \right). \quad (22)$$

By separating variable and integrating in equation (22) and considering the wellbore condition, the formation pressure distribution in the inner zone can be derived:

$$p(r) = p_{wf} + \frac{1}{\alpha_2} \ln \left[1 + \frac{\alpha_2 \mu B Q}{542.87 k_{20} e^{-\alpha_2 (p_0 - p_{wf})} h} \cdot \left(\ln \frac{r}{r_w} - \frac{r^2 - r_w^2}{2r_f^2} \right) \right]. \quad (23)$$

Similarly, due to $r_f^2 - r_w^2 \approx r_f^2$, the above equation can be simplified as

$$p(r) = p_{wf} + \frac{1}{\alpha_2} \ln \left[1 + \frac{\alpha_2 \mu B Q}{542.87 k_{20} e^{-\alpha_2 (p_0 - p_{wf})} h} \cdot \left(\ln \frac{r}{r_w} - \frac{r^2}{2r_f^2} \right) \right]. \quad (24)$$

Substituting the pressure at the closure radius into the above equation, we have the following relationship:

$$p_f = p_{wf} + \frac{1}{\alpha_2} \ln \left[1 + \frac{\alpha_2 \mu B Q}{542.87 k_{20} e^{-\alpha_2 (p_0 - p_{wf})} h} \left(\ln \frac{r_f}{r_w} - \frac{1}{2} \right) \right]. \quad (25)$$

For single-phase flow in unsaturated reservoirs, the pseudo-steady productivity equation of a vertical well considering fracture closure and reservoir pressure drop can be obtained by substituting equation (25) into equation (20):

$$p_{avg} - p_{wf} = -\frac{1}{\alpha_1} \ln \left[1 - \frac{\alpha_1 \mu B Q}{542.87 k_{10} h e^{-\alpha_1 (p_0 - p_{avg})}} \left(\ln \frac{r_e}{r_f} - \frac{3}{4} \right) \right] + \frac{1}{\alpha_2} \ln \left[1 + \frac{\alpha_2 \mu B Q}{542.87 k_{20} e^{-\alpha_2 (p_0 - p_{wf})} h} \left(\ln \frac{r_f}{r_w} - \frac{1}{2} \right) \right]. \quad (26)$$

Equation (26) is an implicit function of productivity with regard to bottom-hole pressure, and we cannot directly obtain the analytical expression of the productivity equation. Although this productivity equation has strong nonlinearity, it can be solved by the Newton iterative method.

Furthermore, the corresponding productivity index under different bottom-hole pressure can be achieved.

Combining equations (19) and (26), the relationship between closed radius and bottom-hole pressure can be obtained:

$$r_f = \exp \left\{ \frac{k_{20} \alpha_1 \left[e^{-\alpha_2 (p_0 - p_f)} - e^{-\alpha_2 (p_0 - p_{wf})} \right] \left(\ln r_e - (3/4) \right) + k_{10} \alpha_2 \left[e^{-\alpha_1 (p_0 - p_{avg})} - e^{-\alpha_1 (p_0 - p_f)} \right] \left(\ln r_w - (1/2) \right)}{k_{10} \alpha_2 \left[e^{-\alpha_1 (p_0 - p_{avg})} - e^{-\alpha_1 (p_0 - p_f)} \right] + k_{20} \alpha_1 \left[e^{-\alpha_2 (p_0 - p_f)} - e^{-\alpha_2 (p_0 - p_{wf})} \right]} \right\}. \quad (27)$$

If $\alpha_1 = \alpha_2 = \alpha_K$, $K_{10} = K_{20} = K_{a0}$, and $r_f = r_w$, equation (26) can be reduced to the case without fracture closure (equation (17)). Furthermore, in the early-time production,

due to minor reservoir pressure changes, equation (26) can also be simplified to

$$p_{avg} - p_{wf} = -\frac{1}{\alpha_1} \ln \left[1 - \frac{\alpha_1 \mu B Q}{542.87 k_{10} h} \left(\ln \frac{r_e}{r_f} - \frac{3}{4} \right) \right] + \frac{1}{\alpha_2} \ln \left[1 + \frac{\alpha_2 \mu B Q}{542.87 k_{20} e^{-\alpha_2 (p_{avg} - p_{wf})} h} \left(\ln \frac{r_f}{r_w} - \frac{1}{2} \right) \right]. \quad (28)$$

When reservoir pressure drops over a period of time, it is more accurate to use equation (26) to calculate the

productivity than to use equation (28). Therefore, equation (26) can be regarded as a general equation, and

equations (17) and (28) are two degradation forms of this equation.

4.1.2. *Two-Phase Flow (Oil Phase and Gas Phase)*. When $p_{wf} < p_b$, the fluid near the wellbore begins to degas.

$$p_{avg} - p_{wf} = -\frac{1}{\alpha_1} \ln \left[1 - \frac{\alpha_1 \mu B Q (p_{avg} - p_{wf})}{542.87 k_{10} h e^{-\alpha_1 (p_0 - p_{avg})} \left\{ (p_{avg} - p_b) + (p_b / 1.8) \left[1 - 0.2 (p_{wf} / p_b) - 0.8 (p_{wf} / p_b)^2 \right] \right\}} \right] \left(\ln \frac{r_e}{r_f} - \frac{3}{4} \right) + \frac{1}{\alpha_2} \ln \left[1 + \frac{\alpha_2 \mu B Q (p_{avg} - p_{wf})}{542.87 k_{20} e^{-\alpha_2 (p_0 - p_{wf})} h \left\{ (p_{avg} - p_b) + p_b / 1.8 \left[1 - 0.2 (p_{wf} / p_b) - 0.8 (p_{wf} / p_b)^2 \right] \right\}} \right] \left(\ln \frac{r_f}{r_w} - \frac{1}{2} \right). \quad (29)$$

Equation (29) is also an implicit function of productivity and can be solved iteratively, and the corresponding productivity index also can be achieved.

4.2. *Saturated Reservoirs*. In saturated reservoirs, initial reservoir pressure is lower than bubble-point pressure. When a well begins to produce in this reservoir, two-phase flow immediately appears near the wellbore. Dissolved gas

Similarly, according to the Vogel equation, for two-phase flow in the unsaturated reservoir, the productivity equation of a vertical well considering fracture closure and reservoir pressure drop can be derived [3]:

drive is the main displacement way, and the productivity equation of the saturated reservoir can still be determined by using the Vogel equation [3].

In equation (29), replacing bubble-point pressure with average reservoir pressure, for two-phase flow in the saturated reservoir, the pseudo-steady productivity equation of a vertical well incorporating fracture closure and reservoir pressure drop can be obtained:

$$p_{avg} - p_{wf} = -\frac{1}{\alpha_1} \ln \left[1 - \frac{\alpha_1 \mu B Q (p_{avg} - p_{wf})}{542.87 k_{10} h e^{-\alpha_1 (p_0 - p_{avg})} p_{avg} / 1.8 \left[1 - 0.2 (p_{wf} / p_{avg}) - 0.8 (p_{wf} / p_{avg})^2 \right]} \right] \left(\ln \frac{r_e}{r_f} - \frac{3}{4} \right) + \frac{1}{\alpha_2} \ln \left[1 + \frac{\alpha_2 \mu B Q (p_{avg} - p_{wf})}{542.87 k_{20} e^{-\alpha_2 (p_0 - p_{wf})} h \bar{p}_R / 1.8 \left[1 - 0.2 (p_{wf} / p_{avg}) - 0.8 (p_{wf} / p_{avg})^2 \right]} \right] \left(\ln \frac{r_f}{r_w} - \frac{1}{2} \right). \quad (30)$$

Similarly, this equation can be solved iteratively and further the productivity index under different bottom-hole pressure can be calculated.

5. Results and Discussion

Based on the above productivity equations derived in fractured reservoirs, a series of inflow performance charts were calculated and plotted to conduct productivity evaluation of vertical wells incorporating fracture closure and reservoir pressure drop. The influence of fracture closure on well productivity was first discussed, and then the effects of some key parameters on inflow performance and productivity index, including fracture closure radius, stress sensitivity in the inner and outer zones and reservoir pressure, were further investigated. Taking North Truva Oilfield in Kazakhstan as an example, we present the values of relevant parameters in Table 1. These investigations can directly provide quantitative reference for optimizing the production

system in fractured reservoirs. The effects of other factors, like reservoir permeability, reservoir thickness, and reservoir radius have been discussed in detail by many researchers [18–20], and we do not discuss them in this study.

5.1. *Productivity Comparison between the Well with Fracture Closure and the Well without Fracture Closure in Fractured Reservoirs*. Figures 2 and 3 display the effect of fracture closure on the vertical well inflow performance and productivity index, respectively. For the case considering stress sensitivity, these curves show the same shape and trend as the results in the literature [20]. As presented in Figures 2 and 3, fracture closure has a great effect on well inflow performance in fractured reservoirs. Compared with the case without fracture closure, both productivity and productivity index of the well with fracture closure decrease significantly at a fixed bottom-hole pressure. When fracture closure occurs near the wellbore, its inflow performance and

TABLE 1: The range of parameter values in this study.

Model parameter	Value
k_{10}	$0.025 \mu\text{m}^2$
k_{20}	$0.01 \mu\text{m}^2$
α_1	$0-0.125 \text{MPa}^{-1}$
α_2	$0-0.125 \text{MPa}^{-1}$
B	$1.525 \text{m}^3/\text{m}^3$
h	15.71 m
μ	0.56 mPa·s
p_0	24.14 MPa
p_{avg}	9.7–24.14 MPa
p_b	20.54 MPa
p_{wf}	0.1–24 MPa
r_w	0.1 m
r_e	325 m
r_f	30–150 m

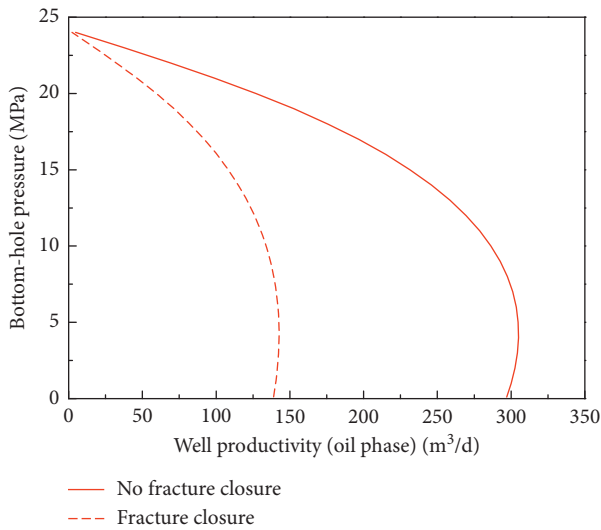


FIGURE 2: Comparison of inflow performance under different fracture closure conditions.

productivity index curves (dash lines) bend closer to the bottom-hole pressure axis (Figure 2) and the slope of the productivity index curve increases (Figure 3). It can be explained by the fact that when fracture closure occurs near the wellbore, the reservoir equivalent permeability decreases and the resistance of the fluid flowing to the wellbore increases, resulting in the decrease of the well productivity and productivity index.

5.2. Influence of Key Parameters on Inflow Performance of Vertical Wells Incorporating Fracture Closure and Reservoir Pressure Drop

5.2.1. *Fracture Closure Radius.* Figures 4 and 5 show the influence of the fracture closure radius on the inflow performance and productivity index of a vertical well in the fractured reservoir, respectively. To investigate the influence of this parameter, we assume that reservoir pressure does not change much and remains constant during production. As

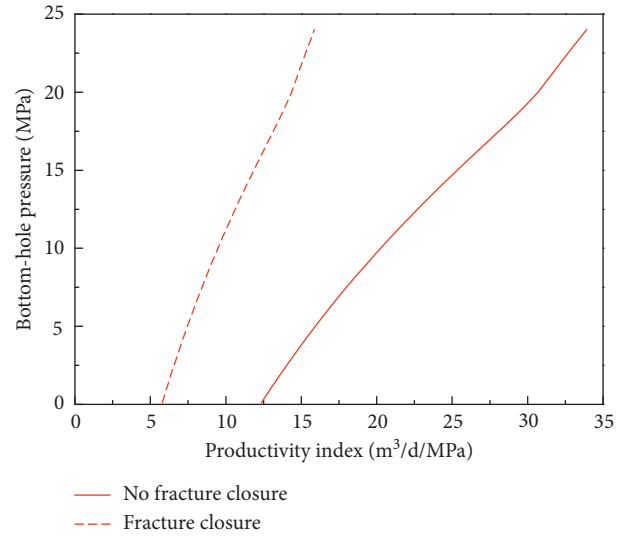


FIGURE 3: Comparison of the productivity index under different fracture closure conditions.

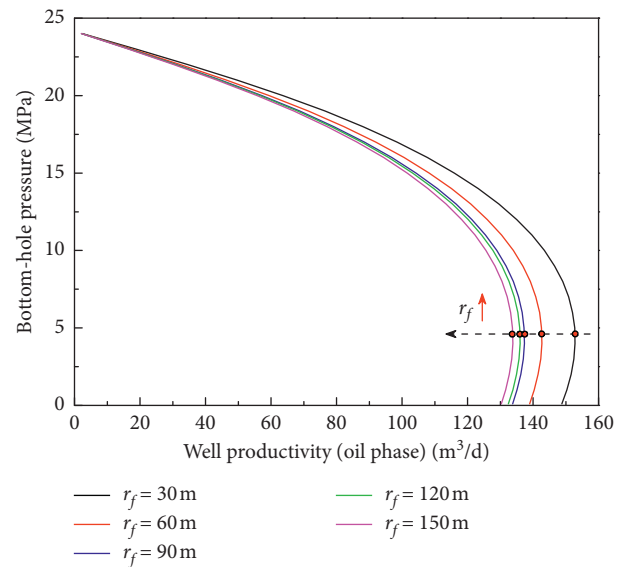


FIGURE 4: Effect of the fracture closure radius on the inflow performance curves of a vertical well with fracture closure in a fractured reservoir.

shown in Figure 4, the fracture closure radius is negatively correlated with well productivity. For a fixed bottom-hole pressure, a decrease of productivity and productivity index can be observed as the closure radius increases. Meanwhile, with an increasing fracture closure radius, the inflow performance curves bend closer to the longitudinal axis (bottom-hole pressure), but the productivity index curves approximately shift to the left side. The expansion of the fracture closure radius means the increase of flow resistance, which causes the decrease of well productivity, which explained the phenomenon observed in Figures 4 and 5. When the fracture closure radius changes, the optimal bottom-hole pressure to achieve the maximum productivity

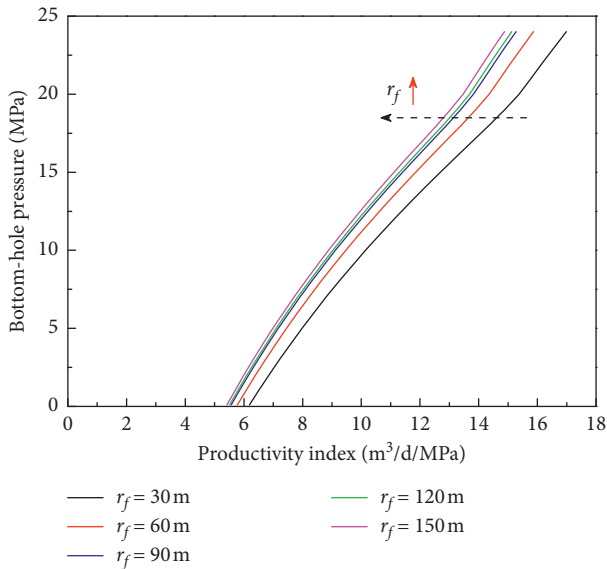


FIGURE 5: Effect of the fracture closure radius on the productivity index curves of a vertical well with fracture closure in a fractured reservoir.

is almost the same and the corresponding inflection point (red dot) just moves to the left slightly.

5.2.2. Stress Sensitivity in the Outer Zone. For a given fracture closure radius and reservoir pressure, Figures 6 and 7 present the effect of stress sensitivity of the outer zone on the inflow performance and productivity index curves of a vertical well with fracture closure, respectively. As demonstrated in these two figures, all the inflow performance curves for different stress sensitivity of the outer zone almost coincide exactly, so it is the same with the productivity index curves. Therefore, it can be concluded that stress sensitivity of the outer zone nearly has no impact on well productivity. Here, we may wonder why stress sensitivity in the outer zone has little effect on the well productivity, and it will be explained later.

5.2.3. Stress Sensitivity in the Inner Zone. Similarly, for a fixed fracture closure radius and average reservoir pressure, Figures 8 and 9 demonstrate the influence of the inner zone's stress sensitivity on the inflow performance and productivity index curves, respectively. It can be seen in Figures 8 and 9 that stress sensitivity of the inner zone has a significant influence on the well productivity. The stress sensitivity in this zone is negatively correlated with the productivity, which is consistent with the case considering stress sensitivity but not considering fracture closure. As the inner zone's stress sensitivity intensifies, the inflow performance curve shows the characteristic of bending to the longitudinal axis (bottom-hole pressure) and a significant drop of productivity and productivity index under the same bottom-hole pressure can be observed. Especially, when stress sensitivity in this zone is severe enough, a significant inflection point can be

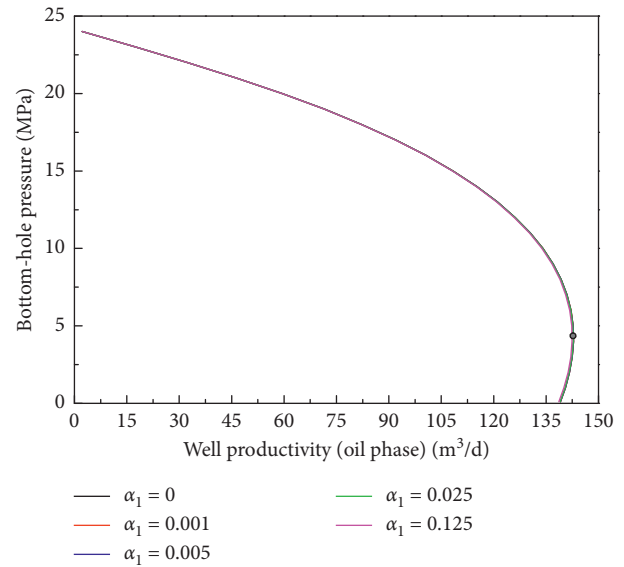


FIGURE 6: Effect of stress sensitivity in the outer zone on the inflow performance curves of a vertical well with fracture closure in a fractured reservoir.

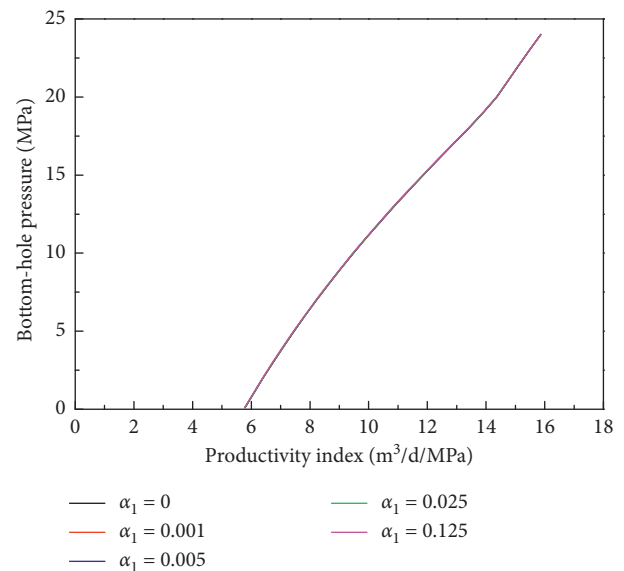


FIGURE 7: Effect of stress sensitivity in the outer zone on the productivity index curves of a vertical well with fracture closure in a fractured reservoir.

observed on the inflow performance curve, which corresponds to the maximum productivity and the optimal bottom-hole pressure. As stress sensitivity goes up, the inflection point (red dot) moves up to the left and the productivity index curve changes from a concave shape to a convex shape.

Through a comparison, we found out that the effects of stress sensitivity of the inner and outer zones on the vertical well's productivity are rather different in this composite model. The inner zone's stress sensitivity has a significant influence on well productivity, while the effect of the outer zone's stress sensitivity is very weak. This kind of difference

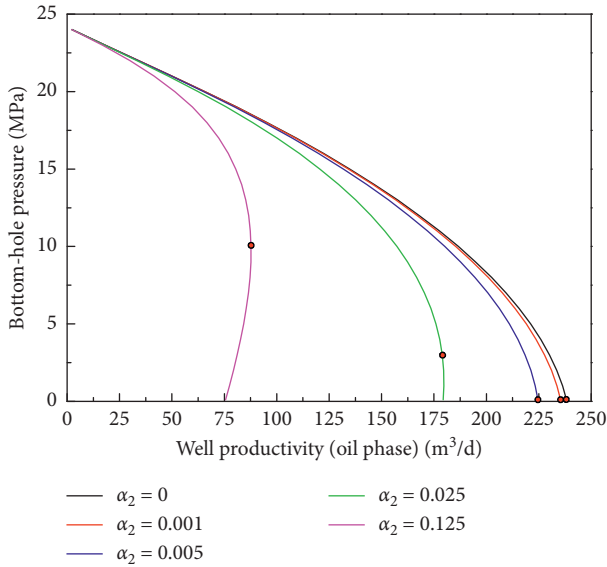


FIGURE 8: Effect of stress sensitivity in the inner zone on the inflow performance curves of a vertical well with fracture closure in a fractured reservoir.

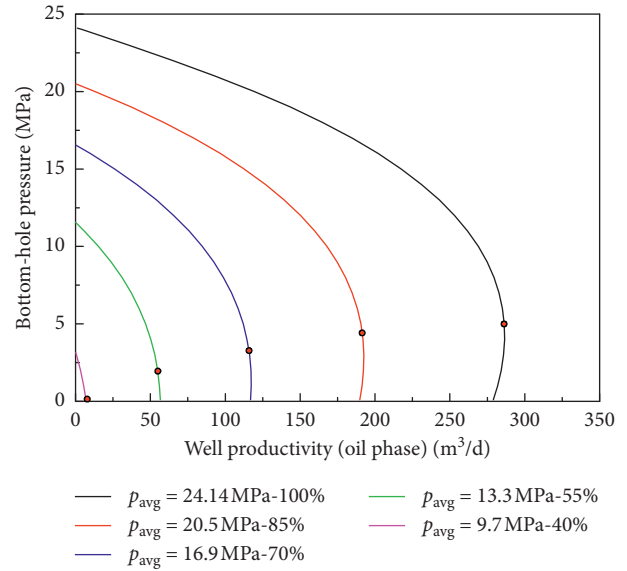


FIGURE 10: Effect of reservoir pressure on the inflow performance of a vertical well with fracture closure and reservoir pressure drop in a fractured reservoir.

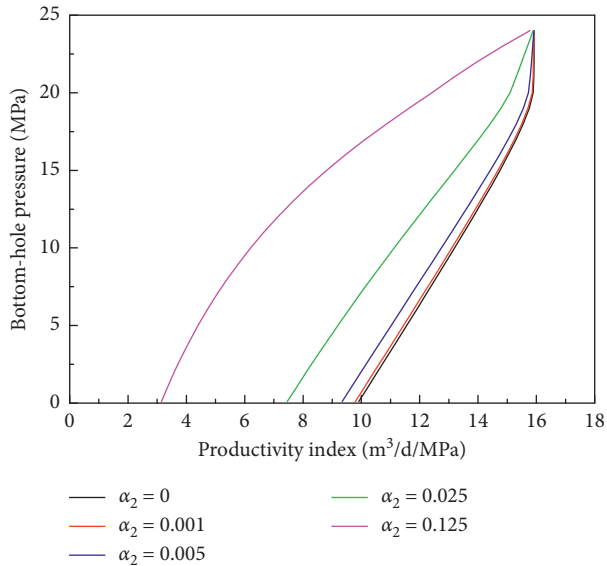


FIGURE 9: Effect of stress sensitivity in the inner zone on the productivity index curves of a vertical well with fracture closure in a fractured reservoir.

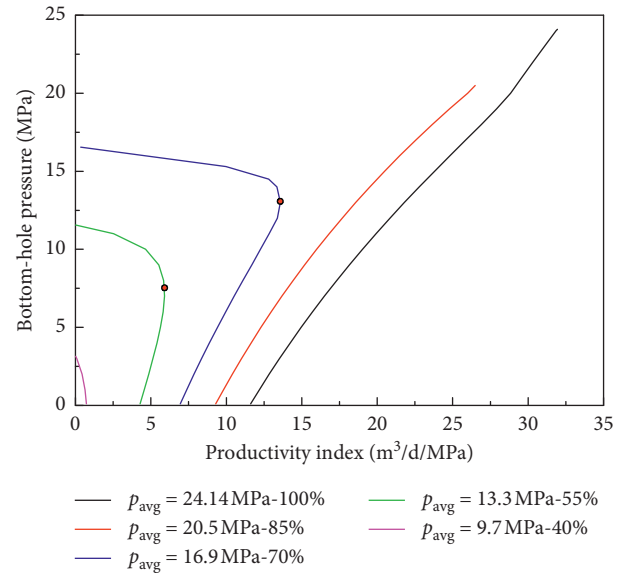


FIGURE 11: Effect of reservoir pressure on the productivity index of a vertical well with fracture closure and reservoir pressure drop in a fractured reservoir.

may be interpreted by the fact that the vertical well productivity mainly depends on the inner zone's flow ability. Under the circumstance, even though the outer zone's flow ability is very poor, it can still meet the liquid demand of the inner zone due to its large seepage area, which also explains the confusion mentioned above.

5.2.4. *Reservoir Pressure.* Here, we use reservoir pressure level to reflect reservoir pressure drop. It represents the ratio of average reservoir pressure to initial reservoir pressure. For a given fracture closure radius, Figures 10 and 11 present the

vertical well's inflow performance and productivity index curves under different reservoir pressure levels, respectively. As displayed in Figure 10, reservoir pressure has a significant impact on the inflow performance and is positively correlated with well productivity. It can also be seen in Figure 10 that the inflow performance curve shrinks toward the coordinate origin and the well productivity reduces significantly under the same bottom-hole pressure when the average reservoir pressure drops. Meanwhile, with the decrease of the reservoir pressure level, the maximum productivity and the optimal bottom-hole pressure decrease gradually and the inflection point moves down to the left

side. When reservoir pressure level is lower than 40%, the well productivity is extremely low. In addition, as reservoir pressure level declines, the productivity index decreases dramatically for a given bottom-hole pressure and the productivity index curve bends toward the bottom-hole pressure axis and gets closer to the coordinate origin. This phenomenon can be explained as follows: when the average reservoir pressure is higher than bubble-point pressure (equal to 20.54 MPa), the fluid in the reservoir does not degas and the productivity index curve is linear; when the average reservoir pressure falls below bubble-point pressure, the productivity index curve bends under the influence of degassing.

6. Conclusions

Through developing a new composite model, this study evaluated the productivity of vertical wells incorporating fracture closure and reservoir pressure drop in fractured reservoirs, including inflow performance and productivity index. According to the results of this investigation, the following conclusions can be drawn:

- (1) By using the idea of composite reservoir, a set of practical pseudo-steady productivity equations for vertical wells considering stress sensitivity, fracture closure, and reservoir pressure drop were derived successfully for unsaturated and saturated fractured reservoirs.
- (2) Fracture closure has a great effect on inflow performance, and the fracture closure radius is negatively correlated with vertical well productivity and productivity index. The optimal bottom-hole pressure to achieve the maximum productivity is almost the same for different fracture closure radii.
- (3) The effects of stress sensitivity of the inner zone and outer zone on vertical well's productivity are rather different. Stress sensitivity in the inner zone has a significant influence on well productivity, while the outer zone's stress sensitivity only has a weak effect on well productivity. With the increase of the inner zone's stress sensitivity, a significant drop of well productivity and productivity index under the same bottom-hole pressure can be observed and the optimal bottom-hole pressure gradually increases.
- (4) Reservoir pressure has a significant effect on inflow performance and is positively correlated with well productivity. The inflow performance curve shrinks toward the coordinate origin, and well productivity is significantly reduced under the same bottom-hole pressure when reservoir pressure drops.
- (5) The related inflow performance curves drawn by using the derived productivity equations can directly provide quantitative reference for the optimization of vertical well's production system in oilfields and then can improve production efficiency.

Nomenclature

k_m :	Permeability of the matrix system, μm^2
k_f :	Permeability of the natural fracture system, μm^2
k_{10} :	Initial equivalent permeability of the outer zone, μm^2
k_{20} :	Initial equivalent permeability of the inner zone, μm^2
k_{a0} :	Initial equivalent permeability throughout the reservoir, μm^2
r_e :	Reservoir radius, m
r_f :	Fracture closure radius, m
r_w :	Wellbore radius, m
h :	Reservoir thickness, m
α_1 :	Stress sensitivity coefficient of the outer zone, MPa^{-1}
α_2 :	Stress sensitivity coefficient of the inner zone, MPa^{-1}
α_k :	Stress sensitivity coefficient throughout the reservoir, MPa^{-1}
p_0 :	Reservoir initial pressure, MPa
p_b :	Reservoir bubble-point pressure, MPa
p_{avg} :	Average reservoir pressure, MPa
p_f :	Pressure at the fracture closure radius, MPa
p_{wf} :	Bottom-hole pressure, MPa
p_e :	Pressure at the closed boundary, MPa
μ :	Fluid viscosity, $\text{mPa}\cdot\text{s}$
B :	Volume factor, m^3/m^3
c_i :	Total compressibility, $1/\text{MPa}$
V :	Total crude oil volume derived out by the elastic energy, m^3
V_f :	Reservoir volume, m^3
t :	Production time, d
q_r :	Flow rate through the cylindrical section at any reservoir radius, m^3/d
v_r :	Flow velocity at any reservoir radius, m/s
Q :	Well surface productivity, m^3/d

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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