Research Article

A Study of Magnetic/Nonmagnetic Nanoparticles Fluid Flow under the Influence of Nonlinear Thermal Radiation

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The present research work scrutinizes numerical heat transfer in convective boundary layer flow having characteristics of magnetic (Fe_3O_4) and nonmagnetic (Al_2O_3) nanoparticles synthesized into two different kinds of Newtonian (water) and non-Newtonian (sodium alginate) convectional base fluids of Casson nanofluid which integrates the captivating effects of nonlinear thermal radiation and magnetic field embedded in a porous medium. The characterization of electrically transmitted viscous incompressible fluid is taken into account within the Casson fluid model. The mathematical formulation of governing partial differential equations (PDEs) with highly nonlinearity is renovated into ordinary differential equations (ODEs) by utilizing the suitable similarity transform that constitutes nondimensional pertinent parameters. The transformed ODEs are tackled numerically by implementing *bvp4c* in MATLAB. A graphical illustration for the purpose of better numerical computations of flow regime is deliberated for the specified parameters corresponding to different profiles (velocity and temperature). To elaborate the behavior of Nusselt and skin friction factor, a tabular demonstration against the distinct specific parameters is analyzed. It is perceived that the velocity gradient of Newtonian fluids is much higher comparatively to non-Newtonian fluids. On the contrary, the thermal gradient of non-Newtonian fluid becomes more condensed than that of Newtonian fluids. Graphical demonstration disclosed that the heat transfer analysis in non-Newtonian (sodium alginate)-based fluid is tremendously influenced comparatively to Newtonian (water)-based fluid, and radiation interacts with the highly denser temperature profile of non-Newtonian fluid in contrast to that of Newtonian fluid. Through such comparative analysis of magnetic or nonmagnetic nanoparticles synthesized into distinct base fluids, a considerable enhancement in thermal and heat transfer analysis is quite significant in many expanding engineering and industrial phenomena.

1. Introduction

The study of non-Newtonian fluids enhanced the substantial attention of researchers due to its extensive utilization in mechanical processing, food industry, paper production, biological engineering, polymer industry, and numerous other concerned industries during the last few decades. In fact, in the mathematical and physical modeling of non-Newtonian fluids, there exist many models which encompass the Walter-B model, the power law model, the Williamson model, and the viscoelastic model that bring forward the analysis of such fluids. However,

these models do not fulfill all the rheological properties of non-Newtonian fluids. The Casson model is one of the most preferred models initiated in 1959 [1], which incorporates shear thinning behavior and yield stress. The classification of Casson fluid in the category of non-Newtonian fluids is related with the relationship between the shear stress-strain and its rheological characteristics. It switches its behavior towards a Newtonian fluid above a critical stress and acts like an elastic solid at low shear strain. The following liquids, i.e., soup, tomato, sauce, and human blood, correspond with the characteristics of Casson fluid [2].

Within the exceeding developments in industrial and engineering sectors such as transportation, production and supply of energy, and electronics, the requirements for the evolvment of heat transfer and higher thermal conductivity have been increased. To overcome the needs for enhancing the thermal conductivity, small-sized solid particles were suspended into regular fluids and a disruption in the fluids was observed as consequences. Furthermore, nanosized particles were used into convectional-based fluids, which cause hike in the thermal conductivity. Such inclusion of microsized particles flourished the concept of nanofluid [3–5]. The nativity of nanofluid is imputed to exhaustive idea of adding nanosized particles (having capacity of 10–100 nm) into base fluids (glycol, ethylene, and oil) to achieve the high thermal conductivity (which is the prominent attribute of nanofluids). In heat transfer fields, nanomaterials (magnetic, nonmetallic, and nanotubes) own a bright future of heat transfer fluids. By dispersing a small amount of nanoparticles, thermal conductivity can be enhanced to a remarkable level. However, it can be managed by changing the nanoparticle size, as it improves with the declination in nanomaterials. Initially, the research in the field of nanomaterials was based on the enhancement of the conductivity of fluids in the numerous engineering field transport engineering (automotive and aerospace), power plant cooling, drilling thermal storage, etc. Later on, in the era of nanotechnology, inventions were significantly enlarged in the fields of inorganic-based nanoparticles, bio-convection, microorganisms, stem cells, etc. [6, 7].

Generally, the formation of nanoparticles consists of titanium dioxide, copper, graphite, aluminium oxide, and silicon monoxide, and these nanoparticles cause elevation of the thermal conductivity than the base fluids. In [8], the dispersion of Cu nanosized particles into ordinary base fluid (ethylene glycol) tremendously enhanced 40% thermal conductivity of fluid. Later on, it was reported that the thermal conductivity of fluids increased more than 20% by the suspension of just [1–5]% nanoparticles. So, the analysis of thermal conductivity was experimentally by Boungiorno's model [9]. Eventually, another model was proposed by the same researcher, and it was perceived that the effects of Brownian diffusion and thermophoresis also have the capability to incline the thermal conductivity [10]. The succession of research on nanofluid can be further found in [11–15]. The synthesis of magnetic nanoparticles broads up a new era of research which is called ferrofluid [16]. These nanoparticles are helpful in the usage of numerous base fluids having roots of applications in many engineerings and industrial fields. Such prescribed particles urge to tackle with the rapidly emerging needs for numerous industries (violetical engineering and energy needs in engineering). To overcome the challenges of waste water treatment processes, magnetic nanoparticles are tested to eliminate the toxins from such processes. These are also beneficial for targeting the specified portion of the human body to demolish the cancer or tumor cells. Some pioneering research about the ferrofluids can be established in [17–25].

The innovations in the field of science are the need of time. This need urges the researchers attention towards

flow in a porous medium. Porous medium has numerous directions of exploration in different fields of engineering. Flow in subterranean porous layers is the main cause of energy transformation, i.e., power plants. So, these phenomenons are beneficial in energy engineering. These are also helpful during energy exploration (petroleum refinery engineering), biomedical engineering, and civil engineering. Different models were proposed for handling of such a phenomenon in which amendments are being made with the passage of time. As customary, the porous media dealt by Darcy and Darcy-modified flow models and homogeneous and isotropic porous media. Modern and acceptable models are structures with coarse porosity and heterogeneous media. The urge for the further new models results in the directions of aerosol transport and porous filters [26, 27].

The phenomenon of boundary layer flow persuades by stretching surface used in many fabricating processes expeditiously. The problem of stretching surface is highly recommended due to its exceptionally range of applications. It is beneficial in numerous industrial and engineering processes such as extrusion of plastic sheets, hot rolling, melt-kneading, glass fibril, manufacturing of plastic and caoutchouc sheets, and refrigeration of wide metallic plate in a bath, which can be brine. By continuous extrusion of the polymer, filaments and polymer sheets are manufactured in industry from a die to a windup roller and kept away at a finite distance. The thin polymer sheet constitutes a continuously moving surface with a nonuniform velocity through an ambient fluid [28–30].

A combination of Newtonian- and non-Newtonian-based fluids along with magnetic and nonmagnetic nanosized particles was firstly explored by A. Hakeem in 2017. Investigations revealed that the effect of magnetic field on magnetic nanoparticles is more influential than on the nonmagnetic nanoparticles. It was also claimed that the specific quantities (Nusselt number and skin friction coefficient) are higher in the phenomenon of non-Newtonian-based nanofluids in comparison of Newtonian-based nanofluids. They extended their investigation towards the two different sorts of Newtonian (water)- and non-Newtonian (sodium alginate)-based fluids of two-dimensional convective flow and heat transfer along a flat plate. The dispersion of nonmagnetic nanoparticle and magnetite Fe_3O_4 nanoparticles in convectional base fluids was taken into account and concluded effects of the slip conditions and thermal radiation. To compensate with the highly emerging needs for nanoparticles, the improvisation of magnetic/nonmagnetic nanoparticles is analyzed by observing the performance of different base fluids on heat and mass transfer [31, 32]. The variation in thermophysical properties of nanoparticles fluid flow (ρ, C_p, k, σ) for two different base fluids (water and sodium alginate) is briefly examined. Due to the prescribed properties of magnetic and nonmagnetic nanoparticles into Newtonian and non-Newtonian base fluids are chosen for the present study. The current research will be beneficial and sustainable in numerous industries and will overcome the emerging needs of science.

2. Mathematical Formulation of the Flow Model

Forced convectional, steady, laminar, and incompressible flow of Newtonian and non-Newtonian base fluids dispersed into magnetic and nonmagnetic nanoparticles of Casson nanofluid is contemplated in the existence of porous medium. The suspended magnetic and nonmagnetic nanoparticles into convectional base fluids are being considered with the assumption of thermal equilibrium. The flow starts off at the dominating edge of the sheet which is inspected as the origin in the frame of reference. The coordinates (x, y) are measured along the stretching sheet in horizontal and vertical direction, respectively, as illustrated in Figure 1. The rheology of incompressible casson nanofluid state can be written down as [33]

$$\tau = \tau_0 + \mu\sigma^*, \quad (1)$$

$$\text{or } \tau_{mn} = \begin{cases} 2\left(\mu_B + \frac{p_\lambda}{\sqrt{2\pi}}\right)e_{mn}, & \pi > \pi_c, \\ 2\left(\mu_B + \frac{p_\lambda}{\sqrt{2\pi_c}}\right)e_{mn}, & \pi < \pi_c, \end{cases} \quad (2)$$

where $\pi = e_{mn}e_{mn}$ indicates that the factor of the deformation rate that is connected to itself by the usual multiplication and the critical value of such product based of non-Newtonian fluid is represented by π_c . e_{mn} is the $(m, n)^{\text{th}}$ factor of the deformation rate; μ_B and p_λ symbolize the plastic dynamic viscosity and yield stress of Casson fluid, respectively.

The free stream or far field velocity of nanofluid is supposed to be U_∞ . Due to small magnetic Reynolds number, the effects of induced magnetic field is negligible and a uniform intensity of magnetic field B_0 is considered. Under these considerations, the boundary layer estimations yield the dimensional form as follows [34, 35]:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (3)$$

$$\rho_{nf}\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu_{nf}\left(1 + \frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} - \mu_{nf}\frac{1}{k}u\left(1 + \frac{1}{\beta}\right) - \sigma_{nf}B^2u, \quad (4)$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho C_p)_{nf}}\frac{\partial q_r}{\partial y}, \quad (5)$$

where components of velocity in horizontal and vertical directions are u and v , T is temperature of fluid, and β is the Casson parameter. The quantities $\mu_{nf}, \nu_{nf} = (\mu_{nf}/\rho_{nf})$, $\rho_{nf}, \alpha_{nf} = k_{nf}/\rho_{nf} (C_p)_{nf}$, and k_{nf} symbolize dynamic viscosity, kinematic viscosity, density, effective thermal diffusivity, and effective thermal conductivity of nanofluid, respectively, which are mentioned below:

$$\mu_{nf} = \mu_f / (1 - \Phi)^{2.5}$$

$$\sigma_{nf} = \alpha_{nf} (\rho C_p)_{nf}$$

$$\sigma = \sigma_s / \sigma_f$$

$$\rho_{nf} = (1 - \Phi)\rho_f + \Phi\rho_s$$

$$(k_{nf}/k_f) = k_s + 2k_f - 2\Phi(k_f - k_s) / (k_s + 2k_f + \Phi(k_f - k_s))$$

$$\sigma_{nf} = \sigma_f \phi_1$$

$$\phi_1 = [1 + (3(\sigma - 1)\phi / ((\sigma + 2) - (\sigma - 1)\phi))]$$

Furthermore, $(\rho C_p)_{nf} = (1 - \Phi)(\rho C_p)_f + \Phi(\rho C_p)_s$ is the capability of specific heat and electrical conductivity of nanosized fluid is $\sigma_{nf} = \alpha_{nf}(\rho C_p)_{nf}$, where ϕ, s , and f refer to the volume fraction, properties of solids, and fluid of nanoparticles of the fluid flow, respectively.

The radiative heating flux q_r is analyzed and expressed using Rosseland approximation in the energy equation [36], i.e.,

$$q_r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y} = -\frac{16\sigma^*}{3k^*} T^3 \frac{\partial T}{\partial y}, \quad (6)$$

where Stefan-Boltzmann constant is σ^* and k^* is the mean absorption coefficient. The variation of temperature is assumed to be high enough to continue nonlinear expansion of T^4 in Taylor series in the negligence of higher order terms. From equation (6), the energy equation (5) takes the form

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{nf}\frac{d^2 T}{dy^2} + \frac{16\sigma^* T^3}{(\rho C_p)_{nf} 3k^*} \frac{d^2 T}{dy^2}. \quad (7)$$

The current mathematical model is associated with the following boundary conditions:

$$\begin{cases} u = U_\infty = ax, v = 0, T = T_w, & \text{at } y = 0, \\ u \rightarrow 0, T \rightarrow T_\infty, & \text{at } y \rightarrow \infty. \end{cases} \quad (8)$$

3. Similarity Transformation and Nondimensionalization

For the mathematical solutions of the problem, similarity transforms are being introduced as follows [37]:

$$\begin{aligned} \eta &= \frac{y}{x} \sqrt{\text{Re}_x}, \\ \psi &= \nu_f \sqrt{\text{Re}_x}, \\ f &= \frac{U_\infty x}{\nu_f} f(\eta), \\ T &= T_\infty + T_0 (bx)\theta(\eta), \end{aligned} \quad (9)$$

where the local Reynolds number Re_x depend on the free stream velocity (U_∞) and kinematic viscosity (ν_f) of the fluid, and the similarity variable η is brought into use. The selection of stream function $\psi(x, y)$ assures to satisfy equation (3):

$$\begin{aligned} u &= \frac{\partial \psi}{\partial y}, \\ v &= -\frac{\partial \psi}{\partial x}. \end{aligned} \quad (10)$$

FIGURE 1: Flow model configuration.

The nondimensional transformed flow model of governing equations (3)–(5) obtained by utilizing the similarity transformation in (9) is as follows:

$$\left(1 + \frac{1}{\beta}\right) f'''' + (1 - \phi)^{2.5} \left[1 - \phi + \phi \left(\frac{\rho_s}{\rho_f}\right)\right] \times \frac{1}{2} f f'' - (1 - \phi)^{2.5} \phi_1 M f' - \left(1 + \frac{1}{\beta}\right) K_1 f' = 0, \quad (11)$$

$$\frac{1}{Pr} \left[\frac{k_{nf}}{k_f} + Rd(1 + ((\theta_w - 1)\theta)^3) \right] \frac{1}{\left[(1 - \phi) + \phi \left((\rho C_p)_s / (\rho C_p)_f \right) \right]} \theta'' + \frac{1}{2} (f \theta' - 2 f' \theta) = 0. \quad (12)$$

The following pertinent parameters are being used during the transmission of the conventional flow model of PDEs into the frame of reference of ODEs:

$$Pr = \left(\frac{\mu C_p}{k} \right)_f,$$

$$M = \left(\frac{\sigma}{\rho} \right)_f B_0^2 \frac{1}{U_\infty}, \quad (13)$$

$$\theta_w = \frac{T_f}{T_\infty},$$

$$Rd = \frac{16 T_\infty^3}{3} \frac{\sigma^*}{k^* k_f},$$

subjected to the following transformed boundary conditions at $\eta \rightarrow 0$ and $\eta \rightarrow \infty$:

$$\begin{cases} f'(0) = 1, f(0) = 0, & \theta(0) = 1, \\ f'(\infty) \rightarrow 0, & \theta(\infty) \rightarrow 0. \end{cases} \quad (14)$$

4. Numerical Method

The nondimensional coupled nonlinear ODEs in the coordination of eminent boundary conditions are handled numerically by the implementation of the *bvp4c* scheme in the utilization of MATLAB and renovated into initial value problem. Such renovation of the boundary value problem is allocated some appropriate restricted values as ($\eta \rightarrow \infty$) for a far field boundary conditions say ($\eta_\infty = 5$). We introduce the new set of dependent specified variables as follows:

$$\begin{aligned}
f &= z_1, \\
f' &= z_2, \\
z_2' &= z_3, \\
\theta &= z_4, \\
z_4' &= z_5, \\
z_3' &= \frac{(1-\phi)^{2.5} [\phi - 1 - \phi(\rho_s/\rho_f)] \times (1/2)z_1z_3 + (1-\phi)^{2.5}\phi_1 M z_2 + K_1(1+(1/\beta))z_2}{(1+(1/\beta))},
\end{aligned} \tag{15}$$

$$z_5' = \left(\frac{\text{Pr} \times (2z_2z_4 - z_1z_5) \times \left(\left[(1-\phi) + \phi \left((\rho C_p)_s / (\rho C_p)_f \right) \right] \right) \times (1/2)}{\left[(k_{nf}/k_f) + \text{Rd}((\theta_w - 1)\theta)^3 \right]} \right), \tag{16}$$

with the associate boundary conditions:

$$\begin{cases} z_2(0) = 1, z_1(0) = 0, & z_4(0) = 1, \\ z_2(\infty) \rightarrow 0, & z_4(\infty) \rightarrow 0. \end{cases} \tag{17}$$

The MATLAB *bvp4c* tool primarily encompasses by default the finite difference code. Generally, this scheme is the fourth-order collocation method. The residual of continuous solution is in charge of the error control and mesh resolution in the grid continuation and tolerance in this method is specified as 10^{-7} .

5. Physical Quantities: Surface Drag Coefficient and Thermal Gradient

In the present problem, the shear stress and rate of heat transfer are indicated by engineering quantities of physical interest skin friction coefficient C_f , and the Nusselt number Nu_x is defined as

$$\begin{aligned}
C_f &= \frac{\tau_{wx}}{\rho_f U_x^2}, \\
\text{Nu}_x &= \frac{x}{k_f} \frac{q_w}{(T_f - T_\infty)}.
\end{aligned} \tag{18}$$

The surface drag coefficient at the wall along the x direction is specified as

$$\tau_{wx} = \left(\mu_{nf} \left(1 + \frac{1}{\beta} \right) \frac{\partial u}{\partial y} \right)_{y=0}, \tag{19}$$

and the surface heat flux which is denoted by q_w is

$$q_w = -k_{nf} \frac{\partial T}{\partial y} \Big|_{y=0} - \frac{16\sigma^*}{3k^*} \left(T^3 \frac{\partial T}{\partial y} \right) \Big|_{y=0}, \tag{20}$$

where local Reynolds number is fixed as

$$\text{Re}_x = \frac{ax^2}{\nu_f}. \tag{21}$$

After utilizing the nondimensional relation, equation (18) takes the form

$$\Rightarrow C_f (\text{Re}_x)^{1/2} = \left(1 + \frac{1}{\beta} \right) f''(0) \frac{1}{(1-\phi)^{2.5}}, \tag{22}$$

$$\Rightarrow (\text{Nu}_x) \text{Re}_x^{-(1/2)} = -\theta'(0) \frac{k_{nf}}{k_f} [1 + \text{Rd}(\theta_w)^3]. \tag{23}$$

6. Results and Discussion

The current work specifically explores the behavior of the magnetic/nonmagnetic nanoparticles (magnetite and aluminium oxide) suspensions into the Newtonian and non-Newtonian base fluids (water and sodium alginate) together with the consideration of heat transfer. Steady, two-dimensional forced convective boundary layer flow of Casson nanofluid due to stretching sheet in the presence of magnetic field effect and thermal radiation suppressed in a porous media is analyzed. Numerical computations are being obtained using MATLAB for many sundry parameters effects on different profiles with the help of *bvp4c* tool, and the consequences are illustrated tremendously numerically and graphically. Table 1 depicts the thermophysical properties of distinct base fluids along with magnetic and nonmagnetic nanoparticles. $C_f (\text{Re}_x)^{1/2}$ and $(\text{Nu}_x) \text{Re}_x^{-(1/2)}$ are listed in Tables 2 and 3 for Newtonian and non-Newtonian base fluids at distinct nondimensional pertinent parameters. It is eminent that variation in the different sundry parameters ϕ and M, K_p, β causes to upsurge and lessen the velocity gradient for both magnetic and nonmagnetic nanoparticles in Newtonian and non-Newtonian base fluids, respectively. While, a decreasing trend is followed up for the thermal gradient. It is to be noted that $C_f (\text{Re}_x)^{1/2}$ of Newtonian fluids is much higher comparatively to non-Newtonian fluids. The thermal gradient of non-Newtonian fluid becomes more condensed than that of Newtonian fluids.

The manifestation of the effect of magnetic field (M) of nanoparticles Al_2O_3 and Fe_3O_4 into suspensions for both

TABLE 1: Thermophysical properties of distinct base fluids and magnetic and nonmagnetic nanoparticles.

Properties of nanofluid	ρ (kgm ⁻³)	C_p (Jkg ⁻¹ K ⁻¹)	k (Wm ⁻¹ K ⁻¹)	σ ((Ω m) ⁻¹)
Water (Newtonian base fluid)	997.1	4179	0.613	5.5×10^{-6}
Sodium alginate (non-Newtonian base fluid)	989	4175	0.6376	2.6×10^{-4}
Nanoparticle (Al ₂ O ₃)	3970	765	40	35×10^6
Nanoparticle (Fe ₃ O ₄)	5180	670	9.7	2.5×10^4

TABLE 2: Variation in velocity gradient $C_f(\text{Re}_x)^{1/2}$ for Newtonian and non-Newtonian base fluids at distinct nondimensional pertinent parameters.

ϕ	M	K_p	β	Water		Sodium alginate	
				Al ₂ O ₃	Fe ₃ O ₄	Al ₂ O ₃	Fe ₃ O ₄
0.05				-1.187141	-1.216334	-1.187925	-1.217333
0.1				-1.182780	-1.233517	-1.184154	-1.235237
0.2				-1.139906	-1.217345	-1.142027	-1.219940
0.3				-1.060873	-1.149444	-1.063316	-1.152392
	0.5			-1.160537	-1.199047	-1.161573	-1.200362
	1.5			-1.672274	-1.697887	-1.672956	-1.698770
	2.5			-2.064541	-2.084847	-2.065081	-2.085549
	3.5			-2.394215	-2.411515	-2.394674	-2.412113
		0.5		-1.579767	-1.607058	-1.580495	-1.607998
		1.5		-2.518318	-2.534707	-2.518753	-2.535274
		2.5		-3.193786	-3.206538	-3.194124	-3.206979
		3.5		-3.749776	-3.760568	-3.750062	-3.760942
			0.5	-0.601767	-0.637716	-0.602745	-0.638929
			1.5	-0.777435	-0.828521	-0.778832	-0.830236
			2.5	-0.842285	-0.898578	-0.843826	-0.900466
			3.5	-0.876324	-0.935300	-0.877939	-0.937278

TABLE 3: Variation in thermal gradient $-(\text{Nu}_x)\text{Re}_x^{-(1/2)}$ for Newtonian and non-Newtonian base fluids at distinct nondimensional pertinent parameters.

ϕ	M	K_p	β	Water		Sodium alginate	
				Al ₂ O ₃	Fe ₃ O ₄	Al ₂ O ₃	Fe ₃ O ₄
0.05				4.090438	4.103566	4.193110	4.207611
0.1				3.896720	3.921343	3.994415	4.021619
0.2				3.549645	3.595005	3.638243	3.688269
0.3				3.240500	3.306772	3.320832	3.393619
	0.5			2.782926	2.822792	2.854064	2.897684
	1.5			2.687366	2.730471	2.758611	2.805490
	2.5			2.613064	2.657883	2.684305	2.732914
	3.5			2.550481	2.596474	2.621653	2.671452
		0.5		2.704799	2.747429	2.776032	2.822433
		1.5		2.526967	2.573357	2.598097	2.648299
		2.5		2.400189	2.448415	2.470923	2.523003
		3.5		2.298292	2.347679	2.368493	2.421773
			0.5	2.907816	2.949634	2.978719	3.024257
			1.5	2.875345	2.914244	2.946297	2.988927
			2.5	2.862978	2.900818	2.933955	2.975531
			3.5	2.856406	2.893691	2.927398	2.968421

Newtonian (water) and non-Newtonian (sodium alginate) conventional base fluids on velocity $f(\eta)$ and temperature $\theta(\eta)$ profiles is given in Figures 2(a), 2(b) and 3(a), 3(b).

In Figures 2(a) and 2(b), the momentum boundary layer thickness depreciates by enhancing the values of magnetic field for both magnetic and nonmagnetic nanoparticles due to

presence of magnetic flux in the perpendicular direction of electrical transmission of nanoparticles fluid flow which cause exertion a resistive drag force called Lorentz force, that opposes the direction of flow and reduce the velocity of fluid. While such retarding force contributes to orchestrate the nanoparticles due to augmentation in kinetic energy, and the temperature profile faces a dominant surge for the increasing

FIGURE 2: Variation in magnetic field on $f'(\eta)$ for (a) Al_2O_3 and (b) Fe_3O_4 . FIGURE 3: Variation in magnetic field on $\theta(\eta)$ for (a) Al_2O_3 and (b) Fe_3O_4 .

values of M . Figures 4 and 5 accurately exhibit the effects of Casson parameter β on different boundary layer profiles. It necessitates that the velocity boundary layer thickness is influenced by the augmented β and gets to diminish for both geometries of nanoparticles. This trend followed up by the fact that β crucially has an ability to remold the properties of fluid such as plasticity and yield stress. The yield stress and β are inversely correlated, and enlarging β reduces the yield stress. Hence, the velocity profile reduces. On the contrary, in Figure 5(a), considerable inclination in the fluid patterns for the increasing values of parameter is beneficial enough in enhancing the thickness of the thermal boundary layer. This is

due to the fact that the viscosity of fluid escalates with an increase in the values of β , and such escalation in viscosity leads the temperature to rise.

Evident results which integrate with porosity parameter K_p are portrayed in Figures 6 and 7. Due to the existence of porous media, the fluid velocity belittles and turns out as the downfall in the thickness of the momentum boundary layer. Physically, the boundary layer has to tackle with the soaking of a immense amount of fluid in the existence of the porosity factor, and it is perceived that the velocity boundary layer decelerates in the whole flow field. Interestingly, the increasing values of

FIGURE 4: Variation in the Casson parameter on $f'(\eta)$ for (a) Al_2O_3 and (b) Fe_3O_4 .

FIGURE 5: Variation in the Casson parameter on $\theta(\eta)$ for (a) Al_2O_3 and (b) Fe_3O_4 .

K_p prompt to relatively high temperature of the fluid due to the fact that porous media exerts a force which have a tendency to enhance the resistance in fluid. Such resistive force tends to slow down the motion of fluid and as a result the thermal resistance rises in prominence that can be easily visualized in Figure 7. In comparison between these two geometries of Al_2O_3 and Fe_3O_4 nanoparticles, it is observed that the resistance of magnetite nanoparticles (Fe_3O_4) is higher than that of the nonmagnetic nanoparticles in connection with the flow field. The effects of variation in velocity and temperature profiles against the volume fraction ϕ are precisely delineated in Figures 8 and

9. The fluid velocity and temperature bear a trend of being enlarged within the increment of ϕ . Such behavior follows up due to the increase in the density of nanoparticles which retards the fluid viscosity and consequently the velocity profile got enriched. The thermal conductivity of water-based magnetite and sodium alginate-based aluminium oxide nanoparticles tend to rise because ϕ causes detection of energy from the boundary layer which results in the acceleration in the thermal state of fluid. Figure 10 shows the behavior of temperature profile along with the mounting values of thermal radiation (R) for Al_2O_3 and Fe_3O_4 . For the different increasing values of pertinent

FIGURE 6: Variation in the porosity parameter on $f'(\eta)$ for (a) Al_2O_3 and (b) Fe_3O_4 . FIGURE 7: Variation in the porosity parameter on $\theta(\eta)$ for (a) Al_2O_3 and (b) Fe_3O_4 .

parameter (R), fluid temperature is seen to be upsurged. Thermal diffusivity achieves an enhancement because such effects of thermal radiation brings a prominence expand in temperature. Moreover, the effects of radiation prompt more heat energy towards the boundary layer, and advancement in the heat can be easily observed in the entire flow region which initiates the thermal nanoparticles profiles to proceed.

Finally, the ramifications of temperature profile with the variation of temperature ratio parameter θ_w for water-based magnetic and sodium alginate-based nonmagnetic nano-sized particles are established in Figure 11. Eventually, the temperature nanoparticles distribution explores an

enrichment with the increasing values of θ_w . Such behavior fits with the physical agreement that the mounted values of θ_w reflects the elevation in surface temperature in contrast with ambient temperature; thus, enhancement in the state of temperature is observed. It is of high consideration that magnetic water-based nanoparticles experienced high-temperature profile than those of nonmagnetic sodium alginate-based nanoparticles.

The skin friction coefficient, which is an exciting physical engineering quantity in evaluating the viscous stress acting on the stretching sheet and Nusselt number, is analyzed graphically in Figures 12 and 13. Figures 12(a), 12(b), 13(a), and 13(b) show the effect of M variations on skin friction

FIGURE 8: Variation in volume fraction on $f'(\eta)$ for (a) Al_2O_3 and (b) Fe_3O_4 . FIGURE 9: Variation in volume fraction on $\theta(\eta)$ for (a) Al_2O_3 and (b) Fe_3O_4 .

coefficient and, in the presence of ϕ for two geometries, face a diminution in surface drag force. It is perceived that the non-Newtonian fluid is much denser than Newtonian fluid. While, comparison between magnetic and nonmagnetic nanoparticles in the existence of magnetic field reveals that both the nanoparticles show more or less the same behavior because induced magnetic field is negligible as compared to applied magnetic field.

Figures 14(a) and 14(b) show the fluctuation in the Nusselt number with varying volume fraction of

nanoparticles against temperature ratio. The magnification in the nanoparticles' volume fraction enriches the Nusselt number for both Newtonian and non-Newtonian convective base fluids. This is due to the fact that an enhancement in volume fraction causes to increase the thermal conductivity of the fluids. The presence and absence of magnetic field diminishes and increases the thermal boundary layer thickness. It is observed that the Nusselt numbers for magnetic nanoparticles are minimum and maximum for nonmagnetic nanoparticles.

FIGURE 10: Variation in the radiation parameter on $f'(\eta)$ for (a) Al_2O_3 and (b) Fe_3O_4 .

FIGURE 11: Variation in the temperature ratio parameter on $\theta(\eta)$ for (a) Al_2O_3 and (b) Fe_3O_4 .

FIGURE 12: Variation in the magnetic field parameter on the skin friction coefficient for (a) Al_2O_3 and (b) Fe_3O_4 against ϕ .

FIGURE 13: Variation in the magnetic field parameter on the Nusselt number for Al_2O_3 and Fe_3O_4 against ϕ for Al_2O_3 and Fe_3O_4 against ϕ .

FIGURE 14: Variation in the volume fraction parameter on the Nusselt number for (a) Al_2O_3 and (b) Fe_3O_4 against temperature ratio.

7. Conclusion

This research work reports a comparative analysis of the phenomenon of Newtonian (water)/non-Newtonian (sodium alginate) base fluids with the magnetic (magnetite)/nonmagnetic (aluminium oxide) nanoparticles. The steady boundary layer flow of Casson nanofluid over a stretching sheet fixed in porosity media in association with the transverse magnetic field and thermal radiation is tremendously analyzed. The governing highly ordered nonlinear coupled PDEs are converted into nondimensional ODEs with the help of appropriate similarity transformation. For the purposes of better numerical computations, the ODEs are being handled by numerical technique, *bvp4c*. The novelty of pertinent parameters is delineated for nondimensional velocity and temperature profiles. The additional remarks of the present study encompass the results as follows:

- (i) The velocity profile shares a diminution with the intensification in the values of transverse magnetic field (M), Casson parameter (β), and porosity parameter (K_p). The Newtonian base fluid is much thicker than the non-Newtonian fluid for both the magnetic and nonmagnetic nanosized particles.
- (ii) The augmentation in the values of sundry parameters' magnetic field, Casson parameter, porosity parameter, volume fraction of nanofluid, radiation parameter, and temperature ratio parameter yields hike in the thermal boundary layer. In temperature profile, the thickness of non-Newtonian fluid becomes more condensed than that of Newtonian fluid. The fluid temperature upsurges in the presence of Lorentz forces.
- (iii) The increment in the radiation parameter possesses much contact with the temperature profile of

non-Newtonian fluid (Casson fluid) as compared to Newtonian fluid.

- (iv) The embedded porous media and magnetic parameter have the propensity to surge the heat transfer in the entire flow field.
- (v) Heat transfer is greatly influenced by the thermal conductivity of nanoparticles, and nonmagnetic nanoparticles entail high thermal conductivity in contrast with the magnetic.
- (vi) It is perceived that the velocity gradient of non-magnetic nanoparticles' Newtonian fluids is slightly higher comparatively to that of magnetic for both types of base fluids. On the contrary, the thermal profile shows reverse trend just opposite to velocity gradient.
- (vii) Magnetic nanosized particles expedites viscosity (resistance) in comparison with nonmagnetic.
- (viii) It was also perceived that Newtonian fluid (water) shares the least ability of heat and mass transfer comparatively to non-Newtonian (sodium alginate).

Nomenclature

Greek Symbols

- β : Casson parameter
 η : Similarity variable
 ∞ : Edge of the boundary layer
 μ : Absolute viscosity
 ν : Kinematic viscosity
 ϕ : Volume fraction
 Ψ : Stream function
 ρ : Capacity of heat
 σ : Chemical reaction rate constant

σ^* : Stefan–Boltzmann constant
 τ_w : Wall shear stress
 θ : Dimensionless temperature
 θ_w : Temperature ratio variable
 U_∞ : Free stream velocity

Symbols

k : Thermal conductivity
 α : Thermal diffusivity
 Pr : Prandtl number
 ι : Differentiation w.r.t η
 v : Velocity field
 a : Positive constant
 B_0 : Magnetic field intensity
 C_f : Velocity gradient
 c_p : Specific heat
 f : Base fluid
 f : Nondimensional stream function
 h : Heat transfer coefficient
 h^* : Heat transfer coefficient
 k : Thermal conductivity
 L : Slip length
 M : Magnetization parameter
 nf : Subscript for nanofluid
 Nu_x : Local Nusselt Number
 p : Pressure
 q_r : Radiative heat flux
 q_w : Wall heat flux
 Rd : Radiation parameter
 Re_x : Local Reynolds Number
 s : Solid nanoparticles
 T : Local temperature of fluid
 T_∞ : Temperature at free stream
 T_f : Reference temperature
 T_w : Temperature at the surface of sheet
 u, v : Velocity components
 u_w : Wall velocity
 x, y : Distance along and normal of sheet.

Data Availability

All the dataset for supporting the results and conclusion is provided within the article.

Additional Points

Highlights. (i) A comparative study of the magnetic and nonmagnetic nanoparticles suspended into Newtonian and non-Newtonian base fluids is being analyzed. (ii) The effects of nonlinear thermal radiation for two different categories of nanofluids, i.e., magnetic and nonmagnetic nanoparticles are observed. (iii) The parameters which help to control the flow rate, i.e., magnetic field and porosity tend to depreciate the thickness of velocity profile of the fluid. (iv) The temperature profile of non-Newtonian fluids endures high contact in the presence of thermal radiation as compared to Newtonian fluid.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References

- [1] N. S. Casson, *Rheology of Dispersed System*, Vol. 84, Pergamon Press, Oxford, UK, 1959.
- [2] K. Ramesh, A. Riaz, and Z. Ahmad Dar, “Simultaneous effects of MHD and Joule heating on the fundamental flows of a Casson liquid with slip boundaries,” *Propulsion and Power Research*, vol. 10, no. 2, pp. 118–129, 2021.
- [3] S. Choi, “Enhancing thermal conductivity of fluids with nanoparticles,” in *Proceedings of the ASME International Mechanical Engineering Congress and Exposition*, pp. 99–105, San Francisco, CA, USA, November 1995.
- [4] K. Yih, “Free convection effect on MHD coupled heat and mass transfer of a moving permeable vertical surface,” *International Communications in Heat and Mass Transfer*, vol. 26, no. 1, pp. 95–104, 1999.
- [5] S. Choi, Z. G. Zhang, W. Yu, F. E. Lockwood, and E. A. Grulke, “Anomalous thermal conductivity enhancement on nanotube suspensions,” *Applied Physics Letters*, vol. 79, pp. 2252–2254, 2001.
- [6] M. M. Bhatti, A. Riaz, and L. Zhang, “Biologically inspired thermal transport on the rheology of Williamson hydro-magnetic nanofluid flow with convection: an entropy analysis,” *Journal of Thermal Analysis and Calorimetry*, vol. 144, no. 6, pp. 2187–2202, 2021.
- [7] A. Riaz, R. Ellahi, and T. Muhammad, “Magnetized Jeffrey nanofluid with energy loss in between an annular part of two micro nonconcentric pipes,” *Energy Sources, Part A: Recovery, Utilization and Environmental Effects*, Taylor & Francis, Abingdon, UK, 2020.
- [8] J. A. Eastman, S. U. S. Choi, S. Li, W. Yu, and L. J. Thompson, “Anomalous increased effective thermal conductivities of ethylene glycolbased nanofluids containing copper nanoparticles,” *Applied Physics Letters*, vol. 78, no. 6, pp. 718–720, 2001.
- [9] J. Buongiorno, “Convective transport in nanofluids,” *Journal of Heat Transfer*, vol. 128, Article ID 240250, 2006.
- [10] J. Buongiorno, “A benchmark study of thermal conductivity of nanofluids,” *Journal of Applied Physics*, vol. 106, Article ID 094312, 2009.
- [11] S. C. Moon, B.-Y. Ryu, J. K. Choi, B. W. Jo, and R. J. Farris, “The morphology and mechanical properties of sodium alginate based electrospun poly(ethylene oxide)nanofibers,” *Polymer Engineering & Science*, vol. 49, no. 1, p. 5259, 2009.
- [12] A. J. Chamkha, A. M. Aly, and M. A. Mansour, “Similarity solution for unsteady heat and mass transfer from a stretching surface embedded in a porous medium with suction/injection and chemical reaction effects,” *Chemical Engineering Communications*, vol. 197, no. 6, Article ID 846858, 2010.
- [13] R. Ellahi, A. Zeeshan, K. Vafai, and H. U. Rahman, “Series solutions for magnetohydrodynamic flow of nonNewtonian nanofluid and heat transfer in coaxial porous cylinder with slip conditions,” *Proceedings of the Institution of Mechanical Engineers, Part N: Journal of Nanoengineering and Nanosystems*, vol. 225, no. 3, Article ID 123132, 2011.
- [14] T. G. Motsumi and O. D. Makinde, “Effects of thermal radiation and viscous dissipation on boundary layer flow of nanofluids over a permeable moving flat plate,” *Physica Scripta*, vol. 86, Article ID 045003, 2012.

- [15] Aurangzaib, A. R. M. Kasim, N. F. Mohammad, and S. Shafie, "Effect of thermal stratification on MHD free convection with heat and mass transfer over an unsteady stretching surface with heat source, Hall current and chemical reaction," *International Journal of Advances in Engineering Sciences and Applied Mathematics*, vol. 4, no. 3, pp. 217–225, 2012.
- [16] L. Colla, L. Fedele, M. Scattolini, and S. Bobbo, "Water-based Fe_2O_3 nanofluid characterization: thermal conductivity and viscosity measurements and correlation," *Advances in Mechanical Engineering*, vol. 4, no. 1, 2012.
- [17] S. Nadeem, R. Ul Haq, and N. S. Akbar, "MHD three-dimensional boundary layer flow of Casson nanofluid past a linearly stretching sheet with convective boundary condition," *IEEE Transactions on Nanotechnology*, vol. 13, no. 1, pp. 1326–1332, 2014.
- [18] M. Qasim, Z. H. Khan, W. A. Khan, and A. Shah, "MHD boundary layer slip flow and heat transfer of ferrofluid along a stretching cylinder with prescribed heat flux," *PLoS One*, vol. 9, no. 1, Article ID e83930, 2014.
- [19] M. Sheikholeslami and D. D. Ganji, "Ferrohydrodynamic and magnetohydrodynamic effects on ferrofluid flow and convective heat transfer," *Energy*, vol. 75, no. 1, pp. 400–410, 2014.
- [20] M. Sheikholeslami, D. D. Ganji, and M. M. Rashidi, "Ferrofluid flow and heat transfer in a semi annulus enclosure in the presence of magnetic source considering thermal radiation," *Journal of the Taiwan Institute of Chemical Engineers*, vol. 47, no. 1, pp. 6–17, 2015.
- [21] A. Gul, I. Khan, S. Shafie, A. Khalid, and A. Khan, "Heat transfer in MHD mixed convection flow of a ferrofluid along a vertical channel," *PLoS One*, vol. 2015, no. 1, pp. 1–14, 2015.
- [22] I. S. Oyelakin, S. Mondal, and P. Sibanda, "Unsteady Casson nanofluid flow over a stretching sheet with thermal radiation, convective and slip boundary conditions," *Alexandria Engineering Journal*, vol. 55, no. 2, pp. 1025–1035, 2016.
- [23] R. Mahmood, N. Kousar, M. Yaqub, and K. Jabeen, "Numerical simulations of the square lid driven cavity flow of bingham fluids using nonconforming finite elements coupled with a direct solver," *Advances in Mathematical Physics*, vol. 2017, Article ID 5210708, 10 pages, 2017.
- [24] A. K. Abdul Hakeem, S. Saranya, and B. Ganga, "Comparative study on Newtonian/non-Newtonian base fluids with magnetic/non-magnetic nanoparticles over a flat plate with uniform heat flux," *Journal of Molecular Liquids*, vol. 230, Article ID 445452, 2017.
- [25] M. Siddiqui, A. Riaz, and I. Khan, "Augmentation of mixed convection heat transfer in a lid-assisted square enclosure utilizing micropolar fluid under magnetic environment: a numerical approach," *Results in Physics*, vol. 18, Article ID 103245, 2020.
- [26] K. Jabeen, M. Mushtaq, and R. M. Akram, "A comparative study of MHD flow analysis in a porous medium by using differential transformation method and variational," *Iteration Method*, vol. 9, pp. 2222–5498, 2019.
- [27] K. Jabeen, M. Mushtaq, and R. M. Akram Muntazir, "Analysis of MHD fluids around a linearly stretching sheet in porous media with thermophoresis, radiation, and chemical reaction," *Mathematical Problems in Engineering*, vol. 2020, Article ID 9685482, 14 pages, 2020.
- [28] K. Jabeen, M. Mushtaq, and R. M. Akram, "Analysis of the MHD boundary layer flow over a nonlinear stretching sheet in a porous medium using semianalytical approaches," *Mathematical Problems in Engineering*, vol. 2020, Article ID 3012854, 9 pages, 2020.
- [29] R. M. Akram Muntazir, M. Mushtaq, and K. Jabeen, "A numerical study of MHD carreau nanofluid flow with gyrotactic microorganisms over a plate, wedge, and stagnation point," *Mathematical Problems in Engineering*, vol. 2021, Article ID 5520780, 22 pages, 2021.
- [30] R. M. Akram Muntazir, M. Mushtaq, S. Shahzadi, and K. Jabeen, "Influence of chemically reacting ferromagnetic carreau nanofluid over a stretched sheet with magnetic dipole and viscous dissipation," *Mathematical Problems in Engineering*, vol. 2021, Article ID 6652522, 12 pages, 2021.
- [31] K. Jabeen, M. Mushtaq, and R. M. A. Muntazir, "Suction and injection impacts on casson nanofluid with gyrotactic microorganisms over a moving wedge," *ASME: Journal of Fluids Engineering*, vol. 144, no. 1, 2021.
- [32] R. M. Muntazir, M. Mushtaq, S. Shahzadi, and K. Jabeen, "MHD nanofluid flow around a permeable stretching sheet with thermal radiation and viscous dissipation," *Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science*, 2021.
- [33] S. Saranya, P. Ragupathi, and B. Ganga, "Non-linear radiation effects on magnetic/non-magnetic nanoparticles with different base fluids over a flat plate," *Advanced Powder Technology*, vol. 29, no. 9, pp. 1977–1990, 2018.
- [34] A. Jamaludin, K. Naganthran, and R. NazarAnuar, "Thermal radiation and MHD effects in the mixed convection flow of Fe_3O_4 -water ferrofluid towards a nonlinearly moving surface," *Processes*, vol. 8, p. 95, 2020.
- [35] N. ThamaraiKannan, S. Karthikeyan, and D. K. Chaudhary, "Significance of MHD radiative non-Newtonian nanofluid flow towards a porous channel: a framework of the casson fluid model," *Journal of Mathematics*, vol. 2021, Article ID 9912239, 15 pages, 2021.
- [36] S. S. Ghadikolaiea, K. H. Hosseinzadeh, and D. Ganji, "Nonlinear thermal radiation effect on magneto Casson nanofluid flow with Joule heating effect over an inclined porous stretching sheet," *Case Studies in Thermal Engineering*, vol. 12, pp. 176–187, 2018.
- [37] T. Hayat, T. Muhammad, and A. Alsaedi, "Magnetohydrodynamic three-dimensional flow of viscoelastic nanofluid in the presence of nonlinear thermal radiation," *Journal of Magnetism and Magnetic Materials*, vol. 385, pp. 222–229, 2015.