Research Article

Implications of Contract-Bargaining Mechanisms for Coordination and Profit Sharing in a Distribution Channel

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The present work models a three-level distribution channel that has a manufacturer, multiple distributors, and multiple retailers under each distributor to analyze channel members' cooperative, semicooperative, and noncooperative decisions for an arbitrary replenishment cycle other than the first in the infinite time horizon. It uses two sequential bargaining processes: forward contract-bargaining (FCB) and backward contract-bargaining (BCB) to eliminate channel conflict and allocate additional profit among channel members. We successfully implement a hybrid contract mechanism that combines wholesale price discount (WPD) and subsidy on holding cost for channel coordination. The concept of Nash bargaining is applied for additional profit sharing. The proposed hybrid contract can fully coordinate the tree-like supply chain and enrich the entire profit of the supply chain at its best. The manufacturer provides WPD to each distributor separately, and each distributor provides a subsidy to each of its retailers independently. Both the sequential bargaining processes are designed in such a way that an upstream channel member always has the opportunity to account for different reservations for its different downstream members. Although each bargaining process eliminates the channel conflict, finds win-win ranges, and distributes surplus profit, the distributors prefer BCB, whereas the manufacturer and the retailers prefer the FCB. Also, without receiving WPD, the distributors have the ability to coordinate the supply chain and find win-win profits by subsidizing the retailers' holding costs. A numerical case is presented to explain the findings of the work.

1. Introduction

With increasing competitive pressures and market globalization the relation among a manufacturer, multiple distributors and multiple retailers of a three-stage conventional distribution channel have undergone enormous changes. When three stages are managed independently, the solution becomes suboptimal because of the differing self-regarding purposes of the channel participants [1]. So, coordination is vital for enhancing profit of the distribution channel [2, 3]. With supply chain coordination, some channel members are impelled to accept some side-payment contracts offered by some other channel members in such a way that objectives of

all participants will be aligned with the finest channel performance [4]. Several classes of coordination contract are used in the literature of supply chain. The coordination contracts the literature found in this regards are vast, and some of these are quantity discount [2], price discounts [5], revenue sharing [6], two-part tariff [7], etc. In reality, it is tougher to coordinate a three-echelon distribution channel with multiple members in some echelons by using a coordination contract as dimension of the solution space increase. It is required to coordinate the channel both vertically and horizontally to get the best performance [2].

Ganeshan [8] analyzed a three-echelon supply chain, where many suppliers supply goods to numerous retailers

through single distributor. Khouja [9] modeled a three-stage supply chain, where at each stage there are several members. Ben-Daya and Al-Nassar [10] extended Khouja's [9] work based on a new assumption that lot-shipment between two different stages can be made before sale of an entire lot is ended. Khouja [11] developed an optimal synchronization algorithm for the management and incentive alignment along the each stage of supply chain. Cardenas-Barron [12] extended the work of Khouja [11] by assuming an n -stage-multi-customer inventory distribution channel. Jaber and Goyal [13] examined the problem of order quantity synchronization in a many buyers-one manufacturer-many retailers supply chain. Jonrinaldi and Zhang [14] considered a multiplayer multiechelon distribution channel and reverse logistics to find the best decisions over a restricted time horizon. In this direction, works of Sana [15], Pal et al. [16, 17], and Crdenas-Barrn and Sana [18] are worth mentioning.

Bargaining states the opportunities where two or more players can get win-win profit in a different manner through collaboration. In reality, bargaining is not only means a relationship among the participants but also involves negotiation over the trade contracts [19, 20]. More than three decades ago, Kohli and Park [21] used the concept of bargaining in a buyer-seller supply chain in an EOQ setting. They investigated how the buyer and the seller negotiate to get mutual benefit over the amount of quantity discount. Several research works used Nash bargaining solution to maximize channel members expected utility functions [19, 20, 22]. Nagarajan and Sošić [23] reviewed basics of cooperative bargaining models in supply chain. Strategic behaviour of the channel members plays an important role to determine interactions among them and determine optimal decisions under different game theoretical perspectives [24–27].

This paper considers a distribution channel consisting of single manufacturer, many distributors, and several retailers corresponding to every distributor. The channel works in an infinite time horizon. A replenishment cycle apart from the first is considered in the modeling, and analysis of the first replenishment cycle is derived independently. Besides exploring centralized, semicentralized, and decentralized decisions, the major aim of the paper is to find the channel best solution by applying a hybrid contract. The hybrid contract involves wholesale price discount (WPD) that the manufacturer offers to each distributor separately and subsidy on holding cost, in which the distributors provide to their retailers independently. Also, the concept of Nash bargaining is used to allocate the additional profit among the channel participants.

Present work contributes to the literature differing from the previous research works are as follows. Firstly, it models a three-level distribution channel to analyze channel members cooperative, semicooperative, and noncooperative decisions for an arbitrary replenishment cycle other than the first in the infinite time horizon. Secondly, unlike earlier models, a hybrid coordination contract is used for channel coordination. The manufacturer provides different WPDs to different distributors. Also the subsidies on holding cost are

dissimilar for different retailers. In the literature of multi-echelon multimember supply chain, coordination contracts were not exclusively implemented to get the best channel performance. Most of these models assumed that members are cooperative and approved to implement centralized decisions. However, those models are failed to address every channel members' win-win profit simultaneously [13, 14]. Thirdly, the win-win ranges for every member of the distribution channel are determined analytically in closed forms. Fourthly, the distributors play a key role in channel coordination and in distribution of surplus profit. When coordination contract and bargaining are applied, channel members' profits depend on the sequence of applications of these on the channel members. Standard practice in marketing demonstrates that the coordination and profit division are settled between two members, one each in two different stages at a time rather than among all the stages and all the members of a supply chain simultaneously. Also, each upstream channel member has diverse arrangement for each of its corresponding downstream channel member. Thus, an upstream member contracts with each of its downstream members individually. To implement coordination contract and bargaining, the paper suggests, namely, forward contract-bargaining (FCB) and backward contract-bargaining (BCB). In FCB, first each distributor and each of its corresponding retailer identify the range of subsidy on holding cost that provides win-win opportunities. Within this range, they settle for a particular subsidy through bargaining. Based on the bargaining outcome, each distributor and the manufacturer find the win-win range for WPD. Finally, the manufacturer bargains with each distributor for the specific WPD that shares the surplus profit. The same coordination contracts and bargaining are applied but in reverse order in BCB process. The procedures are nested because the outcome of the later coordination contract and bargaining are completely dependent on the outcome of the former contract and bargaining.

We use quantitative mathematical modeling for research development. Two nested contract-bargaining procedures along with an iterative method are introduced to find the optimal solution. Finally, analytical findings are examined through numerical simulation. The next section of the paper discusses decentralized, semicentralized, and centralized models. Channel coordination using contract mechanisms and profit distribution using bargaining are analyzed. Section 3 demonstrates a numerical illustration to verify analytical findings of the model. Finally, Section 4 summarizes the model outcomes and proposed various future research possibilities.

2. Model Description and Basic Analysis

The following notations are used in developing the model:

The ij th ($i = 1, 2, \dots, n_j; j = 1, 2, \dots, n$) retailer

D_{ij}^r : demand rate in the decentralized model

h_{ij}^r : holding cost per unit per unit time including interest

s_{ij}^r : setup cost per cycle

π_{ij}^r : profit function in the decentralized model
 π^{rs} : profit function of all retailers' in the semicentralized model
 π_{ij}^{rc} : profit function in the centralized model
 T_{ij}^r : cycle length in the decentralized model
 T^s : cycle length of all retailers' in the semicentralized model
 Q_{ij}^r : order quantity of in the decentralized model
 Q_{ij}^{rs} : order quantity retailer in the semicentralized model
 Q_{ij}^{rc} : order quantity retailer in the centralized model
 p^r : unit product selling price to the customer

The j th ($j = 1, 2, \dots, n$) distributor

D_j^d : demand rate
 h_j^d : holding cost per unit per unit time including interest
 s_j^d : setup cost per cycle
 w^d : unit product wholesale price
 π_j^d : profit function in the decentralized model
 π^{ds} : profit function of all distributors' in the semicentralized model
 π_j^{dc} : profit function in the centralized model
 T_j^d : cycle length in the decentralized model
 Q_j^d : order quantity in the decentralized model
 Q_j^{dc} : order quantity in the centralized model

The manufacturer

D^m : demand rate
 h^m : holding cost per unit per unit time including interest
 s^m : setup cost per cycle.
 w^m : unit product wholesale price
 c^m : manufacturer's production cost
 π^m : profit function in the decentralized model
 π^{ms} : profit function in the semicentralized model
 π^{mc} : profit function in the centralized model
 T^m : cycle length in the decentralized model
 Q^m : order quantity in the decentralized model
 Q^{mc} : order quantity in the centralized model
 α : integer multiple of cycle time for the distributors
 β : integer multiple of cycle time for the manufacturer
 λ_{ij} : percentage of compensation on holding cost (PCHC) of the ij th retailer provided by the j th distributor
 λ_{ij} : lower limit of PCHC
 $\bar{\lambda}_{ij}^b$: upper limit of PCHC in BCB
 $\bar{\lambda}_{ij}^f$: upper limit of PCHC in FCB
 λ_{ij}^b : bargaining PCHC in BCB

λ_{ij}^f : bargaining PCHC in FCB

ρ_j : wholesale price discount (WPD) that the manufacturer provides to its j th distributor

$\bar{\rho}_j$: upper limit of WPD

ρ_j^b : lower limit of WPD in BCB

ρ_j^f : lower limit of WPD in FCB

ρ_j^b : bargaining WPD in BCB

ρ_j^f : bargaining WPD in FCB

Consider a conventional three-stage supply chain consisting of single manufacturer, multiple distributors, and multiretailer associated with each distributor. The channel operates in an infinite time horizon, and we consider an arbitrary replenishment cycle other than the first. The channel structure is depicted in Figure 1.

The manufacturer produces and supplies a product to n distributors. The j th distributor ($j = 1, 2, \dots, n$) transfers the product to its n_{ij} ($i = 1, 2, \dots, n_j$) number of retailers and the retailers sell the product to customers and fulfill the channel demand. Thus, there is $\sum_{j=1}^n \sum_{i=1}^{n_j} n_{ij}$ number of retailers in the channel. Assume that each of n_{ij} ($i = 1, 2, \dots, n_j$; $j = 1, 2, \dots, n$) retailers faces deterministic demand D_{ij}^r ($i = 1, 2, \dots, n_j$; $j = 1, 2, \dots, n$) per unit time. The manufacturer produces the product at a unit cost c^m and sales it to the distributors at a wholesale price w^m . Each distributor supplies the product to its retailers at a price w^d , and retailers sell the product to end customer at a price p_r . The present model has following limitations: a particular retailer can place order to a particular distributor, who is associated with it; that is, for each retailer, there is only one available distributor. Shortages are not allowed. No lead times are considered as the end customer's demand is assumed as constant.

2.1. Decentralized Decision-Making Process. In decentralized structure, members pay no attention to overall performance of the channel. Each member of the channel maximizes own profit function. Retailers first decide their individual order quantities and order cycle lengths. Obtaining responses from its retailers, a distributor finalizes its decision. Finally, the manufacturer makes decision based on the distributor's decisions. Decentralized order quantities of the manufacturer and the distributors in a replenishment cycle are structurally different from the order quantities of the first-order cycle. Initially, the manufacturer produces large amount of inventories to satisfy all the distributors' orders and to avoid stock out or late deliveries. Similarly, in the first-order cycle, a distributor's order quantity consists of initial orders from all of its retailers and safety stock [14, 28, 29]. After the first replenishment cycle, all the replenishment cycles are identical because an upstream channel member already carries safety stock to prevent stock out and it only bears holding cost corresponding to the safety stock. However, the retailers have no such responsibility. We explore the channel members' decisions in the initial-order cycle as well as in any one of the other identical order cycles.

FIGURE 1: Structure of the distribution channel.

2.1.1. Retailers' Decisions. In a replenishment cycle, the ij th retailer's ($i = 1, 2, \dots, n_j$; $j = 1, 2, \dots, n$) inventory level decreases from the initial level $Q_{ij}^{r/ds}$ at the rate D_{ij}^r and it reaches to zero level after time $T_{ij}^{r/ds}$. Then, the next replenishment cycle starts because of no shortage situation. The average profit of the ij th retailer is $\pi_{ij}^{r/ds}$ (sales revenue-purchase cost-holding cost-setup cost) per unit time and can be found as

$$\pi_{ij}^{r/ds} = (p^r - w^d)D_{ij}^r - \frac{s_{ij}^r}{T_{ij}^{r/ds}} - \frac{h_{ij}^r D_{ij}^r T_{ij}^{r/ds}}{2}. \quad (1)$$

The necessary condition $d\pi_{ij}^{r/ds}/dT_{ij}^{r/ds} = 0$ for existence of optimal solution yields

$$T_{ij}^{r/ds} = \sqrt{\frac{2s_{ij}^r}{h_{ij}^r D_{ij}^r}}. \quad (2)$$

Moreover,

$$\frac{d^2 \pi_{ij}^{r/ds}}{dT_{ij}^{r/ds^2}} = -\frac{2s_{ij}^r}{(T_{ij}^{r/ds})^3} < 0. \quad (3)$$

Therefore, $\pi_{ij}^{r/ds}$ is a concave function of $T_{ij}^{r/ds}$ and equation (1) provides global optimum. Substituting optimal value of T_{ij}^r in equation (1), the average optimal decentralized profit and optimal order quantity of the ij th retailer are, respectively, as

$$\pi_{ij}^{r/ds} = (p^r - w^d)D_{ij}^r - \sqrt{2s_{ij}^r h_{ij}^r D_{ij}^r}, \quad (4)$$

$$Q_{ij}^{r/ds} = D_{ij}^r T_{ij}^{r/ds} = \sqrt{\frac{2D_{ij}^r s_{ij}^r}{h_{ij}^r}}. \quad (5)$$

2.1.2. Distributors' Decisions. The j th, ($j = 1, 2, 3, \dots, n$) distributor has n_j retailers. In the first replenishment cycle, it satisfies the demand at an average rate $D_j^d = \sum_{i=1}^{n_j} D_{ij}^r$. Since the channel members operate independently, the distributor

carries a safety stock in such a way that it can satisfy all of its corresponding retailers' demand simultaneously. Failing to it, the distributor will suffer stock out or late deliveries as stated in the studies of Chan and Kingsman [28], Chan et al. [29], and Jonrinaldi and Zhang [14]. The largest possible aggregate order quantity of all the retailers under the j th distributor is $\sum_{i=1}^{n_j} Q_{ij}^{r/ds}$. The optimal first replenishment cycle length and optimal order quantity in the first cycle of the j th distributor are, respectively, as (see Appendix A)

$$T_j^{d1} = \sqrt{\frac{2[s_j^d + w^m \sum_{i=1}^{n_j} Q_{ij}^{r/ds}]}{h_j^d D_j^d}}, \quad j = 1, 2, \dots, n, \quad (6)$$

$$Q_j^{d1} = \sum_{i=1}^{n_j} Q_{ij}^{r/ds} + \sqrt{\frac{2[s_j^d + w^m \sum_{i=1}^{n_j} Q_{ij}^{r/ds}]}{h_j^d} D_j^d}, \quad j = 1, 2, \dots, n. \quad (7)$$

Apart from the first replenishment cycle, all the other cycles in the infinite time horizon are identical. Now, we consider a replenishment cycle in the infinite time horizon other than the first. The j th distributor has no need to replenish the safety stock as it is already in hand. In the initial stage at the beginning of order cycle, products at distributors will be immediately used to satisfy the first order from retailers, but for the other replenishment cycles, there is no guarantee that all the retailers demand orders from its distributor at the beginning. So, the j th ($j = 1, 2, \dots, n$) distributor's general stock will be $T_j^d D_j^d$. Thus, the average profit function of the j th distributor can be found as

$$\begin{aligned} \pi_j^{d/ds} &= (w^d - w^m)D_j^d - \frac{s_j^d}{T_j^{d/ds}} \\ &\quad - h_j^d \sum_{i=1}^{n_j} Q_{ij}^{r/ds} - \frac{h_j^d T_j^{d/ds} D_j^d}{2}, \quad j = 1, 2, \dots, n. \end{aligned} \quad (8)$$

Differentiating $\pi_j^{d/ds}$ with respect to $T_j^{d/ds}$ and equating to zero the optimal cycle length of the j th distributor is found as

$$T_j^{d/ds} = \sqrt{\frac{2s_j^d}{h_j^d D_j^d}}, \quad j = 1, 2, \dots, n. \quad (9)$$

Moreover,

$$\frac{d^2 \pi_j^{d/ds}}{dT_j^{d/ds^2}} = -\frac{2s_j^d}{(T_j^{d/ds})^3} < 0. \quad (10)$$

Thus, $T_j^{d/ds}$ provides global optimum. The optimal order quantity and optimal average profit of the j th distributor are

$$Q_j^{d/ds} = D_j^d T_j^{d/ds} = \sqrt{\frac{2D_j^d s_j^d}{h_j^d}}, \quad j = 1, 2, \dots, n, \quad (11)$$

$$\begin{aligned} \pi_j^{d/ds} = & (w^d - w^m) D_j^d - \frac{s_j^d}{T_j^{d/ds}} \\ & - h_j^d \sum_{i=1}^{n_j} Q_{ij}^{r/ds} - \frac{h_j^d T_j^{d/ds} D_j^d}{2}, \quad j = 1, 2, \dots, n. \end{aligned} \quad (12)$$

2.1.3. Manufacturer's Decision. The manufacturer faces orders from all the distributors. So, the demand at the manufacturer's end is $D^m = \sum_{j=1}^n \sum_{i=1}^{n_j} D_{ij}^r$. Similar to the first replenishment of a distributor, the optimal replenishment cycle length and order quantity of the manufacturer in the first replenishment cycle can be found as (see Appendix B)

$$T^{m*} = \sqrt{\frac{2[s^m + c^m \sum_{j=1}^n Q_j^{d/ds}]}{h^m D^m}}, \quad (13)$$

$$Q^{m*} = \sum_{j=1}^n Q_j^{d/ds} + \sqrt{\frac{2[s^m + c^m \sum_{j=1}^n Q_j^{d/ds}] D^m}{h^m}}. \quad (14)$$

Similar to the distributors, when the manufacturer wants to optimize its decisions after the first cycle, the decisions must be different from the initial one as the safety stock is already in hand. The manufacturer calculates its holding cost for an order cycle other than the initial one assuming that all the distributors must not demand their orders at the beginning of each replenishment cycle. Thus, the average profit function of the manufacturer is

$$\pi^{m/ds} = (w^m - c^m) D^m - \frac{s^m}{T^{m/ds}} + h^m \sum_{j=1}^n Q_j^{d/ds} - \frac{h^m T^{m/ds} D^m}{2}. \quad (15)$$

Using necessary condition, the optimal cycle length of the manufacturer can be found as

$$T^{m/ds} = \sqrt{\frac{2s^m}{h^m D^m}}. \quad (16)$$

Moreover,

$$\frac{d^2 \pi^{mg}}{dT^{m/ds^2}} = -\frac{2s^m}{(T^{m/ds})^3} < 0. \quad (17)$$

Thus, the average profit function of the manufacturer given in equation (15) is a concave function of the cycle length of the manufacturer and equation (16) provides global optimum cycle length of the manufacturer. Optimal production lot and optimal average profit of the manufacturer are, respectively, as

$$Q^{m/ds} = D^m T^{m/ds} = \sqrt{\frac{2D^m s^m}{h^m}}, \quad (18)$$

$$\pi^{m/ds} = (w^m - c^m) D^m - \frac{s^m}{T^{m/ds}} + h^m \sum_{j=1}^n Q_j^{d/ds} - \frac{h^m T^{m/ds} D^m}{2}. \quad (19)$$

2.2. Semicentralized Decision-Making Process. When all participants in a certain stage of the supply chain agree to synchronize decisions of that stage, the situation refers to the semicentralized channel.

2.2.1. Horizontal Coordination among the Retailers. In decentralized channel, all the retailers operate independently. So, different retailers have different optimal order quantities and different replenishment cycles. As a result, anticipating any of its associated retailer's stock out, a distributor maintains a safety stock throughout the replenishment cycle. It incurs additional holding cost to the distributor. This situation can be avoided if the replenishment cycles of the retailers are synchronized, i.e., all the retailers opt for a common replenishment cycle T^s . In such case, a distributor does not need to carry the safety stock.

When all the retailers agree for a common replenishment cycle, average total profit of all the retailers is

$$\pi^{rs} = (p^r - w^d) \sum_{j=1}^n \sum_{i=1}^{n_j} D_{ij}^r - \frac{\sum_{j=1}^n \sum_{i=1}^{n_j} s_{ij}^r}{T^s} - \frac{T^s}{2} \sum_{j=1}^n \sum_{i=1}^{n_j} h_{ij}^r D_{ij}^r. \quad (20)$$

The necessary condition for the existence of optimal solution yields

$$T^{s*} = \sqrt{\frac{2 \sum_{j=1}^n \sum_{i=1}^{n_j} s_{ij}^r}{\sum_{j=1}^n \sum_{i=1}^{n_j} h_{ij}^r D_{ij}^r}}. \quad (21)$$

The optimal order quantity of the ij th retailer can be found as

$$Q_{ij}^{rs} = D_{ij}^r \sqrt{\frac{2 \sum_{j=1}^n \sum_{i=1}^{n_j} s_{ij}^r}{\sum_{j=1}^n \sum_{i=1}^{n_j} h_{ij}^r D_{ij}^r}}, \quad i = 1, 2, \dots, n_j; j = 1, 2, \dots, n. \quad (22)$$

Optimal average profit of the ij th retailer can be found from equation (1) by substituting T^{s*} for T_{ij}^{r*} .

Since, for $T_{ij}^{r/ds}$, ij th retailer's, $i = 1, 2, \dots, n_j$; $j = 1, 2, \dots, n$ is maximum, any deviation of the replenishment cycle length leads to some profit loss of the retailer. So, when the retailers follow a common order cycle through synchronization, their profits will be less than their decentralized profits.

2.2.2. Horizontal Coordination among the Distributors.

In response to the synchronized decision of the retailers, suppose the distributors agree to synchronize their replenishment cycle lengths to a common one. Then, the distributor's common cycle length is an integer multiple, say α of T^{s*} . Thus, average total profit of all the distributors is

$$\pi^{ds} = (w^d - w^m) \sum_{j=1}^n D_j^d - \frac{\sum_{j=1}^n s_j^d}{\alpha T^{s*}} - \frac{(\alpha - 1)T^{s*}}{2} \sum_{j=1}^n h_j^d D_j^d. \quad (23)$$

The distributors can perfectly anticipate the demands of the retailers. So, the distributors do not need to keep the safety stock to prevent stock outs as in the case of decentralized decision making. Also the distributors can use inventories immediately to satisfy the last order from the retailers in the replenishment cycle. Thus, the distributors maximum stock will be $D_j^d (\alpha - 1)T^{s*}$. The holding cost of the distributors will be incurred in its average cost for the time $(\alpha - 1)T^{s*}$. So, the average total profit of the distributors will be greater when compared with the decentralized decision.

2.2.3. Manufacturer's Decision. When there is stagewise synchronization in the first and second echelons of the supply chain, the manufacturer considers a common production cycle length that is an integer multiple of the common replenishment cycle length of the distributors, i.e., $\alpha\beta T^{r*}$, where β is an integer. The profit function of the manufacturer per unit time is

$$\pi^{ms} = (w^m - c^m)D^m - \frac{s^m}{\alpha\beta T^{s*}} - \frac{(\beta - 1)\alpha T^{s*}}{2} h^m D^m. \quad (24)$$

As in the case of the distributors, the manufacturer can anticipate the distributor's first-order quantity and does not need to carry any stocks for stock outs and/or late deliveries. The maximum stock of the manufacturer is $D^m (\beta - 1)\alpha T^{s*}$. So, the manufacturer's profit is less compared to its decentralized profit.

From the above discussion of semicentralized decision making process, the following result can be realized.

Lemma 1. *Stagewise coordination provides higher profits to the distributors and the manufacturer but lower profits to the retailers when compared with decentralized profits.*

2.3. Centralized Decision. In centralized decision, members of the supply chain obey synchronized or integrated decisions to get maximum channel performance. It seems to be operated by a single decision maker who takes all the decisions in the distribution channel. The manufacturer, the distributors, and the retailers adopt a common cycle T . The manufacturer and the distributor follow lot-for-lot policy. Since the manufacturer and the distributors do not carry any inventory, the stock holding cost will be incurred in the retailers' profits only. Total profit of the channel per unit time is

$$\begin{aligned} \pi^c = & (p^r - c^m) \sum_{i=1}^{n_j} \sum_{j=1}^n D_{ij}^r - \frac{\sum_{i=1}^{n_j} \sum_{j=1}^n s_{ij}^r + \sum_{j=1}^n s_j + s_m}{T} \\ & - \frac{T}{2} \sum_{i=1}^{n_j} \sum_{j=1}^n h_{ij}^r D_{ij}^r. \end{aligned} \quad (25)$$

Necessary condition for the existence of optimal solution yields the common cycle length of the channel is

$$T^* = \sqrt{\frac{2(\sum_{i=1}^{n_j} \sum_{j=1}^n s_{ij}^r + \sum_{j=1}^n s_j^d + s_m)}{\sum_{i=1}^{n_j} \sum_{j=1}^n h_{ij}^r D_{ij}^r}}. \quad (26)$$

Moreover,

$$\frac{d^2 \pi^c}{dT^2} = -\frac{2(\sum_{i=1}^{n_j} \sum_{j=1}^n s_{ij}^r + \sum_{j=1}^n s_j^d + s_m)}{(T)^3} < 0. \quad (27)$$

Hence, equation (26) provides the global optimal solution of equation (25).

The optimal order quantities and optimal average profits of the ij th retailer, the j th distributor, and the manufacturer are, respectively, as

$$Q_{ij}^{rc*} = D_{ij}^r T^*, \quad i = 1, 2, \dots, n_j; j = 1, 2, \dots, n, \quad (28)$$

$$Q_j^{dc*} = \sum_{i=1}^{n_j} D_{ij}^r T^*, \quad j = 1, 2, \dots, n, \quad (29)$$

$$Q^{mc*} = \sum_{i=1}^{n_j} \sum_{j=1}^n D_{ij}^r T^*, \quad (30)$$

$$\pi_{ij}^{rc*} = (p^r - w^d) D_{ij}^r - \frac{s_{ij}^r}{T^*} - \frac{h_{ij}^r D_{ij}^r T^*}{2}, \quad i = 1, 2, \dots, n_j; j = 1, 2, \dots, n, \quad (31)$$

$$\pi_j^{dc*} = (w^d - w^m) D_j^d - \frac{s_j^d}{T^*}, \quad j = 1, 2, \dots, n, \quad (32)$$

$$\pi^{mc*} = (w^m - c^m) D^m - \frac{s^m}{T^*}. \quad (33)$$

The optimal total profit of the centralized channel is maximum when compared with the semicentralized or decentralized channel [13, 14, 28, 29]. Since, the manufacturer and the distributors do not have stock holding cost, and their profits are greater than their semicentralized as well as decentralized profits. So, the manufacturer and the distributors prefer centralized decisions. However, $\pi_{ij}^{rs*} - \pi_{ij}^{rc*} < 0$, ($i = 1, 2, \dots, n_j; j = 1, 2, \dots, n$), i.e., the

retailer's profit is less than decentralized decision making. Thus, we have the following lemma.

Lemma 2. *The centralized profits of the retailers' are lower, whereas the distributors and the manufacturers are higher in centralized decision when compared with semicentralized decision.*

From the above discussion, we have the results displayed in Table 1.

Therefore, from Lemmas 1 and 2, and Table 1, we have the following proposition.

Proposition 1. *In a conventional three-echelon distribution channel, the retailers' profits are minimum but the distributors' and the manufacturer's profits are maximum for centralized decision.*

3. Channel Coordination, Ranges of Win-Win Opportunities, and Surplus Profit Division

When centralized decisions are implemented, upstream channel members (the manufacturer and the distributors) follow lot-for-lot policy and their holding costs are zero. On the other hand, holding cost of the retailers increases significantly. As a result, the retailers' profits in centralized decision are less compared to their decentralized profits. Thus, there is no reason that the retailers accept centralized decision. The retailers accept the centralized decision only when they receive some incentives, which ensure at least their decentralized profits. Assume that the distributors provide percentage subsidies on holding cost to each of their corresponding retailers to induce them to order centralized quantities. Because of incentives, the distributors' profits may be lower than their decentralized profits. So, to encourage the distributors, the manufacturer provides different wholesale price discounts to different distributors in such a way that it ensure each distributor's decentralized profit. In reality, an upstream member offers more amenities to those downstream members who have the ability to give more business. Thus, an upstream member affords different percentages of subsidies for its different downstream members independently. The amount of compensation an upstream member offers to a downstream member will depend on the amount of profit it gets from that downstream member's cooperation. In that case, a channel member assumes that there are no other echelon and no other members.

Note that, for channel synchronization, the distributors play a vital role as they maintain the lot streaming between the manufacturer and the retailers. Here, we propose two different approaches to encourage the distributors to play the key role for channel coordination. When one approaches from the manufacturer after satisfying its decentralized profit, a distributor receives some extra profit through negotiation. Decentralized profit plus the surplus profit share is the distributor's intermediate profit. Based on this profit, the distributor approaches to each of its corresponding retailers to provide minimum subsidy and to bargain for surplus share. When one approaches from the retailers, a distributor

first compensates a retailer's loss of profit due to centralized order quantity and then bargains for surplus share. Obviously, in this case, the intermediate profit of a distributor is different from the other approach. Thus, there arise two cases, viz, (i) FCB process and (ii) BCB process.

3.1. FCB Process. The sequence of application of coordination contracts and bargaining among the channel members are as follows:

- (1) The manufacturer considers each of its distributors separately and determines the WPD ranges for win-win profits.
- (2) Within a wholesale price range, the manufacturer bargains with the corresponding distributor for a particular wholesale price discount for surplus profit share. The decentralized profit plus cumulative surplus profit shares from all the distributors is the manufacturer's optimal profit. The decentralized profit plus the surplus profit share is the distributor's intermediate profit.
- (3) Based on this profit, a distributor identifies the ranges of percentage subsidy on holding costs corresponding to each of its retailers.
- (4) Within a range, a distributor bargains with the corresponding retailer for subsidy fraction that determines surplus profit share. The decentralized profit plus surplus profit share is a retailer's optimal cost. Profit of step 2 plus cumulative surplus profit shares corresponding to all of its retailers is the distributor's optimal profit.

The manufacturer has n distributors. The manufacturer considers its profit corresponding to each distributor separately and takes decision. So, the profit component of the manufacturer for j th ($j = 1, 2, \dots, n$) distributor in the centralized channel and decentralized channel is, respectively, as

$$\pi_j^{mc} = \frac{Q_j^{dc*}}{Q^{mc*}} \pi^{mc*}, \quad j = 1, 2, \dots, n, \quad (34)$$

$$\pi_j^{m/ds} = \frac{T^{m/ds} D_j^d}{Q^{m/ds}} \pi^{m/ds}, \quad j = 1, 2, \dots, n. \quad (35)$$

The manufacturer offers WPD to encourage a distributor for accepting centralized decision. The manufacturer can provide WPD to a particular distributor until its profit corresponding to that distributor is not less than corresponding decentralized profit. If the manufacturer gives the maximum $\bar{\rho}_j$ discount on wholesale price to the j th distributor for accepting its centralized order quantity, then

$$(\bar{\rho}_j w^m) Q_j^{dc*} = \pi_j^{mc*} - \pi_j^{m/ds}, \quad (36)$$

i.e.,

$$\bar{\rho}_j = \frac{\pi_j^{mc*} - \pi_j^{m/ds}}{w^m Q_j^{dc*}}, \quad j = 1, 2, \dots, n. \quad (37)$$

TABLE 1: Comparison of channel members' profits in three decision-making scenarios.

Profit	Decentralized	Semicentralized	Centralized
Retailer	Highest	Medium	Lowest
Distributor	Lowest	Medium	Highest
Manufacturer	Lowest	Medium	Highest

Therefore, after providing maximum WPD to all the distributors, the manufacturer's profit will be exactly equal to its decentralized profit.

Similarly, if the j th ($j = 1, 2, \dots, n$) distributor demands minimum ρ_j^f ($0 \leq \rho_j^f \leq 1$) wholesale price discount from the manufacturer, then ρ_j^f can be found as

$$\rho_j^f = \frac{\pi_j^{d/ds} - \pi_j^{dc*}}{w^m D_j^d}, \quad j = 1, 2, \dots, n. \quad (38)$$

Thus, at this point for any $\rho_j \in (\rho_j^f, \overline{\rho_j})$, the j th ($j = 1, 2, \dots, n$) distributor's profit and the manufacturer's profit corresponding to the distributor are win-win.

Now, we use the Nash bargaining product to divide the surplus profit. In the Nash bargaining model, the objective function is the product of the player's gain from cooperation and it will have to be maximized. Each player's benefit is the difference between the negotiated profit and profit under decentralized decision making.

The Nash bargaining product of the manufacturer and the j th ($j = 1, 2, \dots, n$) distributor can be found as

$$\max_{\rho_j^f \leq \rho_j \leq \overline{\rho_j}} [\pi_j^{dc*} - \pi_j^{d/ds} + \rho_j w^m D_j^d] \quad (39)$$

$$[\pi_j^{mc} - \pi_j^{m/ds} - \rho_j w^m D_j^d], \quad j = 1, 2, \dots, n.$$

The optimal value of ρ_j ($j = 1, 2, \dots, n$) is

$$\rho_j^f = \frac{(\pi_j^{mc} - \pi_j^{m/ds}) - (\pi_j^{dc*} - \pi_j^{d/ds})}{2w^m D_j^d}, \quad j = 1, 2, \dots, n. \quad (40)$$

After settling benefit share with the manufacturer, the intermediate profit of the j th ($j = 1, 2, \dots, n$) distributor becomes

$$\pi_j^{df} = (w^d - (1 - \rho_j^f)w^m)D_j^d - \frac{s_j^d}{T^*}, \quad j = 1, 2, \dots, n. \quad (41)$$

The decrement of profit of the j th ($i = 1, 2, \dots, n_j; j = 1, 2, \dots, n$) retailer for accepting Q_{ij}^{rc*} instead of $Q_{ij}^{r/ds}$ is $\pi_{ij}^{rc*} - \pi_{ij}^{r/ds}$. As incentive, the j th

($j = 1, 2, \dots, n$) distributor shares a percentage of holding cost to its associated ($i = 1, 2, \dots, n_j; j = 1, 2, \dots, n$) retailers. If λ_{ij} ($i = 1, 2, \dots, n_j; j = 1, 2, \dots, n$) is the minimum percent of holding cost that j th distributor shares of its ij th retailer, then

$$\pi_{ij}^{r/ds} - \pi_{ij}^{rc*} = \lambda_{ij} \left[\frac{h_{ij}^r D_{ij}^r T^*}{2} \right], \quad i = 1, 2, \dots, n_j; j = 1, 2, \dots, n. \quad (42)$$

Thus, from equation (42), it follows that

$$\lambda_{ij} = \frac{2(\pi_{ij}^{r/ds} - \pi_{ij}^{rc*})}{h_{ij}^r D_{ij}^r T^*}. \quad (43)$$

Profit component of the j th distributor ($j = 1, 2, \dots, n$) corresponding to the ij th retailer is

$$\pi_{ij}^{df} = \frac{Q_{ij}^{rc*}}{Q_j^{dc*}} \pi_j^{df}, \quad i = 1, 2, \dots, n_j; j = 1, 2, \dots, n. \quad (44)$$

Similarly, if the j th ($j = 1, 2, \dots, n$) distributor provides maximum $\overline{\lambda_{ij}^f}$ ($i = 1, 2, \dots, n_j; j = 1, 2, \dots, n$) compensation on holding cost to the ij th retailer, then $\overline{\lambda_{ij}^f}$ can be found as

$$\overline{\lambda_{ij}^f} = \frac{2(\pi_{ij}^{df} - \pi_{ij}^d)}{h_{ij}^r D_{ij}^r T^*}. \quad (45)$$

Thus, for any $\lambda_{ij} \in (\lambda_{ij}, \overline{\lambda_{ij}^f})$, the j th distributor and ij th retailer achieve win-win profit. Now, if the distributor and the retailer bargain for a particular λ_{ij} within this range, then the Nash bargaining product can be found as

$$\max_{\lambda_{ij} \leq \lambda_{ij} \leq \overline{\lambda_{ij}^f}} \left[\pi_{ij}^{rc*} - \pi_{ij}^{r/ds} + \lambda_{ij} \left(\frac{h_{ij}^r D_{ij}^r T^*}{2} \right) \right] \quad (46)$$

$$\left[\pi_{ij}^{df} - \pi_{ij}^d - \lambda_{ij} \left(\frac{h_{ij}^r D_{ij}^r T^*}{2} \right) \right], \quad i = 1, 2, \dots, n_j; j = 1, 2, \dots, n.$$

Optimal value of λ_{ij} is

$$\lambda_{ij}^f = \frac{\pi_{ij}^{df} - \pi_{ij}^d + \pi_{ij}^{r/ds} - \pi_{ij}^{rc*}}{h_{ij}^r D_{ij}^r T^*}, \quad i = 1, 2, \dots, n_j; j = 1, 2, \dots, n. \quad (47)$$

Therefore, optimal profits of the retailers, the distributors, and the manufacturer are

$$\pi_{ij}^{rf*} = \pi_{ij}^{r/ds} + \frac{1}{2} \left[(\pi_{ij}^{df} - \pi_{ij}^d) - (\pi_{ij}^{r/ds} - \pi_{ij}^{rc*}) \right], \quad i = 1, 2, \dots, n_j; j = 1, 2, \dots, n, \quad (48)$$

$$\pi_j^{df*} = \pi_j^{d/ds} + \frac{1}{4} \left[(\pi_j^{mc} - \pi_j^{m/ds}) + (\pi_j^{dc*} - \pi_j^{d/ds}) - 2 \sum_{i=1}^{n_j} (\pi_{ij}^{r/ds} - \pi_{ij}^{rc*}) \right], \quad j = 1, 2, \dots, n, \quad (49)$$

$$\pi^{mf*} = \pi^{m/ds} + \frac{1}{2} \left[(\pi^{mc*} - \pi^{m/ds}) + \sum_{j=1}^n (\pi_j^{dc*} - \pi_j^{d/ds}) \right]. \quad (50)$$

From the above bargaining process, following results can be realized. First, unlike the centralized decision, the ranges of the contract parameters ensure win-win opportunities for all the channel members. Moreover, the channel is coordinated and maximum channel surplus profit is distributed among the channel members. Second, the manufacturer shares the surplus profit for channel coordination with the j th ($j = 1, 2, \dots, n$) distributor equitably. Third, the j th distributor and its i th ($i = 1, 2, \dots, n_j; j = 1, 2, \dots, n$) retailer share the surplus profit equitably.

3.2. BCB Process. Unlike the previous case, here we approach from the retailers to the manufacturer through the distributors. The sequence of events applying contract mechanisms and bargaining is as follows:

- (1) The j th ($j = 1, 2, \dots, n$) distributor and each of its n_j retailers determine the ranges of subsidy on holding cost. The lower limit is the same as the previous case.
- (2) The j th distributor ($j = 1, 2, \dots, n$) bargains with each retailer independently for percentage share of retailer's holding cost within its range and settles for a profit distribution. Decentralized profit plus the surplus profit share is the optimal profit of the ij th retailer. After the process, the distributor's immediate profit is the decentralized profit plus cumulative surplus profit shares from all the retailers.
- (3) Based on this profit, the j th distributor and the manufacturer identify the range of WPD for win-win profits. The lower limit can be determined by assuming that the distributor accepts the WPD as long as its decentralized profit is reserved and the upper limit is the same as the previous case.
- (4) The manufacturer and the j th distributor bargain for profit share. The j th distributor's optimal profit is as follows: decentralized profit + accumulated surplus profit share from its retailers + surplus profit share from the manufacturer. The manufacturer's profit is decentralized profit plus accumulated profits from all the distributors.

The j th distributor's decentralized and centralized profit corresponding to its ij th retailer are, respectively, as

$$\pi_{ij}^{dc} = \frac{Q_{ij}^{rc*}}{Q_j^{dc*}} \pi_j^{dc*}, \quad i = 1, 2, \dots, n_j; j = 1, 2, \dots, n, \quad (51)$$

$$\pi_{ij}^d = \frac{D_{ij}^r T_j^{d/ds}}{Q_j^{d/ds}} \pi_j^{d/ds}, \quad i = 1, 2, \dots, n_j; j = 1, 2, \dots, n. \quad (52)$$

If the j th distributor shares maximum $\bar{\lambda}_{ij}^b$ percent of holding cost to its ij th retailer, then

$$\bar{\lambda}_{ij}^b \left[\frac{h_{ij}^r D_{ij}^r T^*}{2} \right] = \pi_{ij}^{dc} - \pi_{ij}^d, \quad (53)$$

i.e.,

$$\bar{\lambda}_{ij}^b = \frac{2(\pi_{ij}^{dc} - \pi_{ij}^d)}{h_{ij}^r D_{ij}^r T^*}, \quad i = 1, 2, \dots, n_j; j = 1, 2, \dots, n. \quad (54)$$

Therefore, any $\lambda_{ij} \in (\lambda_{ij}, \bar{\lambda}_{ij}^b)$, ($i = 1, 2, \dots, n_j; j = 1, 2, \dots, n$) ensures that the j th distributor's profit only for ij th retailer and ij th retailer's profit are win-win. Now, within this range, the j th distributor and the ij th retailer bargain for a particular compensation fraction that effectively divides the benefit.

Since any $\lambda_{ij} \in (\lambda_{ij}, \bar{\lambda}_{ij}^b)$ resolves channel conflicts, the j th distributor and the ij th retailer bargain to determine an acceptable $\lambda_{ij} \in (\lambda_{ij}, \bar{\lambda}_{ij}^b)$ that will maximize the Nash bargaining product. The Nash bargaining product can be found as

$$\max_{\lambda_{ij} \leq \lambda_{ij} \leq \bar{\lambda}_{ij}^b} \left[\pi_{ij}^{rc*} - \pi_{ij}^{r/ds} + \lambda_{ij} \left(\frac{h_{ij}^r D_{ij}^r T^*}{2} \right) \right] \left[\pi_{ij}^{dc} - \pi_{ij}^d - \lambda_{ij} \left(\frac{h_{ij}^r D_{ij}^r T^*}{2} \right) \right], \quad i = 1, 2, \dots, n_j; j = 1, 2, \dots, n. \quad (55)$$

Optimal value of λ_{ij} is

$$\lambda_{ij}^b = \frac{\pi_{ij}^{dc} - \pi_{ij}^d + \pi_{ij}^{r/ds} - \pi_{ij}^{rc*}}{h_{ij}^r D_{ij}^r T^*}, \quad i = 1, 2, \dots, n_j; j = 1, 2, \dots, n. \quad (56)$$

The j th distributor's intermediate profit is

$$\pi_j^{\text{db}} = (w^d - w^m)D_j^d - \frac{s_j^d}{T^*} - \sum_{i=1}^{n_j} \lambda_{ij}^b \left[\frac{h_{ij}^r D_{ij}^r T^*}{2} \right], \quad j = 1, 2, \dots, n. \quad (57)$$

As soon as the j th distributor finds optimal cumulative profit gain shares from its n_j retailers, it determines the minimum quantity discount that it can accept from the manufacturer. If the manufacturer provides minimum ρ_j^b discount that ensures the j th distributor's decentralized profit after the first bargaining, then

$$\underline{\rho}_j^b = \frac{\pi_j^{d/\text{ds}} - \pi_j^{\text{db}}}{w^m D_j^d}, \quad j = 1, 2, \dots, n. \quad (58)$$

Any $\rho_j \in (\underline{\rho}_j^b, \overline{\rho}_j)$ ensures win-win outcomes for the manufacturer and j th distributor. Now, if the j th distributor

bargains independently with the manufacturer for a $\rho_j \in (\underline{\rho}_j^b, \overline{\rho}_j)$ that will maximize their profit, then the Nash bargaining product can be found as

$$\max_{\underline{\rho}_j^b \leq \rho_j \leq \overline{\rho}_j} [\pi_j^{\text{db}} - \pi_j^{d/\text{ds}} + \rho_j w^m D_j^d] \quad (59)$$

$$[\pi_j^{\text{mc}} - \pi_j^{m/\text{ds}} - \rho_j w^m D_j^d], \quad j = 1, 2, \dots, n.$$

Optimal value of ρ_j is

$$\rho_j^b = \frac{(\pi_j^{\text{mc}} - \pi_j^{m/\text{ds}}) - (\pi_j^{\text{db}} - \pi_j^{d/\text{ds}})}{2w^m D_j^d}, \quad j = 1, 2, \dots, n. \quad (60)$$

Therefore, optimal profits of the retailers, the distributors, and the manufacturer are, respectively, as

$$\pi_{ij}^{\text{rb}*} = \pi_{ij}^{r/\text{ds}} + \frac{1}{2} [(\pi_{ij}^{\text{dc}} - \pi_{ij}^d) - (\pi_{ij}^{r/\text{ds}} - \pi_{ij}^{\text{rc}*})], \quad i = 1, 2, \dots, n_j; j = 1, 2, \dots, n, \quad (61)$$

$$\pi_j^{\text{db}*} = \pi_j^{d/\text{ds}} + \frac{1}{4} \left[2(\pi_j^{\text{mc}} - \pi_j^{m/\text{ds}}) + (\pi_j^{\text{dc}*} - \pi_j^{d/\text{ds}}) - \sum_{i=1}^{n_j} (\pi_{ij}^{r/\text{ds}} - \pi_{ij}^{\text{rc}*}) \right], \quad j = 1, 2, \dots, n, \quad (62)$$

$$\pi^{\text{mb}*} = \pi^{m/\text{ds}} + \frac{1}{4} \left[2(\pi^{\text{mc}*} - \pi^{m/\text{ds}}) + \sum_{j=1}^n (\pi_j^{\text{dc}*} - \pi_j^{d/\text{ds}}) - \sum_{j=1}^n \sum_{i=1}^{n_j} (\pi_{ij}^{r/\text{ds}} - \pi_{ij}^{\text{rc}*}) \right]. \quad (63)$$

Note that the j th distributor's average profit in centralized decision is higher when compared with the profit in decentralized decision, i.e., $\pi_j^{\text{dc}*} > \pi_j^{d/\text{ds}}$, i.e., $\pi_j^{\text{dc}*} - \pi_j^{d/\text{ds}} > 0$ $j = 1, 2, \dots, n$. Then, from equation (38), it follows that $\underline{\rho}_j^f < 0$, since $w^m D_j^d > 0$. Similarly, from equation (58), it follows that $\underline{\rho}_j^b < 0$, because $\pi_j^{d/\text{ds}} < \pi_j^{\text{db}}$. However, in reality, discount on wholesale price cannot be negative. So, the lower limit of quantity discount is zero. Thus, we have the following proposition.

Proposition 2. In both contract-bargaining processes, without receiving any quantity discount, the distributors can coordinate the channel.

3.3. Comparison of Two Contract-Bargaining Processes. From the above two bargaining procedures, two lower limits of WPD and two upper limits of percentage share on holding cost are identified. Note from equation (58) and equation (38) that

$$\underline{\rho}_j^b - \underline{\rho}_j^f = \frac{\pi_j^{\text{dc}*} - \pi_j^{\text{db}}}{w^m D_j^d}, \quad j = 1, 2, \dots, n. \quad (64)$$

However, $\pi_j^{\text{dc}*} > \pi_j^{\text{db}}$. Thus, $\rho_j^b - \rho_j^f > 0$, since $w^m D_j^d > 0$, i.e., $\rho_j^b > \rho_j^f$. Thus, minimum WPD demanded by the j th ($j = 1, 2, \dots, n$) distributor in backward process is higher

when compared with minimum WPD of forward process. Similarly, $\overline{\lambda}_{ij}^b - \overline{\lambda}_{ij}^f < 0$; that is, the minimum compensation provided by the j th ($j = 1, 2, \dots, n$) distributor to its ij th ($i = 1, 2, \dots, n_j; j = 1, 2, \dots, n$) retailer in forward process is higher than that of backward. So, we have the following proposition.

Proposition 3. (i) $\rho_j^f < \rho_j^b$, $j = 1, 2, \dots, n$ and (ii) $\overline{\lambda}_{ij}^f > \overline{\lambda}_{ij}^b$, $i = 1, 2, \dots, n_j; j = 1, 2, \dots, n$

Moreover,

$$\pi_{ij}^{\text{rf}*} - \pi_{ij}^{\text{rb}*} = \frac{1}{2} [\pi_{ij}^{\text{df}} - \pi_{ij}^{\text{dc}}] = \frac{Q_{ij}^{\text{rc}*}}{2Q_j^{\text{dc}*}} [\pi_j^{\text{df}} - \pi_j^{\text{dc}*}] > 0,$$

$$\pi_j^{\text{db}*} - \pi_j^{\text{df}*} = \frac{1}{4} \left[(\pi_j^m - \pi_j^{d/\text{ds}}) + \sum_{i=1}^{n_j} (\pi_{ij}^{r/\text{ds}} - \pi_{ij}^{\text{rc}*}) \right] > 0,$$

$$\pi^{\text{mf}*} - \pi^{\text{mb}*} = \frac{1}{4} \left[\sum_{j=1}^n (\pi_j^{\text{dc}*} - \pi_j^{d/\text{ds}}) + \sum_{j=1}^n \sum_{i=1}^{n_j} (\pi_{ij}^{r/\text{ds}} - \pi_{ij}^{\text{rc}*}) \right] > 0. \quad (65)$$

Thus, we have the following proposition.

Proposition 4. The manufacturer and the retailers want to promote forward bargaining process, but the distributors want the other.

The manufacturer and the distributors divide the surplus profit, which is generated due to the adaptation of centralized decisions through symmetric Nash bargaining product. In forward bargaining process for minimum WPD, a distributor's profit is equal to its decentralized profit and the manufacturer's profit is maximum. On the other hand, in backward bargaining process, a distributor bargains with the manufacturer after providing holding cost compensation to its retailers. Obviously, the demand for WPD amount is higher in the later, i.e., $\rho_j^f < \rho_j^b$. Thus, the bargaining space between manufacturer and the distributors is wider in backward bargaining when compared with the forward bargaining. Since the surplus profit is divided equitably between the manufacturer and a distributor, the distributor gets more opportunity to gain additional profit in forward bargaining. However, the manufacture and the retailers will be wanted to promote BCB process as they get more opportunity in this process.

4. Numerical Results

Assume that a supply chain network with five retailers R_{11} , R_{21} , R_{12} , R_{22} , and R_{32} , two distributors D_1 and D_2 , and a manufacturer M , where the distributor D_1 supplies the product to R_{11} and R_{21} and the distributor D_2 supplies the product to the other three retailers. Parameter values of the manufacturer are $s^m = 12000$, $h^m = 4.5$, $c^m = 130$, and $w^m = 165$, and other parameters are $w^d = 200$ $p = 250$. Parameter values of the retailers and the distributors are shown in Table 2. Based on the solution procedure of Appendix C and the parameter values, the computational results are presented in Tables 3–6.

From Table 3, note that total profit of the channel is maximum in centralized decision making, whereas it is least in decentralized decision making. Optimal channel profit is increased by 14.23% in centralized decision. The manufacturer's profit and all the distributors' total profit are increased by 39.05% and 26.91%, but the sum of the retailers' total profit is decreased by 5.47% when compared with decentralized decisions. Thus, the centralized decisions are not acceptable to the retailers unless they get some incentives from the distributors.

When the contract-bargaining processes are used, the optimal profits are 182643, the same as the centralized channel. Thus, by applying any one of these two processes, it is possible to resolve the channel conflict. Moreover, all channel members' profits are higher when compared with decentralized profits.

Both contract-bargaining processes create win-win opportunities for all the channel members, and the ranges of contract parameters are different in two processes (see Tables 5 and 6).

For example, the ranges of percentage subsidy on holding cost for the R_{22} in backward and forward processes are (0.4159, 1.0445) and (0.4159, 1.175). As indicated, the lower limits are the same. Also note that when the contract-bargaining processes are used, the optimal profits are 182643, the same as the centralized channel. Thus, by

TABLE 2: Input parameters of retailers (R) and distributors (D).

Retailer	s_{ij}^r (\$/order)	h_{ij}^r (\$/unit/year)	D_{ij}^r (items/year)
R_{11}	550	7	300
R_{21}	580	7.5	375
R_{12}	550	6.75	305
R_{22}	600	7.5	400
R_{32}	575	7.25	325
Distributor	s_j^d (\$/order)	h_j^d (\$/unit/year)	
D_1	2200	5	
D_2	2500	5	

TABLE 3: Computational results for decentralized, semicentralized, and centralized policies.

Decision variables	Decentralized	Semicentralized	Centralized
T	—	0.680582	1.78117
α	—	2	—
β	—	1	—
Retailers' total profit	76876	76860	72669
Distributors' total profit	44942	53321	57036
Manufacturer's profit	38070	50859	52938
Channel profit	159888	181040	182643

applying any one of these two processes, it is possible to resolve the channel conflict. Moreover, all channel members' profits are higher when compared with decentralized profits.

From Tables 5 and 6, one may note that the percentage increments in profits of the distributors in backward process are higher. The retailers' and the manufacturer's percentage increments in profits are higher in FCB process. Thus, the distributors prefer backward process, whereas the retailers and the manufacturer prefer the forward. This is quite obvious because the bargaining space in backward process over which the manufacturer and the distributors bargain is wider compared to the forward process. The reverse trend may be observed for percentage subsidy on the retailers' holding cost. Also, the characteristics remain the same for the change in input parameters values (see Figure 2).

The first three subfigures in Figure 2 present the effects of the key parameters (demand, holding cost, and setup cost) of the retailers on the solution space on the percentage share on holding cost in BCB and FCB processes, while the remaining three subfigures in Figure 2 show the effects of the distributors' demand, holding cost, and setup cost on the solution space on WPD in the two bargaining processes. Figure 2(a) depicts that although the ij th retailer's demand has little effect on the limits of percentage share on holding costs, the solution spaces decrease if demand increases for both the BCB and FCB processes. Figure 2(b) shows that the limits of percentage share on holding costs are highly sensitive to the retailer's holding cost. The upper limits of percentage share on holding costs in both the BCB and FCB processes are decreasing with increasing holding costs of the ij th retailer. On the other hand, the lower limit of percentage share on holding cost is increasing with increasing holding

TABLE 4: Computational result for individuals in decentralized, semicentralized, and centralized policies.

Members	Decentralized			Semicentralized			Centralized		
	Time	Q	Profit	Time	Q	Profit	Time	Q	Profit
R_{11}	0.7237	217	13480	0.6806	204	13477	1.7812	534	12821
R_{21}	0.6422	240	16944	0.6806	255	16941	1.7812	668	15920
R_{12}	0.7309	223	13745	0.6806	208	13741	1.7812	543	13108
R_{22}	0.6324	253	18103	0.6806	272	18098	1.7812	712	16991
R_{32}	0.6986	227	14604	0.6806	221	14603	1.7812	579	13828
D_1	1.1418	771	17481	1.3612	918	20860	1.7812	1202	22390
D_2	0.9853	1015	27461	1.3612	1402	32461	1.7812	1834	34646.
M	1.7686	3016	38070	1.3612	2320	50859	1.7812	3036	52938.

TABLE 5: Computational result for backward contract-bargaining process.

Members	λ_{ij}	λ_{ij}^b	$\overline{\lambda_{ij}^b}$	ρ_j^b	ρ_j^b	$\overline{\rho_j^b}$	Profit	% increase in profit
R_{11}	0.3524	0.7594	1.1664	—	—	—	14241	5.64
R_{21}	0.4089	0.7488	1.0886	—	—	—	17795	5.02
R_{12}	0.3476	0.7541	1.1605	—	—	—	14491	5.42
R_{22}	0.4159	0.7302	1.0445	—	—	—	18942	4.63
R_{32}	0.3694	0.7249	1.0805	—	—	—	15350	5.11
D_1	—	—	—	0	0.0192	0.0264	21231	21.45
D_2	—	—	—	0	0.0196	0.0264	33117	20.60
M	—	—	—	—	—	—	47540	24.88

TABLE 6: Computational result for forward contract-bargaining process.

Members	λ_{ij}	λ_{ij}^b	$\overline{\lambda_{ij}^f}$	ρ_j^f	ρ_j^f	$\overline{\rho_j^f}$	Profit	% increase in profit
R_{11}	0.3524	0.8175	1.2826	—	—	—	14350	6.45
R_{21}	0.4089	0.8030	1.1971	—	—	—	17931	5.83
R_{12}	0.3476	0.8266	1.3056	—	—	—	14623	6.39
R_{22}	0.4159	0.7955	1.1750	—	—	—	19116	5.60
R_{32}	0.3694	0.7925	1.2155	—	—	—	15492	6.08
D_1	—	—	—	0	0.0044	0.0264	19338	10.62
D_2	—	—	—	0	0.0053	0.0264	30241	10.12
M	—	—	—	—	—	—	51703	35.81

(a)

(b)

FIGURE 2: Continued.

FIGURE 2: Variations in decision variables with respect to change in input parameter values.

cost of the ij th retailer. In this way, the solution space decreases when holding cost of the ij th retailer increases. Figure 2(c) displays that the limits of percentage share on holding costs are less sensitive on the ij th retailer's setup cost. Solution spaces in both the bargaining processes increase if the ij th retailer's setup cost increases. Figure 2(d) portrays that changes in the j th distributor's demand have a low impact on the limits of WPD and the solution spaces in both the BCB and FCB processes are decreasing with the increasing demand of the j th distributor. Figures 2(e) and 2(f) show that the distributor's holding cost and setup cost have a similar effect on the limits of WPD and the solution spaces in both the BCB and FCB processes increase with increasing holding cost and setup cost of the j th distributor.

The upper limits of percentage subsidy on holding cost in both the processes are higher than one. That is, a distributor may provide more than its holding cost as subsidy to a

retailer, and still its profit is win-win. It occurs when the surplus profit that a distributor receives due to channel coordination is higher than the holding cost of a retailer. The lower limits of WPD in the contract-bargaining processes are zero. It simply demonstrates that, if the distributors do not get discounts from the manufacturer, then also they have the power to coordinate the channel by providing subsidies on holding costs of the retailers'. In that case, they can also enjoy win-win profits until the upper limits are zero.

Some more sensitivity analyses of the optimal average bargaining profit functions are performed and are presented in Figure 3. Figure 3(a) shows that all the channel members' profits increase (or decrease) significantly with the increasing (or decreasing) demand. Profit of the ij th retailer, the j th distributor, and the manufacturer increases, 41.3%, 44.7%, and 43.3%, respectively, when demand increases 40 percent. Figure 3(b) depicts that the manufacturer's profit is

FIGURE 3: Fluctuations in bargaining profits with respect to the retailers input parameter values.

more sensitive to the retailer's setup cost. Forty percent increase in the retailer's setup cost will decrease 3.75%, 3.14%, and 1.26% of profits of the manufacturer, the j th distributor, and the retailer, respectively. Figure 3(c) shows that the distributors' profit is more sensitive to the retailers holding cost. Forty percent increase in the retailers holding cost will decrease 6.56%, 5.51%, and 3.54% of profits of the j th distributor, manufacturer, and the retailer, respectively. Figure 3 indicates that the bargaining profits are highly sensitive for the change in demand at the retailers' end and low sensitivity may be observed for the error in the estimation of setup costs of the channel members and holding costs of the retailers. The same characteristics of the optimal average bargaining profit functions may be observed for the change in the parameter values of the distributors and the manufacturer.

In real-world business scenarios, it is important to maximize the outputs of a supply chain. We have planned and used a hybrid contract for coordination as it is not possible to coordinate this model through a single coordination contract mechanism. Our proposed hybrid contract can fully coordinate the tree-like supply chain. This means the proposed hybrid contract can enrich the entire profit of the supply chain at its best. Moreover, two bargaining processes are implemented to share the additional profits among the channel members. This ensures a win-win outcome for all the channel members of the supply chain.

5. Summary and Concluding Remarks

The paper analyzes about the coordination and surplus profit division in a conventional distribution channel. Using two nested contract-bargaining processes, attempts are made to find cooperative decision which effectively resolve channel conflict and find win-win opportunities for all the channel members. The contract-bargaining processes are very close to business practice because an upstream channel member interacts with each of its downstream channel members independently in one-to-one basis. It is quite obvious that an upstream channel member will provide how much benefit to a downstream channel member that depends on how much it receives from that member. Numerical example suggests that an upstream channel member has different reservations for its corresponding downstream channel members. Earlier studies in this direction have considered an integrated approach through synchronized order quantity or replenishment cycle. As a result, some channel members are benefited from the decision and some are not. Apart from few, earlier studies on two-echelon supply chain coordination have considered a particular benefit split rather than assuming that a channel member may have the right to say something about the benefit share. In this direction, the present paper also extends the literature by assuming that the channel members bargain to divide the channel surplus. However, the centralized, semicentralized, and decentralized decisions are derived in

closed forms and some interesting results are found for a distribution channel; those are summarized below.

Firstly, although centralized decision provides best channel performance, the manufacturer and the distributors are benefited, but the retailers are not. The centralized decision is acceptable to the retailers when they get some incentives from the upstream channel members. Secondly, as incentives, the WPD and subsidy on holding cost of the retailer cut out channel conflict and find win-win profits for the channel members. Thirdly, using Nash bargaining product, it is always possible to find particular profit shares for all the channel members. Fourthly, two contract-bargaining processes are designed in such a way that an upstream channel member always has the opportunity to account different reservations for its different downstream members. Fifthly, the ranges of contract parameters, which provide win-win opportunities, are different in two contract-bargaining processes, i.e., the monetary space over which the channel members' bargain differs, so their profit share as well. As a result, the distributors prefer the backward process, whereas the manufacturer and the retailers prefer the forward. Sixthly, in both contract-bargaining processes, the manufacturer may not provide WPD. That is, without receiving any discount from the manufacturer, the distributors have the power to coordinate the channel by subsidizing its corresponding retailers' holding costs. By doing so, it still can find some profit share because the upper limit of percentage subsidy on holding cost is positive.

Although the model is developed by considering uniform demand, models may be formulated by considering price dependent and stock/price dependent or stochastic demand. In our view, the main results of the paper will not differ for such demand patterns. An important limitation of the paper is the assumption of common cycle approach. A model may be developed by relaxing it. In such case, the model will be more general though computation will be more complex. Instead of considering WPD and subsidy on holding cost, some other coordination contracts may be used in the contract-bargaining processes. The surplus profit is distributed among the channel members equitably in one-to-one basis. The model may be analyzed by considering asymmetric Nash bargaining product.

Appendix

A. The Optimal Decision of the j th Distributor in the First Cycle

The average profit function of the j th ($j = 1, 2, \dots, n$) distributor in the first order cycle is

$$\begin{aligned} \pi_j^{d1} = & (w^d - w^m)D_j^d - \frac{s_j^d}{T_j^{d1}} - h_j^d \sum_{i=1}^{n_j} Q_{ij}^{r/ds} \\ & - \frac{h_j^d [T_j^{d1} D_j^d - \sum_{i=1}^{n_j} Q_{ij}^{r/ds}]}{2} - \frac{w^m \sum_{i=1}^{n_j} Q_{ij}^{r/ds}}{T_j^{d1}}. \end{aligned} \quad (\text{A.1})$$

The second term is purchasing cost of safety stock anticipating the orders from retailers at the same time. The third term above is the holding costs per unit time for a distributor if the distributor is to have zero stock outs. Fourth term is the holding costs for the general stock, $T_j^{d1} D_j^d - \sum_{i=1}^{n_j} Q_{ij}^{r/ds}$ because at the beginning of order cycle, products at distributors will be immediately used to satisfy the first order from retailers. Differentiating π_j^{d1} with respect to T_j^{d1} and equating to zero, we get

$$T_j^{d1} = \sqrt{\frac{2[s_j^d + w^m \sum_{i=1}^{n_j} Q_{ij}^{r/ds}]}{h_j^d D_j^d}}, \quad j = 1, 2, \dots, n \quad (\text{A.2})$$

Thus, the order quantity of the j th distributor in the initial replenishment will be

$$Q_j^{d1} = \sum_{i=1}^{n_j} Q_{ij}^{r/ds} + \sqrt{\frac{2[s_j^d + w^m \sum_{i=1}^{n_j} Q_{ij}^{r/ds}] D_j^d}{h_j^d}}, \quad j = 1, 2, \dots, n. \quad (\text{A.3})$$

Since $d^2 \pi_j^{d1} / dT_j^{d12} = -[2(s_j^d + w^m \sum_{i=1}^{n_j} Q_{ij}^{r/ds})][(T_j^{d1})^3] < 0$, therefore, (A.1) is a concave function in T_j^{d1} , and the optimal solution of (A.2) is a global optimum.

B. The Optimal Decision of the Manufacturer in the First Cycle

The production lot of the manufacturer is

$$Q^{m1} = \sum_{j=1}^n Q_j^{d/ds} + T^m D^m. \quad (\text{B.1})$$

Therefore, the average profit function of the manufacturer is

$$\begin{aligned} \pi^{m1} = & (w^m - c^m)D^m - \frac{\sum_{j=1}^n Q_j^{d/ds} c^m}{T^{m1}} - \frac{s^m}{T^{m1}} \\ & - h^m \sum_{j=1}^n Q_j^{d/ds} - \frac{h^m [T^{m1} D^m - \sum_{j=1}^n Q_j^{d1}]}{2}. \end{aligned} \quad (\text{B.2})$$

Differentiating π^{m1} with respect to T^{m1} and equating to zero, the optimal cycle length of the manufacturer for first cycle is found as

$$T^{m1} = \sqrt{\frac{2[s^m + c^m \sum_{j=1}^n Q_j^{d/ds}]}{h^m D^m}}. \quad (\text{B.3})$$

The optimal production size of the manufacturer for the first cycle is

$$Q^{m1} = \sum_{j=1}^n Q_j^{d/ds} + \sqrt{\frac{2[s^m + c^m \sum_{j=1}^n Q_j^{d/ds}] D^m}{h^m}}. \quad (\text{B.4})$$

Moreover,

$$\frac{d^2 \pi^{m1}}{dT^{m12}} = -\frac{2(s^m + c^m Q^{m1})}{(T^{m1})^3} < 0. \quad (B.5)$$

Therefore, (B2) is a concave function of T^{m1} .

C. Solution Procedure to Find the Optimal Solutions

Solution procedure to find the optimal solution.
Input parameter values.

Procedure 1. //Decentralized decision//

```

TPD = 0,  $D_m = 0$ 
for ( $j = 1$  to  $n$ )
  TPR $_j = 0$ ,  $D_j^d = 0$ 
  for ( $i = 1$  to  $n_j$ )
    calculate  $T_{ij}^{r/ds}$ ,  $Q_{ij}^{r/ds}$  and  $\pi_{ij}^{r/ds}$  using equations (2),
    (4) and (5)
    TPR $_j = \text{TPR}_j + \pi_{ij}^{r/ds}$ 
     $D_j^d = D_j^d + Q_{ij}^{r/ds} / T_{ij}^{r/ds}$ 
    write output  $T_{ij}^{r/ds}$ ,  $Q_{ij}^{r/ds}$ ,  $\pi_{ij}^{r/ds}$  and TPR $_j$ 
    calculate  $T_j^{d/ds}$ ,  $Q_j^{d/ds}$  and  $\pi_j^{d/ds}$  using equations (9),
    (11), and (12)
    TPD = TPD +  $\pi_j^{d/ds}$ 
     $D^m = D^m + Q_j^{d/ds} / T_j^{d/ds}$ 
    write output  $T_j^{d/ds}$ ,  $Q_j^{d/ds}$ ,  $\pi_j^{(d/ds)}$  and TP D
    calculate  $T^{m/ds}$ ,  $Q^{m/ds}$  and  $\pi^{(m/ds)}$  using equations (16),
    (18), and (19)
    write output  $T^{m/ds}$ ,  $Q^{m/ds}$ ,  $\pi^{m/ds}$ 

```

Procedure 2. //Semicentralized decision//

```

calculate  $T_{ij}^{rs}$ ,  $Q_{ij}^{rs}$  and  $\pi_{ij}^{rs}$  using equations (21), (22), and
(20)
 $\alpha = 1$ ,  $A = 0$ 
repeat
  calculate  $\pi^{ds}$  using (21)
   $A = \pi^{ds}$ 
   $\alpha = \alpha + 1$ 
until ( $\pi^{ds} > A$ )
write output  $\alpha$ ,  $A$  as distributors' cost
 $\beta = 1$ ,  $B = 0$ 
repeat
  calculate  $\pi^{ms}$ 
   $B = \pi^{ms}$ 
   $\beta = \beta + 1$ 
until ( $\pi^{ms} > B$ )
write output  $\beta$ ,  $B$  as manufacturer's cost

```

Procedure 3. //Centralized decision//

calculate T^* , Q_{ij}^{rc*} , Q_j^{dc*} , Q^{mc*} , π_{ij}^{rc*} , π_j^{dc*} and π^{mc*}
using equations (26) to (33)

Procedure 4. //Minimum and maximum contract limits for retailers and manufacturer//

```

for ( $j = 1$  to  $n$ )
  for ( $i = 1$  to  $n_j$ )
    calculate  $\lambda_{ij}$  using (43)
    calculate  $\pi_j^{mc*}$ ,  $\pi_j^{m*}$  and  $\bar{\rho}_j$  using equations (34),
    (35), and (37)

```

Procedure 5. //FCB process//

```

for ( $j = 1$  to  $n$ )
  calculate  $\rho_j^f$ ,  $\rho_j^f$  and  $\pi_j^{df}$  using (38), (40), and (41)
  for ( $j = 1$  to  $n$ )
    for ( $i = 1$  to  $n_j$ )
      calculate  $\pi_{ij}^{df}$ ,  $\bar{\lambda}_{ij}^f$  and  $\lambda_{ij}^f$  using (44), (45), and (47)
  for ( $j = 1$  to  $n$ )
    for ( $i = 1$  to  $n_j$ )
      calculate  $\pi_{ij}^{rf*}$  using (48)
      calculate  $\pi_j^{df*}$  using (49)
      calculate  $\pi^{mf*}$  using (50)
      write output  $\lambda_{ij}^f$ ,  $\pi_{ij}^{rf*}$ ,  $\pi_j^{df*}$ ,  $\pi^{mf*}$ 

```

Procedure 6. //BCB process//

```

for ( $j = 1$  to  $n$ )
  for ( $i = 1$  to  $n_j$ )
    calculate  $\pi_{ij}^{dc}$ ,  $\pi_{ij}^{d/ds}$  and  $\bar{\lambda}_{ij}^b$  using (51), (52), and
    (54)
    determine  $\lambda_{ij}^b$  using (56)
    determine  $\rho_j^b$  and  $\rho_j^b$  using (58) and (60)
  for ( $j = 1$  to  $n$ )
    for ( $i = 1$  to  $n_j$ )
      calculate  $\pi_{ij}^{rb*}$  using (61)
      calculate  $\pi_j^{db*}$  using (62)
      calculate  $\pi^{mb*}$  using (63)
      write output  $\lambda_{ij}^b$ ,  $\pi_{ij}^{rb*}$ ,  $\pi_j^{db*}$ ,  $\pi^{mb*}$ 

```

D. List of Abbreviations

FCB: forward contract-bargaining
BCB: backward contract-bargaining
PCHC: percentage of compensation on holding cost
WPD: wholesale price discount.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

There are no conflicts of interest in the manuscript.

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