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Research Article

Direct Position Determination for Augmented Coprime Arrays via Weighted Subspace Data Fusion Method

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Direct position determination (DPD) for augmented coprime arrays is investigated in this paper. Augmented coprime array expands degree of freedom and array aperture and improves positioning accuracy. Because of poor stability and noise sensitivity of the subspace data fusion (SDF) method, we propose two weighted subspace data fusion (W-SDF) algorithms for direct position determination. Simulation results show that two W-SDF algorithms have a prominent promotion in positioning accuracy than SDF, Capon, and propagator method (PM) algorithm for augmented coprime arrays. SDF based on optimal weighting (OW-SDF) is slightly better than SDF based on SNR weighting (SW-SDF) in positioning accuracy. The performance for DPD of the W-SDF method with augmented coprime arrays is better than that of the W-SDF method with uniform arrays.

1. Introduction

Wireless location technology is a prominent research area in present positioning. Two-step positioning is the most commonly used in passive positioning. By utilizing the arrival time, arrival angle, and arrival frequency difference, two-step positioning constructs a mathematical model to realize positioning [1, 2]. However, there are many shortcomings in traditional two-step positioning methods. In the process of positioning, because two-step positioning experienced more intermediate processing steps, the corresponding positioning accuracy is affected [3]. In order to avoid the problem of intermediate processing steps in the two-step positioning method, in recent years, many scholars have proposed new positioning method—direct position determination [4]. Weiss proposed direct position determination in 2004 firstly [5]. Direct position determination (DPD) estimates the target position without any location intermediate parameters [6]. Because of the direct use of the original observation data, DPD makes use of the target information and effectively avoids the steps of the location intermediate parameters [7].

For reducing the computational complexity, Demmissie proposed a subspace data fusion (SDF) with higher computational efficiency in 2008 [8], which extends the ultrahigh resolution multiple signal classification angle estimation algorithm in the field of array signal processing to direct positioning. Multiple arrays receive signals from multiple different positions through fusing the received signals of multiple arrays based on the spatial spectrum estimation theory [9, 10]. Then, the SDF algorithm constructs the loss function and obtains the position estimation of emitter. Although the SDF algorithm also needs grid search, it only needs one 2D or 3D grid search in the effective space to get the position estimation of all emitters [11]. The traditional SDF algorithm based on the direct position determination algorithm does not consider the heteroscedasticity of the observation error [12–15]. So, the proposed weighted SDF method for DPD makes most of the eigenvalues and eigenvectors of the covariance matrix eigenvalue

decomposition and combines with the augmented coprime array to obtain the asymptotic accuracy. The optimal position estimation performance is achieved, and the source resolution is improved [16–21].

For the problem of limited degree of freedom of uniform array, there are many research studies in traditional coprime array for direction of arrival (DOA) estimation [3, 17, 22–25]. Compared with traditional coprime arrays, augmented coprime array can use the same number of real array elements to generate more virtual arrays in the same range. And, it has a longer continuous virtual element part. Based on the existing research foundation of array signal processing introduced above, direct position determination extends from uniform array to sparse array. Now, augmented coprime array for DPD is worth studying. Single augmented coprime array or multiple augmented coprime arrays is constructed for the DPD model, and then a continuous virtual array model is constructed by spatially smoothing [14, 26–28]. So, the location loss function is constructed based on subspace data fusion for augmented arrays. This paper mainly studies the weighted subspace data fusion method based on multiple augmented coprime arrays.

We summarize the main contributions as follows:

(1) We propose two weighted subspace data fusion (W-SDF) algorithms for direct position determination to solve poor stability and noise sensitivity of the SDF algorithm. We assign a weight to the projection result at each observation position. We balance the orthogonal projection to obtain small error and high robustness loss function.

(2) We introduce multiple augmented coprime arrays into the direct position determination model and then combine with spatial smoothing subspace data fusion. We use augmented coprime arrays to increase spatial freedom and recognize more sources.

(3) We use the weighted subspace data fusion method for DPD. The proposed algorithm does not need estimation steps of intermediate parameters and avoids second loss of information.

The structure of this paper is as follows. In Section 2, we introduce some basic concepts of augmented coprime array and scene of direct position determination. Section 3 depicts the proposed W-SDF method. Section 4 depicts performance analysis about W-SDF algorithms with augmented coprime array for DPD. In Section 5, we simulate the proposed W-SDF algorithms for multiple coprime arrays and compare it with other algorithms under different arrays and different elements. And, Section 6 summarizes this paper.

Notations. $(\cdot)^*$ represents the conjugate, $(\cdot)^T$ represents the transposition, and $(\cdot)^H$ represents the conjugate transpose. The symbol $\mathrm{vec}(\cdot)$ represents the received covariance matrix virtualization, and symbol $\otimes$ represents the Kronecker product. I_n represents an $n \times n$ identity matrix, and $E(\cdot)$ represents the mathematical expectation.

2. Preliminaries

In this chapter, we introduce some basic concepts of augmented coprime array and scene of direct position determination.

2.1. Array Model. In order to use virtual array for direct position determination, augmented coprime linear array is introduced. Figure 1 shows augmented coprime linear array and the number of elements of two subarrays is $2M$ and N. The augmented coprime array element spacing is $N\ d$ and $M\ d$, M and N are coprime, and $M < N$.

2.2. Multiple Arrays Combination Positioning Model. We use the positioning scene in Figure 2. Assume that there are Q uncorrelated far-field narrow-band sources in the known two-dimensional X-Y plane. There are L observation stations with L augmented coprime arrays placed along the X-axis. The target sources are $\mathbf{p}_q = [x_q, y_q]^T\ (q = 1, 2, \ldots, Q)$. There are L observation stations expressed as $\mathbf{u}_l = [x_{ul}, y_{ul}]^T\ (l = 1, 2, \ldots, L)$. There are $D\,(D = 2M + N - 1)$ array elements on every observation station.

Assume that all the Q emitter signals are far-field narrow-band signals with wavelength of λ. In practice, according to the free space propagation loss model, when the signals from the same source are incident on the array at different positions, the received signal strength of the array is often different. Assuming that the power of all emitter signals is W_q and the power of the signal from the qth emitter received at the observation position $\mathbf{u}_l = [x_{ul}, y_{ul}]^T\ (l = 1, 2, \ldots, L)$ is $W_{l,q}$, the path propagation loss coefficient can be expressed as follows:

$$b_{l,q} = \sqrt{\frac{W_{l,q}}{W_q}}, \tag{1}$$

$s_{l,q}(k)$ is recorded as the qth radiation source, and the array output signal of the lth observation station at the kth $(k = 1, 2, 3, \ldots, K)$ fast beat time is obtained as follows [7]:

$$\mathbf{r}_l(k) = \sum_{q=1}^{Q} b_{l,q}\mathbf{a}_l\left(\mathbf{p}_q\right)s_{l,q}(k) + \mathbf{n}_l(k), \tag{2}$$

where $\mathbf{n}_l(k)$ denotes the noise vector of the lth observation station and $\mathbf{a}_l(\mathbf{p}_q)$ is the direction vector, which is determined by the angle of arrival $\theta_l(\mathbf{p}_q)$ [7] as follows:

$$\theta_l\left(\mathbf{p}_q\right) = \arctan\frac{\mathbf{x}_{ul}(1) - \mathbf{p}_q(1)}{\mathbf{y}_{ul}(2) - \mathbf{p}_q(2)}. \tag{3}$$

Equation (2) can be expressed as

$$\mathbf{r}_l(k) = \mathbf{A}_l(\mathbf{p})\mathbf{s}_l(k) + \mathbf{n}_l(k), \tag{4}$$

where

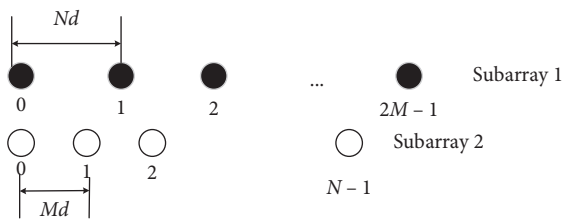

Figure 1: Augmented coprime array model.

$$\begin{aligned}
\mathbf{A}_l(\mathbf{p}) &= \left[\mathbf{a}_1(\mathbf{p}_1), \mathbf{a}_2(\mathbf{p}_2), \ldots, \mathbf{a}_2(\mathbf{p}_Q)\right]^T,\\
\mathbf{A}_l(\mathbf{p}_q) &= \left[\mathbf{a}_{l,1}^T(\mathbf{p}_q), \mathbf{a}_{l,2}^T(\mathbf{p}_q)\right]^T,\\
\mathbf{a}_{l,1}(\mathbf{p}_q) &= \left[1, e^{-j2\pi N\, d \sin\ \theta_l(\mathbf{p}_q)}, \ldots, e^{-j2\pi(2M-1)N\, d \sin\ \theta_l(\mathbf{p}_q)}\right]^T,\\
\mathbf{a}_{l,2}(\mathbf{p}_q) &= \left[1, e^{-j2\pi M\, d \sin\ \theta_l(\mathbf{p}_q)}, \ldots, e^{-j2\pi(N-1)M\, d \sin\ \theta_l(\mathbf{p}_q)}\right]^T,\\
\mathbf{s}_l(k) &= \left[\mathbf{s}_{l,1}(k), \mathbf{s}_{l,2}(k), \ldots, \mathbf{s}_{l,Q}(k)\right]^T,\\
\mathbf{p} &= \left[\mathbf{p}_1^T, \mathbf{p}_2^T, \ldots, \mathbf{p}_Q^T\right]^T,\\
\mathbf{n}_l(k) &= \left[\mathbf{n}_{l,1}(k), \mathbf{n}_{l,2}(k), \ldots, \mathbf{n}_{l,D}(k)\right]^T.
\end{aligned} \tag{5}$$

3. The Proposed Algorithm

3.1. SDF Direct Position Determination Algorithm Based on Augmented Coprime Array. We can obtain the covariance matrix of array output signal from equation (4).

$$\mathbf{R}_l = E\left[\mathbf{r}_l(k)\mathbf{r}_l^H(k)\right]. \tag{6}$$

We vectorize $\mathbf{R}_l$ as follows:

$$\tilde{\mathbf{z}} = \mathrm{vec}(\mathbf{R}_l) = \mathbf{H}_l(\mathbf{p})\boldsymbol{\mu} + \sigma_n^2\mathbf{I}_n, \tag{7}$$

where $\mathbf{H}_l(\mathbf{p}) = \mathbf{A}^* \odot \mathbf{A} = [\mathbf{a}(\mathbf{p}_1)\otimes\mathbf{a}(\mathbf{p}_2), \mathbf{a}(\mathbf{p}_2)\otimes\mathbf{a}(\mathbf{p}_2), \ldots, \mathbf{a}(\mathbf{p}_2)\otimes\mathbf{a}(\mathbf{p}_q)]$ is the direction matrix of the virtual array, $\boldsymbol{\mu} = [\sigma_1^2, \sigma_2^2, \ldots, \sigma_q^2]^T$ is a single snapshot signal vector, and $\mathbf{I}_n = \mathrm{vec}(\mathbf{I})$, in which $\mathbf{I}$ is the identity matrix. In order to facilitate processing, we need use $\tilde{\mathbf{z}}$ to sort by phase and the remove redundancy and then get the vector $\mathbf{z}$; vector $\mathbf{z}$ is the receiving signal of the augmented matrix virtual array.

Because the spatial smoothing algorithm needs the continuity of the array elements, therefore, the vector $\mathbf{z}_1$ can be obtained by intercepting the continuous virtual elements of $\mathbf{z}$.

The intercepted virtual array is a virtual array in range $[-(MN+M-1), MN+M-1]$ and long uniform linear array with element spacing of d. The number of array elements is $2MN+2M-1$.

The basic idea of the spatial smoothing algorithm is to divide the equidistant linear array into several overlapping subarrays. If the subarrays have the same structure, their covariance matrices are added to replace the original covariance matrix.

As shown in Figure 3, we divide the intercepted virtual array into $MN+M$ overlapping subarrays. Each subarray contains $MN+M$ elements. The element position of the ith subarray is

$$\{(i+1+n)d,\ n = 0, 1, \ldots, MN+M-1\}. \tag{8}$$

The received signal matrix is from line $MN+M+1-i$ to line $2MN+2M+1-i$ of $\mathbf{z}_1$, which is denoted as $\mathbf{z}_{hi}$, and the covariance matrix is constructed [27].

$$\mathbf{R}_i = \mathbf{z}_{hi}\mathbf{z}_{hi}^H. \tag{9}$$

The covariance matrix of all $MN+M$ submatrices is summed, and the mean value is calculated to obtain the spatial smooth covariance matrix:

$$\mathbf{R}_l = \frac{1}{MN+M}\sum_{i=1}^{MN+M}\mathbf{R}_i. \tag{10}$$

Because signal and noise are independent of each other, matrix $\mathbf{R}_l$ eigenvalue decomposition can be divided into signal space and noise subspace as follows:

$$\mathbf{R}_l = [\,\mathbf{U}_l^s\ \ \mathbf{U}_l^n\,]\sum_l[\,\mathbf{U}_l^s\ \ \mathbf{U}_l^n\,]^H. \tag{11}$$

According to the orthogonal property of signal subspace and noise subspace, the projection of steering vector to noise subspace is zero only when the steering vector of array $\mathbf{a}_l(\mathbf{p})$ is composed of real emitter position parameters $\mathbf{p}_q$. Using this property, the noise subspace projection results of the steering vector to the Lth observation positions are added to construct the following loss function:

$$f_{\mathrm{SDF}}(\mathbf{p}) = \sum_{l=1}^{L}\mathbf{a}_l^H(\mathbf{p})\mathbf{U}_l^n(\mathbf{U}_l^n)^H\mathbf{a}_l(\mathbf{p}). \tag{12}$$

Obviously, the loss function processes the projection results at all observation positions equally. When one of the L spectral functions has poor performance, the loss function is vulnerable to interference; that is, the performance of the traditional SDF based on the direct localization algorithm is affected by the heteroscedasticity of orthogonal projection errors from different observation positions.

For another, SDF only uses the noise subspace resulting in vulnerable to the external factor, such as few snapshots and low SNR. Because of these factors, positioning performance is restricted. Aiming at the problem of poor stability and noise sensitivity of the SDF method, this chapter considers to obtain the loss function with small error and high robustness by balancing the orthogonal projection error. The W-SDF method makes most of all the data to improve the positioning accuracy. So, we assign a weight to the projection result at each observation position, and we construct the following loss function:

$$f_{W-\mathrm{SDF}}(\mathbf{p}) = \sum_{l=1}^{L}w_l\mathbf{a}_l^H(\mathbf{p})\mathrm{U}_l^n(\mathbf{U}_l^n)^H\mathbf{a}_l(\mathbf{p}), \tag{13}$$

where w_l is the weight of the lth observation position.

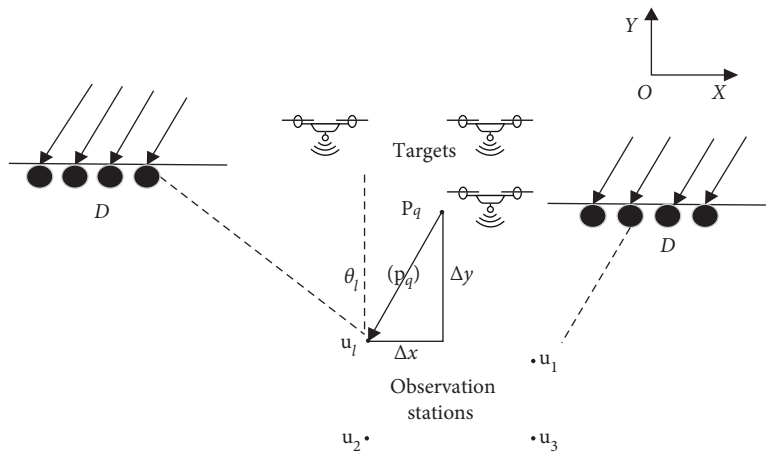

Figure 2: Multiple arrays combination positioning scene.

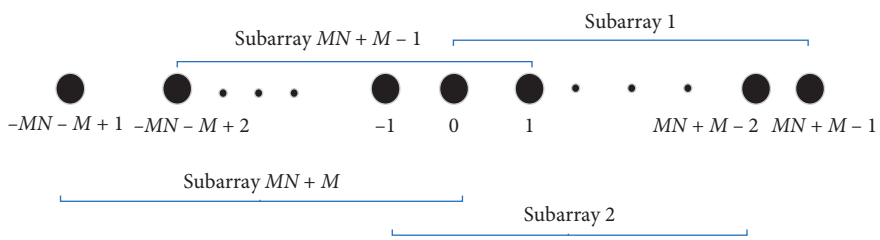

Figure 3: Spatial smoothing diagram.

3.2. Direct Position Determination Based on SNR Weighted SDF. According to the principle of power allocation based on water injection principle, we allocate more power into the channel with good quality and we allocate less power into the channel with poor quality. So, we can obtain the maximum channel capacity. Similarly, in order to reduce the total projection error, we need to design a weight that makes it increase when error decreases. Because high SNR leads to small positioning error and low SNR results in large positioning error. So, in this section, we propose SNR weighted subspace data fusion (SW-SDF) for DPD.

Under the assumption that the noises are uncorrelated and the signals and noises are independent each other, the form of the covariance matrix can be rewritten by substituting equation (4) into (11):

$$\widehat{R}_l = \frac{1}{K}\sum_{k=1}^{K}\left(\sum_{q=1}^{Q} b_{l,q}^2 W_q \mathbf{a}_l(\mathbf{p})\mathbf{a}_l^H(p) + \sigma_n^2 \mathbf{I}_{V\times V}\right), \tag{14}$$

where $\mathbf{I}_{V\times V}$ is identity matrix of $V \times V$, in which $V = MN + M$. The same array receives different power of different radiation sources. And, different arrays receive different power from the same array. These all depend on the signal power W_q and unknown parameters $b_{l,q}$. We assumed that the noise power is constant in the whole observation process, so the SNR of different observation positions is proportional to $b_{l,q}^2 W_q$, that is, $W_{l,q}$; this value is unknown in practical application.

The received signal covariance matrix can be decomposed into two parts:

$$\widehat{R}_l = \mathbf{R}_s + \mathbf{R}_n = \mathbf{A}_l(\mathbf{p})\mathrm{diag}\left(\left[W_{l,1},\ldots,W_{l,Q}\right]\right)\mathbf{A}_l^H(\mathbf{p}) + \sigma_n^2\mathbf{I}_{V\times V}. \tag{15}$$

Under the same assumption, the eigenvalue can be expressed as

$$\lambda_{l,i} = \begin{cases} \sigma_{y_i}^2 + \sigma_n^2, & 1 \le i \le Q, \\ \sigma_n^2, & Q+1 \le i \le V, \end{cases} \tag{16}$$

is large nonzero eigenvalues of $\mathbf{R}_s$, $\sigma_{y_i}^2$ denotes the power Wl,q of the received signal. It is assumed that the noise power is constant in the observation process, and its specific value is unknown in practice. According to equation (16), the estimated value of noise power can be calculated by $V - Q$ smaller eigenvalue as follows:

$$\widehat{\sigma}_{nl}^2 = \frac{1}{V-Q}\sum_{i=Q+1}^{V}\lambda_{l,i}. \tag{17}$$

Due to the small deviation between the estimated value and the real value, the estimated values of the noise power at different observation positions are approximately equal. According to the estimated value of the noise power, the l th observation station can obtain the power as follows:

$$\widehat{W}_l = \sum_{i=1}^{Q}\left(\lambda_{l,i} - \widehat{\sigma}_{nl}^2\right). \tag{18}$$

According to the previous analysis, the position with large SNR of the received signal will produce smaller error. So, we give larger weight to the position, that is, the SNR of

the received signal for this position. Therefore, the loss function of the direct position determination algorithm based on SNR weighting can be constructed as follows:

$$f_{\mathrm{SW-SDF}}(\mathbf{p}) = \sum_{l=1}^{L} \frac{W_l}{\widehat{\sigma}_{nl}^2} \left\| \left(\mathbf{U}_l^n\right)^H \mathbf{a}_l(\mathbf{p}) \right\|^2. \tag{19}$$

By searching the front Q minimum values of equation (19), the high precision emitter position estimation results can be obtained.

3.3. Direct Position Determination Based on Optimal Weighting. In the previous chapter, we introduce the SW-SDF algorithm for DPD that can effectively reduce the total projection error of L observation positions, but the SW-SDF algorithm does not achieve the minimum of the total projection error. So, it is not optimal. In this section, the optimal weighted subspace data fusion (OW-SDF) algorithm for DPD is proposed. The projection error between the steering vector and the noise subspace obtained from the first observation position is defined as

$$\boldsymbol{\xi}_l = \left(\mathbf{U}_l^n\right)^H \mathbf{a}_l(\mathbf{p}). \tag{20}$$

Then, the optimal weighted direct position determination problem can be expressed as finding an optimal weight T^* and emitter position estimation $\widehat{p}$ to minimize the total projection error, that is, the optimal weighted direct position determination problem:

$$\widehat{p}, T^* = \arg\min_{\mathbf{p},\mathbf{W}} \left\| T^{1/2} \boldsymbol{\xi} \right\|^2, \tag{21}$$

where $\xi = [\xi_1^H, \xi_2^H, \ldots, \xi_L^H]^H$ is projection error of all observation positions.

According to reference [15], the projection error vector ξ_l is a variable of zero mean Gaussian distribution, and its covariance matrix has the following form:

$$\begin{aligned} E\left(\boldsymbol{\xi}_i \boldsymbol{\xi}_j^H\right) &= \mathbf{a}_l^H(\mathbf{p}) \boldsymbol{\Lambda}_l \mathbf{a}_l(\mathbf{p}) \boldsymbol{\delta}_{i,j} \mathbf{I}_{(V-Q)\times(V-Q)}, \\ E\left(\boldsymbol{\xi}_i \boldsymbol{\xi}_j^T\right) &= 0_{(V-Q)\times(V-Q)}, \quad \text{for } \forall i, j, \end{aligned} \tag{22}$$

where $\mathbf{I}_{(V-Q)\times(V-Q)}$ and $\mathbf{I}_{(V-Q)\times(V-Q)}$ are $V \times V$ identity matrix and $(V-Q) \times (V-Q)$ zero matrix, $\delta_{i,j}$ is an impulse variable, if and only if $i = j$, $\delta_{i,j} = 1$, the other cases $\delta_{i,j}$ are zero, and the matrix Λ_l has the following form:

$$\boldsymbol{\Lambda}_l = \frac{1}{K} \mathbf{U}_l^s \mathrm{diag}\left(\left[\frac{\lambda_{l,1}}{\sigma_n^2} + \frac{\sigma_n^2}{\lambda_{l,1}} - 2, \ldots, \frac{\lambda_{l,Q}}{\sigma_n^2} + \frac{\sigma_n^2}{\lambda_{l,Q}} - 2 \right] \right) \left(\mathbf{U}_l^s\right)^H. \tag{23}$$

It can be seen from equation (22) that the subvectors ξ_l of the error vector ξ are independent of each other, so we can get that the covariance matrix of the error vector ξ is a $(V-Q)L \times (V-Q)L$ matrix, and each matrix $E(\xi_l \xi_l^H), l = 1, 2, \ldots, L$ is expressed as

$$\mathrm{cov}(\boldsymbol{\xi}) = E\left(\boldsymbol{\xi}\boldsymbol{\xi}^H\right) = \mathrm{diagblk}\left(\left[E\left(\boldsymbol{\xi}_1 \boldsymbol{\xi}_1^H\right), \ldots, E\left(\boldsymbol{\xi}_L \boldsymbol{\xi}_L^H\right) \right] \right). \tag{24}$$

By substituting equations (22) and (23) into equation (24), the solution of the optimal weight can be obtained

$$T^* = \mathrm{diag}\left(\left[t_1^*, \ldots, t_L^* \right] \right) \otimes \mathbf{I}_{(V-Q)\times(V-Q)}, \tag{25}$$

where

$$t_l^*(\mathbf{p}) = \frac{1}{\sum_{q=1}^{Q} g_{l,q} \left\| \left(\mathbf{U}_l^s\right)^H \mathbf{a}_l(\mathbf{p}) \right\|^2}, \tag{26}$$

where $g_{l,q}$ is the weight of the signal from the radiation source Q in the received signal of l th array and $g_{l,q}$ is related to SNR and can be expressed as

$$g_{l,q} = \left(\rho_{l,q} + \frac{1}{\rho_{l,q}} - 2 \right)^{-1}, \tag{27}$$

where $\rho_{l,q} = 1 + \mathrm{SNR}_{l,q} = \lambda_{l,q}/\sigma_n^2$. According to equation (27), the optimal weight not only considers the difference between the received signal SNR but also considers the noise subspace and search grid points.

According to equation (25), the loss function of the direct position determination algorithm based on optimal weight can be constructed as follows:

$$f_{\mathrm{OW-SDF}}(\mathbf{p}) = \sum_{l=1}^{L} \frac{\left\| \left(\mathbf{U}_l^n\right)^H \mathbf{a}_l(\mathbf{p}) \right\|^2}{\left\| \mathrm{diag}\left(\left[g_{l,1}^{1/2}, \ldots, g_{l,Q}^{1/2} \right] \right) \left(\mathbf{U}_l^s\right)^H \mathbf{a}_l(\mathbf{p}) \right\|^2}. \tag{28}$$

By searching the front Q minimum values of equation (28), the high precision emitter position estimation results can be obtained.

3.4. The Procedure of the Proposed Algorithm. We summarize several steps about W-SDF algorithms as follows:

Step 1. Construct the sources model and augmented coprime array positioning model for DPD.

Step 2. Adopt the vector and spatial smoothing method for receiving signals.

Step 3. Calculate the covariance matrix to get noise subspace $\mathbf{U}_l^n$. Assign a weight to the projection result at each observation position.

Step 4. Construct the cost function $f_{W-\mathrm{SDF}}(\mathbf{p})$. The coordinate corresponding to the peak value is the position estimation value $(\widehat{x}_q, \widehat{y}_q)$.

4. Performance Analysis

4.1. Achievable DOFs. In this paper, we use augmented coprime array which increases spatial degree of freedom than uniform linear array. The DOFs of the augmented coprime array are $2MN + 2M - 1$, and the DOFs of the uniform linear array are $2M + N - 1$. It can be obviously

seen that the degree of freedom of augmented coprime array is higher than freedom of uniform linear array.

4.2. Computational Complexity. The computational complexity of the proposed two weighted direct position determination algorithms is compared with SDF direct position determination for augmented coprime array, which only considers the number of complex multiplication. The computational complexity of W-SDF and SDF algorithms is related to the following parameters: L denotes the number of observation positions, Q denotes the number of targets, D denotes the number of array elements, and K denotes the number of snapshots; we divide X direction into L_x equal parts in global search and divide Y direction into L_y equal parts.

For the direct position determination algorithm SW-SDF and OW-SDF, the complexity of the algorithm includes the following aspects: the calculation of the covariance matrix of dimension receiving signal $O(KD^2)$, the decomposition of eigenvalue of the covariance matrix of the dimension receiving signal $O(V^3)$, and the calculation of spectral peak value of each searching grid point $O(V^2(V-Q)+V^2+V)$. In addition, the weight calculation in the SW-SDF algorithm does not need to increase the additional computational complexity. The OW-SDF algorithm computational complexity is $O(LKD^2+ LV^3+ LL_xL_y(V^2(V-Q)+2V^2+2V))$. In summary, the computational complexity of the three direct positioning algorithms is shown in Table 1.

Figure 4 shows the comparison of the complexity of several algorithms with the number of search points in X (or Y) direction under specific parameters. The simulation parameters are set as follows: the number of observation positions $L = 5$, the number of radiation sources $Q = 3$, two subarrays are $M = 3$ and $N = 5$, the number of augmented array elements $D = 10$, the number of snapshots $K = 100$, the number of array elements after smoothing is $V = 18$, and the number of search points along X and Y directions, with the range of 100 to 1000. The computational complexity of the PM algorithm is slightly lower than that of SDF and SW-SDF algorithms. The computational complexity of the Capon algorithm is higher than that of other algorithms. Compared with the SDF method, SW-SDF can improve the positioning performance without increasing the complexity. The complexity of the OW-SDF algorithm is slightly higher than that of the SW-SDF algorithm, but the positioning performance is greatly improved. We will explain this in detail in the subsequent simulation analysis.

4.3. Advantage. Based on the above research, we make a list of advantages about W-SDF algorithms for coprime array:

(1) The proposed W-SDF algorithms do not need any parameter estimation step, avoid the secondary loss of information, and effectively improve the positioning accuracy.

(2) The proposed W-SDF algorithms use augmented coprime array characteristics. Compared with the algorithm of uniform array, there is a significant improvement in DOF. The spatial freedom of the array can be expanded, and the number of identified sources is increased.

(3) We assign a weight to the projection result at each observation position and construct the loss function. Aiming at the problem of poor stability and noise sensitivity of the SDF method, this paper considers to obtain the loss function with small error and high robustness by balancing the orthogonal projection error and makes most of the data to improve the positioning accuracy.

5. Simulation Results

5.1. Simulations Results versus Proposed Algorithm. There are 5 augmented coprime arrays located at 5 observation stations. Each station has an augmented coprime array. The locations of observations are $\mathbf{U}_1 = [-1000\,\text{m}, -500\,\text{m}]$, $\mathbf{U}_1 = [-500\,\text{m}, -500\,\text{m}]$, $\mathbf{U}_3 = [0\,\text{m}, -500\,\text{m}]$, $\mathbf{U}_4 = [500\,\text{m}, -500\,\text{m}]$, and $\mathbf{U}_5 = [1000\,\text{m}, -500\,\text{m}]$. Multiple targets are $\mathbf{P}_1 = [100\,\text{m}, 100\,\text{m}]$, $\mathbf{P}_2 = [300\,\text{m}, 300\,\text{m}]$, and $\mathbf{P}_3 = [700\,\text{m}, 700\,\text{m}]$, and Figure 5 denotes direct position determination cost function with the SNR weighting algorithm. Figure 6 denotes the direct position determination scatter diagram with the SNR weighting algorithm. Figure 7 denotes direct position determination with the optimal weighting algorithm. Figure 8 denotes the direct position determination scatter diagram with the optimal weighting algorithm.

In these simulation experiments, the performance of the proposed W-SDF method is analyzed by calculating the root mean square error (RMSE), and it can be expressed as

$$\text{RMSE} = \frac{1}{Q}\sqrt{\frac{1}{\text{MC}}\sum_{mc=1}^{\text{MC}}\sum_{q=1}^{Q}\left\|\mathbf{p}_q - \widehat{\mathbf{p}}_{q,mc}\right\|^2}, \tag{29}$$

where MC is the number of the Monte Carlo (MC) simulation test, Q is the number of target sources, $\widehat{\mathbf{p}}_{q,mc}$ denotes the mc th Monte Carlo estimated value of the location of the qth target, and $\mathbf{p}_q$ is the real value of the qth target.

5.2. RMSE Results versus Comparison of W-SDF Algorithms with Other Algorithms. This paper simulates the comparison of different algorithms with augmented coprime array. The number of augmented coprime array elements is $(M, N) = (3, 5)$, and there are multiple targets $\mathbf{P}_1 = [100\,\text{m}, 100\,\text{m}]$, $\mathbf{P}_2 = [300\,\text{m}, 300\,\text{m}]$, and $\mathbf{P}_3 = [700\,\text{m}, 700\,\text{m}]$. The snapshot number is 100. Each station has an augmented coprime array. Figure 9 shows that the performance for DPD of the W-SDF algorithm is better than that of the SDF and PM algorithm for augmented coprime array. OW-SDF is slightly better than SW-SDF in positioning accuracy. CRB for augmented coprime array is simulated in Figure 9.

Table 1: Computational complexity of different algorithms.

Algorithms	Computational complexity	Running time (s)
SDF	$O(LKD^2 + LV^3 + LL_xL_y(V^2(V-Q)+V^2+V))$	32.867873
SW-SDF	$O(LKD^2 + LV^3 + LL_xL_y(V^2(V-Q)+V^2+V))$	32.180425
OW-SDF	$O(LKD^2 + LV^3 + LL_xL_y(V^2(V-Q)+2V^2+2V))$	36.953023
Capon	$O(LKD^2 + LL_xL_y(V^3+V^2+V))$	101.878995
PM	$O(LKD^2 + L(2Q^2V + QV(V-Q)+Q^3) + LL_xL_y(V^2(V-Q)+V^2+V))$	23.165267

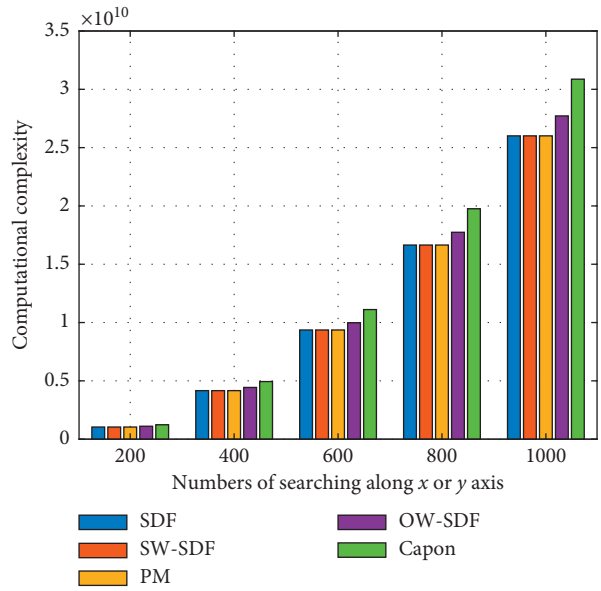

Figure 4: Comparison of different algorithms in computational complexity.

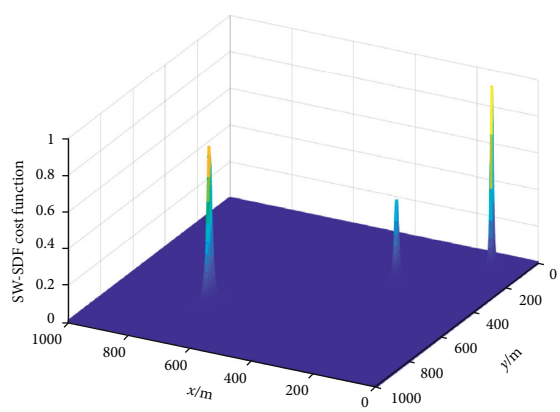

Figure 5: SW-SDF direct position determination.

5.3. RMSE Results versus Comparison of W-SDF Algorithms under Different Snapshot Numbers for Augmented Coprime Array. This paper simulates the comparison of different algorithms with different snapshot numbers for augmented coprime array. The number of augmented coprime array elements is $(M, N) = (3, 5)$, and there are multiple targets $\mathbf{P}_1 = [100\,\text{m}, 100\,\text{m}]$, $\mathbf{P}_2 = [300\,\text{m}, 300\,\text{m}]$, and $\mathbf{P}_3 = [700\,\text{m}, 700\,\text{m}]$. Each station has an augmented coprime array. Figure 10 shows that as the number of snapshots increases, the performance for DPD with augmented coprime array of W-SDF algorithms is better than that of SDF, Capon, and PM algorithms.

5.4. RMSE Results versus Comparison of W-SDF Algorithms under Different Arrays. This paper simulates the comparison of different algorithms with augmented coprime array and uniform array. The number of augmented coprime array elements is $(M, N) = (3, 5)$, and there are multiple targets $\mathbf{P}_1 = [100\,\text{m}, 100\,\text{m}]$, $\mathbf{P}_2 = [300\,\text{m}, 300\,\text{m}]$, and $\mathbf{P}_3 = [700\,\text{m}, 700\,\text{m}]$. Each station has an augmented coprime array. The snapshot number is 100. Figure 11 shows that as SNR increases, the performance of W-SDF algorithms for DPD with augmented coprime array is better than that of W-SDF and SDF algorithms with uniform array.

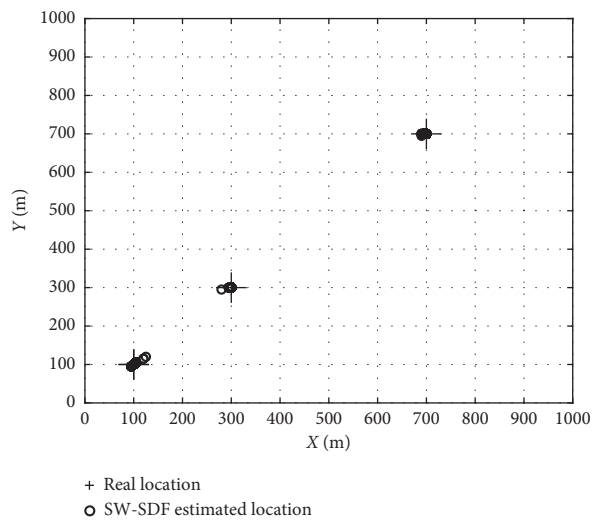

Figure 6: SW-SDF direct position determination scatters.

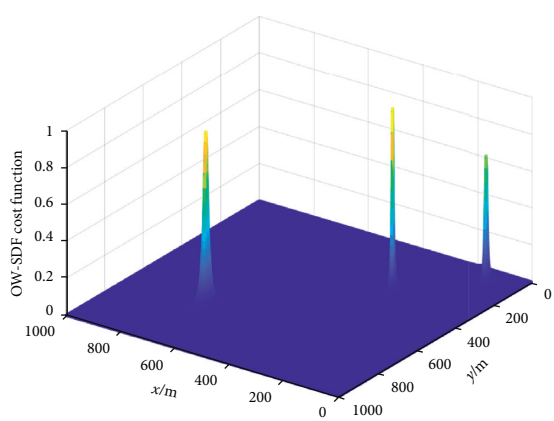

Figure 7: OW-SDF direct position determination.

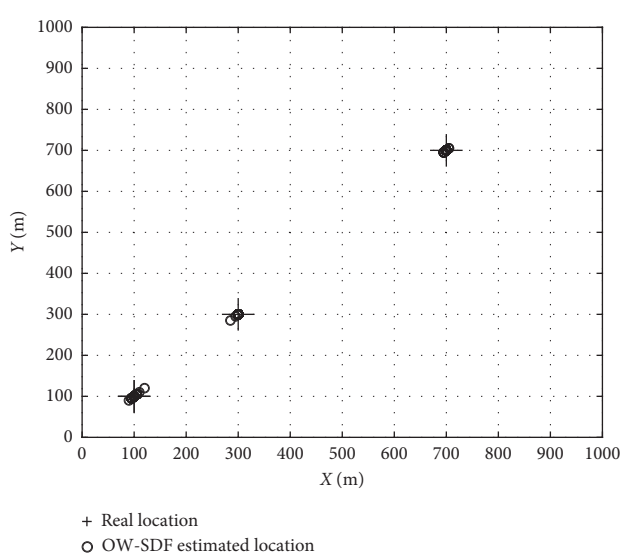

Figure 8: OW-SDF direct position determination scatters.

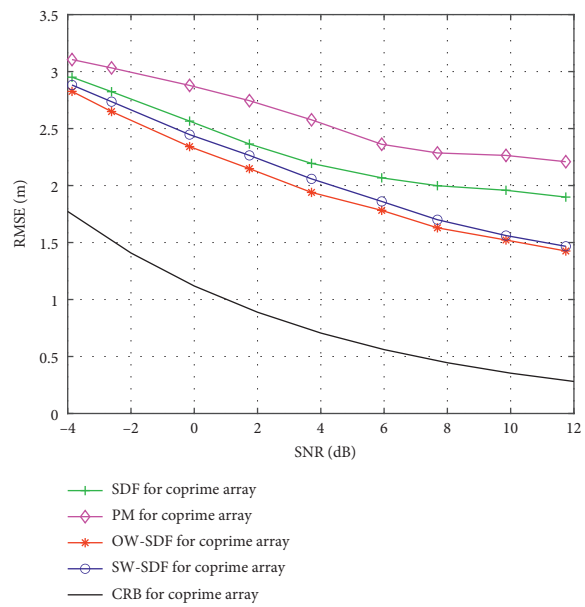

Figure 9: Comparison of different algorithms for coprime array.

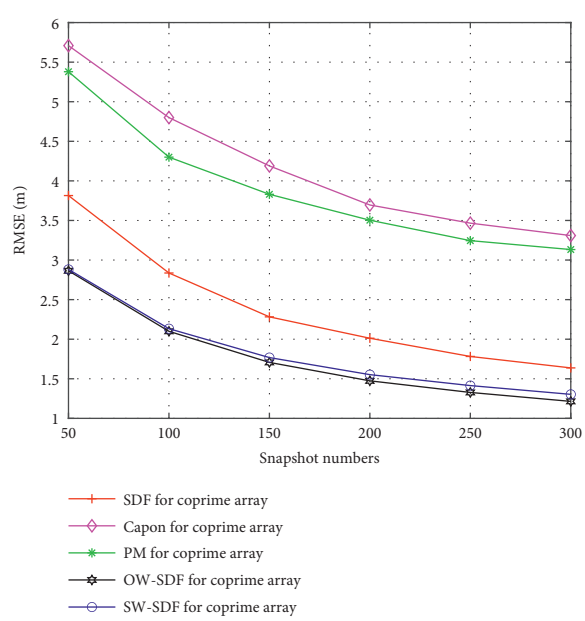

Figure 10: Comparison of different algorithms with snapshot numbers.

5.5. RMSE Results versus Comparison of W-SDF Algorithms under Different Elements. This paper simulates the comparison of different algorithms with different elements. There are multiple targets $\mathbf{P}_1 = [100\,\text{m}, 100\,\text{m}]$, $\mathbf{P}_2 = [300\,\text{m}, 300\,\text{m}]$, and $\mathbf{P}_3 = [700\,\text{m}, 700\,\text{m}]$. Each station has an augmented coprime array. Set the number of elements $(M1, N1) = (3, 5)$, $(M2, N2) = (3, 7)$, and $(M3, N3) = (5, 7)$. The snapshot number is 100. In Figure 12, simulation results show that performance of the proposed W-SDF algorithms is better with increment of elements.

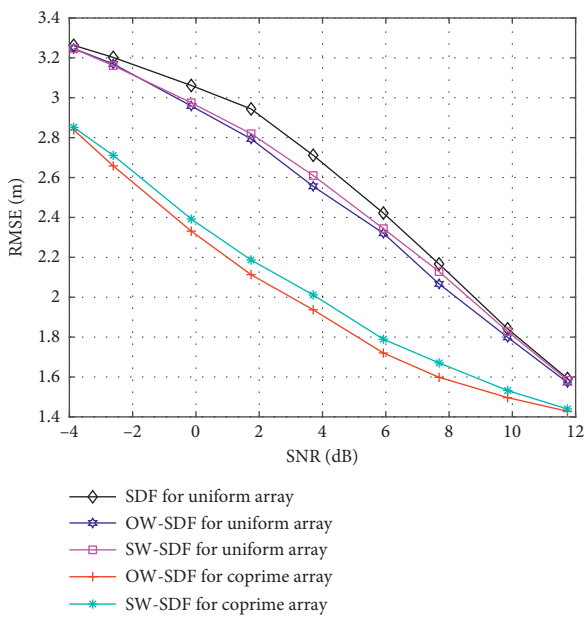

Figure 11: Comparison of algorithms with uniform array and coprime array.

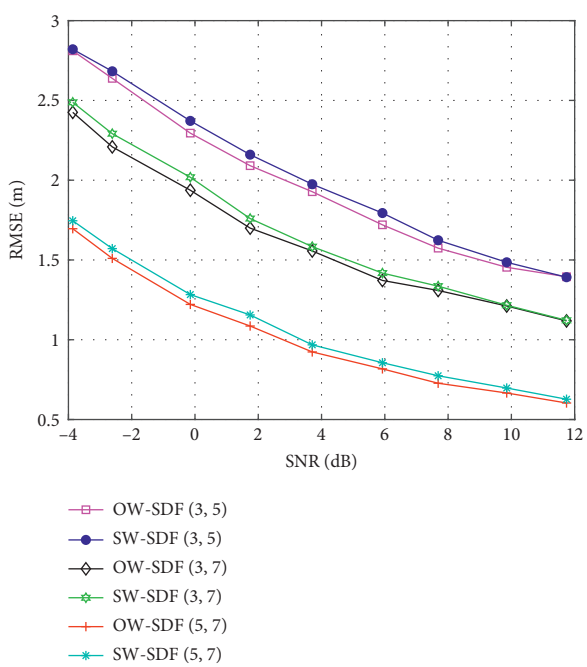

Figure 12: Comparison of W-SDF with different elements.

6. Conclusion

We introduce multiple augmented coprime arrays into the direct position determination model, which increases spatial freedom and position accuracy. We assign a weight to the projection result at each observation position to obtain better positioning accuracy. Simulation results show that two W-SDF algorithms have a prominent promotion in positioning accuracy than SDF, Capon, and PM algorithms for augmented coprime arrays. OW-SDF is slightly better than SW-SDF in positioning accuracy. The performance for DPD of the W-SDF method with augmented coprime arrays is better than that of the W-SDF method with uniform arrays.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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