Research Article

Closed-Form Solution of a Rational Difference Equation

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In this paper, we study the solution of the difference equation

$$\Omega_{m+1} = \frac{\Omega_{m-1}}{1 + a\Omega_{m+1}^2}, \quad m = 0, 1, \ldots,$$

(1)

respectively. DeVault et al. [4] examined

$$\Omega_{m+1} = \frac{A}{\Omega_m} + \frac{1}{\Omega_{m-2}}, \quad m = 0, 1, \ldots$$

(2)

Elsayed [5] dealt with

$$\Omega_{m+1} = a + \frac{b\Omega_{m-1} + c\Omega_{m-2}}{d\Omega_{m-2} + e\Omega_{m-2}}, \quad m = 0, 1, \ldots$$

(3)

respectively. For some more results concerning difference equations, we refer the reader to [10–21].

In this work, we deal with the following nonlinear difference equation:

$$\Omega_{m+1} = \frac{\Omega_{m-3}}{1 + \prod_{t=0}^{\infty} \Omega_{m-(qt+1)}}, \quad m = 0, 1, \ldots$$

(5)

where $$\Omega_{m-(qt+6)}, \Omega_{m-(qt+5)}, \ldots, \Omega_{m-1}, \Omega_0 \in (0, \infty)$$ is investigated.
2. Main Results

Let \( \Omega \) be the unique equilibrium of equation (5); then,
\[
\tilde{\Omega} = \frac{\Omega}{1 + \Omega \cdot \Omega \cdot \Omega \cdot \Omega \cdot \Omega} \Rightarrow \Omega + \tilde{\Omega} = \tilde{\Omega} = 0 \Rightarrow \Omega = 0,
\]
so \( \Omega = 0 \) is obtained. For every \( q \geq 0 \) and \( m > q \), notation \( i = \frac{q}{m} \) means \( i = q, q + 1, \ldots, m \).

**Theorem 1.** For (5), the following statements are true:

(a) The sequences \( \{ \Omega(t) \} \) are decreased, and \( a_1, a_2, \ldots, a_{7q+3} \geq 0 \) exists such that
\[
\lim_{n \to \infty} \Omega(7q+3)n-(7q+6)+t = a_{1+t}, \quad t = 0, 7q+6.
\]

(b) In view of equation (5),
\[
\Omega(7q+3)n+(7q+6) = \frac{\Omega(7q+3)n+(7q+6)}{1 + \sum_{i=0}^{5} \Omega(7q+3)n-(q+i)|t|}.
\]

Then,
\[
\lim_{n \to \infty} \Omega(7q+3)n+(7q+6) = \lim_{n \to \infty} \frac{\Omega(7q+3)n+(7q+6)}{1 + \sum_{i=0}^{5} \Omega(7q+3)n-(q+i)|t|}.
\]

(c) If there exists \( n_0 \in \mathbb{N} \) such that
\[
\sum_{t=0}^{5} \Omega(n-t)=\Omega_{n+1} \leq \Omega_{n+1} \leq \sum_{t=0}^{5} \Omega(n-t)=\Omega_{n+1}
\]
for all \( n \geq n_0 \), then
\[
\lim_{n \to \infty} \Omega_{n}=0.
\]

(d) We can generate the following formulas:
\[
\Omega(7q+3)n+(7q+6) = \frac{\Omega(7q+3)n+(7q+6)}{1 + \sum_{i=0}^{5} \Omega(7q+3)n-(q+i)|t|}.
\]

(e) If \( \Omega(7q+3)n+(7q+6)+1 |t| = a_{1+t} \neq 0 \) then \( \Omega(7q+3)n+(7q+6)+1 \).

If \( \Omega(7q+3)n+(7q+6)+1 \) then \( \Omega(7q+3)n+(7q+6)+1 \) as \( n \to \infty \).

\[
\Omega_{n+1} \leq \Omega_{n} \leq \Omega_{n+1}
\]
for all \( n \geq n_0 \), then
\[
\lim_{n \to \infty} \Omega_{n}=0.
\]

Proof

(a) Firstly, for all \( n \in \mathbb{N} \), from (5), one gets
\[
\lim_{n \to \infty} \Omega(7q+3)n+(7q+6)+t = a_{1+t}, \quad t = 0, 7q+6.
\]

(b) In view of equation (5),
\[
\Omega(7q+3)n+(q+i)|t| = \frac{\Omega(7q+3)n+(q+i)|t|}{1 + \sum_{i=0}^{5} \Omega(7q+3)n-(q+i)|t|}.
\]

Then,
\[
\lim_{n \to \infty} \Omega(7q+3)n+(q+i)|t| = \lim_{n \to \infty} \frac{\Omega(7q+3)n+(q+i)|t|}{1 + \sum_{i=0}^{5} \Omega(7q+3)n-(q+i)|t|}.
\]

(c) If there exists \( n_0 \in \mathbb{N} \) such that
\[
\sum_{t=0}^{5} \Omega(n-t)=\Omega_{n+1} \leq \Omega_{n+1} \leq \sum_{t=0}^{5} \Omega(n-t)=\Omega_{n+1}
\]
for all \( n \geq n_0 \), then
\[
\lim_{n \to \infty} \Omega_{n}=0.
\]

(d) Subtracting \( \Omega_{n-(7q+6)} \) from both sides in (5), we have
\[
\Omega_{n+1} - \Omega_{n-(7q+6)} = \frac{1}{1 + \sum_{i=0}^{5} \Omega_{n-(q+i)|t|}}.
\]
and the following formula, for \( n \geq q+1 \),
\[
\Omega(n-t)=\Omega_{n-(7q+6)} \leq \Omega_{n-(7q+6)} \leq \Omega(n-t).
\]

From (20), we get
\[
\Omega_{n+1} - \Omega_{n-(6q+6)} = \frac{1}{1 + \sum_{i=0}^{5} \Omega_{n-(q+i)|t|}}.
\]
\[ \Omega_{(7q+7)n+1} - \Omega_{-(7q+6)} = (\Omega_1 - \Omega_{-(7q+6)}) \sum_{h=0}^{n} \prod_{k=1}^{7h} \frac{1}{\Omega_{(q+1)(-q+1)t-q} + 1}. \]

\[
\overbrace{\cdots}^{\theta}
\]

So,

\[ \Omega_{(7q+7)n+q+1} - \Omega_{-(6q+6)} = (\Omega_{q+1} - \Omega_{-(6q+6)}) \sum_{h=0}^{n} \prod_{k=1}^{7h} \frac{1}{\Omega_{(q+1)(-q+1)t-q} + 1}. \]

\[
\overbrace{\cdots}^{\theta}
\]

Also,

\[ \Omega_{(7q+7)n+2q+2} - \Omega_{-(5q+5)} = (\Omega_{q+1} - \Omega_{-(5q+5)}) \sum_{h=0}^{n} \prod_{k=1}^{7h+1} \frac{1}{\Omega_{(q+1)(-q+1)t-q} + 1}. \]

\[
\overbrace{\cdots}^{\theta}
\]

\[ \Omega_{(7q+7)n+2q+3} - \Omega_{-(5q+4)} = (\Omega_{q+1} - \Omega_{-(5q+4)}) \sum_{h=0}^{n} \prod_{k=1}^{7h+2} \frac{1}{\Omega_{(q+1)(-q+1)t-q} + 1}. \]

\[
\overbrace{\cdots}^{\theta}
\]

\[ \Omega_{(7q+7)n+3q+3} - \Omega_{-(4q+4)} = (\Omega_{q+1} - \Omega_{-(4q+4)}) \sum_{h=0}^{n} \prod_{k=1}^{7h+2} \frac{1}{\Omega_{(q+1)(-q+1)t-q} + 1}. \]
Moreover,

\[
\Omega_{(7q+7)n+3q+4} - \Omega_{-(4q+3)} = \left(\Omega_1 - \Omega_{-(7q+6)}\right) \sum_{h=0}^{n} \prod_{k=1}^{\frac{7h+3}{5}} \frac{1}{\Omega_{(q+1)k-(q+1)t-q} + 1}
\]

(24)

On the contrary,

\[
\Omega_{(7q+7)n+4q+4} - \Omega_{-(3q+3)} = \left(\Omega_{q+1} - \Omega_{-(6q+6)}\right) \sum_{h=0}^{n} \prod_{k=1}^{\frac{7h+4}{5}} \frac{1}{\Omega_{(q+1)k-(q+1)t-q} + 1}
\]

(25)

Also,

\[
\Omega_{(7q+7)n+5q+5} - \Omega_{-(2q+2)} = \left(\Omega_{q+1} - \Omega_{-(6q+6)}\right) \sum_{h=0}^{n} \prod_{k=1}^{\frac{7h+5}{5}} \frac{1}{\Omega_{(q+1)k-(q+1)t-q} + 1}
\]

(26)
Moreover,

\[
\Omega_{(7q+7)m+6q+7} - \Omega_{-r} = \left( \Omega_{1} - \Omega_{-(7q+6)} \right) \sum_{h=0}^{n} \prod_{k=1}^{7h+6} \frac{1}{\prod_{t=0}^{r} \Omega_{(q+1)k-(q+1)r-q} + 1}
\]

(27)

Now, we get the above formulas:

\[
\Omega_{(7q+7)m+7q+7} - \Omega_{0} = \left( \Omega_{q+1} - \Omega_{-(q+6)} \right) \sum_{h=0}^{n} \prod_{k=1}^{7h+6} \frac{1}{\prod_{t=0}^{r} \Omega_{(q+1)k-(q+1)r + 1}}
\]

where \( r = 0, 6 \) and \( s = 0, q \) hold.

(e) Suppose that \( a_1 = a_{q+2} = a_{2q+3} = a_{3q+4} = a_{4q+5} = a_{5q+6} = a_{6q+7} = 0 \). By (d), we produce the following formulas:

\[
\lim_{n \to \infty} \Omega_{(7q+7)m+1} = \lim_{n \to \infty} \Omega_{-(7q+6)} \left( 1 - \frac{\prod_{t=0}^{5} \Omega_{-(q+1)t-q}}{\prod_{t=0}^{5} \Omega_{(q+1)t-(q+1)t-q} + 1} \right)
\]

\[
\times \sum_{h=0}^{\infty} \prod_{k=1}^{7h} \frac{1}{\prod_{t=0}^{r} \Omega_{(q+1)k-(q+1)t-q} + 1}
\]

(29)

\[
a_1 = \Omega_{-(7q+6)} \left( 1 - \frac{\prod_{t=0}^{5} \Omega_{-(q+1)t-q} \Omega_{-11+q}}{\prod_{t=0}^{5} \Omega_{(q+1)t-(q+1)t-q} + 1} \right) \sum_{h=0}^{\infty} \prod_{k=1}^{7h} \frac{1}{\prod_{t=0}^{r} \Omega_{(q+1)k-(q+1)t-q} + 1}
\]

\[
a_1 = \frac{\prod_{t=0}^{5} \Omega_{-(q+1)t-q} + 1}{\prod_{t=0}^{5} \Omega_{(q+1)t-(q+1)t-q} + 1} = \sum_{h=0}^{\infty} \prod_{k=1}^{7h} \frac{1}{\prod_{t=0}^{r} \Omega_{(q+1)k-(q+1)t-q} + 1}
\]
Similarly,

\[
\lim_{n \to \infty} \Omega_{(7q+7)n+q+2} = \lim_{n \to \infty} \Omega_{-(6q+5)} \left( 1 - \frac{\prod_{r=0}^{6} (\Omega_{-(q+1)r-q}/\Omega_{-(6q+5)})}{\prod_{r=0}^{5} \Omega_{-(q+1)r-q} + 1} \right) \\
\times \sum_{h=0}^{n} \prod_{k=1}^{7h+1} \frac{1}{\prod_{r=0}^{5} \Omega_{-(q+1)k-(q+1)r-q} + 1},
\]

\[
a_{q+2} = \Omega_{-(6q+5)} \left( 1 - \frac{\prod_{r=0}^{6} (\Omega_{-(q+1)r-q}/\Omega_{-(6q+5)})}{\prod_{r=0}^{5} \Omega_{-(q+1)r-q} + 1} \right) \\
\times \sum_{i=0}^{\infty} \prod_{k=1}^{7h+1} \frac{1}{\prod_{r=0}^{5} \Omega_{-(q+1)k-(q+1)r-q} + 1},
\]

\[
a_{q+2} = 0 \Rightarrow \prod_{r=0}^{6} \Omega_{-(q+1)r-q} + 1 = \sum_{h=0}^{\infty} \prod_{k=1}^{7h+1} \frac{1}{\prod_{r=0}^{5} \Omega_{(q+1)k-(q+1)r-q} + 1}
\]

Similarly,

\[
\lim_{n \to \infty} \Omega_{(7q+7)n+2q+3} = \lim_{n \to \infty} \Omega_{-(5q+4)} \left( 1 - \frac{\prod_{r=0}^{6} (\Omega_{-(q+1)r-q}/\Omega_{-(5q+4)})}{\prod_{r=0}^{5} \Omega_{-(q+1)r-q} + 1} \right) \\
\times \sum_{h=0}^{n} \prod_{k=1}^{7h+2} \frac{1}{\prod_{r=0}^{5} \Omega_{-(q+1)k-(q+1)r-q} + 1},
\]

\[
a_{2q+3} = \Omega_{-(5q+4)} \left( 1 - \frac{\prod_{r=0}^{6} (\Omega_{-(q+1)r-q}/\Omega_{-(5q+4)})}{\prod_{r=0}^{5} \Omega_{-(q+1)r-q} + 1} \right) \\
\times \sum_{h=0}^{\infty} \prod_{k=1}^{7h+2} \frac{1}{\prod_{r=0}^{5} \Omega_{-(q+1)k-(q+1)r-q} + 1},
\]

\[
a_{2q+3} = 0 \Rightarrow \prod_{r=0}^{6} \Omega_{-(q+1)r-q} + 1 = \sum_{h=0}^{\infty} \prod_{k=1}^{7h+2} \frac{1}{\prod_{r=0}^{5} \Omega_{(q+1)k-(q+1)r-q} + 1}
\]
Similarly,

\[
\lim_{n \to \infty} \Omega_{(7q+7)\tau + 3q-4} = \lim_{n \to \infty} \Omega_{-(4q+3)} \left( 1 - \frac{\prod_{t=0}^{6}(\Omega_{-(q+1)\tau-q}/\Omega_{-(4q+3)})}{\prod_{t=0}^{5} \Omega_{-(q+1)\tau-q} + 1} \right) 
\]

\[
\times \sum_{h=0}^{n} \prod_{k=1}^{7h+3} \frac{1}{\Omega_{-(q+1)k-(q+1)\tau-q} + 1} 
\]

\[
da_{3q+4} = \Omega_{-(4q+3)} \left( 1 - \frac{\prod_{t=0}^{6}(\Omega_{-(q+1)\tau-q}/\Omega_{-(4q+3)})}{\prod_{t=0}^{5} \Omega_{-(q+1)\tau-q} + 1} \right) 
\]

\[
\times \sum_{h=0}^{n} \prod_{k=1}^{7h+3} \frac{1}{\Omega_{-(q+1)k-(q+1)\tau-q} + 1} 
\]

\[
da_{3q+4} = 0 \Rightarrow \frac{\prod_{t=0}^{6}(\Omega_{-(q+1)\tau-q}/\Omega_{-(4q+3)})}{\prod_{t=0}^{5} \Omega_{-(q+1)\tau-q} + 1} = \sum_{h=0}^{n} \prod_{k=1}^{7h+3} \frac{1}{\Omega_{-(q+1)k-(q+1)\tau-q} + 1} 
\]

Similarly,

\[
\lim_{n \to \infty} \Omega_{(7q+7)\tau + 4q-5} = \lim_{n \to \infty} \Omega_{-(3q+2)} \left( 1 - \frac{\prod_{t=0}^{6}(\Omega_{-(q+1)\tau-q}/\Omega_{-(3q+2)})}{\prod_{t=0}^{3} \Omega_{-(q+1)\tau-q} + 1} \right) 
\]

\[
\times \sum_{h=0}^{n} \prod_{k=1}^{7h+4} \frac{1}{\Omega_{-(q+1)k-(q+1)\tau-q} + 1} 
\]

\[
da_{4q+5} = \Omega_{-(3q+2)} \left( 1 - \frac{\prod_{t=0}^{6}(\Omega_{-(q+1)\tau-q}/\Omega_{-(3q+2)})}{\prod_{t=0}^{3} \Omega_{-(q+1)\tau-q} + 1} \right) 
\]

\[
\times \sum_{h=0}^{n} \prod_{k=1}^{7h+4} \frac{1}{\Omega_{-(q+1)k-(q+1)\tau-q} + 1} 
\]

\[
da_{4q+5} = 0 \Rightarrow \frac{\prod_{t=0}^{6}(\Omega_{-(q+1)\tau-q}/\Omega_{-(3q+2)})}{\prod_{t=0}^{3} \Omega_{-(q+1)\tau-q} + 1} = \sum_{h=0}^{n} \prod_{k=1}^{7h+4} \frac{1}{\Omega_{-(q+1)k-(q+1)\tau-q} + 1} 
\]
Similarly,

$$\lim_{n \to \infty} \Omega_{(7q+7)n+5q+6} = \lim_{n \to \infty} \Omega_{-(2q+1)} \left( 1 - \frac{\prod_{t=0}^{6} \Omega_{-(q+1)t-q}/\Omega_{-(2q+1)}}{\prod_{t=0}^{5} \Omega_{-(q+1)t-q} + 1} \right) \times \sum_{h=0}^{n} \prod_{k=1}^{7h+5} \frac{1}{\prod_{r=0}^{5} \Omega_{-(q+1)r-q} + 1},$$

$$a_{5q+6} = \Omega_{-(2q+1)} \left( 1 - \frac{\prod_{t=0}^{6} \Omega_{-(q+1)t-q}/\Omega_{-(2q+1)}}{\prod_{t=0}^{5} \Omega_{-(q+1)t-q} + 1} \right) \times \sum_{h=0}^{n} \prod_{k=1}^{7h+5} \frac{1}{\prod_{r=0}^{5} \Omega_{-(q+1)r-q} + 1},$$

(34)

Similarly,

$$\lim_{n \to \infty} \Omega_{(7q+7)n+6q+7} = \lim_{n \to \infty} \Omega_{-(7q+7)n+6q+7} \left( 1 - \frac{\prod_{t=0}^{6} \Omega_{-(q+1)t-q}/\Omega_{-(2q+1)}}{\prod_{t=0}^{5} \Omega_{-(q+1)t-q} + 1} \right) \times \sum_{h=0}^{n} \prod_{k=1}^{7h+6} \frac{1}{\prod_{r=0}^{5} \Omega_{-(q+1)r-q} + 1},$$

$$a_{6q+7} = \Omega_{-(7q+7)n+6q+7} \left( 1 - \frac{\prod_{t=0}^{6} \Omega_{-(q+1)t-q}/\Omega_{-(2q+1)}}{\prod_{t=0}^{5} \Omega_{-(q+1)t-q} + 1} \right) \times \sum_{h=0}^{n} \prod_{k=1}^{7h+6} \frac{1}{\prod_{r=0}^{5} \Omega_{-(q+1)r-q} + 1},$$

(35)

$$a_{6q+7} = 0 \Rightarrow \frac{\prod_{t=0}^{5} \Omega_{-(q+1)t-q}}{\prod_{t=0}^{5} \Omega_{-(q+1)t-q} + 1} = \sum_{h=0}^{n} \prod_{k=1}^{7h+6} \frac{1}{\prod_{r=0}^{5} \Omega_{-(q+1)r-q} + 1},$$

From equations (29) and (30),

$$\frac{\prod_{t=0}^{5} \Omega_{-(q+1)t-q} + 1}{\prod_{t=0}^{5} \Omega_{-(q+1)t-q}} = \sum_{h=0}^{n} \prod_{k=1}^{7h+6} \frac{1}{\prod_{r=0}^{5} \Omega_{-(q+1)r-q} + 1} >$$

(36)

$$\frac{\prod_{t=0}^{5} \Omega_{-(q+1)t-q} + 1}{\prod_{t=0}^{5} \Omega_{-(q+1)t-q} + 1} = \sum_{h=0}^{n} \prod_{k=1}^{7h+6} \frac{1}{\prod_{r=0}^{5} \Omega_{-(q+1)r-q} + 1}$$
Thus, $\Omega_{-(7q+6)} > \Omega_{-(6q+5)}$.

$$\frac{\prod_{f=0}^{5} \Omega_{-(q+1)r-q}}{\prod_{f=0}^{6} \Omega_{-(q+1)r-q}^2} = \sum_{h=0}^{\infty} \prod_{k=1}^{7k+1} \frac{1}{\prod_{r=0}^{7r+1} \Omega_{-(q+1)r-q}^k r-q + 1}$$

(37)

Thus, $\Omega_{-(6q+5)} > \Omega_{-(5q+4)}$.

$$\frac{\prod_{f=0}^{5} \Omega_{-(q+1)r-q}}{\prod_{f=0}^{6} \Omega_{-(q+1)r-q}^3} = \sum_{h=0}^{\infty} \prod_{k=1}^{7h+2} \frac{1}{\prod_{r=0}^{7r+2} \Omega_{-(q+1)r-q}^k r-q + 1}$$

(38)

Thus, $\Omega_{-(5q+4)} > \Omega_{-(4q+3)}$.

$$\frac{\prod_{f=0}^{5} \Omega_{-(q+1)r-q}}{\prod_{f=0}^{6} \Omega_{-(q+1)r-q}^4} = \sum_{h=0}^{\infty} \prod_{k=1}^{7h+3} \frac{1}{\prod_{r=0}^{7r+3} \Omega_{-(q+1)r-q}^k r-q + 1}$$

(39)

Thus, $\Omega_{-(4q+3)} > \Omega_{-(3q+2)}$.

$$\frac{\prod_{f=0}^{5} \Omega_{-(q+1)r-q}}{\prod_{f=0}^{6} \Omega_{-(q+1)r-q}^5} = \sum_{h=0}^{\infty} \prod_{k=1}^{7h+4} \frac{1}{\prod_{r=0}^{7r+4} \Omega_{-(q+1)r-q}^k r-q + 1}$$

(40)

Thus, $\Omega_{-(3q+2)} > \Omega_{-(2q+1)}$.

$$\frac{\prod_{f=0}^{5} \Omega_{-(q+1)r-q}}{\prod_{f=0}^{6} \Omega_{-(q+1)r-q}^6} = \sum_{h=0}^{\infty} \prod_{k=1}^{7h+5} \frac{1}{\prod_{r=0}^{7r+5} \Omega_{-(q+1)r-q}^k r-q + 1}$$

(41)
Figure 1: Dynamics of equation (5) with initial conditions in Example 1.

Figure 2: Dynamics of equation (5) with initial conditions in Example 2.
Thus, $\Omega_{-(2q+1)} > \Omega_{-(q)}$.

\[\Omega_{-(7q+6)} > \Omega_{-(6q+5)} > \Omega_{-(5q+4)} > \Omega_{-(4q+3)} > \Omega_{-(3q+2)} > \Omega_{-(2q+1)} > \Omega_{-(q)}\]  \hspace{1cm} (42)

3. Examples

In this section, we consider some numerical examples.

**Example 1.** Assume that, for $q = 1$, we get

$$\Omega_{m+1} = \frac{(\Omega_{-13}/(1 + \Omega_{-2} \Omega_{-3} \Omega_{-4} \Omega_{-5} \Omega_{-6} \Omega_{-7} \Omega_{-8} \Omega_{-9} \Omega_{-10}))}{\Omega_{-13} = 10, \Omega_{-12} = 9, \Omega_{-11} = 2, \Omega_{-10} = 28, \Omega_{-9} = 27, \Omega_{-8} = 6.5, \Omega_{-7} = 5.5, \Omega_{-6} = 24, \Omega_{-5} = 23, \Omega_{-4} = 22, \Omega_{-3} = 21, \Omega_{-2} = 5, \Omega_{-1} = 4, \Omega_{0} = 3.}$$

Then, we have the graph in Figure 1.

**Example 2.** If we select the initial conditions as follows,

$$\Omega_{-13} = 29, \Omega_{-12} = 28, \Omega_{-11} = 27, \Omega_{-10} = 26, \Omega_{-9} = 25, \Omega_{-8} = 24, \Omega_{-7} = 23, \Omega_{-6} = 22, \Omega_{-5} = 21, \Omega_{-4} = 20, \Omega_{-3} = 19, \Omega_{-2} = 18, \Omega_{-1} = 17, \Omega_{0} = 16,$$

then we have the graph in Figure 2.

Data Availability

All the data utilized in this article have been included, and the sources from where they were adopted are cited accordingly.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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