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Research Article

Applied Mathematical Modelling and Heat Transport Investigation in Hybrid Nanofluids under the Impact of Thermal Radiation: Numerical Analysis

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Nanofluids are solid-liquid mixtures that have a dispersion of nanometer-sized particles in conventional base fluids. The flow and heat transmission in an unstable mixed convection boundary layer are affected by the thermal conductivity and dynamic viscosity uncertainty of a nanofluid over a stretching vertical surface. There is time-dependent stretching velocity and surface temperature instability in both the flow and temperature fields. It is possible to scale the governing partial differential equations and then solve them using ordinary differential equations. Cu and Al_2O_3 nanofluids based on water are among the possibilities being investigated. An extensive discussion has been done on relevant parameters such as the unsteadiness parameter and the mixed convection parameter's effect on solid volume fraction of nanoparticles. In addition, alternative nanofluid models based on distinct thermal conductivity and dynamic viscosity formulas are examined for their flow and heat transmission properties. On the basis of the comparison, it is concluded that the results are spot on for steady state flow.

1. Introduction

When it comes to classic heat transfer fluids such as water, motor oil, ethylene glycol, and others, "nanofluid" has historically been defined as the distribution of nanometer-sized solid particles in these fluids. Many studies investigated how nanofluid size, shape, concentration, and other thermophysical parameters affect the pace at which heat is transferred. The interesting thermal properties and possible uses of this study area have sparked a lot of attention among scientists. In spite of this, a new generation of nanofluids has recently been developed using two different types of nanoparticles dispersed in a base fluid called "hybrid nanofluid". As a result of the synergistic effects of hybrid nanofluid, it can be used to improve the beneficial aspects of individual nanoparticles while minimizing the drawbacks of employing them independently.

Compared to other nanofluids, hybrid nanofluids are a relatively new class of nanofluids with several potential uses in a wide range of disciplines related to heat transfer, including microelectronics and microfluidics as well as transportation and manufacturing, medicine, and defense. It is possible for hybrid nanofluids to provide a wide range of advantages by dispersing nanoparticles properly. Compared to traditional fluid flow, nanofluid has a greater heat transfer rate. Hybrid nanofluid is added to make it even more effective. The current work is concerned with an issue of this nature and examines the features of heat transport over a stretching sheet. There have been numerous experimental studies done on hybrid nanofluid that have led to new technological concepts. As an example, researchers Suresh et al. [1] looked at how to synthesize the hybrid nanofluid (Al_2O_3–Cu/water). Momin [2] states that he performed an Inclined Tube Laminar Flow Experiment with (Al_2O_3–Cu/water) hybrid nanofluid [3]. According to the researchers, a hybrid Al_2O_3–Cu/water nanofluid affects heat transport. Suresh et al. [4] examined the turbulent heat transfer and pressure drop properties of water-based hybrid nanofluids (Al_2O_3–Cu/water). Using an Al_2O_3 Cu/water hybrid nanofluid as an electronic heat sink was investigated by Selvakumar and Suresh [5]. Gireesha et al. [6] found that a sinusoidal corrugated enclosure's heat transfer performance improved when hybrid nanofluid was used. Using a (Al_2O_3–Cu/water) hybrid nanofluid, Takabi and Shokouhmand [7] looked at how heat transmission and flow properties changed in a turbulent environment. To better understand how forced convection occurs in a flat plate solar collector, researchers Wang et al. [8] tested out a nanofluid that contained double nanoparticles. Acharya [9] examined the effectiveness of Al_2O_3 and Cu nanoparticles in a water-based hybrid suspension in laminar convection.

The issues surrounding MHD and nanofluids have grown in industrial importance during the last few decades. MHD power generators and electromechanical systems are used in numerous technical procedures. High temperatures create magnetic accelerators, so understanding hydromagnetic nanofluid flows is critical when developing new equipment. In the beginning, Sparrow and Cess [10] looked at how a magnetic field affected natural convection heat transfer. Chakrabarti and Gupta [11] examined the flow and transport of heat over a stretching sheet using hydromagnetic flow. According to Vajravelu [12], on a moving surface, hydromagnetic convective flow has been studied in great depth. Hydromagnetic flows past a stretched sheet include the notion of nanofluid presented for the increase of heat transmission. These fluids' advantages include increased heat transmission and stability as well as effective thermal conductivity. Higher energy economy, improved performance, and lower operating costs are all benefits of nanofluid thermal conductivity. Choi [13] demonstrated that adding a modest number of nanoparticles to standard heat transfer liquids increased the thermal conductivity by up to double the amount of the base fluid. Flow problems involving Newtonian heating have numerous practical applications in industry and engineering. In electrical devices, computer power supplies, and automobile engine cooling systems like radiators, these types of thermal flows determine the influence on the device. Thermal energy storage systems, nuclear power reactors, and gas turbines all use Newtonian heating.

Because of its many uses, boundary layer flows with Newtonian heating have been extensively researched by researchers in a variety of physical situations. Makinde [14] investigated the effects of viscosity dissipation and Newtonian heating on Sakiadis nanofluid flow. According to Yacob et al. [15], they looked at boundary layer flow in a nanofluid past a stretching/shrinking surface in external uniform shear flow with a convective surface boundary condition. The MHD-driven convection was discovered to be responsible for the laminar boundary layer flow of alumina-water nanofluid on a moving permeable flat plate [16]. To better understand nanofluid flow and heat transfer at a three-dimensional stagnation point, Bachok et al. [17] undertook this investigation. According to Attia [18], he looked into the flow of a three-dimensional hydromagnetic stagnation point toward a stretching sheet while creating heat. Khan et al. [11] conducted research to better understand the flow and heat transfer in three dimensions over a nonlinearly stretched sheet. Using a permeable stretched sheet and a convective boundary condition, Mansur and Ishak [19] showed that a nanofluid may flow and transmit heat in three dimensions.

In order to build on previous discoveries, the current study tackles the problem of unsteady flow and heat transmission in a stretching Al_2O_3–Cu/water hybrid nanofluid sheet. Water with Al_2O_3 and Cu nanoparticles is used to create a hybrid nanofluid. A similarity transformation is used to turn the governing equations and boundary conditions into an ordinary differential equations system. The boundary value problem solver in Mathematica is used to numerically solve the equations (bvp4c). Numerous influences on flow and heat transfer are shown graphically in this illustration. These numbers were checked against those from other studies to ensure they were accurate. To the best of the authors' knowledge, no one has ever examined hybrid nanofluid flow across a stretching sheet before.

2. Problem Formulation

An unstable, laminar, mixed convection boundary layer flow is generated by stretching a viscous and incompressible hybrid nanofluid sheet vertically (see Figure 1). Many different nanoparticles can be found in this water-based hybrid nanofluid such as copper, silver, aluminum oxide, and titanium dioxide. By assuming no heat transport between surfaces, we may rule out convection or convection-diffusion effects. Thermophysical properties of a nanofluid are shown in Table 1. The fluid and heat flows remain constant at time t 0. At time $t = 0$, a stretched sheet moves at a velocity through an instable fluid and heat flow Uw (x, t). Linear relationships exist between the temperature of the sheet Tw(x, t), and its decline over time is predicted by an inverse square law. An alternative Cartesian coordinate system uses a sheet as the starting point and continues upwards from there, whereas the y-axis is parallel to and positive in relation to the sheet's surface when compared to a fluid-based coordinate system. The terms "assisting flow" and "opposing flow" are interchangeable when discussing sheet heating and cooling. Thermal buoyancy and stiffening-induced flow work together to benefit the helping flow, but the opposite is true for the opposing flow.

Even if we assume the boundary layer approximation is true, in the case of nanofluids, the fundamental conservation equations for unsteady mass, momentum, and thermal energy are written as follows (see [21, 22]):

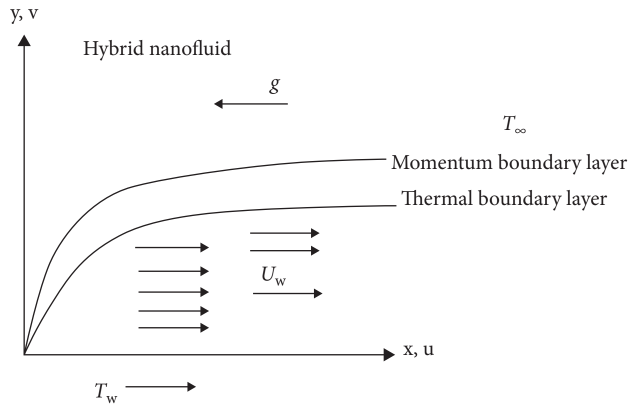

Figure 1: Schematic diagram of the problem.

Table 1: Thermophysical properties of H_2O, (Al_2O_3), and (Cu) (see [20]).

Properties	Pure water H_2O	Alumina (Al_2O_3)	Copper (Cu)
$\rho\,(kg/m^3)$	997.1	3970	8933
$C_p\,(J/kgk)$	4179	765	385
$k\,(Wm^{-1}k^{-1})$	0.6130	40	401
$\beta \times 10^5 k^{-1}$	21	0.850	1.670

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \quad (1)$$

$$\rho_{hnf}\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=\mu_{hnf}\frac{\partial^2 u}{\partial y^2}+g(\rho\beta)_{hnf}(T-T_\infty), \quad (2)$$

$$\left(\rho C_p\right)_{hnf}\left(\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=K_{hnf}\frac{\partial^2 T}{\partial y^2}-\frac{\partial q_r}{\partial y}, \quad (3)$$

with boundary conditions

$$\begin{aligned} u &= U_w,\\ v &= 0,\\ T &= T_w,\\ &\text{at } y=0,\\ u &\longrightarrow 0,\\ T &\longrightarrow T_\infty,\\ &\text{at } yn\infty, \end{aligned} \quad (4)$$

$u=\partial\psi/\partial y$, and $v=-\partial\psi/\partial x$, where ψ is dented stream function. We know that u and v are velocity components of the stream function defined as

$$\begin{aligned} u &= \frac{ax}{1-ct}F'(\eta)\\ v &= -\sqrt{\frac{av_f}{1-ct}}F(\eta). \end{aligned} \quad (5)$$

Equations (2) and (3) are used in $(\rho C_p)_{hnf}$, ρ_{hnf} is denoted as the density which is effective of the hybrid nanofluid, and μ_{hnf} is the dynamic viscosity, g denotes the gravity acceleration of the hybrid nanofluid, and β_{hnf} is thermal expansion of hybrid nanofluid. Now, for the hybrid nanofluids, as defined by the expression for ρ_{hnf}, $(\rho C_p)_{hnf}$, and $(\rho\beta)_{hnf}$, we have

$$\begin{aligned} \rho_{hnf} &= (1-\phi_2)\left\{(1-\phi_1)+\phi_1\frac{\rho_{s1}}{\rho_f}\right\}+\phi_2\frac{\rho_{s2}}{\rho_f},\\ \left(\rho C_p\right)_{hnf} &= (1-\phi_1)(1-\phi_2)+\phi_1(1-\phi_2)\frac{\left(\rho C_p\right)_{s1}}{\left(\rho C_p\right)_f}+\phi_2\frac{\left(\rho C_p\right)_{s2}}{\left(\rho C_p\right)_f},\\ (\rho\beta)_{hnf} &= \left\{(1-\phi_2)(1-\phi_1)+(1-\phi_2)\phi_1\frac{(\rho\beta)_{s1}}{(\rho\beta)_f}\right\}+\phi_2\frac{(\rho\beta)_{s2}}{(\rho\beta)_f}. \end{aligned} \quad (6)$$

The thermal conductivity and dynamic viscosity of hybrid nanofluid are defined as

$$\begin{aligned} A_2 &= \frac{\left(\left(K_{s1}+k_f\right)-2\phi_1\left(K_f-K_{s1}\right)\right)}{\left(\left(K_{s1}+2f\right)+\phi_1\left(K_f-K_{s1}\right)\right)},\\ \frac{K_{hnf}}{K_f} &= \frac{\left(\left(K_{s2}+2A_2\right)-\phi_2\left(A_2-K_{s2}\right)\right)}{\left(\left(K_{s2}+2A_2\right)+\phi_2\left(A_2-K_{s2}\right)\right)},\\ \mu_{hnf} &= \frac{\mu_f}{(1-\phi_1)^{2\cdot 5}(1-\phi_2)^{2\cdot 5}}, \end{aligned} \quad (7)$$

where μ_{hnf} is the dynamic viscosity of the base hybrid nanofluid, ϕ_1 and ϕ_2 are the volume fraction for the hybrid nanoparticles, and K_{bf} and K_{hnf} are the thermal conductivities and hybrid nanoparticles, respectively. The hybrid nanofluids of properties presented in the above determinations are considered from H_2O, and the average amounts temperature are properties of hybrid nanoparticles. Consider that the velocity and temperature of the surface of the stretching sheet are defined as

$$U_w(x,t) = \frac{ax}{1-ct},$$
$$T_w(x,t) = T_\infty + \frac{bx}{(1-ct)^2}. \tag{8}$$

q_r symbolized the radiative temperature flux:

$$q_r = -\left(\frac{4\sigma^*}{3k^*}\right)\frac{\partial T^4}{\partial y}. \tag{9}$$

We can define the dimensionless function F and θ as

$$\eta = \sqrt{\frac{a}{v_f(1-ct)}}y,$$
$$\psi = \sqrt{\frac{av_f}{(1-ct)}}xF(\eta), \tag{10}$$
$$\theta(\eta) = \frac{T-T_\infty}{T_w-T_\infty},$$

where η is similarity variable, θ is the dimensionless heat transfer, and $A = ca^{-1}$ is a parameter that measures the unsteadiness. So, equation (1) is identically satisfied. The transformation of mass, momentum, and energy equations are obtained as

$$F'''(\eta) + (1-\phi_1)^{2.5}(1-\phi_2)^{2.5}\left[\left\{(1-\phi_2)\left((1-\phi_1)+\phi_1\frac{\rho_{s1}}{\rho_f}\right)+\phi_2\frac{\rho_{s2}}{\rho_f}\right\}\left\{F(\eta)F''(\eta)-(F'(\eta))^2 - A\left(F(\eta)+\frac{1}{2}\eta F'''(\eta)\right)\right\}\right]$$
$$+\left\{(1-\phi_2)\left((1-\phi_1)+\phi_1\frac{(\rho\beta)_{s1}}{(\rho\beta)_f}\right)+\phi_2\frac{(\rho\beta)_{s2}}{(\rho\beta)_f}\right\}\gamma\theta(\eta) = 0, \tag{11}$$

$$\frac{1}{\left(\rho C_p\right)_{hnf}\Pr}\left(\frac{K_{hnf}}{K_f} + R\,d\left(1+(\theta_w-1)\right)^3\right)\theta''(\eta) + \frac{A\eta}{2}\theta'(\eta) + F(\eta)\theta'(\eta) = 0. \tag{12}$$

The transformed boundary conditions are as follows. At $\eta \longrightarrow 0$,

$$F(0) = 0,$$
$$\theta(0) = 1, \tag{13}$$
$$F'(0) = 1.$$

At $\eta \longrightarrow \infty$,

$$F'(\eta) \longrightarrow 0,$$
$$\theta(\eta) \longrightarrow 0, \quad \text{at}\ \eta \longrightarrow \infty, \tag{14}$$

where Re is the Reynolds number, Gr is the Grashof number, and γ is the buoyancy parameter given by

$$\mathrm{Re}_x = \frac{U_w x}{v_f},$$
$$Gr_x = \frac{g\beta_f(T_w-T_\infty)}{v_f^2}x^3, \tag{15}$$
$$\gamma = Gr_x \mathrm{Re}_x^{-2}.$$

The assisting and opposing flows correspond to gamma greater than zero or gamma less than zero, respectively.

Furthermore, $\Pr = v_f/\propto_f$ represents the Prandtl number and also the relationship between the density and specific heat for the hybrid nanofluids. The shear stresses, skin friction, and Nusselt number are defined by the following mathematical relations:

$$\tau_w = \mu_{hnf},$$
$$C_f = \frac{2\tau_w}{\rho_f U_w^2}, \tag{16}$$
$$Nu_x = \frac{xq_w}{K_f(T_w-T_\infty)},$$

where q_w is the hotness transfer from the sheet defined as follows:

$$q_w = -K_{hnf}\left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{17}$$

Applying transformation in the nondimensional parameter, we get

$$C_f \mathrm{Re}_2^{1/2} = 2\left(\frac{\mu_{hnf}}{\mu_f}\right)F''(0),$$
$$Nu_x \mathrm{Re}_2^{-1/2} = -\frac{K_{hnf}}{K_f} + R\,d\left(1+\left(\theta_w-1\right)\right)^3\theta'(0). \quad (18)$$

3. Solution Procedure

We considered unsteady flow and temperature transfer of hybrid nanofluids described in equations. Mathematical relationships signifying the unsteady flow of hybrid nanofluid are represented in equations (7) and (8). The calculated method is called the shooting method to convert higher-order ODEs to first-order ODEs. The following transformations are obtained:

$$\begin{aligned}
y_1 &= F,\\
y_2 &= F',\\
y_3 &= F'',\\
y_4 &= F'''.\\
y_5 &= \theta,\\
y_6 &= \theta',\\
y_7 &= \theta'',\\
y_1' &= F' = y_2,\\
y_2' &= F'' = y_3,\\
y_3 &= F''' = y_4.\\
y_5' &= \theta' = y_6,\\
y_6' &= \theta'' = y_7\\
y_3' &= \left[\left(1-\phi_1\right)^{2.5}\left(1-\phi_2\right)^{2.5}\left\{\begin{array}{c} nA_1\left(A\left(y_1+\frac{1}{2}\eta y_3\right)+y_2^2-y_1y_3\right)+ \\ n\left\{\left(1-\phi_1\right)\left[\left(1-\phi_2\right)+\phi_1\frac{(\rho\beta)_{s1}}{(\rho\beta)_f}\right]+\phi_2\frac{(\rho\beta)_{s2}}{(\rho\beta)_f}\right\}\gamma y_5\end{array}\right\}\right],\\
y_6' &= \frac{A_4\mathrm{Pr}}{\left(A_3+R\,d\left(1+\left(\theta_w-1\right)y_5\right)^3\right)}\left[y_1y_6-\frac{1}{2}A\eta y_6\right].
\end{aligned} \quad (19)$$

For the sake of comparison, we have also solved the same problem by using the R-K-4 method (coupled with shooting technique) and the results are compared in Table 2. Both solutions show an excellent agreement with each other. These solutions are calculated for $=\theta_w = R\,d = 1\cdot 2$ and $\phi 1 = \phi 2 = 0$, and the Prandtl number is taken to be 6.2.

4. Graphical Outcomes and Discussion

We will use graphs to show how various emergent parameters such as velocity and temperature distributions are affected. The hybrid Cu–Al_2O_3 water-based nanofluid is shown in nanofluid-based diagrams. Special attention is being paid to the comparison of nanofluids and hybrid

Table 2: Comparison between the present result and [23].

γ	A	Ref [23]	Current result
0.2	0.4	4.675	4.6751
0.4		3.458	3.4583
0.6		3.123	3.1232
0.8	0.0	2.537	2.5371
	0.5	3.828	3.8283
	1.0	3.342	3.3422

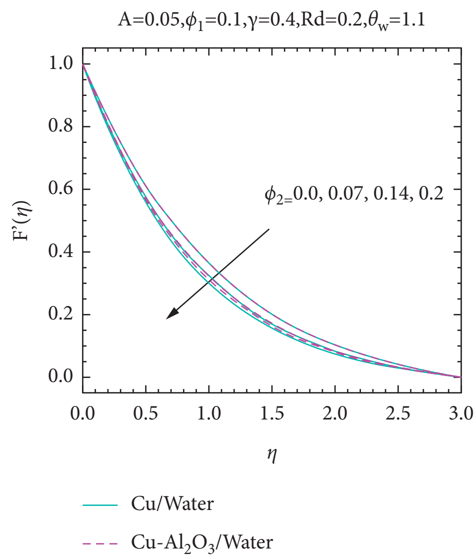

Figure 2: Variation of ϕ_2.

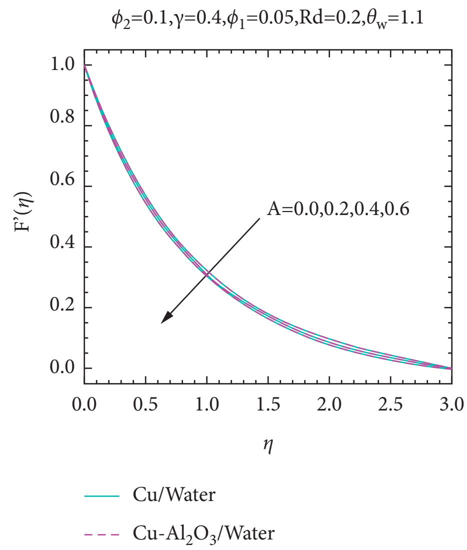

Figure 3: Effect of A.

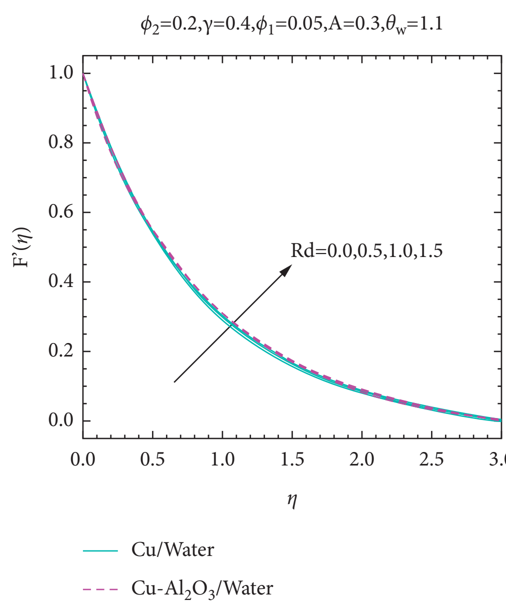

Figure 4: Variation of $R\,d$.

nanofluids. In the following diagrams, static lines depict nanofluid outcomes, whereas pointed lines depict hybrid nanofluid results. Thermal radiation Rd, buoyancy parameter γ, and temperature difference parameter θ_w are presented in Figures 2–11 for specific hybrid volumetric fractions $\varnothing_2$, respectively. Hybrid volumetric fractions $\varnothing_2$ have an impact on velocity, as shown in Figure 2. Figure 2 shows that hybrid volumetric fractions $\varnothing_2$ contribute to reducing velocity, as can be shown. Because the fluid movement is caused by the expansion of the surface, any changes in the fluid flow on the expanding surface will slow the flow.

Figure 3 shows the increasing velocity in the opposite direction. Velocity drops as unsteadiness parameter A grows, as can be seen in the figure. However, this computation shows that the decrease in velocity for hybrid nanofluids is significantly greater. As the thermal radiation Rd increases, the fluid flow becomes more intense, as seen in Figure 4, which shows the effect on the velocity profile. The boundary layer is getting thicker at the same time. In terms of physics, the curved surface's pliability aids fluid flow over it. In hybrid nanofluids, the increase in speed is a little more noticeable. Figure 5 shows how the buoyancy parameter gamma affects the velocity of a hybrid nanofluid. This image shows that the hybrid nanofluid flows faster than the nanofluid for a large buoyancy parameter γ of Cu and Cu–Al_2O_3.

Figure 6 depicts the temperature differential parameter θ_w as it changes over time. In terms of temperature, θ_w makes a big improvement. θ_w must be greater than 1 because it is the ratio of wall and free surface temperatures. As θ_w increases, the temperature should rise and the thermal boundary layer should thicken. Stretch and shrink temperatures are clearly separated by a wide margin of error. Similar emergent factors have different effects on the temperature profile, as seen in Figures 7–11. The graphs show that nanofluids and mixed nanofluids can be more accurately represented. In addition, five different types of variables are shown in each illustration. The efficiency of nanofluids and hybrid nanofluids is shown by solid and pointed lines, respectively. To show increasing parameters, the dotted and solid lines have been used. These parameters include hybrid volumetric fractions $\varnothing_2$, unsteadiness, and thermal Radiation Rd, as well as buoyancy γ and temperature difference θ_w.

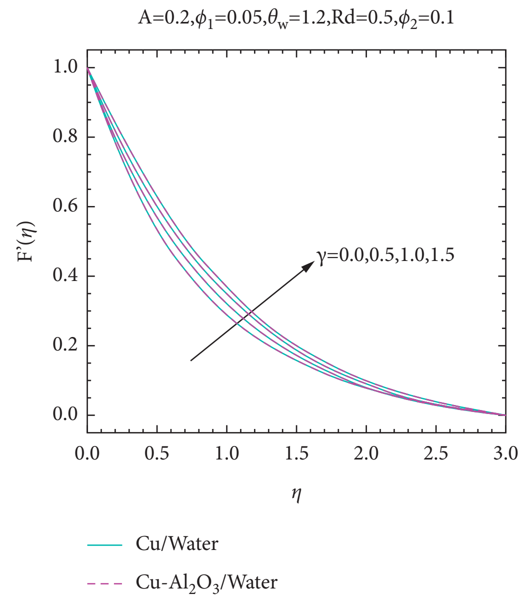

Figure 5: Variation of γ.

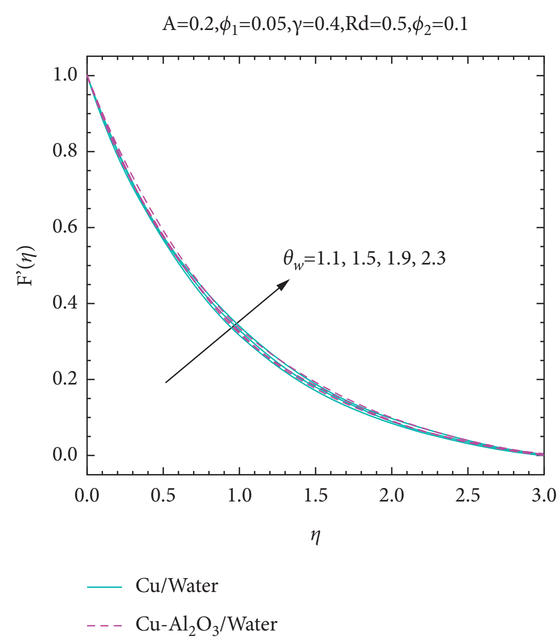

Figure 6: Variation of θ_w.

It is shown in Figure 7 that unsteadiness parameter A is affected by temperature. This graph shows that as temperature exponent A rises, so does the temperature. This is clear. Because the increase in A's value improves the conduction's results, the temperature rises as a result. In the field, this rise is broad and contributes to the air heat flux. Figure 8 depicts the effect of hybrid volumetric fractions $\varnothing_2$ on nanofluid and hybrid nanofluid temperatures, showing that as hybrid volumetric fractions $\varnothing_2$ increase, the thickness of the surface and the thermal limit layer decreases. 2 contributes to slow down of both fluid flows and temperature rise because it has a hybrid volumetric $\varnothing_2$ effect.

The magnitude of the temperature for hybrid nanofluids is higher than for nanofluids at hybrid volumetric fractions $\varnothing_2$ based on this result. Figure 9 depicts the temperature operation for the buoyancy parameter γ. As the buoyancy parameter γ increases, so does the temperature. As can be shown in Figure 10, the unsteadiness parameter A of a

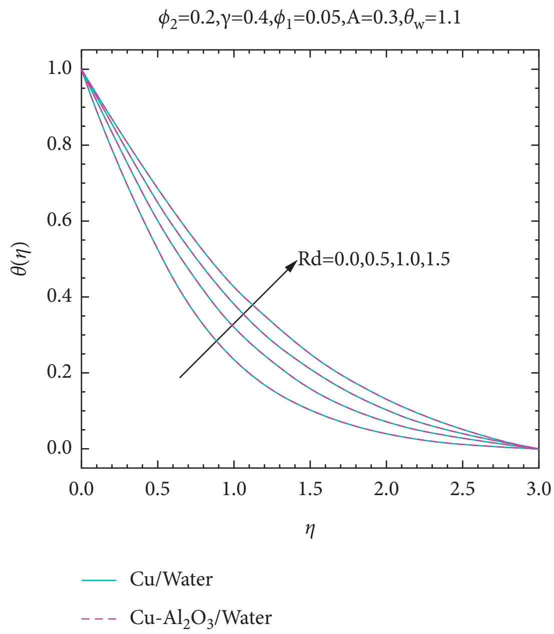

Figure 7: Variation of Rd.

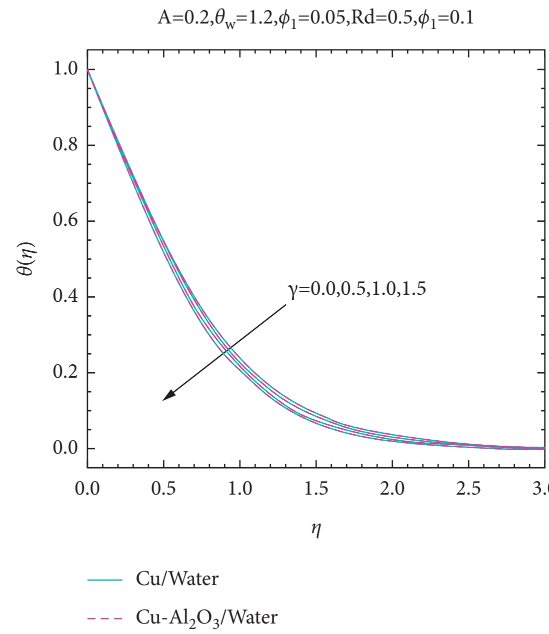

Figure 9: Variation of γ.

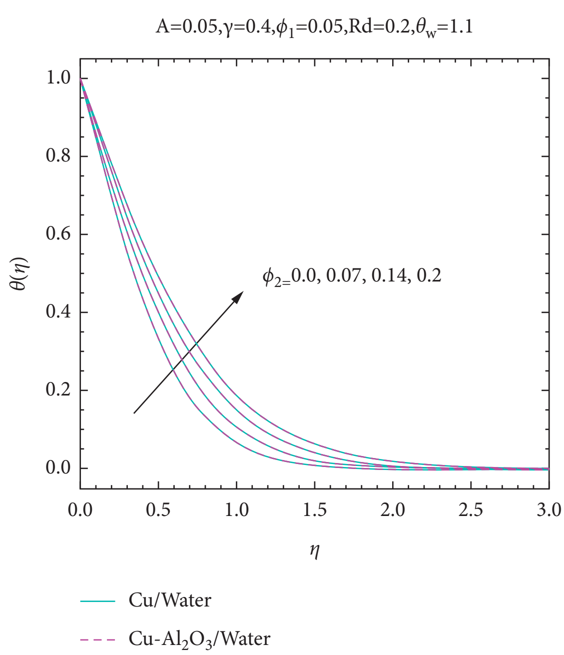

Figure 8: Variation of ϕ_2.

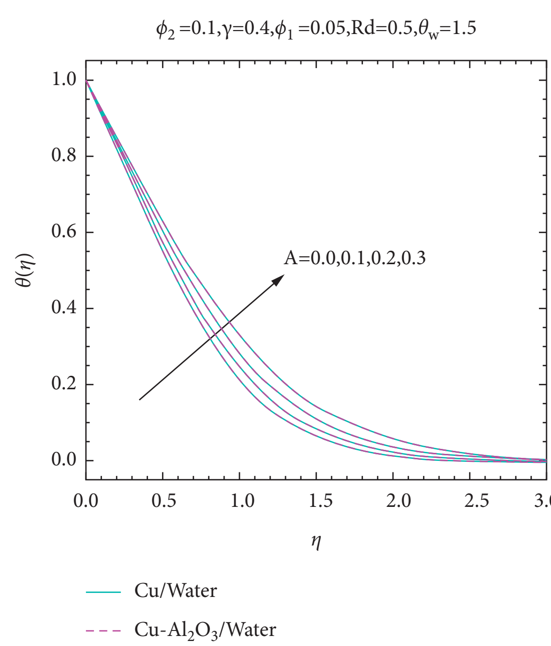

Figure 10: Variation of A.

hybrid nanofluid has an impact on the temperature. This graph indicates that hybrid nanofluids move faster than nanofluids for large unsteadiness parameter A. θ_w is a temperature difference parameter. The results are shown in Figure 11. This graph showed that raising the parameter θ_w changed the temperature distribution. In other words, because it is hotter up there, more heat is being radiated from the ground, which in turn makes for a thicker, more stable thermal layer. Hybrid nanofluids may also be affected by this temperature increase.

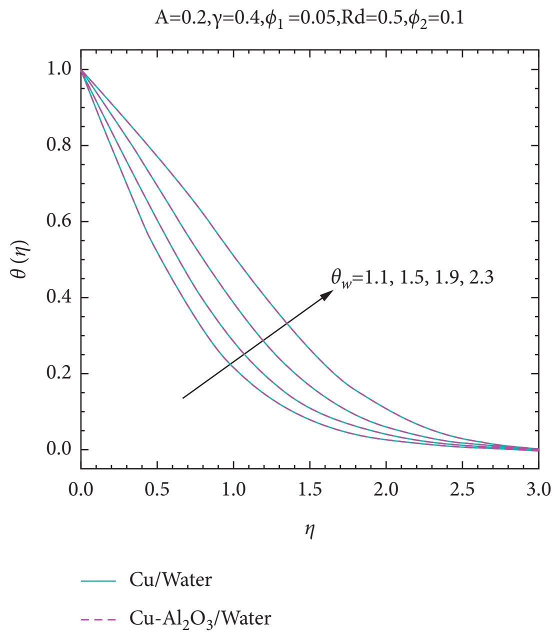

Figure 11: Variation of θ_w.

5. Conclusions

An unstable mixed convective boundary layer in two dimensions has been researched to see how well a stretching vertical surface transfers heat and flow while employing nanofluids. We talked about the impact of the governing parameters $\varnothing_2$, A, Rd, γ, andθ_w on the system. Mathematica's ND Solver software was utilized. Numerical analysis was carried out to give the flow problem a physical imagination for different assessments of the flow quality parameters. The results for $F'(\eta)$ and $\theta(\eta)$ are represented visually. Here are some of the more significant and interesting takeaways from this article:

(i) The increase of nanoparticle volumetric fractions $\varnothing_2$ of Cu/water and Al_2O_3 − Cu/water decreases both the momentum and thermal boundary layer thickness

(ii) $F'(\eta)$ decreases and $\theta(\eta)$ increases with rising values of A

(iii) To begin with, we discovered that $\theta(\eta)$ magnifies for greater $\varnothing_2$ values, whereas $F'(\eta)$ responds in the opposite manner

(iv) There is an inverse relationship between the percentage of trustworthy nanoparticulate content and both velocity and temperature fields γ

(v) Increasing the thermal radiation Rd has an impact on the velocity and temperature fields

Data Availability

The study was based on numerical technique and no data were used in findings of the study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

[1] N. Acharya and F. Mabood, "On the hydrothermal features of radiative Fe3O4-graphene hybrid nanofluid flow over a slippery bended surface with heat source/sink," *Journal of Thermal Analysis and Calorimetry*, vol. 143, no. 2, pp. 1273–1289, 2021.

[2] G. G. Momin, "Experimental investigation of mixed convection with Al_2O_3/water & hybrid nanofluid in inclined tube for laminar flow," *International Journal of Scientific & Technology Research*, vol. 2, pp. 195–202, 2013.

[3] T. Hayat, F. Shah, M. I. Khan, M. I. Khan, and A. Alsaedi, "Entropy analysis for comparative study of effective Prandtl number and without effective Prandtl number via γAl2O3-H2O and γAl2O3-C2H6O2 nanoparticles," *Journal of Molecular Liquids*, vol. 266, pp. 814–823, 2018.

[4] S. Suresh, K. P. Venkitaraj, M. S. Hameed, and J. Sarangan, "Turbulent heat transfer and pressure drop characteristics of dilute water based Cu-Al_2O_3 hybrid nanofluids," *Journal of Nanoscience and Nanotechnology*, vol. 14, Article ID 25632572, 2014.

[5] P. Selvakumar and S. Suresh, "Use of ${\rm Al}_{2}{\rm O}_{3}\hbox{--}{\rm Cu}$/Water hybrid nanofluid in an electronic heat sinkfluid in an electronic heat sink," *IEEE Transactions on Components, Packaging, and Manufacturing Technology*, vol. 2, no. 10, pp. 1600–1607, 2012.

[6] B. J. Gireesha, G. Sowmya, M. I. Khan, and H. F. Öztop, "Flow of hybrid nanofluid across a permeable longitudinal moving fin along with thermal radiation and natural convection," *Computer Methods and Programs in Biomedicine*, vol. 185, Article ID 105166, 2020.

[7] B. Takabi and H. Shokouhmand, "Effects of Al2O3-Cu/water hybrid nanofluid on heat transfer and flow characteristics in turbulent regimeffects of Cu-Al2O3/water hybrid nanofluid on heat transfer and flow characteristics in turbulent regime," *International Journal of Modern Physics C*, vol. 26, no. 4, Article ID 1550047, 2015.

[8] J. Wang, M. I. Khan, W. A. Khan, S. Z. Abbas, and M. I. Khan, "Transportation of heat generation/absorption and radiative heat flux in homogeneous-heterogeneous catalytic reactions of non-Newtonian fluid (Oldroyd-B model)," *Computer Methods and Programs in Biomedicine*, vol. 189, Article ID 105310, 2020.

[9] N. Acharya, "Spectral quasi linearization simulation on the hydrothermal behavior of hybrid nanofluid spraying on an inclined spinning disk," *Partial Differential Equations in Applied Mathematics*, vol. 4, Article ID 100094, 2021.

[10] K. Vajravelu, "Hydromagnetic convection at a continuous moving surface," *Acta Mechanica*, vol. 72, no. 3-4, pp. 223–232, 1988.

[11] R. Ahmad and W. A. Khan, "Unsteady heat and mass transfer magnetohydrodynamic (MHD) nanofluid flow over a stretching sheet with heat source-sink using quasi-linearization techniquefluid flow over a stretching sheet with heat source-sink using quasi-linearization technique," *Canadian Journal of Physics*, vol. 93, no. 12, pp. 1477–1485, 2015.

[12] K. Kumar and S. Bandari, "Melting heat transfer in boundary layer stagnation-point flow of a nanofluid towards a stretching-shrinking sheetfluids with nanoparticles," *Canadian Journal of Physics*, vol. 92, no. 12, pp. 1703–1708, 2014.

[13] M. A. Mansour, R. A. Mohamed, M. M. Abd-Elaziz, and S. E. Ahmed, "Numerical simulation of mixed convection flows in a square lid-driven cavity partially heated from below

using nanofluid," *International Communications in Heat and Mass Transfer*, vol. 37, no. 10, pp. 1504–1512, 2010.

[14] S. Choi, "Enhancing thermal conductivity of fluids with nanoparticle," in *Proceedings of the Developments and Applications of Non-newtonian Flows, ASME FED*, D. A. Signer, Ed., pp. 99–105pp. 99–, San Francisco, CA, USA, November 1995.

[15] L. Sunder and K. V. Sharma, "Numerical analysis of heat transfer and friction factor in a circular tube with Al2O3 nanofluid," *International Journal of Dynamics of Fluids*, vol. 4, pp. 121–129, 2008.

[16] S. Sureh, K. Venkitaraj, p. Selvakumar, and M. Chandrasekar, "Effect of Cu-Al_2O_3/water hybrid nanofluid in heat transfer," *Experimental Thermal and Fluid Science*, vol. 38 9, pp. 54–60, 2012.

[17] W. A. Khan and I. Pop, "Boundary-layer flow of a nanofluid past a stretching sheet," *International Journal of Heat and Mass Transfer*, vol. 53, no. 11-12, pp. 2477–2483, 2010.

[18] A. Chakrabarti and A. S. Gupta, "Hydromagnetic flow and heat transfer over a stretching sheet," *Quarterly of Applied Mathematics*, vol. 37, no. 1, pp. 73–78, 1979.

[19] H. I. Andersson and B. S. Aarseth, "Heart transfer in a liquid film on an unsteady stretching surface," *International Journal of Thermal Sciences*, vol. 50, pp. 385–393, 2000.

[20] M. J. Nine, B. Munkhbayar, M. S. Rahman, H. Chung, and H. Jeong, "Highly productive synthesis process of well dispersed Cu2O and Cu/Cu2O nanoparticles and its thermal characterization," *Materials Chemistry and Physics*, vol. 141, no. 2-3, pp. 636–642, 2013.

[21] M. Sheikholeslami, T. Hayat, and A. Alsaedi, "MHD free convection of Al2O3-water nanofluid considering thermal radiation: a numerical study," *International Journal of Heat and Mass Transfer*, vol. 96, pp. 513–524, 2016.

[22] M. Sheikholeslami and M. M. Rashidi, "Non-uniform magnetic field effect on nanofluid hydrothermal treatment considering Brownian motion and thermophoresis effects," *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, vol. 38, no. 4, pp. 1171–1184, 2016.

[23] A. Mahdy, "Unsteady mixed convection boundary layer flow and heat transfer of nanofluids due to stretching sheet," *Nuclear Engineering and Design*, vol. 249, pp. 248–255, 2012.