Research Article

Classification of All Single Traveling Wave Solutions of Fractional Perturbed Gerdjikov–Ivanov Equation

Zhao Li ^{1,2} and Tianyong Han^{1,2}

¹College of Computer Science, Chengdu University, Chengdu 610106, China

²Key Laboratory of Pattern Recognition and Intelligent Information Processing, Institutions of Higher Education of Sichuan Province, Chengdu University, Chengdu 610106, China

Correspondence should be addressed to Zhao Li; lizhao10.26@163.com

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The fractional perturbed Gerdjikov–Ivanov (pGI) equation plays a momentous role in nonlinear fiber optics, especially in the application of photonic crystal fibers. Constructing traveling wave solutions to this equation is a very challenging task in physics and mathematics. In the current article, our main purpose is to give the classifications of traveling wave solutions of the fractional pGI equation. These results can help physicists to further explain the complex fractional pGI equation.

1. Introduction

The nonlinear Schrödinger equation (NLSE) is usually used to describe many problems in nonlinear optics, quantum mechanics, fluid dynamics, plasmas, and so on [1–5].

Among them, NLSE with cubic nonlinearity, namely, the perturbed Gerdjikov–Ivanov (pGI) equation, is the most studied [6–11], which is given by

$$i \frac{\partial u}{\partial t} + a \frac{\partial^2 u}{\partial x^2} + b|u|^4 u = i \left[cu^2 \frac{\partial \bar{u}}{\partial x} + \delta_1 \frac{\partial u}{\partial x} + \delta_2 \frac{\partial(|u|^2 u)}{\partial x} + \theta \frac{\partial |u|^2}{\partial x} u \right], \quad (1)$$

where $u(t, x)$ is a complex valued wave profile with respect to time and space variables. $\bar{u}$ is the complex conjugation of u . c stands for the nonlinear dispersion coefficient. δ_1 , δ_2 , and θ are known parameters.

In recent years, with the development of fractional derivative, the fractional perturbed Gerdjikov–Ivanov equation has been widely concerned by many researchers in the following form:

$$iD_t^\alpha u + aD_x^{2\alpha} u + b|u|^4 u = i \left[cu^2 D_x^\alpha \bar{u} + \delta_1 D_x^\alpha u + \delta_2 D_x^\alpha (|u|^2 u) + \theta D_x^\alpha |u|^2 u \right], \quad 0 < \alpha < 1, \quad (2)$$

where $u = u(t, x)$ represents unknown functions about time t and space x , respectively. $\bar{u}$ is the complex conjugation of u .

$D_t^\alpha u$ and $D_x^\alpha u$ are the conformable fractional derivative [12–14].

In recent years, the traveling wave solution of the fractional pGI equation has attained a lot of interest in the field of nonlinear science. Some explicit traveling wave solutions of the fractional pGI equation have been reported [15–17]. In Reference [15], Li and his collaborators studied the space-time fractional pGI equation by the fractional H-expansion method. In Reference [16], Ghanbari and Dumitru obtained the optical solutions of the fractional GI equation by using the generalized exponential rational function method. In Reference [17], Zulfiqar and Ahmad explored the optical solution of the fractional pGI equation by the Tanh method and Tanh-coth method, respectively. The main purpose of this paper is to give the classifications of traveling wave solutions of the fractional pGI equation by using the complete discrimination system method for polynomial, which is an important method to find the exact traveling wave solution of the fractional partial differential equation (FPDE). In general, the FPDE can be simplified to the following complete discriminant system by traveling wave transformation:

$$\Psi'(\xi) = F(\Psi, \phi_1, \dots, \phi_m), \quad (3)$$

where $\phi_1, \dots, \phi_m$ are parameters. $F(\Psi, \phi_1, \dots, \phi_m)$ is third-order polynomial, fourth-order polynomial, or fifth-order polynomial. Then, integrating equation (3) once, we obtain

$$\xi - \xi_0 = \int \frac{d\Psi}{\sqrt{F(\Psi, \phi_1, \dots, \phi_m)}}, \quad (4)$$

where ξ_0 is the integration constant. The readers can refer to [2, 4, 18] for details of this method.

The structure of this paper is as follows. In Section 2, we simplify equation (2) to the nonlinear ordinary differential equation by fractional traveling wave transformation. Then, the complete discrimination system is used to construct the classification of all single traveling wave solutions of the fractional pGI equation. In Section 3, we give a summary.

2. Traveling Wave Solutions of System (2)

Consider the fractional traveling wave transformation as follows:

$$\begin{aligned} u(t, x) &= U(\xi)e^{i\eta}, \\ \xi &= \frac{x^\alpha}{\alpha} - v \frac{t^\alpha}{\alpha}, \\ \eta &= -k \frac{x^\alpha}{\alpha} + w \frac{t^\alpha}{\alpha}, \end{aligned} \quad (5)$$

where v is the phase component, w represents the wave-number, and k is the frequency.

Substituting (5) into (2) and separating the real and imaginary part, we obtain

$$aU'' - (w + ak^2 + \delta_1 k)U + k(c - \delta_2)U^3 + bU^5 = 0, \quad (6)$$

and

$$\begin{aligned} v &= -\delta_1 - 2ak, \\ \theta &= -\frac{1}{2}(c + 3\delta_2). \end{aligned} \quad (7)$$

Multiplying equation (6) by U' and integrating it with respect to ξ , we obtain

$$\frac{a}{2}(U')^2 + \frac{b}{6}U^6 + \frac{k(c - \delta_2)}{4}U^4 - \frac{w + ak^2 + k\delta_1}{2}U^2 + d = 0, \quad (8)$$

where d is the integral constant. Next, we fix $U = \sqrt{\psi}$, then equation (8) can be converted into

$$(\psi')^2 = -\frac{4b}{3a}\psi^4 - \frac{2k(c - \delta_2)}{a}\psi^3 + \frac{4(w + ak^2 + k\delta_1)}{a}\psi^2 - \frac{8d}{a}\psi. \quad (9)$$

(1) If $d = 0$, equation (9) can be simplified as

$$(\psi')^2 = \psi^2(b_3\psi^2 + b_2\psi + b_1), \quad (10)$$

where $b_3 = -(4b/3a)$, $b_2 = -(2k(c - \delta_2)/a)$, and $b_1 = (4(w + ak^2 + k\delta_1)/a)$. Then, equation (10) can be written in the following integral form:

$$\int \frac{d\psi}{\psi \sqrt{b_3\psi^3 + b_2\psi + b_1}} = \pm 2(\xi - \xi_0). \quad (11)$$

Denote $\Delta = b_2^2 - 4b_1b_3$, where Δ is the discriminant of polynomial $b_3\psi^3 + b_2\psi + b_1$. According to the complete discriminant method of second-order polynomials, the classification of all solutions of equation (11) has the following three cases:

Case 1.1. Suppose that $\Delta = 0$. From equation (11), we can get

$$\pm \frac{b_2}{b_3} \sqrt{|b_3|} (\xi - \xi_0) = \ln \left| \frac{\psi - (b_2/2b_3)}{\psi} \right|. \quad (12)$$

Case 1.2. Suppose that $\Delta > 0$. When $b_3 > 0$, we have from (11)

$$\begin{aligned}
\pm\sqrt{b_3}(\xi - \xi_0) &= \frac{1}{\sqrt{\gamma_1\gamma_2}} \ln \frac{\left[\sqrt{(-\gamma_2)(\psi - \gamma_1)} - \sqrt{(-\gamma_1)(\psi - \gamma_2)} \right]^2}{|\psi|}, \\
\pm\sqrt{b_3}(\xi - \xi_0) &= \frac{1}{\sqrt{\gamma_1\gamma_2}} \ln \frac{\left[\sqrt{\gamma_2(\psi - \gamma_1)} - \sqrt{\gamma_1(\psi - \gamma_2)} \right]^2}{|\psi|}, \\
\pm\sqrt{b_3}(\xi - \xi_0) &= \frac{1}{\sqrt{-\gamma_1\gamma_2}} \arcsin \frac{(-\gamma_2)(\psi - \gamma_1) + (-\gamma_1)(\psi - \gamma_2)}{|\psi||\gamma_1 - \gamma_2|}.
\end{aligned} \tag{13}$$

When $b_3 < 0$, we have from (11)

$$\begin{aligned}
\pm\sqrt{-b_3}(\xi - \xi_0) &= \frac{1}{\sqrt{-\gamma_1\gamma_2}} \ln \frac{\left[\sqrt{(-\gamma_2)(\psi + \gamma_1)} - \sqrt{\gamma_1(\psi - \gamma_2)} \right]^2}{|\psi|}, \\
\pm\sqrt{-b_3}(\xi - \xi_0) &= \frac{1}{\sqrt{-\gamma_1\gamma_2}} \ln \frac{\left[\sqrt{\gamma_2(-\psi + \gamma_1)} - \sqrt{(-\gamma_1)(\psi - \gamma_2)} \right]^2}{|\psi|}, \\
\pm\sqrt{-b_3}(\xi - \xi_0) &= \frac{1}{\sqrt{\gamma_1\gamma_2}} \arcsin \frac{\left[\sqrt{(-\gamma_2)(\psi + \gamma_1)} - \sqrt{\gamma_1(\psi - \gamma_2)} \right]^2}{|\psi|},
\end{aligned} \tag{14}$$

where $\gamma_1 = (-b_2 + \sqrt{\Delta}/2b_3)$ and $\gamma_2 = (-b_2 - \sqrt{\Delta}/2b_3)$.

Case 1.3. Suppose that $\Delta < 0$. Then, we have

$$\pm\sqrt{b_1}(\xi - \xi_0) = \frac{1}{\sqrt{b_1^2}} \ln \left| \frac{-\left(1/2b_2\sqrt{b_1}\right)\psi + \sqrt{b_1} - \sqrt{b_3\psi^2 - b_2\psi + b_1}}{\psi} \right|, \quad (b_1 > 0). \tag{15}$$

(2) If $d \neq 0$, equation (9) can be simplified as

$$(\psi')^2 = \psi \left[-\frac{4b}{3a}\psi^3 - \frac{2k(c - \delta_2)}{a}\psi^2 + \frac{4(w + ak^2 + k\delta_1)}{a}\psi - \frac{8d}{a} \right]. \tag{16}$$

Then, equation (16) can be written in the following integral form:

$$\int \frac{d\psi}{\sqrt{\varepsilon\psi(\psi^3 + p\psi^2 + q\psi + r)}} = \pm 2\sqrt{\frac{b\varepsilon}{3a}}(\xi - \xi_0), \tag{17}$$

where $\varepsilon = \pm 1$, $p = (3b/k(c - \delta))$, $q = -(3(w + ak^2 + k\delta_1)/b)$, and $r = (6d/b)$.

Denote

$$\begin{cases} \Delta = -27\left(\frac{2p^3}{27} + r - \frac{qr}{3}\right)^2 - 4\left(q - \frac{p^2}{3}\right)^3, \\ D_1 = q - \frac{p^2}{3}, \end{cases} \tag{18}$$

where Δ is the discriminant of polynomial $G(\psi) = \psi^3 + p\psi^2 + q\psi + r$. According to the complete discriminant method of third-order polynomials, the classification of all solutions of equation (11) has the following four cases:

Case 2.1. Assume that $\Delta = 0$, $D_1 < 0$. Denote $G(\psi) = (\psi - \beta_1)^2(\psi - \beta_2)$, where $\beta_1 \neq \beta_2$ and $\beta_2 > 0$.

If $\varepsilon = 1$, $ab < 0$, $\beta_1 > \beta_2$, and $\psi > \beta_2$ or $\beta_1 < 0$ and $\psi < 0$, then the implicit solution of equation (17) is

$$\pm 2\sqrt{\frac{b}{3a}}(\xi - \xi_0) = \frac{1}{\beta_1(\beta_1 - \beta_2)} \ln \frac{[\sqrt{\beta_1(\psi - \beta_2)} - \sqrt{\psi(\beta_1 - \beta_2)}]^2}{|\psi - \beta_1|}. \quad (19)$$

If $\varepsilon = 1$, $ab < 0$, $\beta_1 > \beta_2$, and $\psi < 0$ or $\beta_1 < 0$ and $\psi < \beta_2$, then the implicit solution of equation (17) is

$$\pm 2\sqrt{\frac{b}{3a}}(\xi - \xi_0) = \frac{1}{\beta_1(\beta_1 - \beta_2)} \ln \frac{[\sqrt{-\beta_1(\psi - \beta_2)} - \sqrt{\psi(\beta_2 - \beta_1)}]^2}{|\psi - \beta_1|}. \quad (20)$$

If $\varepsilon = 1$, $ab < 0$, and $\beta_2 > \beta_1 > 0$, then the implicit solution of equation (17) is

$$\pm 2\sqrt{\frac{b}{3a}}(\xi - \xi_0) = \frac{1}{\beta_1(\beta_2 - \beta_1)} \arcsin \frac{\beta_1(\psi - \beta_2) + \psi(\beta_1 - \beta_2)}{\beta_2|\psi - \beta_1|}. \quad (21)$$

If $\varepsilon = -1$, $ab > 0$, $\beta_1 > \beta_2$, and $\psi > \beta_2$ or $\beta_1 < 0$ and $\psi < 0$, then the implicit solution of equation (17) is

$$\pm 2\sqrt{\frac{b}{3a}}(\xi - \xi_0) = \frac{1}{\beta_1(\beta_2 - \beta_1)} \ln \frac{[\sqrt{\beta_1(-\psi + \beta_2)} - \sqrt{\psi(\beta_1 - \beta_2)}]^2}{|\psi - \beta_1|}. \quad (22)$$

If $\varepsilon = -1$, $ab > 0$, $\beta_1 > \beta_2$, and $\psi < 0$ or $\beta_1 < 0$ and $\psi < \beta_2$, then the implicit solution of equation (17) is

$$\pm 2\sqrt{\frac{b}{3a}}(\xi - \xi_0) = \frac{1}{\beta_1(\beta_2 - \beta_1)} \ln \frac{[\sqrt{-\beta_1(-\psi + \beta_2)} - \sqrt{\psi(\beta_1 - \beta_2)}]^2}{|\psi - \beta_1|}. \quad (23)$$

If $\varepsilon = -1$, $ab > 0$, and $\beta_2 > \beta_1 > 0$, then the implicit solution of equation (17) is

$$\pm 2\sqrt{\frac{b}{3a}}(\xi - \xi_0) = \frac{1}{\beta_1(\beta_1 - \beta_2)} \arcsin \frac{\beta_1(-\psi + \beta_2) + \psi(\beta_2 - \beta_1)}{\beta_2|\psi - \beta_1|}. \quad (24)$$

Case 2.2. Assume that $\Delta = 0$ and $D_1 = 0$. Denote $G(\psi) = (\psi - \beta_1)^3$, where β_1 is the real constant.

If $\varepsilon = 1$, $\psi > \beta_1$, and $\psi > 0$ or $\psi < \beta_1$ and $\psi < 0$ then the explicit solution of equation (2) is

$$u_1(t, x) = \pm \left[\frac{\beta_1}{-(b/3a)\beta_1^2((x^\alpha/\alpha) - v(t^\alpha/\alpha)) - 1} + \beta_1 \right]^{1/2} e^{i(-k(x^\alpha/\alpha) + w(t^\alpha/\alpha))}. \quad (25)$$

If $\varepsilon = -1$, $\psi > \beta_1$, and $\psi < 0$ or $\psi < \beta_1$ and $\psi > 0$, then the explicit solution of equation (2) is

$$u_2(t, x) = \pm \left[\frac{\beta_1}{(b/3a)\beta_1^2((x^\alpha/\alpha) - v(t^\alpha/\alpha)) - 1} + \beta_1 \right]^{1/2} e^{i(-k(x^\alpha/\alpha) + w(t^\alpha/\alpha))}. \quad (26)$$

Case 2.3. Assume that $\Delta > 0$ and $D_1 < 0$. Denote $G(\psi) = (\psi - \beta_1)(\psi - \beta_2)(\psi - \beta_3)$, where $0 < \beta_1 < \beta_2 < \beta_3$.

If $\varepsilon = 1$, $\psi < 0$, or $\psi > \beta_3$, then we make transformation $\psi = (\beta_2\beta_3\sin^2 \vartheta - \beta_2\beta_3/\beta_3\sin^2 \vartheta - \beta_2)$, and we obtain

$$\pm \sqrt{-\frac{b}{3a}}(\xi - \xi_0) = \frac{1}{\sqrt{\beta_2(\beta_3 - \beta_1)}} \int \frac{d\vartheta}{\sqrt{1 - m_1^2 \sin^2 \vartheta}}, \quad (27)$$

where $m_1^2 = (\beta_3(\beta_1 - \beta_2)/\beta_2(\beta_1 - \beta_3))$. Then, the explicit solution of equation (2) is

$$u_3(t, x) = \pm \left[\frac{\beta_2\beta_3\sin^2 \left(\sqrt{-(b/3a)\beta_2(\beta_3 - \beta_1)}((x^\alpha/\alpha) - v(t^\alpha/\alpha)), m_1 \right) - \beta_2\beta_3}{\beta_3\sin^2 \left(\sqrt{-(b/3a)\beta_2(\beta_3 - \beta_1)}((x^\alpha/\alpha) - v(t^\alpha/\alpha)), m_1 \right) - \beta_2} \right]^{1/2} e^{i(-k(x^\alpha/\alpha) + w(t^\alpha/\alpha))}. \quad (28)$$

If $\varepsilon = 1$ and $\beta_1 < \psi < \beta_2$, then we make transformation $\psi = (-\beta_1\beta_2/(\beta_2 - \beta_1)\sin^2 \vartheta - \beta_2)$. Similarly, the explicit solution of equation (2) is

$$u_4(t, x) = \pm \left[\frac{-\beta_1\beta_2}{(\beta_2 - \beta_1)\sin^2 \left(\sqrt{-(b/3a)\beta_2(\beta_3 - \beta_1)}((x^\alpha/\alpha) - v(t^\alpha/\alpha)), m_1 \right) - \beta_2} \right]^{1/2} e^{i(-k(x^\alpha/\alpha) + w(t^\alpha/\alpha))}. \quad (29)$$

If $\varepsilon = -1$ and $\beta_2 < \psi < \beta_3$, then we make transformation $\psi = (\beta_1(\beta_3 - \beta_2)\sin^2 \vartheta - \beta_2(\beta_3 - \beta_1)/(\beta_3 -$

$\beta_2)\sin^2 \vartheta - (\beta_3 - \beta_1))$, and the explicit solution of equation (2) is

$$u_5(t, x) = \pm \left[\frac{\beta_1(\beta_3 - \beta_2)\sin^2 \left(\sqrt{-(b/3a)\beta_2(\beta_3 - \beta_1)}((x^\alpha/\alpha) - v(t^\alpha/\alpha)), m_2 \right) - \beta_2(\beta_3 - \beta_1)}{(\beta_3 - \beta_2)\sin^2 \left(\sqrt{-(b/3a)\beta_2(\beta_3 - \beta_1)}((x^\alpha/\alpha) - v(t^\alpha/\alpha)), m_2 \right) - (\beta_3 - \beta_1)} \right]^{1/2} e^{i(-k(x^\alpha/\alpha) + w(t^\alpha/\alpha))}, \quad (30)$$

where $m_2^2 = (\beta_1(\beta_2 - \beta_3)/\beta_2(\beta_1 - \beta_3))$.

If $\varepsilon = -1$ and $0 < \psi < \beta_1$, then we make transformation $\psi = (\beta_1 \beta_3 \sin^2 \vartheta / \beta_1 \sin^2 \vartheta - (\beta_1 - \beta_3))$. Similarly, the explicit solution of equation (2) is

$$u_6(t, x) = \pm \left[\frac{\beta_1 \beta_2 \operatorname{sn}^2 \left((x^\alpha / \alpha) - v(t^\alpha / \alpha) / 2, \sqrt{(b/3a)\beta_2(\beta_3 - \beta_1)}, m_2 \right)}{\beta_1 \operatorname{sn}^2 \left((x^\alpha / \alpha) - v(t^\alpha / \alpha) / 2, \sqrt{(b/3a)\beta_2(\beta_3 - \beta_1)}, m_2 \right) - (\beta_1 - \beta_3)} \right]^{1/2} e^{i(-k(x^\alpha / \alpha) + w(t^\alpha / \alpha))}. \quad (31)$$

Case 2.4. Assume that $\Delta < 0$. Denote $G(\psi) = (\psi - \beta_1)[(\psi - \beta_2)^2 + \gamma]$, where $\beta_1, \beta_2, \gamma > 0$ are real constants. Then, we make transformation $\psi = (d_1 \cos \vartheta + d_2 / d_3 \cos \vartheta + d_4)$, where $d_1 = (\beta_1 / 2)(d_3 - d_4)$, $d_2 = -(\beta_1 / 2)(d_3 - d_4)$, $d_3 = \beta_1 - \beta_2 -$

(γ / m) , $d_4 = \beta_1 - \beta_2 - \gamma m$, $m = D \pm \sqrt{D^2 + 1}$, and $D = (\gamma^2 - \beta_2(\beta_1 - \beta_2)) / \beta_2 \gamma$.

If $\varepsilon = 1$ and $m = D - \sqrt{D^2 + 1}$, then the explicit solution of equation (2) is

$$u_7(t, x) = \pm \left[\frac{d_1 \operatorname{cn}((x^\alpha - vt^\alpha / \alpha) \sqrt{2(b/3a)\gamma m \beta_1 / mm_3}, m_3) + d_2}{d_3 \operatorname{cn}((x^\alpha - vt^\alpha / \alpha) \sqrt{2(b/3a)\gamma m \beta_1 / mm_3}, m_3) + d_4} \right]^{1/2} e^{i(-k(x^\alpha / \alpha) + w(t^\alpha / \alpha))}. \quad (32)$$

If $\varepsilon = -1$ and $m = D + \sqrt{D^2 + 1}$, then the explicit solution of equation (2) is

$$u_8(t, x) = \pm \left[\frac{d_1 \operatorname{cn}((x^\alpha - vt^\alpha / \alpha) \sqrt{-2(b/3a)\gamma m \beta_1 / mm_3}, m_3) + d_2}{d_3 \operatorname{cn}((x^\alpha - vt^\alpha / \alpha) \sqrt{-2(b/3a)\gamma m \beta_1 / mm_3}, m_3) + d_4} \right]^{1/2} e^{i(-k(x^\alpha / \alpha) + w(t^\alpha / \alpha))}, \quad (33)$$

where $m_3^2 = (1/1 + m^2)$.

3. Conclusion

In the paper, the traveling wave solution of the fractional pGI equation is studied by using the complete discrimination system method. Firstly, we reduce equation (2) to the nonlinear ordinary differential equation by fractional traveling wave transformation. Then, the complete discrimination system is used to construct the classification of all single traveling wave solutions of the fractional pGI equation. Among them, the implicit solutions have not been reported in other literatures. Compared with the existing literature [15–17], our solutions are more abundant, including rational function solutions, Jacobian function solutions, and implicit solutions. It is worth mentioning that the complete discrimination system method is an effective and powerful tool for constructing the traveling wave solution fractional pGI equation. Moreover, the method can be employed to construct the traveling wave solution of general FPDE.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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