Research Article

Quantile-Based Estimation of Liu Parameter in the Linear Regression Model: Applications to Portland Cement and US Crime Data

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In multiple linear regression models, the multicollinearity problem mostly occurs when the explanatory variables are correlated among each other. It is well known that when the multicollinearity exists, the variance of the ordinary least square estimator is unstable. As a remedy, Liu in [1] developed a new method of estimation with biasing parameter d . In this paper, we have introduced a new method to estimate the biasing parameter in order to mitigate the problem of multicollinearity. The proposed method provides the class of estimators that are based on quantile of the regression coefficients. The performance of the new estimators is compared with the existing estimators through Monte Carlo simulation, where mean squared error and mean absolute error are considered as evaluation criteria of the estimators. Portland cement and US Crime data is used as an application to illustrate the benefit of the new estimators. Based on simulation and numerical study, it is concluded that the new estimators outperform the existing estimators in certain situations including high and severe cases of multicollinearity. 95% mean prediction interval of all the estimators is also computed for the Portland cement data. We recommend the use of new method to practitioners when the problem of high multicollinearity exists among the explanatory variables.

1. Introduction

The commonly used method of estimation in multiple linear regression models (MLRM) is the method of ordinary least squares (OLS) [1]. The results obtained through this method might be misleading when the problem of multicollinearity is present among the explanatory variables [2]. To overcome such problem, Ridge regression (RR) and Liu regression suggested by [1, 3], respectively, are the two widely used alternative methods. Liu estimator (LE) is usually preferred over RR, because it is the linear function of its biasing parameter d [4].

The optimal value of biasing parameter d in Liu regression plays an important role in minimizing the variance. Many researchers have suggested several estimators for estimating d . Few of them are [4–6]. In this paper, the performance of some existing LEs is investigated, and a new method called as quantile based estimation of Liu or biasing parameter d is proposed. Also, the new estimators are compared with the existing ones through a Monte Carlo simulation based on mean squared error (MSE) and mean absolute error (MAE) performance criteria. The rest of the article is written as follows. The model estimation, newly proposed, and existing LEs are discussed in Section 2. The

simulation design and results are discussed in Section 3. Section 4 includes two empirical applications to demonstrate the benefits of the new method. The conclusion of the paper is given in Section 5.

2. Statistical Methodology

Consider the following MLRM:

$$y = X\beta + \varepsilon, \quad (1)$$

where y is the vector of random response variable of order $(n \times 1)$, X shows the fixed design matrix of explanatory variables with order $(n \times p)$ and β is the $(p + 1) \times 1$ vector of population regression coefficients. ε is the vector of stochastic or random errors with order $(n \times 1)$ and is distributed as normal with mean $E(\varepsilon) = 0$ and variance covariance matrix $E(\varepsilon\varepsilon') = \sigma^2 I_n$, I_n is an $(n \times n)$ identity matrix. The vector of OLS estimators for β is given below:

$$\hat{\beta}_{OLS} = (X'X)^{-1}X'Y. \quad (2)$$

The OLS estimator is unbiased and more efficient than all other unbiased estimators [2]. However, in the presence of multicollinearity, OLS estimator becomes inefficient and provide large variance [4]. To circumvent such situation, numerous biased estimation methods are available, which provide smaller MSE than OLS, and LE is one of them. The Liu estimator defined by [1] is given as

$$\hat{\beta}_{LIU} = (X'X + I)^{-1}(X'X + dI)\hat{\beta}_{OLS}, \quad 0 \leq d \leq 1. \quad (3)$$

In the presence of multicollinearity, $\hat{\beta}_{LIU}$ provides the smaller MSE than OLS [7]. The optimal choice of Liu parameter d plays a vital role in minimizing the MSE of $\hat{\beta}_{LIU}$ [8]. Some existing LEs for the biasing parameter d are given in the following subsection.

2.1. Some Existing Liu Estimators. Consider the canonical form of model (1):

$$y = Z\alpha + \varepsilon, \quad (4)$$

where $Z = XD$ and $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_p)' = D'\beta$, D is an orthogonal matrix such that $D'D = I$ and $Z'Z = D'X'XD = \Lambda$, $\Lambda = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_p)$ consists of the eigenvalues of the $X'X$ matrix. Note here that $\text{MSE}(\hat{\alpha}) = \text{MSE}(\hat{\beta})$ so it suffices to consider the canonical form only. The OLS estimator can be defined in canonical form as follows:

$$\hat{\alpha}_{OLS} = \Lambda^{-1}Z'y. \quad (5)$$

The LE is defined as

$$\hat{\alpha}_{LIU} = (\Lambda + I)^{-1}(\Lambda + dI)\hat{\alpha}_{OLS}, \quad (6)$$

The first estimator for d was suggested by [1] and is given below:

$$\hat{d}_j = \frac{\hat{\alpha}_j^2 - \hat{\sigma}^2}{(\hat{\sigma}^2/\lambda_j) + \hat{\alpha}_j^2}, \quad (7)$$

where $\hat{\alpha}_j$ is the j^{th} element of $\hat{\alpha}$, an OLS estimator of α . $\hat{\sigma}^2$ is the unbiased estimator of population error variance σ^2 and λ_j is the j^{th} eigenvalue of the matrix $X'X$. Liu in [1] also suggested the following estimator:

$$D1 = \max\left(0, \text{abs}\left(\frac{\hat{\alpha}_{\max}^2 - \hat{\sigma}^2}{(1/\lambda_{\max}) + \hat{\alpha}_{\max}^2}\right)\right). \quad (8)$$

Shukur et al. [6] considered the idea of [5, 9] and suggested the following four estimators:

$$\begin{aligned} D2 &= \max\left(0, \text{median}\left(\frac{\hat{\alpha}_j^2 - \hat{\sigma}^2}{(1/\lambda_j) + \hat{\alpha}_j^2}\right)\right), \\ D3 &= \max\left(0, \frac{1}{p} \sum_{j=1}^p \left(\frac{\hat{\alpha}_j^2 - \hat{\sigma}^2}{(1/\lambda_j) + \hat{\alpha}_j^2}\right)\right), \\ D4 &= \max\left(0, \max\left(\frac{\hat{\alpha}_j^2 - \hat{\sigma}^2}{(1/\lambda_j) + \hat{\alpha}_j^2}\right)\right), \\ D5 &= \max\left(0, \min\left(\frac{\hat{\alpha}_j^2 - \hat{\sigma}^2}{(1/\lambda_j) + \hat{\alpha}_j^2}\right)\right). \end{aligned} \quad (9)$$

Based on the work of [10, 11], we proposed five new quantile based LEs, which are defined in the following section.

2.2. Proposed Method. Let $(\hat{d}_1, \hat{d}_2, \dots, \hat{d}_p)$ be the realized values of equation (7) and it can be written in the ascending order of magnitude as

$$\hat{d}_{(1)} \leq \hat{d}_{(2)} \leq \dots \leq \hat{d}_{(p)}, \quad (10)$$

where $\hat{d}_{(1)} = \min(\hat{d}_1, \hat{d}_2, \dots, \hat{d}_p)$ and $\hat{d}_{(p)} = \max(\hat{d}_1, \hat{d}_2, \dots, \hat{d}_p)$. The set $\{\hat{d}_{(1)}, \hat{d}_{(2)}, \dots, \hat{d}_{(p)}\}$ is the order statistics for $(\hat{d}_1, \hat{d}_2, \dots, \hat{d}_p)$ and $\hat{d}_{(j)}$, $j = 1, 2, \dots, p$, is the j^{th} ordered observation. Now let $\hat{d}_\gamma$, $0 < \gamma < 1$, be the $100\gamma^{\text{th}}$ quantile of $\{\hat{d}_{(1)}, \hat{d}_{(2)}, \dots, \hat{d}_{(p)}\}$, and then the new proposed quantile estimator is

$$\hat{d}_\gamma = \{\hat{d}_{(j)}\}_\gamma = \frac{(\hat{\alpha}_{(j)}^2)_\gamma - \hat{\sigma}^2}{(\hat{\sigma}^2/(\lambda_{(j)})_\gamma) + (\hat{\alpha}_{(j)}^2)_\gamma}, \quad (11)$$

such that

$$P(\hat{d} < \hat{d}_\gamma) = \gamma, \quad (12)$$

where “ γ ” is the quantile probability, $\hat{\alpha}_{(j)}$ is the j^{th} ordered element of $\hat{\alpha}$, an OLS estimator of α , and $\lambda_{(j)}$ is the j^{th} ordered eigenvalue of the matrix $X'X$. The quantile

probability generally depends on the degree of multicollinearity in order to obtain the minimum MSE or MAE. The new estimator $\hat{d}_\gamma$, defined in equation (11) depends on the quantile probability whose value is selected according to the degrees of multicollinearity. So the new proposed estimator is more robust to high or severe degrees of multicollinearity. Since the range of LE must be between zero and one, therefore we rewrite the equation (11) as

$$D\gamma = \max(0, \hat{d}_\gamma) = \max\left(0, \frac{(\hat{\alpha}_{(j)}^2)_\gamma - \hat{\sigma}^2}{(\hat{\sigma}^2/(\lambda_{(j)})_\gamma) + (\hat{\alpha}_{(j)}^2)_\gamma}\right). \quad (13)$$

Equation (13) satisfies the interval condition for Liu parameter d suggested by [1]. In order to present the role of quantile probability, we choose some specific values for “ γ ” as: 0 (minimum), 0.25 (first quartile), 0.50 (median), 0.75 (third quartile) and 1 (maximum). The mathematical form of new LEs, denoted here by D6, D7, D8, D9, and D10, is given below:

$$\begin{aligned} D6 &= \max(0, \hat{d}_{0.0}) = \max\left(0, \frac{(\hat{\alpha}_{(j)}^2)_{0.0} - \hat{\sigma}^2}{(\hat{\sigma}^2/(\lambda_{(j)})_{0.0}) + (\hat{\alpha}_{(j)}^2)_{0.0}}\right), \\ D7 &= \max(0, \hat{d}_{0.25}) = \max\left(0, \frac{(\hat{\alpha}_{(j)}^2)_{0.25} - \hat{\sigma}^2}{(\hat{\sigma}^2/(\lambda_{(j)})_{0.25}) + (\hat{\alpha}_{(j)}^2)_{0.25}}\right), \\ D8 &= \max(0, \hat{d}_{0.50}) = \max\left(0, \frac{(\hat{\alpha}_{(j)}^2)_{0.50} - \hat{\sigma}^2}{(\hat{\sigma}^2/(\lambda_{(j)})_{0.50}) + (\hat{\alpha}_{(j)}^2)_{0.50}}\right), \\ D9 &= \max(0, \hat{d}_{0.75}) = \max\left(0, \frac{(\hat{\alpha}_{(j)}^2)_{0.75} - \hat{\sigma}^2}{(\hat{\sigma}^2/(\lambda_{(j)})_{0.75}) + (\hat{\alpha}_{(j)}^2)_{0.75}}\right), \\ D10 &= \max(0, \hat{d}_{1.0}) = \max\left(0, \frac{(\hat{\alpha}_{(j)}^2)_{1.0} - \hat{\sigma}^2}{(\hat{\sigma}^2/(\lambda_{(j)})_{1.0}) + (\hat{\alpha}_{(j)}^2)_{1.0}}\right). \end{aligned} \quad (14)$$

3. The Design of an Experiment

This section covers the Monte Carlo simulation experiment, performance criterion measures, and results and discussion.

3.1. The Monte Carlo Simulation. The performance of LEs is compared in this section through the simulation study. Following [12–14], the explanatory variables are generated as

$$x_{ij} = (1 - \rho^2)^{1/2} z_{ij} + \rho z_{i(j+1)}, \quad i = 1, 2, \dots, n, j = 1, 2, \dots, p, \quad (15)$$

where ρ is the degree or level of multicollinearity between the explanatory variables, which are given as 0.80, 0.90, 0.99, and 0.999. z_{ij} are the random numbers obtained from the standard normal distribution.

The n observations on the response variable are computed as

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \dots + \beta_p x_{ip} + \varepsilon_i, \quad i = 1, 2, \dots, n, \quad (16)$$

where $\varepsilon_i \sim N(0, \sigma^2)$, σ^2 is the error variance. β_0 is considered to be identically zero. Following [9], the eigenvector corresponding to maximum eigenvalue of the $X'X$ matrix is taken as the vector of regression coefficients. Following [4–6], the different factors we choose to vary in our study are given below:

Error variance: $\sigma^2 = 0.1, 1, 5, 10$

Sample size: $n = 25, 50, 75, 100, 200, 400$

Explanatory variables: $p = 4, 8, 16, 32$

Multicollinearity: $\rho = 0.80, 0.90, 0.99, 0.999$

3.2. Performance Evaluation Criteria. Following [4], MSE and MAE criteria are used to judge the performance of the different LEs. Estimated MSE (EMSE) and MAE (EMAE) are defined as

$$\begin{aligned} \text{MSE} &= \frac{\sum_{i=1}^M (\hat{\beta}_i - \beta)' (\hat{\beta}_i - \beta)}{M}, \\ \text{MAE} &= \frac{\sum_{i=1}^M |\hat{\beta}_i - \beta|}{M}, \end{aligned} \quad (17)$$

where $\hat{\beta}_i$ is the estimated value of β . M shows the simulation runs. In this study, we choose $M = 5000$. The EMSE simulation results are presented in Tables 1–6 and EMAE in Tables 7, 8. The results are discussed in the following section.

3.3. Results and Discussion. The EMSE and EMAE values of the new and existing Liu estimators are presented in Tables 1–6 and 7, 8 respectively. The performance of the LEs is evaluated by varying the values of factors such as multicollinearity, error variance, sample size, and the explanatory variables. A general remark from the literature is that these factors have the significant effect on the simulation design. The effect of each factor on EMSE and EMAE of estimators is discussed below:

The first factor we considered is the effect of multicollinearity on the EMSE and EMAE of estimators. The EMSE and EMAE of all the estimators increase by increasing the degree of multicollinearity generally except D5–D8. Estimators D5–D8 first increase when multicollinearity increases from mild to high and decrease for severe multicollinearity. It is evident from these tables that the LE is always superior to the OLS estimator. For mild multicollinearity ($\rho = 0.80$), D5 outperforms others. Estimators D6 and D7 are the close competitors to D5. When the degree of multicollinearity is considered to be high ($\rho = 0.99$) or severe ($\rho = 0.999$), then the proposed estimators D6 and D7 outperform generally. However, the performance of D6 is good among others, because it yields lowest EMSE and EMAE when the degree of multicollinearity is high or severe.

TABLE 1: EMSE with $\epsilon \sim N(0, 1)$ and $p = 4$.

n	25				50			
ρ	0.80	0.90	0.99	0.999	0.80	0.90	0.99	0.999
OLS	0.8647	1.7642	17.9023	169.9837	0.3396	0.7572	8.5342	89.7533
D1	0.5808	1.1134	15.4244	166.5402	0.2760	0.5175	6.4910	86.3338
D2	0.5242	0.6865	0.8420	1.5461	0.2775	0.4702	0.7127	0.5065
D3	0.5342	0.7047	1.0837	3.2359	0.2780	0.4723	0.8117	0.9502
D4	0.6343	0.8677	2.9571	21.7347	0.2897	0.5008	1.3389	5.0789
D5	0.4644	0.6017	0.5689	0.2561	0.2677	0.4492	0.6269	0.1773
D6	0.4714	0.5933	0.5274	0.1105	0.2712	0.4535	0.6150	0.1360
D7	0.5056	0.6316	0.5480	0.1876	0.2778	0.4640	0.6196	0.1625
D8	0.5104	0.6340	0.6784	0.9397	0.2787	0.4620	0.6500	0.3115
D9	0.5353	0.8218	11.5467	158.7430	0.2867	0.4982	4.2448	79.6463
D10	0.5808	1.1134	15.4244	166.5402	0.2760	0.5175	6.4910	86.3338
n	75				100			
ρ	0.80	0.90	0.99	0.999	0.80	0.90	0.99	0.999
OLS	0.1857	0.3867	3.9896	41.5707	0.1148	0.2416	2.5878	26.2797
D1	0.1629	0.2964	2.7042	38.5442	0.1051	0.1994	1.6566	23.5315
D2	0.1657	0.2951	0.7072	0.4358	0.1070	0.2016	0.6437	0.4751
D3	0.1660	0.2950	0.7485	0.6318	0.1073	0.2016	0.6664	0.5933
D4	0.1717	0.3061	0.9309	2.0486	0.1103	0.2081	0.7625	1.3133
D5	0.1611	0.2837	0.6765	0.2965	0.1048	0.1953	0.6259	0.3961
D6	0.1625	0.2863	0.6698	0.2790	0.1054	0.1965	0.6205	0.3856
D7	0.1667	0.2946	0.6758	0.2902	0.1078	0.2022	0.6287	0.3924
D8	0.1679	0.2948	0.6786	0.3515	0.1087	0.2030	0.6265	0.4277
D9	0.1700	0.3067	1.6765	32.9259	0.1092	0.2090	1.0733	18.8356
D10	0.1629	0.2964	2.7042	38.5442	0.1051	0.1994	1.6566	23.5315

Bold values indicate the minimum MSE.

TABLE 2: EMSE with $\epsilon \sim N(0, 1)$ and $p = 8$.

n	25				50			
ρ	0.80	0.90	0.99	0.999	0.80	0.90	0.99	0.999
OLS	4.4661	10.0978	111.2283	1110.9940	0.6621	1.3407	13.5747	137.7026
D1	3.2642	8.2668	107.8316	1107.1216	0.5340	0.9594	11.0166	133.4576
D2	1.4005	1.5967	1.4873	2.8012	0.5435	0.8599	1.6739	0.7777
D3	1.5054	1.7422	2.3441	8.7056	0.5464	0.8634	1.7928	1.2676
D4	2.6862	3.2757	17.9696	158.0872	0.5924	0.9438	2.8294	8.7950
D5	1.0341	1.2624	1.0926	0.2678	0.5083	0.7991	1.5932	0.5333
D6	1.0015	1.2246	1.0602	0.2167	0.5130	0.8019	1.5877	0.5213
D7	1.1855	1.3696	1.0863	0.2744	0.5348	0.8416	1.6162	0.5393
D8	1.4120	1.4951	1.2728	1.8072	0.5557	0.8691	1.6141	0.6823
D9	1.3468	1.5446	5.1341	39.8563	0.5520	0.8541	1.7414	1.6307
D10	3.2642	8.2668	107.8316	1107.1216	0.5340	0.9594	11.0166	133.4576
n	75				100			
ρ	0.80	0.90	0.99	0.999	0.80	0.90	0.99	0.999
OLS	0.5387	1.1324	12.0519	121.2389	0.4350	0.8792	8.8955	89.0391
D1	0.4280	0.7986	9.7917	117.2642	0.3596	0.6425	6.8484	85.0153
D2	0.4386	0.7090	1.6120	0.7809	0.3692	0.6086	1.6144	0.8232
D3	0.4389	0.7104	1.6829	1.1080	0.3670	0.6061	1.6616	1.0510
D4	0.4707	0.7656	2.3031	5.7538	0.3806	0.6293	2.0561	3.7974
D5	0.4086	0.6603	1.5626	0.6425	0.3503	0.5779	1.5802	0.7294
D6	0.4112	0.6611	1.5565	0.6341	0.3521	0.5795	1.5770	0.7237
D7	0.4340	0.7003	1.5860	0.6470	0.3674	0.6054	1.6029	0.7332
D8	0.4445	0.7148	1.5767	0.7260	0.3710	0.6105	1.5930	0.7895
D9	0.4457	0.7076	1.6199	1.1489	0.3698	0.6040	1.6037	1.0074
D10	0.4280	0.7986	9.7917	117.2642	0.3596	0.6425	6.8484	85.0153

Bold values indicate the minimum MSE.

Secondly, the effect of sample size on the EMSE and EMAE of estimators is considered. In general, the EMSE and EMAE of OLS and LE decrease by raising the sample size.

The proposed estimators D6 and D7 exhibit lowest EMSE in most of the cases, while D5 remains the closest competitor to the proposed estimators.

TABLE 3: EMSE with $\varepsilon \sim N(0, 5)$ and $p = 4$.

n		25				50			
ρ	0.80	0.90	0.99	0.999	0.80	0.90	0.99	0.999	
OLS	41.4642	42.9732	454.5090	4496.5388	18.2290	18.9241	211.7116	2227.7276	
D1	26.1760	39.7862	403.0533	4426.9805	13.3709	23.2564	170.5487	2157.6404	
D2	137.0247	174.2478	188.2420	1675.7301	20.7851	37.1738	66.7591	492.4123	
D3	378.0522	1374.5314	39523.4204	490286.3305	36.1051	153.7916	8463.1000	129718.4494	
D4	2975.1719	15340.5083	594331.0632	7470388.8610	168.1168	1275.1605	123401.6391	1962914.4830	
D5	15.7533	23.3267	57.2993	306.0084	8.3171	13.8682	28.0594	74.3255	
D6	32.8340	37.9098	22.6465	4.1954	11.7765	19.9355	22.8880	4.7488	
D7	82.1737	109.0638	54.0141	156.5435	17.1275	31.1904	30.4916	40.7833	
D8	104.7290	129.5769	93.6694	1064.4169	18.6407	34.2817	37.5308	262.5660	
D9	42.6720	53.2101	348.4940	4338.9415	18.0778	34.8008	140.4900	2082.4728	
D10	26.1760	39.7862	403.0533	4426.9805	13.3709	23.2564	170.5487	2157.6404	
n		75				100			
OLS	6.4770	9.1141	99.9004	1013.3213	4.8499	5.8779	62.7563	665.4290	
D1	7.7849	14.6692	77.4999	950.8600	4.9080	10.3165	50.7675	608.5832	
D2	10.7290	20.9964	42.3556	174.1792	6.3392	13.6072	34.4089	92.2086	
D3	12.9579	41.9918	2191.7497	41734.7737	6.6077	19.7573	996.4616	21399.1122	
D4	35.4892	219.4767	29783.5059	627203.7990	12.8567	71.3084	12681.4377	319880.7932	
D5	4.8090	7.9526	25.3509	27.4556	3.1361	5.5455	21.0400	19.9151	
D6	5.8656	10.1178	22.3379	8.5072	3.5901	6.5644	19.0367	10.9167	
D7	8.7665	16.8892	33.6034	19.3000	5.2500	10.8209	30.1756	16.6966	
D8	9.5607	19.1424	32.5339	84.0075	5.6506	12.3200	30.7390	45.0526	
D9	9.5700	21.1117	68.9550	883.0060	5.6113	13.6654	53.8102	547.8611	
D10	7.7849	14.6692	77.4999	950.8600	4.9080	10.3165	50.7675	608.5832	

Bold values indicate the minimum MSE.

TABLE 4: EMSE with $\varepsilon \sim N(0, 5)$ and $p = 8$.

n		25				50			
ρ	0.80	0.90	0.99	0.999	0.80	0.90	0.99	0.999	
OLS	113.7344	248.4263	2749.9318	27301.9092	26.4210	32.8776	338.0425	3442.7954	
D1	91.8201	208.6214	2672.6189	27216.2742	21.7582	32.8855	279.0388	3346.8484	
D2	1062.0091	773.0738	646.7100	5747.9291	54.8393	89.7872	111.6563	382.6700	
D3	4249.4455	8958.2538	94691.2091	889078.7675	74.4252	196.1502	3581.3148	55459.1032	
D4	103592.5974	346235.2477	5504480.4751	52208065.6888	465.9665	2876.8675	177596.7782	3166224.5262	
D5	48.2987	63.8589	72.7158	188.9864	15.0735	23.2842	50.1511	27.8849	
D6	45.0039	43.3329	29.5639	5.7589	20.3380	30.2377	48.8018	15.3856	
D7	509.4831	426.5375	85.5024	186.3474	41.4855	71.1363	84.7333	36.4997	
D8	1111.9509	860.3599	261.2728	3877.2132	56.5530	95.5960	78.9714	236.9419	
D9	510.7864	355.2202	1286.3553	16591.8930	44.5999	71.2858	97.6326	1105.5173	
D10	91.8201	208.6214	2672.6189	27216.2742	21.7582	32.8855	279.0388	3346.8484	
n		75				100			
OLS	23.6804	28.8332	293.2024	2994.0666	20.9489	22.2814	228.8529	2206.8140	
D1	19.3794	30.4080	240.8266	2903.6818	16.4682	26.3007	181.9324	2116.7986	
D2	47.5362	78.4761	88.1685	214.7922	27.6423	46.4392	72.6255	126.2054	
D3	62.5967	164.4174	2542.1039	34753.5135	34.4166	84.5315	1510.1974	22352.1191	
D4	354.7812	2201.6875	124050.8858	1975140.4297	169.1426	951.9774	69486.0127	1269566.1336	
D5	11.7509	19.5163	46.6486	23.8021	9.8660	16.3002	44.8918	24.1901	
D6	14.4069	22.3938	43.4837	17.2655	11.3559	18.3690	43.5199	19.7012	
D7	38.5554	66.2074	77.8238	29.5669	25.4996	43.4169	70.5102	28.8082	
D8	46.3333	80.2369	71.0713	130.2053	27.0422	46.7123	64.5158	83.4833	
D9	42.9726	71.1838	68.5353	629.6861	24.5740	40.8675	56.6078	323.1200	
D10	19.3794	30.4080	240.8266	2903.6818	16.4682	26.3007	181.9324	2116.7986	

Bold values indicate the minimum MSE.

In the third case, we varied the number of explanatory variables from 4 to 8. It is found that EMSE and EMAE of all the estimators increase. But the performance pattern

remains the same as in the case of multicollinearity and sample size. It is seen that the increase in the EMSE of OLS estimator is relatively higher than all LEs. LE with Liu

TABLE 5: EMSE with $\varepsilon \sim N(0, 0.1)$.

n	200				400			
p	16							
ρ	0.80	0.90	0.99	0.999	0.80	0.90	0.99	0.999
OLS	0.0035	0.0075	0.0793	0.8021	0.0019	0.0039	0.0410	0.4163
D1	0.0035	0.0073	0.0736	0.7226	0.0019	0.0039	0.0386	0.3744
D2	0.0031	0.0056	0.0247	0.0275	0.0017	0.0033	0.0167	0.0343
D3	0.0031	0.0057	0.0263	0.0392	0.0018	0.0033	0.0175	0.0416
D4	0.0035	0.0073	0.0736	0.7103	0.0019	0.0039	0.0386	0.3728
D5	0.0031	0.0056	0.0247	0.0275	0.0017	0.0033	0.0167	0.0343
D6	0.0031	0.0056	0.0247	0.0275	0.0017	0.0033	0.0167	0.0343
D7	0.0031	0.0056	0.0247	0.0275	0.0017	0.0033	0.0167	0.0343
D8	0.0031	0.0056	0.0247	0.0275	0.0017	0.0033	0.0167	0.0343
D9	0.0031	0.0056	0.0247	0.0275	0.0017	0.0033	0.0167	0.0343
D10	0.0035	0.0073	0.0736	0.7226	0.0019	0.0039	0.0386	0.3744
p	32							
OLS	0.0099	0.0213	0.2197	2.1819	0.0039	0.0080	0.0822	0.8277
D1	0.0097	0.0207	0.2088	2.0873	0.0038	0.0079	0.0791	0.7841
D2	0.0073	0.0122	0.0550	0.0544	0.0034	0.0061	0.0323	0.0730
D3	0.0074	0.0124	0.0568	0.0667	0.0034	0.0061	0.0330	0.0798
D4	0.0097	0.0207	0.2088	2.0284	0.0038	0.0079	0.0791	0.7765
D5	0.0073	0.0122	0.0550	0.0544	0.0034	0.0061	0.0323	0.0730
D6	0.0073	0.0122	0.0550	0.0544	0.0034	0.0061	0.0323	0.0730
D7	0.0073	0.0122	0.0550	0.0544	0.0034	0.0061	0.0323	0.0730
D8	0.0073	0.0122	0.0550	0.0544	0.0034	0.0061	0.0323	0.0730
D9	0.0073	0.0122	0.0550	0.0544	0.0034	0.0061	0.0323	0.0730
D10	0.0097	0.0207	0.2088	2.0873	0.0038	0.0079	0.0791	0.7841

TABLE 6: EMSE with $\varepsilon \sim N(0, 10)$.

n	200				400			
p	16							
ρ	0.80	0.90	0.99	0.999	0.80	0.90	0.99	0.999
OLS	35.3073	74.6252	788.6547	8050.3116	18.7928	39.7571	409.0704	4084.6535
D1	58.7745	99.4727	627.2726	7630.1947	36.0942	67.2299	331.9748	3737.2599
D2	330.6592	776.5051	1029.5021	871.0269	105.0624	257.7099	608.7140	579.6858
D3	410.9069	1320.7036	20406.3241	321323.4893	117.7264	367.6435	7379.0438	119656.0579
D4	3950.1774	34835.8967	3510745.9992	75009924.6589	696.5349	6218.5817	1113803.0191	27530620.4314
D5	36.9002	64.4541	291.8689	320.6374	19.6933	36.4964	200.5263	365.5992
D6	47.4151	84.1486	283.5081	297.1206	21.8618	41.6713	182.9172	356.2603
D7	304.0021	719.7100	1131.0888	501.2894	95.5070	233.0943	605.4684	524.1439
D8	337.4555	819.1739	1101.8653	505.4253	109.5028	274.5622	662.2097	444.3032
D9	273.7871	640.8076	674.2769	1735.7151	96.5125	240.9880	515.3343	613.2459
D10	58.7745	99.4727	627.2726	7630.1947	36.0942	67.2299	331.9748	3737.2599
p	32							
OLS	99.4473	209.8515	2186.3350	22167.5823	38.4568	80.6980	810.6738	8312.4423
D1	126.8498	204.5474	1926.1034	21620.9156	65.4947	110.6764	676.0445	7859.4889
D2	2962.5019	4694.9656	2240.2809	1777.2502	417.3801	990.8754	1302.9078	1189.0352
D3	3901.4652	7511.5308	23463.7648	234740.7023	471.1671	1287.3019	7116.4822	67892.5484
D4	54753.9636	314083.2155	12238618.7632	201244100.9571	3495.3559	29928.2260	2792922.9331	56051950.0538
D5	82.4224	146.5039	582.2643	583.8353	37.0539	66.1789	367.3434	756.0658
D6	103.4789	156.2744	569.0643	558.9004	41.3191	73.8115	335.2977	747.2898
D7	2545.6092	4065.9192	2422.9089	921.0806	362.5921	851.4343	1237.2237	1099.3145
D8	3486.9055	5656.6705	2660.4023	973.9301	453.9594	1086.7720	1448.3273	922.6883
D9	2728.5450	4209.1945	1468.1516	4644.7662	439.1082	1067.4070	1219.6257	1188.9104
D10	126.8498	204.5474	1926.1034	21620.9156	65.4947	110.6764	676.0445	7859.4889

parameter D6 exhibits lowest EMSE and EMAE for both the cases of explanatory variables.

Finally, the performance of estimators is compared by varying error variance. When the error variance increases

from one to five, the EMSE of all the estimators deteriorates, except the new estimator D6, which is least affected.

The concluded remarks from Tables 1–8 are that the new estimators D6 and D7 have shown better performance than

TABLE 7: EMAE with $\varepsilon \sim N(0, 1)$ and $p = 4$.

n	25				50			
ρ	0.85	0.95	0.99	0.999	0.85	0.95	0.99	0.999
OLS	1.4382	2.0404	6.4201	19.8598	0.9155	1.3542	4.5402	14.6991
D1	1.0641	1.3822	4.7235	16.1845	0.7610	1.0081	3.1781	12.1143
D2	1.0385	1.1961	1.2746	1.4109	0.7634	0.9807	1.1983	0.9127
D3	1.0467	1.2090	1.4512	2.0672	0.7642	0.9826	1.2830	1.2601
D4	1.1205	1.3141	2.1683	4.7916	0.7774	1.0073	1.5970	2.6147
D5	0.9892	1.1298	1.0873	0.6554	0.7524	0.9617	1.1384	0.6004
D6	0.9964	1.1292	1.0482	0.4918	0.7559	0.9657	1.1293	0.5412
D7	1.0260	1.1607	1.0684	0.5986	0.7638	0.9757	1.1334	0.5802
D8	1.0370	1.1665	1.1648	1.1060	0.7668	0.9764	1.1543	0.7462
D9	1.0611	1.2761	3.9183	15.5864	0.7797	1.0166	2.5083	11.4210
D10	1.0641	1.3822	4.7235	16.1845	0.7610	1.0081	3.1781	12.1143
n	75				100			
OLS	0.6747	0.9665	3.0577	9.8898	0.5313	0.7610	2.4557	7.8329
D1	0.5821	0.7682	1.9853	7.7947	0.4697	0.6249	1.5687	5.9675
D2	0.5863	0.7686	1.1998	0.9061	0.4732	0.6280	1.1416	0.9495
D3	0.5868	0.7685	1.2344	1.0959	0.4736	0.6281	1.1600	1.0734
D4	0.5947	0.7801	1.3601	1.8000	0.4788	0.6359	1.2304	1.5178
D5	0.5800	0.7568	1.1777	0.7717	0.4691	0.6205	1.1267	0.8766
D6	0.5817	0.7591	1.1737	0.7498	0.4703	0.6217	1.1241	0.8647
D7	0.5877	0.7682	1.1795	0.7634	0.4745	0.6288	1.1316	0.8719
D8	0.5903	0.7701	1.1807	0.8280	0.4765	0.6309	1.1299	0.9061
D9	0.5949	0.7885	1.6346	6.9604	0.4778	0.6414	1.3634	5.1163
D10	0.5821	0.7682	1.9853	7.7947	0.4697	0.6249	1.5687	5.9675

Bold values indicate the minimum MAE.

TABLE 8: EMAE with $\varepsilon \sim N(0, 1)$ and $p = 8$.

n	25				50			
ρ	0.85	0.95	0.99	0.999	0.85	0.95	0.99	0.999
OLS	4.1906	6.1205	19.7503	62.4992	1.7593	2.4556	7.7250	24.4799
D1	3.0025	4.4984	16.1106	52.3829	1.5064	1.9679	6.1153	21.7759
D2	2.3971	2.6431	2.5434	2.6882	1.5212	1.9215	2.7498	1.8304
D3	2.4615	2.7378	3.1069	4.6326	1.5245	1.9244	2.8452	2.3196
D4	2.9882	3.3961	6.4900	16.6139	1.5729	1.9923	3.4669	5.2519
D5	2.1371	2.3851	2.1607	1.0517	1.4824	1.8654	2.6814	1.5286
D6	2.1188	2.3539	2.1130	0.9423	1.4871	1.8695	2.6771	1.5090
D7	2.2661	2.4884	2.1534	1.0739	1.5117	1.9056	2.7041	1.5362
D8	2.4127	2.5829	2.3384	2.2730	1.5348	1.9302	2.7010	1.7221
D9	2.3726	2.6012	4.0392	8.9855	1.5352	1.9225	2.7893	2.5095
D10	3.0025	4.4984	16.1106	52.3829	1.5064	1.9679	6.1153	21.7759
N	75				100			
OLS	1.5476	2.2020	7.0421	22.2763	1.4011	1.9755	6.2091	19.5233
D1	1.3214	1.7562	5.5081	19.6549	1.2196	1.6162	4.7581	17.2197
D2	1.3369	1.7191	2.6936	1.8412	1.2325	1.6031	2.6968	1.9021
D3	1.3373	1.7202	2.7541	2.1951	1.2300	1.6007	2.7365	2.1581
D4	1.3702	1.7668	3.1670	4.3349	1.2459	1.6227	3.0216	3.7254
D5	1.3043	1.6741	2.6490	1.6660	1.2100	1.5725	2.6663	1.7887
D6	1.3068	1.6760	2.6436	1.6530	1.2117	1.5744	2.6640	1.7807
D7	1.3320	1.7116	2.6724	1.6716	1.2304	1.6000	2.6880	1.7933
D8	1.3433	1.7245	2.6638	1.7737	1.2347	1.6051	2.6795	1.8615
D9	1.3482	1.7222	2.6931	2.1898	1.2349	1.6012	2.6846	2.0897
D10	1.3214	1.7562	5.5081	19.6549	1.2196	1.6162	4.7581	17.2197

Bold values indicate the minimum MAE.

the other existing LEs in terms of smaller EMSE and EMAE, in situations of high multicollinearity disregarding the values of other factors. The proposed estimators also outperform the OLS estimator substantially. Therefore, based

on EMSE and EMAE criterion, we can infer that the new estimator D6 is more efficient and is the best choice for the practitioners in the presence of high and severe multicollinearity.

4. Applications

Since the simulation evidence is not enough to judge the performance of the proposed estimators, because the study is usually conducted assuming some ideal conditions, in practice, the ideal conditions may not be met. Therefore, contrary to the previous section, we used two real applications taken from the books of [2, 15] in order to compare the performance of our proposed estimators in practical situations.

4.1. Portland Cement Data. The first numerical example used in this study is the Portland cement dataset taken from [15] to compare the performance of new estimators in applied scenario. Some authors named this data as Hald's or chemical dataset and has been widely used in other literature; see [16–22]. The dataset consists of 13 observations and is given in Table 9. The response variable (y) is to study the heat evolved after 180 days of curing and measured in calories/gram of cement with 40% water at 35°C (95°F). Besides, the response variable (y), the four explanatory variables considered are the clinker compounds (CALCD) defined as

- (i) X_1 : Tricalcium aluminate ($3\text{CaO} * \text{Al}_2 \text{O}_3$)
- (ii) X_2 : Tricalcium silicate ($3\text{CaO} * \text{SiO}_2$)
- (iii) X_3 : Tetracalcium aluminoferrite ($4\text{CaO} * \text{Al}_2 \text{O}_3 * \text{Fe}_2 \text{O}_3$)
- (iv) X_4 : Dicalcium silicate ($2\text{CaO} * \text{SiO}_2$)

The model is defined as

$$y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \varepsilon. \quad (18)$$

Condition number (CN) is used to measure the severity of multicollinearity among explanatory variables [23]. It can be defined as

$$\text{CN} = \sqrt{\frac{\lambda_{\max}}{\lambda_{\min}}}, \quad (19)$$

where $\lambda_{\max}$ and $\lambda_{\min}$ are the maximum and minimum eigenvalues of the matrix $X'X$, respectively. Following [2], a rule of thumb is that multicollinearity is moderate if the CN is between 10 and 30, high if it is between 30 and 100, and severe when it is greater than 100. The eigenvalues and CN for this dataset are computed and presented in Table 10. From Table 10, it is concluded that high multicollinearity exists among the explanatory variables. The Shapiro-Wilk (W) normality test is used to test the normality of response variable. We obtain the value test statistic $W = 0.96967$ and p -value = 0.8903, which shows that the response variable is normal at 5% level of significance. MSE of OLS and Liu estimators from [1] can be written as

TABLE 9: Portland cement data [15].

Observation	x_1	x_2	x_3	x_4	y
1	1	29	15	52	74.3
2	11	56	8	20	104.3
3	7	26	6	60	78.5
4	7	52	6	33	95.9
5	11	55	9	22	109.2
6	11	31	8	47	87.6
7	1	31	22	44	72.5
8	2	54	18	22	93.1
9	3	71	17	6	102.7
10	11	66	9	12	113.3
11	10	68	8	12	109.4
12	1	40	23	34	83.8
13	21	47	4	26	115.9

TABLE 10: Eigenvalues and condition number of Portland cement data.

λ_1	λ_2	λ_3	λ_4	CN
2.2357 (55.89)	1.5761	0.1866	0.0016	37.38064

Parenthesis value gives the maximum percentage of first eigenvalue.

$$\text{MSE}(\hat{\alpha}) = \sigma^2 \sum_{j=1}^p \frac{1}{\lambda_j},$$

$$\text{MSE}(\hat{\alpha}(d)) = \sigma^2 \sum_{j=1}^p \frac{(\lambda_j + d)^2}{\lambda_j(\lambda_j + 1)^2} + (d - 1)^2 \sum_{j=1}^p \frac{\hat{\alpha}_j^2}{(\lambda_j + 1)^2}. \quad (20)$$

The estimated values for d , coefficients, and MSE of estimators are presented in Table 11. This table shows that the LEs have smaller MSE than OLS. However, among the LEs, estimator D5 and new estimators D6 and D7 outperform others; therefore, they are highly efficient among others.

4.2. Prediction Interval. In this section, 95% mean prediction interval of all the estimators is computed from the Portland cement dataset. We consider the following values of explanatory variables: $X_0 = (X_{10}, X_{20}, X_{30}, X_{40}) = (23, 79, 27, 65)$.

100 (1 - α) % mean prediction interval for the response variable is defined as

$$\begin{aligned} \text{OLS: } \hat{y}_0 \pm t_{1-(\alpha/2),v} \sqrt{\text{var}(\hat{y}_0)}, \\ \text{LIU: } \tilde{y}_0 \pm t_{1-(\alpha/2),v} \sqrt{\text{var}(\tilde{y}_0)}, \end{aligned} \quad (21)$$

where $\hat{y}_0 = \hat{\beta}_1 X_{10} + \hat{\beta}_2 X_{20} + \hat{\beta}_3 X_{30} + \hat{\beta}_4 X_{40}$, $\tilde{y}_0 = \tilde{\beta}_1 X_{10} + \tilde{\beta}_2 X_{20} + \tilde{\beta}_3 X_{30} + \tilde{\beta}_4 X_{40}$, $\hat{\beta}$ and $\tilde{\beta}$ are the OLS and Liu

TABLE 11: Estimated values for d , MSE and regression coefficients of Portland cement data.

Estimators	d	MSE	$\hat{\beta}_1$	$\hat{\beta}_2$	$\hat{\beta}_3$	$\hat{\beta}_4$
OLS	—	16.4515	-0.65696	-0.00831	0.30277	0.38774
D1	0.46099	3.6085	-0.64423	-0.00808	0.25239	0.18274
D2	0.02676	0.2726	-0.63398	-0.00790	0.21180	0.01759
D3	0.12868	0.4937	-0.63639	-0.00795	0.22133	0.05635
D4	0.46099	3.6085	-0.64423	-0.00808	0.25239	0.18274
D5	0.00020	0.2716	-0.63336	-0.00789	0.20932	0.00749
D6	0.00004	0.2716	-0.63335	-0.00789	0.20930	0.00743
D7	0.00588	0.2698	-0.63349	-0.00789	0.20985	0.00965
D8	0.07531	0.3349	-0.63513	-0.00792	0.21634	0.03605
D9	0.24428	1.1617	-0.63912	-0.00799	0.23213	0.10032
D10	0.46099	3.6085	-0.64423	-0.00808	0.25239	0.18274

Bold values indicate the minimum MSE.

TABLE 12: 95% mean prediction interval for the Portland cement data.

Estimators	Lower	Upper	Difference
OLS	265.5974	300.8197	566.4171
D1	130.2433	136.7162	266.9595
D2	-21.2485	4.5604	25.8089
D3	-46.8037	35.5516	82.3553
D4	130.2433	136.7162	266.9595
D5	-14.6542	-3.4506	11.2037
D6	-14.6155	-3.4977	11.1178
D7	-16.0524	-1.7492	14.3033
D8	-33.4124	19.3140	52.7264
D9	-75.8272	70.7413	146.5684
D10	130.2433	136.7162	266.9595

Bold values indicate the best prediction interval.

TABLE 13: Eigenvalues and condition number of US Crime data.

λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	λ_8	CN
4.0851 (51.06)	1.8390	0.9739	0.5119	0.3522	0.1536	0.0786	0.0057	26.77096

Parenthesis value gives the maximum percentage of first eigenvalue.

estimators, respectively. $t_{1-(\alpha/2)}$ is the $(1 - (\alpha/2))$ quantile from the Student's t-distribution with $(v = n - p)$ degrees of freedom. $\text{var}(\hat{y}_0) = \hat{\sigma}^2 X_0' (X'X)^{-1} X_0$ and $\text{var}(\tilde{y}_0) = \hat{\sigma}^2 X_0' Q_d (X'X)^{-1} Q_d' X_0$, $Q_d = (X'X + I)^{-1} (X'X + dI)$. For details, see [2, 4]. The results for the 95% mean prediction interval are given in Table 12. It can be seen from the table that the new estimator D6 gives the best mean prediction interval among all other estimators.

4.3. US Crime Data. The US crime dataset is taken from the book of [2] to explain the crime rate in relation to the eight socioeconomic variables in 47 states in the United States for the year 1960. Response variable and eight explanatory variables are defined as follows:

- (i) y = crime rate, number of offenses reported to police per million population
- (ii) X_1 = mean number of years of schooling times 10 for persons age 25 or older
- (iii) X_2 = 1960 per capita expenditure on police by state and local government
- (iv) X_3 = 1959 per capita expenditure on police by state and local government

TABLE 14: MSE of US Crime data.

Estimators	US crime data
OLS	72.5334
D1	35.5803
D2	3.2247
D3	6.9318
D4	59.4088
D5	3.0783
D6	3.0777
D7	3.1067
D8	3.1654
D9	3.2047
D10	35.5803

Bold values indicate the minimum MSE.

- (v) X_4 = number of males per 1000 females
- (vi) X_5 = state population size in hundred thousands
- (vii) X_6 = unemployment rate of urban males per 1000 of age 35–39
- (viii) X_7 = median value of transferable goods and assets or family income in tens of dollars
- (ix) X_8 = the number of families per 1000 earnings 1/2 the median income

Table 13 gives the eigenvalues and condition number for these data, which shows that moderately high multicollinearity is present. The value of test statistic from Shapiro-Wilk normality test is $W = 0.99379$ with p -value = 0.9967, which shows that the response variable is normal at 5% level of significance. MSE results are tabulated in Table 14. From Table 14, we see that MSE of LEs outperforms OLS. Among all LEs, the proposed estimator D6 showed smallest MSE, which supports the simulation results given in Section 3.

Therefore, based on simulation results and illustrative examples, we recommend the use of LE with Liu parameter D6 and D7 to practitioners in presence of high or severe multicollinearity.

5. Concluding Remarks

In this paper, we introduced a new quantile based method to estimate the Liu parameter in order to minimize the variance and circumvent the problem of multicollinearity. Extensive Monte Carlo simulations were carried out to evaluate the performance of estimators with MSE and MAE criteria by varying the values of different factors such as sample sizes, number of explanatory variables, error variances, and multicollinearity. Results from the simulation study revealed that the LE performed generally better than the traditional OLS estimator.

The LE is a robust choice than the OLS when the problem of multicollinearity is present. Also, the proposed estimator D6 performs better than other considered estimators in many evaluated instances particularly when the problem of multicollinearity is extremely high. Estimator D5 is the closest competitor to D6. Furthermore, the benefits of the new estimators are evidently confirmed in the two empirical applications. Based on the simulation results along with the real applications, we conclude that the Liu method with proposed estimator D6 is the best choice for practitioners to overcome the problem of multicollinearity.

Data Availability

Data used in this research is taken from Portland cement limited available online at: H. Woods, H. H. Steinour, and H. R. Starke, "Effect of composition of Portland cement on heat evolved during hardening," *Ind. Eng. Chem.*, vol. 24, no. 11, pp. 1207–1214, 1932. Research codes will be provided on personal request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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