Research Article

Metric Dimension on Path-Related Graphs

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Graph theory has a large number of applications in the fields of computer networking, robotics, Loran or sonar models, medical networks, electrical networking, facility location problems, navigation problems etc. It also plays an important role in studying the properties of chemical structures. In the field of telecommunication networks such as CCTV cameras, fiber optics, and cable networking, the metric dimension has a vital role. Metric dimension can help us in minimizing cost, labour, and time in the above discussed networks and in making them more efficient. Resolvability also has applications in tricky games, processing of maps or images, pattern recognitions, and robot navigation. We defined some new graphs and named them s -middle graphs, s -total graphs, symmetrical planar pyramid graph, reflection symmetrical planar pyramid graph, middle tower path graph, and reflection middle tower path graph. In the recent study, metric dimension of these path-related graphs is computed.

1. Introduction and Preliminary Results

Term metric dimension was firstly revealed by mathematician named Slater [1]. He explained the study of sonar or Loran model station by this concept. Harary, another mathematician, with the help of Melter, independently described the same concept of metric dimension [2].

Consider G is a simple, undirected, and finite graph and $T = \{s_1, s_2, \dots, s_f\}$ is ordered subset of vertex set of G , so $r(s|T)$ is a representation of the vertex s with respect to T which is f -tuple $(d(l, s_1), d(l, s_2), d(l, s_3), \dots, d(l, s_f))$. If each element of vertex set possesses unique representation with T , then we conclude that T is resolving set of S . If T is the smallest resolving set for G , then its cardinality becomes metric dimension of G and can be symbolized as $\dim(G)$.

Path graph is the only graph with metric dimension 1 [3]. Metric dimension of cycle graph is 2. Resolvability also has applications in tricky games, processing of maps or images, pattern recognitions, and networking robot navigation [4]. More details of applications of metric dimension can be seen

in [5–10]. It is suggested to study the following references for better comprehension and details of metric dimension of graphs [11–16].

A lot of work has been carried out by many authors on metric dimension of path-related graphs. In 2012, Ali et al. computed constant metric dimension of middle path graph and power of path graph [10]. In 2017, Alholi et al. also put contribution in this field and found constant metric dimension [17]. Shahida and Sunitha found constant metric dimension of join of any two graphs [18]. In 2016, Peng et al. calculated metric dimension of Kenser graphs [19].

Ali et al., in 2012, worked on metric dimension of total graph of path graph and found it to be constant [10]. In 2019, Pan et al. calculated the unbounded metric dimension of splitting graphs of path graph and cycle graph [20]. In 2021, Nawaz et al. proved that metric dimension of path power three and four is unbounded; also, they proved some results on edges of power of path and power of total graph [21].

Our research is in continuation with their work, and we computed the metric dimension of s -middle path graphs and

s -total path graphs. These graphs have bounded metric dimension. In the paper under study, we also calculated metric dimension of other path-related graphs such as middle tower path graph, reflection middle tower path graph, symmetrical planar pyramid path graph, and reflection symmetrical planar pyramid path graph.

1.1. Metric Dimension of s -Middle Graph. We generalized the concept of middle of a graph and extended this idea to s vertices. We took two particular cases for path graph and with the help of these results and finally computed the metric dimension of s -middle graph of the path graph.

Definition 1. For undirected simple (planar or nonplanar) graph G with $V(G)$ and $E(G)$ as vertex and edge set, respectively, $V(G) = \{v_1, v_2, \dots, v_l\}$ and $E(G) = \{e_1, e_2, \dots, e_u\}$. We associate for every edge in edge set s vertices, forming another graph possessing vertex set represented by $V(s\text{-Mid}(G))$ which is given by $V(s\text{-Mid}(G)) = V(G) \cup (\cup_{u=1}^s V^u)$, and if there are l vertices in G and number of edges is u , then $s\text{-Mid}(G)$ contains $l + su$ vertices. Each v_g in $V(s\text{-Mid}(G))$ is adjacent to v_h in $V(s\text{-Mid}(G))$:

- (1) $v_g \sim v_g^u$ if v_g is incident to e_g in G
- (2) $v_g^u \sim v_f^u$ if e_g is incident to e_f in G

Example 1. For a particular case of $s\text{-Mid}(G)$, we discussed two-middle path graph in which exactly two vertices were assigned against each edge. We take $G = P_q$ (path graph containing q vertices). We associate h_l and s_l so that $V^* = \{h_1, h_2, h_3, \dots, h_{q-1}\}$, $V^{**} = \{s_1, s_2, s_3, \dots, s_{q-1}\}$.

Thus, the vertex set of Two-Mid(P_q) is $V(\text{Two-Mid}(P_q)) = V(P_q) \cup V^*(P_q) \cup V^{**}(P_q)$. Each $v_l \sim v_m$ in Two-Mid(P_q):

- (1) $v_l \sim s_i$ and $v_l \sim h_l$ are incident to e_l in P_q
- (2) $s_l \sim s_m$ and $h_l \sim h_m$ if and only if e_l is adjacent to e_m in P_q

Two-middle path graph and three-middle path graph are explained in Figures 1 and 2.

Theorem 1. $\dim(\text{Two-Mid}(P_q))$ is 2, where $q \geq 3$.

Proof. Two-Mid(P_q) is not path when $q \geq 3$, so its metric dimension must be greater than 1. The vertex set for Two-Mid(P_q) is $V(\text{Two-Mid}(P_q))$ and is given by $V(\text{Two-Mid}(P_q)) = \{v_1, v_2, v_3, \dots, v_q\} \cup \{h_1, h_2, h_3, \dots, h_{q-1}\} \cup \{s_1, s_2, s_3, \dots, s_{q-1}\}$.

Assume $W = \{s_1, s_{q-1}\}$ as a resolving set for Two-Mid(P_q) so that

FIGURE 1: Two-middle path graph.

FIGURE 2: Three-middle path graph.

$$\begin{aligned}
 r(v_l|W) &= \begin{cases} (l, q-1), & l = 1, \\ (l-1, q-l), & l = 2, 3, 4, \dots, q-1, \\ (l-1, 1), & l = q, \end{cases} \\
 r(s_j|W) &= (j-1, q-j-1), \quad \forall j = 1, 2, 3, \dots, q-1, \\
 r(h_j|W) &= \begin{cases} (2, q-1), & j = 1, \\ (j, q-j), & j = 2, 3, 4, \dots, q-2, \\ (j, 2), & j = q-1. \end{cases}
 \end{aligned} \tag{1}$$

When $l \neq g$ and v_l and v_g have the same representation for W , contradiction arises, i.e.,

$$\begin{aligned}
 r(v_l|W) &= r(v_g|W) \\
 \Rightarrow (l-1, q-l) &= (g-1, q-g) \\
 \Rightarrow l &= g \text{ (contradiction).}
 \end{aligned} \tag{2}$$

Similarly, when $l \neq g$,

$$\begin{aligned}
 r(s_l|W) &= r(s_g|W) \\
 \Rightarrow (l-1, q-l-1) &= (g-1, q-g-1) \\
 \Rightarrow l &= g \text{ (contradiction).}
 \end{aligned} \tag{3}$$

When $l \neq g$,

$$\begin{aligned} r(h_l|W) &= r(h_q|W) \\ \Rightarrow (l, q-l) &= (g-1, q-g-1) \\ \Rightarrow 1 &= g-1 \text{ and } 1 = g+1 \text{ (contradiction).} \end{aligned} \quad (4)$$

When $l \neq g$,

$$\begin{aligned} r(v_l|S) &= r(s_g|S) \\ \Rightarrow (l, q-l) &= (g, g-l) \\ \Rightarrow 1 &= g \text{ (contradiction).} \end{aligned} \quad (5)$$

When $l \neq g$,

$$\begin{aligned} r(h_l|W) &= r(s_g|W) \\ \Rightarrow (l, q-l) &= (g, q-1) \\ \Rightarrow 1 &= g \text{ (contradiction).} \end{aligned} \quad (6)$$

Hence, Two-Mid (P_q) possesses W as its smallest resolving set, so its metric dimension is 2. $\square$

Theorem 2. $\dim(\text{Three-Mid}(P_q))$ is 3, where $q \geq 3$.

Proof. The vertex set for Three-Mid(P_q) is $V(\text{Three-Mid}(P_q))$ and is given by

$$\begin{aligned} V(\text{Three-Mid}(P_q)) &= \{v_1, v_2, v_3, \dots, v_q\} \\ &\quad \cup \{u_1, u_2, u_3, \dots, u_{q-1}\} \\ &\quad \cup \{w_1, w_2, w_3, \dots, w_{q-1}\} \\ &\quad \cup \{m_1, m_2, m_3, \dots, m_{q-1}\}. \end{aligned} \quad (7)$$

As Three-Mid(P_q) is not a path graph, so it possesses metric dimension greater than 1.

Assume $W = \{u_1, v_1, m_1\}$ as a resolving set for Three-Mid(P_q) so that

$$\begin{aligned} r(v_g|W) &= \begin{cases} (1, 0, 1), & g = 1, \\ (g-1, g, g-1), & g = 2, 3, 4, \dots, q, \end{cases} \\ r(u_g|W) &= \begin{cases} (0, 1, 2), & g = 1, \\ (g-1, g, g), & g = 2, 3, 4, \dots, q-1, \end{cases} \\ r(w_g|W) &= \begin{cases} (2, 1, 2), & g = 1, \\ (g, g, g), & g = 2, 3, 4, \dots, q-1, \end{cases} \\ r(m_g|W) &= \begin{cases} (2, 1, 0), & g = 1, \\ (g, g, g-1), & g = 2, 3, 4, \dots, q-1. \end{cases} \end{aligned} \quad (8)$$

When $l \neq g$ and v_l and v_g have the same representation with respect to w , a contradiction arises, i.e.,

$$\begin{aligned} r(v_l|W) &= r(v_g|W) \\ \Rightarrow (l-1, l, l-1) &= (g-1, q, g-1) \\ \Rightarrow l &= g \text{ (contradiction).} \end{aligned} \quad (9)$$

Similarly, when $l \neq g$,

$$\begin{aligned} r(u_l|W) &= r(u_g|W) \\ \Rightarrow (l-1, l, l) &= (g-1, g, g) \\ \Rightarrow l &= g \text{ (contradiction).} \end{aligned} \quad (10)$$

When $l \neq g$,

$$\begin{aligned} r(w_l|W) &= r(w_g|W) \\ \Rightarrow (l, l, l) &= (g, g, g) \\ \Rightarrow l &= g \text{ (contradiction).} \end{aligned} \quad (11)$$

When $l \neq g$,

$$\begin{aligned} r(m_l|W) &= r(m_g|W) \\ \Rightarrow (l, l, l-1) &= (g, g, g-1) \\ \Rightarrow l &= g \text{ (contradiction).} \end{aligned} \quad (12)$$

When $l \neq g$,

$$\begin{aligned} r(v_l|W) &= r(u_g|W) \\ \Rightarrow (l-1, l, l-1) &= (g-1, g, g) \\ \Rightarrow l &= g \text{ (contradiction).} \end{aligned} \quad (13)$$

When $l \neq g$,

$$\begin{aligned} r(v_l|W) &= r(w_g|W) \\ \Rightarrow (l-1, l, l-1) &= (g, g, g) \\ \Rightarrow l &= g \text{ (contradiction).} \end{aligned} \quad (14)$$

When $l \neq g$,

$$\begin{aligned} r(u_l|W) &= r(w_g|W) \\ \Rightarrow (l-1, l, l-1) &= (g, g, g) \\ \Rightarrow l &= g \text{ (contradiction).} \end{aligned} \quad (15)$$

When $l \neq g$,

$$\begin{aligned} r(u_l|W) &= r(w_g|W) \\ \Rightarrow (l-1, l, l) &= (g, g, g) \\ \Rightarrow l &= g \text{ (contradiction).} \end{aligned} \quad (16)$$

When $l \neq g$,

$$\begin{aligned} r(u_l|W) &= r(m_g|W) \\ \Rightarrow (l-1, l, l) &= (g, g, g-1) \\ \Rightarrow l &= g \text{ (contradiction).} \end{aligned} \quad (17)$$

When $l \neq g$,

$$\begin{aligned} r(w_l|W) &= r(m_g|W) \\ \Rightarrow (l, l, l) &= (g, g, g-1) \\ \Rightarrow l &= g \text{ (contradiction).} \end{aligned} \quad (18)$$

Hence, W is the resolving set for Three-Mid(P_q). Now, we shall show that no resolving set having cardinality 2 exists.

Let $T_1 = \{v_l, v_g\}$ be metric basis of Three – Mid(P_q); then, $r(u_j|T_1) = r(m_j|T_1) = r(w_j|T_1)$, so uniqueness is violated.

Let $T_2 = \{u_l, u_g\}$ be metric bases of Three – Mid(P_q); then, $r(w_j|T_2) = r(m_j|T_2)$, so uniqueness is violated.

Let $T_3 = \{w_l, w_g\}$ be metric bases of Three – Mid(P_q); then, $r(u_j|T_3) = r(m_j|T_3)$, so uniqueness is violated.

Let $T_4 = \{m_l, m_g\}$ be metric bases of Three – Mid(P_q); then, $r(w_j|T_4) = r(u_j|T_4)$, so uniqueness is violated.

Let $T_5 = \{v_l, w_g\}$ be metric bases of Three – Mid(P_q); then, $r(u_j|T_5) = r(m_j|T_5)$, so uniqueness is violated.

Let $T_6 = \{v_l, u_g\}$ be metric bases of Three – Mid(P_q); then, $r(w_j|T_6) = r(m_j|T_6)$, so uniqueness is violated.

Let $T_7 = \{v_l, m_g\}$ be metric bases of Three – Mid(P_q); then, $r(w_j|T_7) = r(u_j|T_7)$, so uniqueness is violated.

Let $T_8 = \{u_l, w_g\}$ be metric bases of Three – Mid(P_q); then, $r(v_j|T_8) = r(m_{j-1}|T_8)$, so uniqueness is violated.

Let $T_9 = \{u_l, m_g\}$ be metric bases of Three – Mid(P_q); then, $r(v_j|T_9) = r(m_{j-1}|T_9)$, so uniqueness is violated.

Let $T_{10} = \{w_l, m_g\}$ be metric bases of Three – Mid(P_q); then, $r(v_j|T_{10}) = r(u_{j-1}|T_{10})$, so uniqueness is violated.

So, there are no metric bases of order 2. Hence, W is the resolving set of order 3 and metric dimension of Three – Mid(P_q) is 3. $\square$

Theorem 3. Metric dimension of s -Middle (P_q) is s .

Proof. The vertex set of s -Middle (P_q) is $\{v_1, v_2, v_3, \dots, v_p\} \cup \{w_g^m\}$, where $g = 1, 2, 3, \dots, p-1$ and $m = 1, 2, 3, \dots, s$. Let $W = \{w_1^1, w_1^2, w_1^3, \dots, w_1^s\}$ be the resolving set of the s -Middle (P_q); then, $r(v_g|W) = (g, g-1, g-1, \dots, g-1)_{(s\text{-tuple})}$, where $g = 1, 2, 3, \dots, p$:

$$r(w^{(h)_g}|W) = \begin{cases} (g-1, g, g, \dots, g)_{(s\text{-tuple})}, & g = 1, 2, 3, \dots, p-1; h = 1, \\ (g, g-1, g, \dots, g)_{(s\text{-tuple})}, & g = 1, 2, 3, \dots, p-1; h = 2, \\ (g, g, g-1, \dots, g)_{(s\text{-tuple})}, & g = 1, 2, 3, \dots, p-1; h = 3, \\ \vdots \\ (g, g, g, \dots, g-1)_{(s\text{-tuple})}, & g = 1, 2, 3, \dots, p-1; h = s, \end{cases} \quad (19)$$

When $l \neq m$ and v_l and v_m possess the same representation for W , a contradiction arises, i.e.,

$$\begin{aligned} r(v_l|W) &= r(v_m|W) \\ &\Rightarrow (l, l-1, l-1, \dots, l-1)_{(s\text{-tuple})} \\ &= (m, m-1, m-1, \dots, m-1)_{(s\text{-tuple})} \\ &\Rightarrow l = m \text{ (contradiction).} \end{aligned} \quad (20)$$

Similarly, when $l \neq m$,

$$\begin{aligned} r(w_l^h|W) &= r(w_m^h|W) \\ &\Rightarrow (l, l, l, \dots, l-1_{h^{\text{th}} \text{ position}}, l, \dots, l)_{(s\text{-tuple})} \\ &= (m, m, m, \dots, m-1_{h^{\text{th}} \text{ position}}, m, \dots, m)_{(s\text{-tuple})} \\ &\Rightarrow l = m \text{ (contradiction).} \end{aligned} \quad (21)$$

When $l \neq m$,

$$\begin{aligned} r(w_g^l|W) &= r(w_g^m|W) \\ &\Rightarrow (g, g, g, \dots, g-1_{l^{\text{th}} \text{ position}}, g, \dots, g)_{(s\text{-tuple})} \\ &= (g, g, g, \dots, g-1_{m^{\text{th}} \text{ position}}, g, \dots, g)_{(s\text{-tuple})} \\ &\Rightarrow g-1 = g \text{ (contradiction).} \end{aligned} \quad (22)$$

When $l \neq m$,

$$\begin{aligned} u \neq v \quad r(w_l^u|W) &= r(w_m^v|W) \\ &\Rightarrow (l, l, l, \dots, l-1_{(u^{\text{th}} \text{ position})}, l, \dots, l)_{(s\text{-tuple})} \\ &= (m, m, m, \dots, m-1_{(v^{\text{th}} \text{ position})}, m, \dots, m)_{(s\text{-tuple})} \\ &\Rightarrow l-1 = m \text{ and } l = m-1 \text{ (contradiction).} \end{aligned} \quad (23)$$

Now, we shall show that there is no resolving set with $s - 1$ elements that exists. So, on the contrary, let a set with $s - 1$ elements be a resolving set for s -Middle P_q . Then, particularly, for $s = 3$, there exists a resolving set of $3 - 1 = 2$ elements. So, metric dimension of Three-Middle P_q is not 2, and it leads to contradiction.

So, there is no resolving set with $s - 1$ elements, and hence, s -Middle (P_q) has metric dimension s . $\square$

1.2. s -Total Graph of a Path. We discuss here more generalized concept of total graph of some graphs. We shall apply the very concept for $s = 2, 3$ on the path graph and use these results to find metric dimension of s -total graph of the path graph.

Definition 2. For undirected simple (planar or nonplanar) graph G with vertex set $V(G)$ and $E(G)$ as the edge set, $V(G) = \{v_1, v_2, \dots, v_s\}$ and $E(G) = \{e_1, e_2, \dots, e_u\}$. We associate for every edge in edge set s vertices, forming another graph possessing vertex set represented by $V(s - T(G))$ which is given by $V(s - T(G)) = V(G) \cup (\cup_{u=1}^s V^u)$, and if the total number of vertices in G is r and number of edges is u , then s -Total (G) contains $r + su$ vertices. Each v_g in $V(s - T(G))$ is adjacent to v_h in $V(s - T(G))$:

- (1) $v_g \sim v_f$ if $v_g \sim v_f$ in G
- (2) $v_g \sim v_g^\mu$ if v_g is incident to e_g in G
- (3) $v_g^\mu \sim v_f^\mu$ if e_g is incident to e_f in G

Two-total graph of a path and Three-total graph of a path are explained in Figures 3 and 4.

Theorem 4. $\dim(Two - T(p_q))$ is 2, where $q \geq 2$.

Proof. Two - $T(P_q)$ is not the path when $q \geq 2$, so its metric dimension must be greater than 1. The vertex set for Two - $T(P_q)$ is $V(Two - T(P_q))$ and is given by $V(Two - T(P_q)) = \{v_1, v_2, v_3, \dots, v_q\} \cup \{h_1, h_2, h_3, \dots, h_{q-1}\} \cup \{s_1, s_2, s_3, \dots, s_{q-1}\}$.

Assume $W = \{h_1, h_{q-1}\}$ as a resolving set for Two - $T(P_q)$ so that

$$r(v_l|W) = \begin{cases} (l, q-1), & j = 1, \\ (l-1, q-l), & j = 2, 3, 4, \dots, q-1, \\ (l-1, 1), & l = q, \end{cases}$$

$$r(s_h|W) = (l-1, q-l-1) \quad \forall j = 1, \dots, q-1, \quad (24)$$

$$r(h_j|W) = \begin{cases} (2, q-1) & l = 1, \\ (l, q-l), & q = 2, 3, 4, \dots, q-2, \\ (l, 2), & l = q-1. \end{cases}$$

When $u \neq r$ and h_u and h_r possess the same representation for W , a contradiction arises, i.e.,

FIGURE 3: Two-total graph of the path graph.

FIGURE 4: Three-total graph of the path graph.

$$\begin{aligned} r(h_u|W) &= r(h_r|W) \\ \Rightarrow (u-1, q-u-1) &= (r-1, q-r) \\ \Rightarrow l &= g(\text{contradiction}). \end{aligned} \quad (25)$$

Similarly, when $u \neq r$,

$$\begin{aligned} r(s_u|W) &= r(s_r|W) \\ \Rightarrow (u-1, q-u) &= (r-1, q-r) \\ \Rightarrow u &= r(\text{contradiction}). \end{aligned} \quad (26)$$

When $u \neq r$,

$$\begin{aligned} r(h_u|W) &= r(s_r|W) \\ \Rightarrow (u, q-u) &= (r, q-r) \\ \Rightarrow u &= r(\text{contradiction}). \end{aligned} \quad (27)$$

When $u \neq r$,

$$\begin{aligned} r(v_u|W) &= r(v_r|W) \\ \Rightarrow (u-l, q-u) &= (r-1, q-r) \\ \Rightarrow u &= r(\text{contradiction}). \end{aligned} \quad (28)$$

Hence, Two - $T(P_q)$ possesses W as its smallest resolving set, so its metric dimension is 2. $\square$

Theorem 5. $\dim(Three - T(p_q))$ is 3, where $q \geq 3$.

Proof. The vertex set for $\text{Three} - T(P_q)$ is $V(\text{Three} - T(P_q))$ and is given by $V(\text{Three} - T(P_q)) = \{v_1, v_2, v_3, \dots, v_q\} \cup \{u_1, u_2, u_3, \dots, u_{q-1}\} \cup \{w_1, w_2, w_3, \dots, w_{q-1}\} \cup \{m_1, m_2, m_3, \dots, m_{q-1}\}$. As $\text{Three} - T(P_q)$ is not a path graph, so it possesses metric dimension greater than 1.

Assume $W = \{u_l, v_l, m_l\}$ as a resolving set for $\text{Three} - T(P_q)$ so that

$$\begin{aligned} r(v_g|W) &= \begin{cases} (1, 0, 1), & g = 1, \\ (g-1, g-1, g-1), & g = 2, 3, 4, \dots, q-1, \end{cases} \\ r(u_g|W) &= \begin{cases} (0, 1, 2), & g = 1, \\ (g-1, g, g), & g = 2, 3, 4, \dots, q-1, \end{cases} \\ r(w_g|W) &= \begin{cases} (2, 1, 2), & g = 1, \\ (g, g, g), & g = 2, 3, 4, \dots, q-1, \end{cases} \\ r(m_g|W) &= \begin{cases} (2, 1, 0), & g = 1, \\ (g, g, g-1), & g = 2, 3, 4, \dots, q-1. \end{cases} \end{aligned} \quad (29)$$

When $l \neq g$ and v_l and v_g possess the same representation for W , a contradiction arises, i.e.,

$$\begin{aligned} r(v_l|W) &= r(v_g|W) \\ \Rightarrow (l-1, l, l-1) &= (g-1, g-1, g-1) \\ \Rightarrow l &= g \text{ (contradiction).} \end{aligned} \quad (30)$$

Similarly, when $g \neq l$,

$$\begin{aligned} r(u_g|W) &= r(u_l|W) \\ \Rightarrow (g-1, g, g) &= (l-1, l, l) \\ \Rightarrow g &= l \text{ (contradiction).} \end{aligned} \quad (31)$$

When $l \neq g$,

$$\begin{aligned} r(w_l|W) &= r(w_g|W) \\ \Rightarrow (l, l, l) &= (g, g, g) \\ \Rightarrow l &= g \text{ (contradiction).} \end{aligned} \quad (32)$$

When $l \neq g$,

$$\begin{aligned} r(m_l|W) &= r(m_g|W) \\ \Rightarrow (l, l, l-1) &= (g, g, g-1) \\ \Rightarrow l &= g \text{ (contradiction).} \end{aligned} \quad (33)$$

When $l \neq g$,

$$\begin{aligned} r(v_l|W) &= r(u_g|W) \\ \Rightarrow (l-1, l-1, l-1) &= (g-1, g, g) \\ \Rightarrow l &= g \text{ (contradiction).} \end{aligned} \quad (34)$$

When $l \neq g$,

$$\begin{aligned} r(u_l|W) &= r(w_g|W) \\ \Rightarrow (l-1, l, l) &= (g, g, g) \\ \Rightarrow l &= g \text{ (contradiction).} \end{aligned} \quad (35)$$

When $l \neq g$,

$$\begin{aligned} r(u_l|W) &= r(m_g|W) \\ \Rightarrow (l-1, l, l) &= (g, g, g-1) \\ \Rightarrow l &= g \text{ (contradiction).} \end{aligned} \quad (36)$$

When $l \neq g$,

$$\begin{aligned} r(w_l|W) &= r(m_g|W) \\ \Rightarrow (l, l, l) &= (g, g, g-1) \\ \Rightarrow l &= g \text{ (contradiction).} \end{aligned} \quad (37)$$

Hence, W is resolving set for $\text{Three-Total}(P_q)$. Now, we shall show that no resolving set having cardinality 2 exists.

Let $T_1 = \{v_l, v_g\}$ be a resolving set of $\text{Three} - T(P_q)$; then, $r(u_j|T_1) = r(m_j|T_1) = r(w_j|T_1)$, so uniqueness is violated.

Let $T_2 = \{u_l, u_g\}$ be metric bases of $\text{Three} - T(P_q)$; then, $r(w_j|T_2) = r(m_j|T_2)$, so uniqueness is violated.

Let $T_3 = \{w_l, w_g\}$ be metric bases of $\text{Three} - T(P_q)$; then, $r(u_j|T_3) = r(m_j|T_3)$, so uniqueness is violated.

Let $T_4 = \{m_l, m_g\}$ be metric bases of $\text{Three} - T(P_q)$; then, $r(w_j|T_4) = r(u_j|T_4)$, so uniqueness is violated.

Let $T_5 = \{v_l, w_g\}$ be metric bases of $\text{Three} - T(P_q)$; then, $r(u_j|T_5) = r(m_j|T_5)$, so uniqueness is violated.

Let $T_6 = \{v_l, u_g\}$ be metric bases of $\text{Three} - T(P_q)$; then, $r(w_j|T_6) = r(m_j|T_6)$, so uniqueness is violated.

Let $T_7 = \{v_l, m_g\}$ be metric bases of $\text{Three} - T(P_q)$; then, $r(w_j|T_7) = r(u_j|T_7)$, so uniqueness is violated.

Let $T_8 = \{u_l, w_g\}$ be metric bases of $\text{Three} - T(P_q)$; then, $r(v_j|T_8) = r(m_{j-1}|T_8)$, so uniqueness is violated.

Let $T_9 = \{u_l, m_g\}$ be metric bases of $\text{Three} - T(P_q)$; then, $r(v_j|T_9) = r(w_{j-1}|T_9)$, so uniqueness is violated.

Let $T_{10} = \{w_l, m_g\}$ be metric bases of $\text{Three} - T(P_q)$; then, $r(v_j|T_{10}) = r(u_{j-1}|T_{10})$, so uniqueness is violated.

So, there are no metric bases of order 2. Hence, W is the resolving set of order 3 and metric dimension of $\text{Three} - T(P_q)$ is 3. $\square$

Theorem 6. Metric dimension of $s - \text{Total}(P_q)$ is s .

Proof. The vertex set of $s - \text{Total}(P_q)$ is $\{v_1, v_2, v_3, \dots, v_p\} \cup \{w_g^m\}$, where $g = 1, 2, 3, \dots, p-1$ and $m = 1, 2, 3, \dots, s$. Let $W = \{w_1^1, w_1^2, w_1^3, \dots, w_1^s\}$ be the resolving set of the $s - \text{Total}(P_q)$; then, $r(v_g|W) = (g, g-1, g-1, \dots, g-1)_{(s-\text{tuple})}$, where $g = 1, 2, 3, \dots, p$,

$$r(w_g^h|W) = \begin{cases} (g-1, g, g, \dots, g)_{(s\text{-tuple})}, & g = 1, 2, 3, \dots, p-1; h = 1, \\ (g, g-1, g, \dots, g)_{(s\text{-tuple})}, & g = 1, 2, 3, \dots, p-1; h = 2, \\ (g, g, g-1, \dots, g)_{(s\text{-tuple})}, & g = 1, 2, 3, \dots, p-1; h = 3, \\ \cdot \\ \cdot \\ \cdot \\ (g, g, g, \dots, g-1)_{(s\text{-tuple})}, & g = 1, 2, 3, \dots, p-1; h = s. \end{cases} \quad (38)$$

When $g \neq m$ and v_g and v_m possess the same representation for W , a contradiction arises, i.e.,

$$\begin{aligned} r(v_g|W) &= r(v_m|W) \\ &\Rightarrow (g, g-1, g-1, \dots, g-1)_{(s\text{-tuple})} \\ &= (m, m-1, m-1, \dots, m-1)_{(s\text{-tuple})} \\ &\Rightarrow g = m \text{ (contradiction)}. \end{aligned} \quad (39)$$

Similarly, when $l \neq m$,

$$\begin{aligned} r(w_l^h|W) &= r(w_m^h|W) \\ &\Rightarrow (l, l, l, \dots, l - 1_{h^{\text{th}} \text{ position}}, l, \dots, l)_{(s\text{-tuple})} \\ &= (m, m, m, \dots, m - 1_{h^{\text{th}} \text{ position}}, m, \dots, m)_{(s\text{-tuple})} \\ &\Rightarrow l = m \text{ (contradiction)}. \end{aligned} \quad (40)$$

When $l \neq m$,

$$\begin{aligned} r(w_g^l|W) &= r(w_m^m|W) \\ &\Rightarrow (g, g, g, \dots, g - 1_{l^{\text{th}} \text{ position}}, g, \dots, g)_{(s\text{-tuple})} \\ &= (g, g, g, \dots, g - 1_{m^{\text{th}} \text{ position}}, g, \dots, g)_{(s\text{-tuple})} \\ &\Rightarrow g - 1 = g \text{ (contradiction)}. \end{aligned} \quad (41)$$

When $l \neq m$ and $u \neq v$,

$$\begin{aligned} r(w_l^u|W) &= r(w_m^v|W) \\ &\Rightarrow (l, l, l, \dots, l - 1_{(u^{\text{th}} \text{ position})}, l, \dots, l)_{(s\text{-tuple})} \\ &= (m, m, m, \dots, m - 1_{(v^{\text{th}} \text{ position})}, m, \dots, m)_{(s\text{-tuple})} \\ &\Rightarrow l - 1 = m \text{ and } l = m - 1 \text{ (contradiction)}. \end{aligned} \quad (42)$$

Now, we shall show that there is no resolving set with $s-1$ elements exists. So, on the contrary, let a set with $s-1$ elements be a resolving set for s -Total P_q . Then, particularly, for $s=3$, there exists a resolving set of $3-1=2$ elements. So, metric dimension of Three-Total P_q is not 2, and it leads to contradiction.

So, there is no resolving set with $s-1$ elements, and hence, s -Total (P_q) has metric dimension s . $\square$

1.3. Middle Tower Path Graph. Middle tower path graph is a special graph represented by Middle Tower_s, which is related to the path graph, and later, we extended its concept to its reflection.

Definition 3. An undirected, simple, and planar graph possessing vertex set $V = \{V_g^t \mid 1 \leq t \leq s, 1 \leq g \leq t\}$, where $t = 1, 2, 3, \dots, s$. Each vertex of Middle Tower_s is adjacent in the following manner.

- (1) $v_g^t \sim v_g^{t+1}$ when $g = 1, 2, 3, \dots, t$ and $t = 1, 2, 3, \dots, s-1$
- (2) $v_g^t \sim v_{g+1}^{t+1}$ when $g = 1, 2, 3, \dots, t$ and $t = 1, 2, 3, \dots, s-1$
- (3) $v_g^t \sim v_{g+1}^t$ when $g = 1, 2, 3, \dots, t$ and $t = 1, 2, 3, \dots, s-1$

Middle tower path graph is explained in Figure 5.

Theorem 7. Metric dimension of Middle Tower_s is 1, for $s=2$.

Proof. As Middle Tower₂ is a path graph (by definition) and every path graph has metric dimension 1, so metric dimension of this graph is 1. $\square$

Theorem 8. Metric dimension of Middle Tower_s is 2, for $s \geq 3$.

Proof. As Middle Tower_s is not path graph; hence, it possesses metric dimension ≥ 2 . Vertex set of Middle Tower_s is denoted by V (Middle Tower_s) and is given by $V(\text{Middle Tower } P_s) = \cup_{p=1}^s \{h_1^p, h_2^p, h_3^p, \dots, h_q^p\}$, where $q = 1, 2, 3, \dots, s-p+1$. We take $W = \{h_1^1, h_1^s\}$ resolving set of Middle Tower_s; then, representation $r(h_q^m|W) = (q+m-2, s-m)$, $q = 1, 2, 3, \dots, s-m+1$ and $m = 1, 2, 3, \dots, s$.

When $g \neq r$ and v_g and v_r possess the same representation for W , a contradiction arises, i.e.,

FIGURE 5: Middle tower path graph.

$$\begin{aligned}
 r(h_q^m|W) &= r(h_r^m|W) \\
 \Rightarrow (q+m-2, s-m) &= (r+m-2, s-m) \\
 \Rightarrow q &= r \text{ (contradiction)}.
 \end{aligned} \quad (43)$$

Similarly, when $m \neq n$,

$$\begin{aligned}
 r(h_q^m|W) &= r(h_q^n|W) \\
 \Rightarrow (q+m-2, s-m) &= (q+n-2, s-n) \\
 \Rightarrow m &= n \text{ (contradiction)}.
 \end{aligned} \quad (44)$$

When $m \neq n$ and $q \neq d$,

$$\begin{aligned}
 r(h_q^m|W) &= r(h_d^n|W) \\
 \Rightarrow (q+m-2, s-m) &= (d+n-2, s-n) \\
 \Rightarrow m &= n \text{ and } q = d \text{ (contradiction)}.
 \end{aligned} \quad (45)$$

So, W is the smallest resolving set for Middle Tower P_s and its metric dimension is 2. $\square$

1.4. Reflection Middle Tower Path Graph. Mirror image of Middle Tower P_s is named as Reflection Middle Tower P_s and is denoted by $rl(\text{Tower}_s)$.

Definition 4. Simple undirected graph possessing $V(rl(\text{Tower}_s)) = \{V_{(1)g}^t | 1 \leq t \leq s, 1 \leq g \leq t\} \cup \{V_{(2)g}^t | 1 \leq t \leq s, 1 \leq g \leq t\}$, where $t = 1, 2, 3, \dots, s$. Each vertex of $rl(\text{Tower}_s)$ is adjacent in the following manner.

- (1) $v_g^{(i)t} \sim v_g^{(i)t+1}$ when $i = 1, 2, g = 1, 2, 3, \dots, t$, and $t = 1, 2, 3, \dots, s-1$
- (2) $v_g^{(i)t} \sim v_{g+1}^{(i)t+1}$ when $i = 1, 2, g = 1, 2, 3, \dots, t$, and $t = 1, 2, 3, \dots, s-1$
- (3) $v_g^{(i)t} \sim v_{g+1}^{(i)t}$ when $i = 1, 2, g = 1, 2, 3, \dots, t$, and $t = 1, 2, 3, \dots, s-1$
- (4) $v_g^{(1)t} \sim v_{g+1}^{(2)t-1}$ when $g = 1, 2, 3, \dots, t-1$ and $t = s$
- (5) $v_{g+1}^{(1)t} \sim v_g^{(2)t-1}$ when $g = 1, 2, 3, \dots, t-1$ and $t = s$

Reflection middle tower path graph is explained in Figure 6.

Theorem 9. Metric dimension of $rl(\text{Tower}_s)$ is 2, for $s \geq 3$.

Proof. The vertex set of $rl(\text{Tower}_s)$ is $V(rl(\text{Tower}_s)) = (\cup_{g=2}^{s-1} \{l_1^{(1)g}, l_2^{(1)g}, l_3^{(1)g}, \dots, l_j^{(1)g}\} \cup (\cup_{g=2}^{s-2} \{l_1^{(2)g}, l_2^{(2)g}, l_3^{(2)g}, \dots, l_j^{(2)g}\}))$, where $j = 1, 2, 3, \dots, s-g+1$.

Assume $W = \{l_1^{(1)g}, l_2^{(1)g}\}$ as the resolving set; then,

$$r(l^{(1)1}_j|W) = \begin{cases} (s-1, s-j+1), & j = 1, 2, 3, \dots, s-1, \\ (s-1, 0), & j = s. \end{cases} \quad (46)$$

$$\begin{aligned}
 r(l^{(1)q}_j|W) &= (s-q, s-j), \quad \text{where } j = 1, 2, 3, \dots, s-1 \text{ and } q = 2, 3, 4, \dots, s. \\
 r(l^{(2)q}_j|W) &= (s-q-2, s-j), \quad \text{where } j = 1, 2, 3, \dots, q \text{ and } q = 2, 3, 4, \dots, s.
 \end{aligned}$$

When $m \neq n$ and l_m and l_n possess the same representation for W , a contradiction arises, i.e.,

$$\begin{aligned}
 r(l_m^{(1)1}|W) &= r(l_n^{(1)1}|W) \\
 \Rightarrow (s-1, s-m+1) &= (s-1, s-n+1) \\
 \Rightarrow m &= n \text{ (contradiction)}.
 \end{aligned} \quad (47)$$

Similarly, when $m \neq n$,

$$\begin{aligned}
 r(l_m^{(1)h}|W) &= r(l_n^{(1)h}|W) \\
 \Rightarrow (s-h, s-m) &= (s-h, s-n) \\
 \Rightarrow m &= n \text{ (contradiction)}.
 \end{aligned} \quad (48)$$

When $m \neq n$,

$$\begin{aligned}
 r(l_p^{(1)m}|W) &= r(l_p^{(1)n}|W) \\
 \Rightarrow (s-m, s-1) &= (s-h, s-1) \\
 \Rightarrow m &= n \text{ (contradiction)}.
 \end{aligned} \quad (49)$$

When $m \neq n$ and $u \neq v$,

$$\begin{aligned}
 r(l_u^{(1)m}|W) &= r(l_v^{(1)n}|W) \\
 \Rightarrow (s-m, s-l) &= (s-n, s-v) \\
 \Rightarrow m &= n \text{ and } u = v \text{ (contradiction)}.
 \end{aligned} \quad (50)$$

FIGURE 6: $rl(\text{Tower}_s)$ graph.

When $m \neq n$,

$$\begin{aligned} r(l_m^{(2)g}|W) &= r(l_n^{(2)g}|W) \\ \Rightarrow (s + g - 2, s - m) &= (s + g - 2, s - n) \\ \Rightarrow m &= n \text{ (contradiction).} \end{aligned} \quad (51)$$

When $m \neq n$ and $u \neq v$,

$$\begin{aligned} r(l_u^{(2)m}|W) &= r(l_v^{(2)n}|W) \\ \Rightarrow (s + m - 2, s - u) &= (s + n - 2, s - v) \\ \Rightarrow m &= n \text{ and } u = v \text{ (contradiction).} \end{aligned} \quad (52)$$

When $m \neq n$,

$$\begin{aligned} r(l_c^{(2)m}|W) &= r(l_c^{(2)n}|W) \\ \Rightarrow (s + m - 2, s - c) &= (s + n - 2, s - c) \\ \Rightarrow m &= n \text{ (contradiction).} \end{aligned} \quad (53)$$

Hence, W is the smallest resolving set and metric dimension of $rl(\text{Tower}_s)$ is 2. $\square$

1.5. Symmetrical Planar Pyramid Graph. A graph is obtained by joining the neighbouring vertices of Middle Tower P_s in its base and is represented by SPP_s . If there are s vertices in the base of SPP_s , then its vertex set contains $s(s-1)/2$ vertices.

Definition 5. We define SPP_s with s vertices in base possesses vertex set $\cup_{t=1}^s \{V_p^t\}$, where $t = 1, 2, 3, \dots, s$ and $p \leq t$.

Each vertex of tower_s is adjacent in the following manner.

- (1) $v_p^t \sim v_p^{t+1}$ when $p = 1, 2, 3, \dots, s-1$ and $t = 1, 2, 3, \dots, s-1$ and $p \leq t$
- (2) $v_p^t \sim v_{p+1}^{t+1}$ when $p = 1, 2, 3, \dots, s-1$ and $t = 1, 2, 3, \dots, s-1$
- (3) $v_p^t \sim v_{p+1}^t$ when $p = 1, 2, 3, \dots, s$ and $t = 1, 2, 3, \dots, s-1$

Symmetrical planar pyramid graph is explained in Figure 7.

Theorem 10. Metric dimension of SPP_s is 2.

Proof. The SPP has vertex set $V(\text{SPP}_s) = \cup_{t=1}^s \{V_p^t\}$, where $p = 1, 2, 3, \dots, s$ and $t = 1, 2, 3, \dots, s$. Also, $p \leq t$.

We take $W = \{v_1^1, v_s^s\}$ resolving set of Middle SPP_s and then representation $r(v_p^t|W) = (p + j - 1, s - t + j)$ $p = 1, 2, 3, \dots, s$ and $t = 1, 2, 3, \dots, s$. Also, $p \leq t$.

When $l \neq g$ and v_l and v_g possess the same representation for W , a contradiction arises, i.e.,

$$\begin{aligned} r(v_l^t|W) &= r(v_g^t|W) \\ \Rightarrow (l + j - 1, s - t + j) &= (g + j - 1, s - t + j) \\ \Rightarrow l &= g \text{ (contradiction).} \end{aligned} \quad (54)$$

Similarly, when $l \neq g$,

$$\begin{aligned} r(v_p^l|W) &= r(v_p^g|W) \\ \Rightarrow (p + j - 1, s - l + j) &= (p + j - 1, s - g + j) \\ \Rightarrow l &= g \text{ (contradiction).} \end{aligned} \quad (55)$$

When $l \neq g$ and $u \neq v$,

FIGURE 7: Symmetrical planar pyramid graph.

$$\begin{aligned}
 r(v_u^l | W) &= r(v_v^g | W) \\
 \Rightarrow (l + j - 1, s - u + j) &= (g + j - 1, s - v + j) \\
 \Rightarrow l &= g, u = v \text{ (contradiction)}.
 \end{aligned} \quad (56)$$

So, W is the smallest resolving set for SPP_s and its metric dimension is 2. $\square$

1.6. Reflection Symmetrical Planar Pyramid Graph. Mirror image of SPP_s is named as reflection symmetrical planar pyramid graph and is denoted by $rl(SPP_s)$.

Definition 6. Simple undirected graph possessing vertex set $V(rl(SPP_s)) = (\cup_{t=1}^s V_p^{(1)t}) \cup (\cup_{t=2}^{s-1} V_p^{(2)t})$, where $p = 1, 2, 3, \dots, s$ and $t = 1, 2, 3, \dots, s$; also, $s \leq t$. Each vertex of $rl(SPP_s)$ is adjacent in the following manner.

- (1) $v_p^{(i)t} \sim v_p^{(i)t+1}$ when $p = 1, 2, 3, \dots, s$, $t = 1, 2, 3, \dots, s-1$, $p \leq t$, and $i = 1, 2$
- (2) $v_p^{(i)t} \sim v_{p+1}^{(i)t+1}$ when $p = 1, 2, 3, \dots, s$, $t = 1, 2, 3, \dots, s-1$, $p \leq t$, and $i = 1, 2$
- (3) $v_p^{(i)t} \sim v_p^{(i)t}$ when $p = 1, 2, 3, \dots, s$, $t = 1, 2, 3, \dots, s-1$, $p \leq t$, and $i = 1, 2$
- (4) $v_p^{(1)t} \sim v_p^{(2)t-1}$ when $t = s$ and $g = 2, 3, 4, \dots, s$
- (5) $v_{p+1}^{(1)t} \sim v_p^{(2)t-1}$ when $t = s$ and $p = 1, 2, 3, \dots, s-1$

FIGURE 8: Reflection symmetrical planar pyramid graph.

Reflection symmetrical planar pyramid graph is explained in Figure 8.

Theorem 11. Metric dimension of $rl(SPP_s)$.

Proof. The vertex set of $rl(SPP_s)$ is $V(rl(SPP_s)) = (\cup_{t=2}^s V_p^{(1)t}) \cup (\cup_{t=1}^{s-1} V_p^{(2)t})$, where $p = 1, 2, 3, \dots, s$ and $t = 1, 2, 3, \dots, s$; also, $s \leq t$.

Assume $W = \{v_1^{(1)}, v_s^{(s)}\}$ as the resolving set; then,

$$\begin{aligned}
 r(v^{(1)t_p} | W) &= \{ (p - j - 1, s - (t + 1J)), \quad p = 1, 2, 3, \dots, s, t = 1, 2, 3, \dots, s, p \leq t \text{ and } j = t - p, \\
 r(v^{(2)t_p} | W) &= \{ (s - t + p - 1, 2s - t - 1), \quad j = 1, 2, 3, \dots, s - 1.
 \end{aligned} \quad (57)$$

When $l \neq g$ and v_l and v_g possess the same representation for W , a contradiction arises, i.e.,

$$\begin{aligned} r(v_l^{(1)t}|W) &= r(v_g^{(1)t}|W) \\ \Rightarrow (l + j - 1, s - t + j) &= (g + j - 1, s - g + j) \\ \Rightarrow l &= g \text{ (contradiction).} \end{aligned} \quad (58)$$

Similarly, when $l \neq g$,

$$\begin{aligned} r(v_p^{(1)l}|W) &= r(v_p^{(1)g}|W) \\ \Rightarrow (p + j - 1, s - l + j) &= (p - j - 1, s - g + j) \\ \Rightarrow l &= g \text{ (contradiction).} \end{aligned} \quad (59)$$

When $l \neq g$,

$$\begin{aligned} r(v_l^{(1)u}|W) &= r(v_g^{(1)p}|W) \\ \Rightarrow (l + j - 1, s - u + j) &= (g + j - 1, s - p + j) \\ \Rightarrow l &= g \text{ and } u = p \text{ (contradiction).} \end{aligned} \quad (60)$$

When $l \neq g$,

$$\begin{aligned} r(v_l^{(2)t}|W) &= r(v_g^{(2)t}|W) \\ \Rightarrow (s - t + l - 1, 2s - t - l) &= (s - t + g, 2s - t - 1) \\ \Rightarrow l &= g \text{ (contradiction).} \end{aligned} \quad (61)$$

When $l \neq g$,

$$\begin{aligned} r(v_p^{(2)t}|W) &= r(v_p^{(2)g}|W) \\ \Rightarrow (s - l + p - 1, 2s - t - 1) &= (s - g + p - 1, 2s - t - 1) \\ \Rightarrow l &= g \text{ (contradiction).} \end{aligned} \quad (62)$$

When $l \neq g$ and $u \neq p$,

$$\begin{aligned} r(v_l^{(2)u}|W) &= r(v_g^{(2)p}|W) \\ \Rightarrow (s - u + l - 1, 2s - u - 1) &= (s - v + g - 1, 2s - p - 1) \\ \Rightarrow l &= g \text{ and } u = p \text{ (contradiction).} \end{aligned} \quad (63)$$

Hence, W is the smallest resolving set and metric dimension of $rl(SPP_s)$ is 2. $\square$

2. Conclusions

In this dissertation, some new graphs as $s - \text{Mid}(G)$, $s - \text{Total}(G)$, Middle Tower_s, SPP_s , $rl(\text{Tower}_s)$, and $rl(SPP_s)$ are defined and metric dimension of these graphs is also computed. All these graphs possess constant and bounded metric dimension. This research work may lead to the following open problems.

Open problem 1: compute metric dimension of symmetrical pyramid in space.

Open problem 2: to find metric dimension of $s - \text{Mid}(C_n)$

Open problem 3: to find metric dimension of $s - T(C_n)$

Data Availability

No data were used to support this study.

Disclosure

The paper has not been published elsewhere, and it will not be submitted anywhere else for publication.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

Each author has equal contribution.

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