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Research Article

Multiarea Economic Dispatch Using Evolutionary Algorithms

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Multiarea economic dispatch (MAED) is a vital problem in the present power system to allocate the power generation through dispatch strategies to minimize fuel cost. In economic dispatch, this power generation distribution always needs to satisfy the following constraints: generating limit, transmission line, and power balance. MAED is a complex and nonlinear problem and cannot be solved with classical techniques. Many metaheuristic methods have been used to solve economic dispatch problems. In this study, the dynamic particle swarm optimization (DPSO) and grey wolf optimizer (GWO) have been used to solve the MAED problem for single-area 3 generation units, a two-area system with four generating units, and four areas with 40-unit system. The hunting and social behaviors of grey wolves are implemented to obtain optimal results. During the optimization search, this algorithm does not require any information regarding the objective function's gradient. The tunable parameters of the original PSO that are three parameters are dynamically controlled in this work that provides the efficient cost values in less execution time although satisfying all the MAED problem's diverse constraints. In this study, the authors also implemented the GWO algorithm with two tunable parameters, and its execution is straightforward to implement for the MAED problem.

1. Introduction

The application of economic dispatch (ED) in the operation of the modern power system has a great significance. System demand is economically allocated between different multiarea generators by considering all constraints [1]. ED for multiple areas has paid limited attention. Large and small utilities have many constraints to transmit power through tie lines. All utilities and power pools have different generation characteristics and load patterns in modern power sectors, including spinning reserves. Therefore, the main objective of ED is to minimize the fuel cost of all the generators and satisfy all the constraints such as power balance, losses, and generation limits. In the deregulated environment, the generator with the lowest cost should operate with its maximum capacity and transmit more power to those areas consisting of more expensive generating units. MAED is a form of ED model that satisfies multiple constraints simultaneously. In this study, the significance of power balance, generation limits, and transmission line constraints in the optimal scheduling of power generation has been considered.

Previous ED problem was solved by lambda-iteration method, gradient method, reduced gradient method, NR method, and other methods, such as participation factor method and binary-weighted method [1]. However, these methods required considerable computation effort to solve the ED problem. In multiareas, large interconnected power system ED problem becomes more complex with different cost characteristics. Therefore, to overcome these shortcomings, metaheuristics methodologies can be used. Many such methods were implemented in ED by various researchers. In this respect, evolutionary algorithms, such as simulated annealing (SA) [2], genetic algorithm (GA) [3],

evolutionary programming (EP) [4], artificial neural network (ANN) [5], ant colony optimization (ACO) [6], particle swarm optimization (PSO) [7], artificial immune system (AIS) [8], differential evolution (DE) [9], bacterial foraging algorithm (BFA) [10], and biogeography-based optimization (BBO) [11], have been successfully applied to have complex ED problem without any limitation in size and condition of cost curves. Shoults et al. [12] have made ED by considering import and export constraints for the single-area and three-area problems. Yalcinoz and Short [13] have considered transmission constraints for two areas' power systems and applied the ANN approach for the ED problem. Seiffert [14] has used linear programming methodologies. The author calculated the incremental cost for each area, and according to total cost power, cost and tie line values were adjusted. The problem with this method was not feasible on large interconnected power systems. Chen and Chen [15] have used the direct space method to solve MAED problems. The author built an MAGS algorithm to establish a relation between dependability and system security. The power system of Taiwan has been selected for this work by the author. Manoharan et al. [16] have applied the evaluation algorithm (EA) and Karush–Kuhn–Tucker (KKT) conditions based on optimal confirmation to the MAED problem. KKT-trained variables have been applied to the results obtained by EAs to check optimality. The obtained results using the KKT criterion were compared with linear programming (LP) and dynamic programming (DP) results. The authors concluded that this technique provides better CPU time and standard deviation. Sharma et al. [17] presented differential evolution with a time-varying mutation technique to solve MAED by considering tie line capacity constraints. Venkatakrishnan et al. [18] applied the GWO method to solve the ED problems by considering thermal valves. Evolutionary-based optimization methods are becoming more popular for ED problems due to their advantages, such as the absence of convexity assumptions, better search capability, and simplicity. Many such methods reported in the literature are neural network (NN), tabu search (TS), simulating annealing (SA) [2], particle swarm optimization [7], genetic algorithm (GA) [3], harmony search (HS) [16], ant colony optimization (ACO) [16], and differential evolution (DE) [19]. Some researchers have proposed comprehensive reviews of metaheuristic optimization methods to solve ED problems. These reviews suggested that PSO and DE techniques are more popular for solving ED problems due to their simplicity, fast convergence rate, greater flexibility to search optimum global points, and easy implementation.

However, all evolutionary techniques required a suitable balance between global search (GS) and local search (LS). Few researchers have focused on convergence time, optimal parameters tuning, premature convergence, and so on. Researchers have attempted to handle these issues using various strategies, such as modified evolutionary techniques [20] and hybridization of algorithms [21, 22]. Jain and Pandit [23] have implemented the PSO technique to solve the MAED problem. The authors have modified the PSO technique for the general search to avoid premature convergence. However, AI techniques are becoming popular for nonconvex, multimodal, discontinuous optimization problems for which traditional methods cannot provide a solution. Manoharan et al. [16] have applied EP with the LMO approach to solving the ED problem. The authors reported that the EP-LMO approach has better accuracy and convergence rate than EP. This study focuses on applying DPSO and GWO with optimal mutation to provide an accurate and feasible solution for the ED problems. The main drawback of classical approaches is knocking at local optima and may not offer the best solution. Second, all classical methods are based on the assumption that their objective function to be handled is continuous and differentiable, whereas the practical power system is more complex. Contemporary intelligent techniques have the advantage of being versatile in handling qualitative constraints. Still, their main drawback is that the computational time increases exponentially as the size of the problem increases, and time to convergence is uncertain (convergences are guaranteed). Metaheuristics approaches have been applied to overcome these shortcomings.

Apart from aforementioned papers, there are also some recent studies on multiarea economic dispatch problems, which are hybridization of differential evolution (DE) with immunized ant colony optimization (ACO) [24], electro search optimization approach (ESOA) [25, 26], uncertainty of MAED problem using Monte Carlo simulation [27], water wave optimization (WWO) [28], krill herd algorithm (KHA) [29], dynamic dispatch in wind-based power system using chaotic grasshopper optimization algorithm (CGOA) [30], hybridization of chaotic particle swarm optimization (CPSO) and genetic algorithm (GA) is (HCPSOGA) [31], squirrel search optimization (SSO) [32], complete review of metaheuristics on MAED problem [33], salp swarm algorithm (SSA) on stochastic nature of wind [34], improved grasshopper optimization algorithm (GOA) [35], Coulomb's and Franklin's law-based optimizer [36], fast convergence evolutionary programming (FCEP) [37], artificial bee colony (ABC) [38], nature-inspired optimization (NIO) [39], and hybridization of shuffled frog leaping algorithm with PSO considering emissions on MAED problems [40].

The study also compares the solution obtained using the PSO and GWO method to find the appropriate application and accuracy of these techniques. According to the authors' knowledge, proposed DPSO with all variants of PSO and GWO techniques have not been applied yet by considering valve point loading on these test systems.

2. Problem Formulation

The objective function is to minimize the total fuel cost of generation between all interconnected areas by considering all the constraints. Valve point loading (VPL) directly affects the objective function and produces distortion in heat rate characteristics in ED problems. The introduction of VPL results in the objective function nonconvex, discontinuous, and result in multiple minima of the cost function. Therefore, in the present objective part, the VPL is modeled as a

sinusoidal function [38] in the input-output cost function to rectify the effect of VPL, and it is given as

$$\min F\left(P_{Gij}\right) = \sum_{i=1}^{M}\sum_{j=1}^{N_{Gi}}\left(a_{ij} + b_{ij}P_{Gij} + c_{ij}P_{Gij}^{2}\right) + \left|e_{ij}\sin\left(f_{ij}\left(P_{Gij}^{\min} - P_{Gij}\right)\right)\right|, \quad (1)$$

where P_{Gij} is the power generation of ith to jth units; a_{ij}, b_{ij}, and c_{ij} are the fuel cost coefficients; and e_{ij} and f_{ij} are the fuel cost coefficients of the ith to jth units of the VPL model.

Tie line power flow between areas plays a significant role in deciding the operating cost in multiarea power systems. Taking into consideration the cost of transmission through each tie line, the objective function of MAED is given in the following equation.

The objective function of MAED is stated as follows:

Constraints:

Area power balance: the maximum power generation through all available generators is equal to the demand P_{Di}. In ED, the area power balance constraints, each area power should meet with generation.

$$\sum_{j=1}^{N_{Gi}} P_{Gij} = P_{Di} + \sum_{m,i\neq m} P_{Tim}, \quad i \in \{1, 2, \ldots, M\}. \quad (2)$$

Generator constraints: in generating limit constraint, the output of each unit should satisfy the upper and lower limits of generations.

$$P_{Gij}^{\min} \leq P_{Gij} \leq P_{Gij}^{\max}. \quad (3)$$

Tie line constraints: the flows through tie line should also be in the maximum and minimum limit range. These limits of power flow are important and are stated as follows:

$$\begin{aligned} P_{T\min} \leq P_{Tm,j} \leq P_{T\max}, &\quad m = 1, 2, \ldots, M, j = 1, 2, \ldots, M, j \neq m, \\ P_{T\min} \leq P_{Tj,m} \leq P_{T\max}, &\quad m = 1, 2, \ldots, M, j = 1, 2, \ldots, M, j \neq m. \end{aligned} \quad (4)$$

3. Solution Technique

In the modern era, the computer-aided application in power systems has been increased. The application of evolutionary soft computing techniques in power systems has become more popular for solving optimization problems. The popularity of these techniques in complex power systems is increased due to their ease of implementation and reliable operation. Moreover, the modern interconnected power system has mixed types, that is, highly nonlinear cost characteristics functions. Solving these mixed-type functions by classical methods like Newton Raphson and lambda-iteration methods is difficult and inaccurate. Therefore, the application of metaheuristic methodologies to solve these types of problems is very popular. Among all evolutionary techniques, PSO [41–44] is more popular than other methods in the literature. The PSO can handle large dimension, nonconvex, nonlinear, and multiconstraint problems efficiently due to their random search technique. Despite all these advantages of PSO, with an increase in the size and complexity of the problem, PSO is sometimes stuck in local optimum solutions. PSO approach has three tunable parameters, that is, w, C_1, and C_2. Here, C_1 and C_2 are mainly random numbers. So, these parameters sometimes faced problems in handling the composite functions and struck at local optima. Therefore, a strong variant of PSO is proposed to tackle this problem. DPSO tackled all these problems because all these parameters are tuned dynamically and depend on system parameters like maximum iteration and classical PSO parameters such as velocity, position, gbest, and pbest.

The PSO method starts by selecting a population of auxiliary solutions and searching for optima via the aid of modernizing solutions. The particle's velocity has a significant impact on particular social, cognitive, and initial components. The rule for updating particle velocity demands a proper balance between the social and cognitive properties of the swarm required. Initial domination of cognitive part over social part is must to secure by exploration of search space. However, subordination of social part over cognitive is needed to propel all solutions towards global optima to enhance local exploitation. Therefore, an explored control equation is propounded for regulating particle velocity dynamically by taking constriction variables e_1 and e_2. Likewise, the cognitive and social parts are updated by considering RMS experience and preceding experience, respectively [45].

During the application of PSO, the position and velocity of each particle are updated as follows:

$$S_n^{t+1} = S_n^t + V_n^{t+1} \times \Delta t, \quad (5)$$

where Δt is the time step of 1 second.

The inertia weight is given as

$$W = W_{\min} + \frac{\left(W_{\max} - W_{\min}\right) \times \left(\mathrm{itr}_{\max} - \mathrm{itr}\right)}{\mathrm{itr}_{\max}}. \quad (6)$$

3.1. Proposed DPSO. In this study, the proposed DPSO, W, is modified by exponentially decaying function η to avoid premature convergence. $W = e^{(-\eta \ln k_w)}$, where $k_w = (w_{\min}/w_{\max})$ and $\eta = \mathrm{itr}/\mathrm{itr}_{\max}$, and the factor K_w be chosen in respect of inertia weight's bounds maximum and minimum limit. In this paper, the value of kw is the ratio of maximum and minimum bound of the inertia weight [45].

3.1.1. Proceeding Experience. Update RMS experience and acceleration coefficients and parameters of constriction factor approach where

$$\xi_1 = e^{(-\mu_1\eta)}; \xi_2 = k.e^{(\mu_2\eta)}; k = \frac{\xi_1 \cdot c_{1b}}{\xi_2 \cdot c_2}, \tag{7}$$

in which k is the proposed social and cognitive coefficients. For the identical value of these factors, $\eta = \eta_t$. All other factors valued are stated in Table 1. The flowchart for the proposed DPSO and constraint-handling management for MAED are depicted in Figures 1 and 2, respectively (Algorithm 1).

3.2. Grey Wolf Optimizer. Grey wolf optimization (GWO) algorithm is a metaheuristic optimization recently developed by Mirjalili et al. in 2014. The algorithm is inspired by the hierarchy behaviors of grey wolves and imitates the hunting phenomena of grey wolves. Despite the various advantages of metaheuristic algorithms like those applied on nonconvex functions as system complexity increases, the GWO algorithm is free from input parameters initialization. GWO approach has two tunable parameters, a and c. So, exploration and exploitation in search space become faster. In the present algorithm, the first fittest solution is alpha (α), beta (β) is second, delta (δ) is third, and other are followers, that is, omega (ω). Wolves follow the behavior of encircling prey, pursuing, hunting, tracking, approaching, and so on [46].

3.2.1. Mathematical Modelling of GWO. In the present section, the detailed mathematical modelling of the algorithm using the social hierarchy model of wolves and group hunting of prey is presented.

3.2.2. Social Hierarchy Model. In the social hierarchy model, the fittest solution is assumed as alpha (α) wolf or the leader (first) wolf; the next solution is the beta (β) wolf or second-best solution, the delta (δ) wolf is the third-best solution among all, and the remaining solutions are omega (ω) wolves.

3.2.3. Encircling the Prey. During hunting, the grey wolves encircle the prey, and the following equations are used to model the process [46].

$$\vec{C} = \left|\vec{B} \cdot \overrightarrow{X_p(t)} - \overrightarrow{X(t)}\right|, \tag{8}$$

$$\overrightarrow{X(t+1)} = \overrightarrow{X_p(t)} - \vec{A} \cdot \vec{C}, \tag{9}$$

where t is the present iteration of the objective problem, $\overrightarrow{X_p(t)}$ represents the available position vector of the prey, $\overrightarrow{X(t)}$ represents grey wolf's position vector and the coefficient vectors $\vec{A}$, and it is calculated using the following equations.

Table 1: Parameters are taken into account to deal with Test System 3.

Parameter	Value
Total power demand (MW)	10500
Tie line limit (MW)	200/100
Area load demand (%)	15/40/30/15
Population size	80
$w_{\max}$	0.9
$w_{\min}$	0.1
c_{1b}	2
c_{1p}	0.5
μ_1	5
μ_2	3.9
η_t	2/3
k	4
$itr_{\min}/itr_{\max}$	1/1000

$$\begin{aligned} \vec{A} &= 2\vec{a} \cdot \vec{r}_1 - \vec{a}, \\ \vec{C} &= 2 \cdot \vec{r}_2, \end{aligned} \tag{10}$$

where $\vec{r}_1$ and $\vec{r}_2$ are random numbers between [0, 1], $a = 2 - 2\,(\text{itr}/\max\text{itr})$, where itr = present iteration and max itr = maximum number of iteration.

Here, a is decreasing linearly from 2 to 0 during each iteration. Generally, the grey wolf updates their position randomly in solution space around the prey using equations (8) and (9). This concept can be implemented for n dimensions search space.

As $\vec{A}$ is a function of $\vec{a}$ and a random vector of range [0 1]. Value of $\vec{a}$ decreases from 2 to 0 as the iteration number increases. The fluctuations range of $\vec{A}$ also decreased by $\vec{a}$. So, when the values of $\vec{A}$ are in the range of [−1 1], the new position of search agent has a position between position of prey and its current position. For $|\vec{A}| < 1$, the search agents converge toward the optimal location.

In the GWO technique, positions of search agents are updated to correspond to alpha, beta, and delta. They deviate from searching for prey and assemble to assail prey. For $|\vec{A}| > 1$, the search agents diverge from an optimal local solution to find an optimal global solution. This highlights exploration and permits the GWO algorithm to troll it globally. As the value of $\vec{a}$ decrease linearly from [2–0], so the $\vec{A}$ mainly emphasizes exploration during initial iterations. But value of $\vec{C}$ varies in [0–2] randomly, during initial as well as final iterations. So $\vec{C}$ emphasizes exploration in last iterations also for $\vec{C} > 1$.

3.2.4. Hunting. All grey wolves can capture the site and location of prey during hunting, and the positions of the wolf are updated around the prey using the following equations:

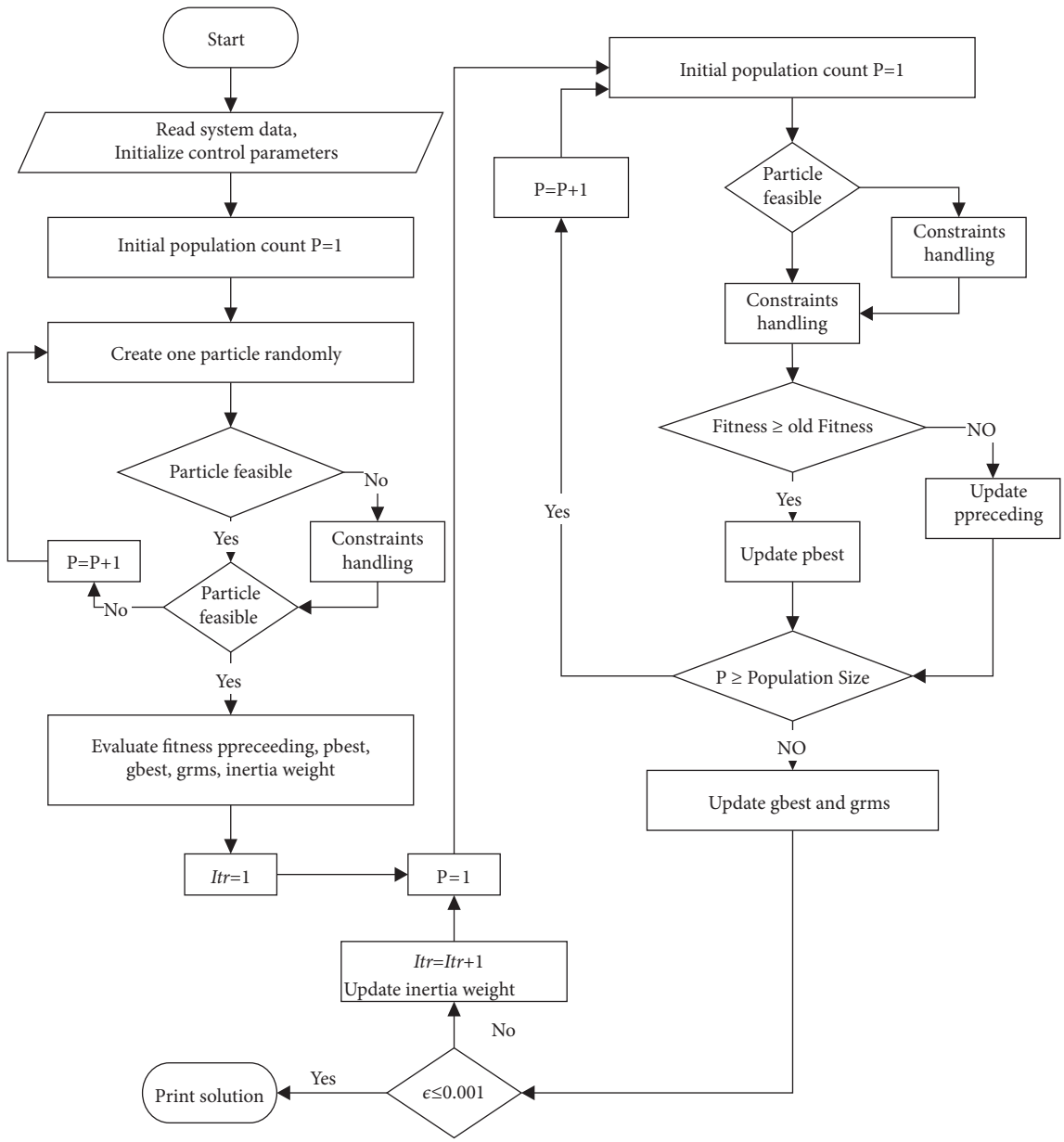

Figure 1: Flowchart for MAED using DPSO.

$$\vec{D}_\alpha = \left|\vec{C}_1 \cdot \vec{X}_\alpha(t) - \vec{X}(t)\right|,$$
$$\vec{D}_\beta = \left|\vec{C}_2 \cdot \vec{X}_\beta(t) - \vec{X}(t)\right|, \quad (11)$$
$$\vec{D}_\delta = \left|\vec{C}_3 \cdot \vec{X}_\delta(t) - \vec{X}(t)\right|,$$

$$\vec{X}_1(t) = \vec{X}_\alpha(t) - \vec{A}_1 \cdot \vec{D}_\alpha,$$
$$\vec{X}_2(t) = \vec{X}_\beta(t) - \vec{A}_2 \cdot \vec{D}_\beta, \quad (12)$$
$$\vec{X}_3(t) = \vec{X}_\delta(t) - \vec{A}_3 \cdot \vec{D}_\delta,$$

$$\vec{X}(t+1) = \frac{\left(\vec{X}_1(t) + \vec{X}_2(t) + \vec{X}_3(t)\right)}{3}, \quad (13)$$

where $\vec{X}_\alpha(t)$, $\vec{X}_\beta(t)$, and $\vec{X}_\delta(t)$ are the position of first-, second-, and third-best fitness value. $\vec{D}_a$, $\vec{D}_\beta$, and $\vec{D}_\delta$ are determined as above equations.

3.2.5. Implementation of GWO for MAED Problem. The implementation of the GWO algorithm to solve the ELD complex problem with VPL is described in Figure 3 as follows (see Algorithm 2).

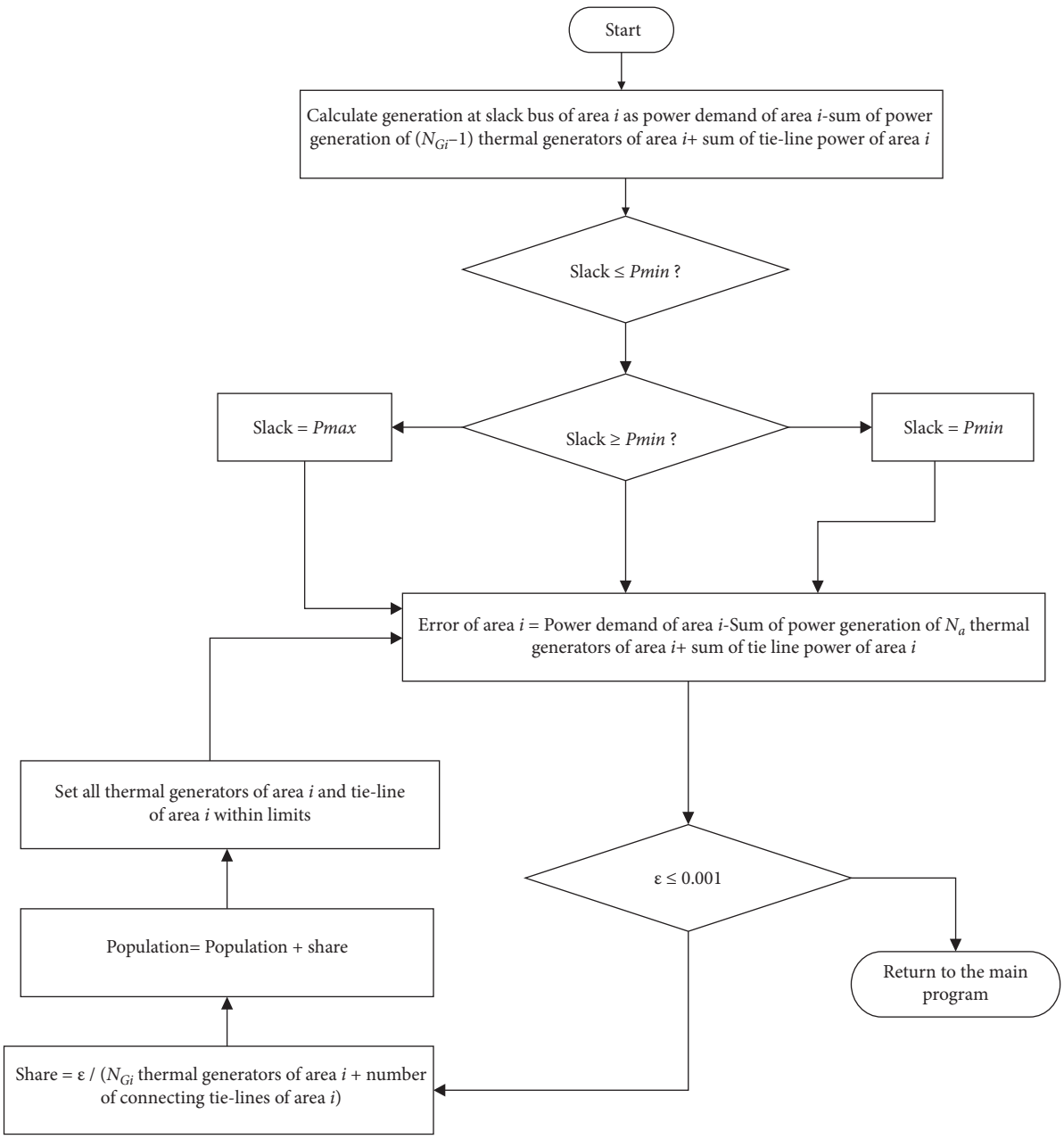

Figure 2: Constraint management algorithm of DPSO.

Step 1: enter the system data and initialize all control parameters and new particles randomly.
Step 2: check the feasibility of the current particle; if it is not feasible, then run the constraint management algorithm.
Step 3: make an increment in population count by 1. Now check the population, if it is less than its maximum value, go back to Step 1.
Step 4: calculate fitness function through (equation (1)), preceding, grms, inertia weight, and constriction function via (equations (5)–(7)).
Step 5: initialize iteration count.
Step 6: repeat Steps 1 and 2. Update preceding for the current particle. Then repeat Step 4.
Step 7: update grms and gbest.
Step 8: make an increment in iteration count by 1. If iteration did not reach its maximum value, repeat Step 8.
Step 9: print final results.

Algorithm 1: Particle encoding and initialization methodology algorithm.

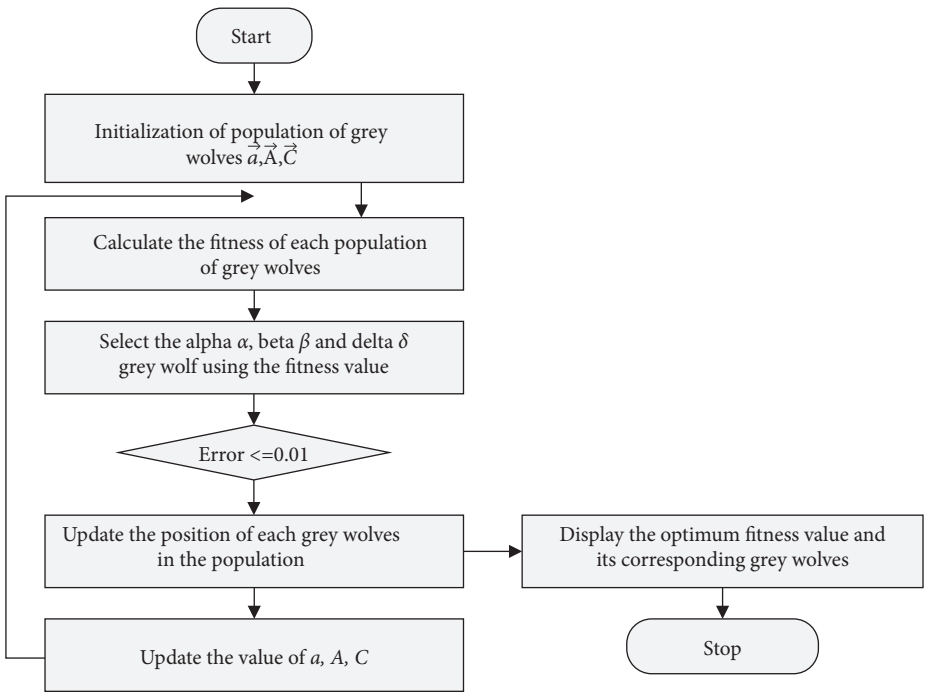

Figure 3: Flowchart for handling MAED problem by grey wolf optimizer.

Step 1: assign grey wolf population X_k ($k = 1, 2, \ldots, N$); N = no of generators
Step 2: initialize a, A, and C
Step 3: compute the objective function values for each search agent
Step 4: find all solutions and initializ the best solution with them
Step 5: X_α = select first leader (archive)
  X_β = select second leader (archive)
  X_δ = select third leader (archive)
  $z = 1$;
Step 6: **while** ($z < Max\ iterations$)
Step 7: **for** every variable
  Update position of present search solution by equations (8)–(13)
  end for
Step 8: modernize a, A, and C
  Compute fitness values of every solution
  Then find other solutions
    Update the solution with reference to best solutions
Step 9: **If** particle checking is full
  Run the grid mechanism to omit one of the present archive members
  Add the new solution to the archive **end if**
Step 10: **If** any new added solutions to the best solution are located shell to a hypercube
  Update the population with other new solution(s) **end if**
Step 11: X_α = select leader (archive)
  Exclude alpha from the archive temporarily to avoid selecting the same leader
  X_β = select leader (archive)
  Exclude beta from the archive temporarily to avoid selecting the same leader
    X_δ = select leader (archive)
    Add back alpha and beta to the archive
    $t = t + 1$
    end while
Step 12: return **archive**

Algorithm 2: Algorithm for handling MAED problem by grey wolf optimizer.

4. Results and Discussion

The constraints considered in this study made MAED problem much more complex and difficult to solve than the classical ED problem. DPSO and GWO techniques are used and tested for the MAED problem on three systems having different sizes and complexities. The performance of both DPSO and GWO variants is compared.

4.1. Description of the Test Systems

4.1.1. Test System 1: Single-Area Problem. The first type of system consists of only a single area with three generator units and no tie line connection as shown in Figure 4. Generator cost coefficients are as follows: fixed cost for 3 generators (a) is 561, 310, and 78; running cost (b) is 7.92, 7.85, and 7.97; and maintenance cost (c) is 0.001562, 0.00194, and 0.00482, respectively. This case study's upper and lower generator limit is [600, 400, and 200] and [150, 100, and 50]. The data for the test system is taken from [1].

In Table 2, results are taken by varying load demand, and results are compared from the classical method, that is, lambda-iteration method.

It is revealed from the results that all constraints are satisfied within their limits. In the above case study, power violation = zero means all constraints are satisfied and no power loss occurs. From Figure 5, it is seen that the GWO approach converges faster compared to the PSO approach.

4.1.2. Test System 2: Two-Area Problem with 1 Tie Line. The two areas with four generator units are tied through a single tie line shown in Figure 6. Generator cost coefficients adopted from literature [17, 45] are as follows: fixed cost for 4 generators (a) is 561, 310, 78, and 250; running cost (b) is 7.92, 7.85, 7.97, and 7.5; and maintenance cost (c) is 0.001562, 0.00194, 0.00482, and 0.00181 respectively. This case study's upper and lower generator limit is [600, 400, 200, and 340] and [150, 100, 50, and 70]. In this case study, the load is varied, and the corresponding tie line limit is also varied. The initial value for C_1 and C_2 is 1.8 and 0.2, respectively. The final value for C_1 and C_2 is 0.2 and 1.9, respectively. The results are concluded after 500 iterations for both methods.

From Table 3, it is seen that power mismatch is zero, and the system satisfies all the constraints within the prespecified limits.(Table 4)..

The results show the effectiveness of the GWO technique over DPSO and ABCO techniques. The generation cost and execution time are less in the case of GWO as compared to DPSO and ABCO. The DPSO saves Rs. 12.4 over ABCO approach and in the case of GWO Rs. 12.7 over ABCO.

From Figure 7, it is seen that the GWO approach converges faster compared to the PSO approach.

4.1.3. Test System 3: Four-Area System. In this system, four areas with ten generator units in each area are considered for generation, including all constraints. All the generating units included valve point loading coefficients. The areas are fully interconnected, that is, the power can flow between any two areas. Hence, the system has four areas, each consisting of 10 generators and connected with three tie lines, as shown in Figure 8. The total demand for this case is 10,500 MW. In this case study, Area 1 shares 15% load demand, Area 2 shares 40% load demand, Area 3 shares 30% load demand, and Area 4 shares 15% load demand. The tie line limit from Area 1 to Area 2, from Area 1 to Area 3, and from Area 2 to Area 3 or vice versa is taken as 200 MW and that for the remaining, each tie line is taken as 100 MW. Along with this, other data related to this case study is mentioned in Table 1. The cost coefficient data for 40 generators are adapted from [47].

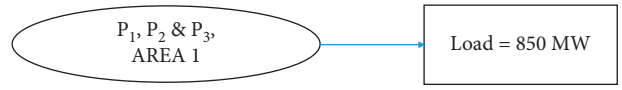

Figure 4: Test System 1: problem descriptions of 1 area and 3 generator units.

The dispatch of the current MAED problem consists of power generation of each generator for every area and the power flowing through the tie line given in the system. The dispatch schedule of the system, for the best run with minimum cost, is presented below in Table 5.

The DPSO and GWO are successfully applied to MAED in MATLAB. The scheduled dispatch of the problem specified earlier is recorded for twenty-five test runs. The total cost and CPU time taken for each dispatch have been presented in Table 6. The analysis of these runs is also done, and the results obtained are compared with other methods reported in Table 7. The Wilcoxon rank-sum test is performed on cost values of both the approaches that are obtained in 25 runs in MATLAB using the command rank-sum and that command prompt return the p value of a two-sided Wilcoxon rank-sum test equal to 2.4170×10^{-6} and h value return equal to 1 that indicates a rejection of the null hypothesis at the 5% significance level. The struct format returns zval and rank-sum values as 4.7150 and 881, respectively, for the obtained cost values.

Table 8 shows the power flow between each pair of areas. Every entry corresponds to the power flowing in the respective tie line. As can be seen below, the diagonal entries will always be zero because no tie line flows possible within the area. The MATLAB program has been run for various combinations of iteration count and population size. The cost convergence curve is a plot of fuel cost obtained versus iteration count. The curve has been plotted for iteration count and population size being one thousand and one hundred, respectively. This curve has been shown in Figure 9.

It is visible that the convergence curves obtained by solving Test System 3 using DPSO and GWO are shown in Figure 9. It is initially observed that the rate of decrease of the cost value is significant but slows down later, and GWO shows better results after complete iterations.

From Table 7, it is concluded that the GWO approach is better in terms of operating cost, execution time, and higher efficiency, keeping in mind all the constraints so that the power mismatch and violation are zero.

Table 2: Results comparison of GWO, PSO, and classical method on 25 statistical runs.

P_D (power demand) (MW)	850 (MW)			1000 (MW)		
Method	GWO	PSO	Classical method [1]	GWO	PSO	Classical method [1]
Cost (Rs/hr)	8194.4	8194.4	8194.45	9583.1	9583.2	9583.1
P1 (MW)	391.84	392.52	393.2	463.11	458.06	462.11
P2 (MW)	337.59	334.07	334.6	392.05	396.46	394.25
P3 (MW)	120.57	123.41	122.2	144.84	145.48	143.64
CPU mean time	5.89	11.85	—	7.75	12.31	---
Power violation	0.00	0.00	0.00	0.00	0.00	0.00

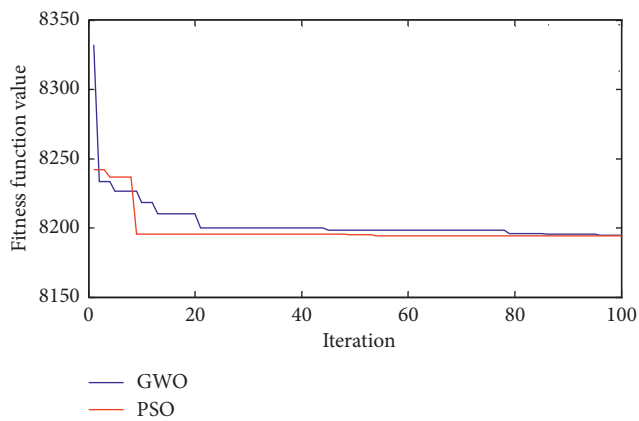

Figure 5: Convergence case of Test System 1 for both techniques.

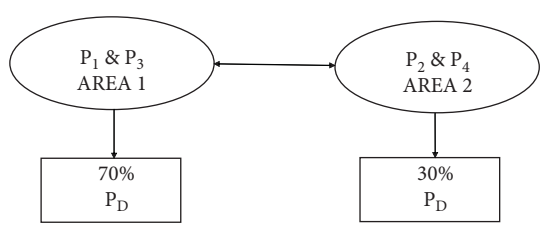

Figure 6: Test System 2: problem descriptions of 2 areas and four generator units.

Table 3: Power dispatch of 4 units according to power demand.

Power (MW)	$P_D = 1000$ (MW) (DPSO)	$P_D = 1000$ (MW) (GWO)	$P_D = 1120$ (MW) (DPSO)	$P_D = 1120$ (MW) (GWO)
P_1	381.73	381.59	444.95	422.5
P_2	195.58	194.66	215.93	211.42
P_3	118.3	118.46	139.05	167.63
P_4	304.39	305.33	320.07	318.45
Tie line power flow (MW)	199.97	199.96	200	193.87

Table 4: Cost of 4 generating units according to demand $P_D = 1120$ MW.

Method	Average cost (Rs/h)	Best cost (Rs/h)	Worst cost (Rs/h)	Mean time (CPU sec)	Standard deviation
DPSO	10,605.14439	10,605	10,605.3425	0.264437	0.063260
GWO	10,604.891	10,604.45	10,605.120	0.250588	0.056321
ABCO [24]	10,617.5431	10,608.6781	10,664.3588	4.3594	27.8354

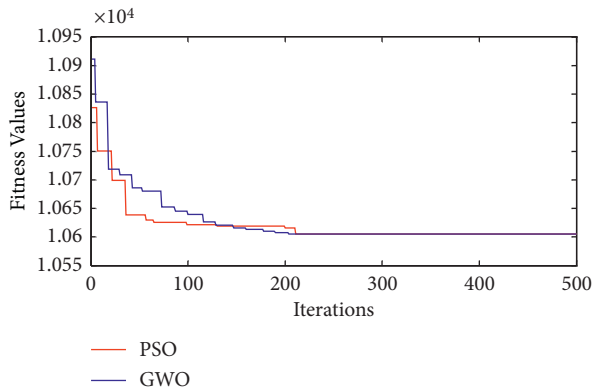

Figure 7: Convergence case of Test System 2 for both techniques.

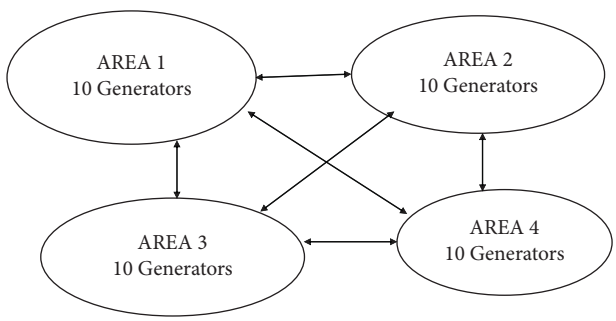

Figure 8: Problem descriptions of 4 areas, 40 generator units, and six tie lines.

Table 5: Results of power dispatch of 40 generators by both the approaches.

Generators	DCPSO (MW)	GWO (MW)	Generators	DCPSO (MW)	GWO (MW)
P_1	110.5595	114	P_{21}	523.1689	514.147
P_2	110.5595	114	P_{22}	523.1689	514.147
P_3	116.5595	66.015	P_{23}	523.1689	534.211
P_4	186.5595	83.204	P_{24}	523.1689	534.211
P_5	93.5595	97	P_{25}	523.1689	468.288
P_6	136.5595	74.3246	P_{26}	523.1689	468.288
P_7	260.2644	240.556	P_{27}	10	10
P_8	296.5595	280.241	P_{28}	10	10
P_9	296.5595	274.65	P_{29}	10	10
P_{10}	93.5595	130	P_{30}	47	97
P_{11}	94	216.98	P_{31}	190	190
P_{12}	94	205.18	P_{32}	190	190
P_{13}	125	312.94	P_{33}	190	190
P_{14}	486.610	418.54	P_{34}	168.744	200
P_{15}	486.610	422.66	P_{35}	168.744	200
P_{16}	486.610	422.66	P_{36}	168.744	200
P_{17}	486.610	500	P_{37}	110	110
P_{18}	486.610	500	P_{38}	110	110
P_{19}	536.610	550	P_{39}	110	110
P_{20}	536.610	550	P_{40}	320.744	110

Table 6: Run analysis of 25 runs of MAED using DPSO and GWO.

S. no.	Cost (Rs/h) of DPSO	Cost (Rs/h) of GWO	CPU time (s) of DPSO	CPU time (s) of GWO
1	123,738.3254	123,721.122	104.67	97.213
2	123,812.5644	123,712.325	61.53	90.534
3	123,877.6792	123,723.631	137.05	109.054
4	123,882.7808	123,792.833	115.45	94.658
5	123,759.8648	123,612.173	65.20	91.983
6	123,818.2971	123,623.243	59.45	87.449
7	123,738.3254	123,125.108	148.94	91.023
8	123,843.1507	123,612.793	147.37	88.998
9	123,776.3162	123,625.108	148.21	86.480
10	123,777.5035	123,593.503	155.19	87.001
11	123,847.6935	123,693.034	147.91	86.802
12	123,798.0078	123,572.307	61.02	89.112
13	123,717.4591	123,661.283	138.72	87.124
14	123,657.4212	123,724.197	140.33	87.501
15	123,599.2091	123,622.873	140.62	92.043
16	123,823.4058	123,599.024	133.54	87.009
17	123,764.0492	123,625.263	129.55	90.641
18	123,830.4886	123,656.898	106.70	87.608
19	123,881.4434	123,597.098	94.81	90.700
20	123,881.6	123,770.425	122.99	94.321
21	123,705.8138	123,711.371	76.46	93.112
22	123,722.2477	123,707.126	93.86	93.001
23	123,800.1607	123,605.183	108.31	86.992
24	123,935.3665	123,787.937	98.42	92.861
25	123,782.9852	123,591.523	119.06	90.992
Average	123,790.8864	123,642.695	114.21	90.986
Minimum	123,599.2091	123,572.307	59.45	86.48
Maximum	123,935.3665	123,792.833	148.94	109.05
Standard deviation	77.1306	66.65618	31.0525	4.779

Table 7: Result comparison of the identical problem by various techniques earlier and by both techniques.

Method	Best cost	Avg. cost	Worst cost	Mean CPU time (s)
ABCO [24]	1,24,009.4	—	—	126.9
DE [24]	—	1,24,544.1	—	134.8
EP [24]	—	1,24,574.5	—	144.5
HCPSOGA [31]	1,23,531.2	—	—	190.58
RCGA [28]	1,28,046.50	—	—	—
PSO [35]	1,28,403	—	—	—
IGOA [35]	1,23,273	—	—	—
FPA [34]	1,23,999.2	—	—	—
DPSO	1,23,599.20	1,23,790.88	1,23,935.36	114.2
GWO	1,23,125.108	1,23,642.695	1,23,792.833	90.986

Note: "—": values are not given in papers.

Table 8: Tie line results of Test System 3.

Tie line limit (MW)	By DCPSO (MW)	By GWO (MW)
1–2	107.5098	198.12
1–3	47.4277	−1.0910
1–4	7.8029	−99.9093
2–3	−186.610	−1.09910
2–4	−86.61	−99.9093
3–4	−73.1689	8.1111

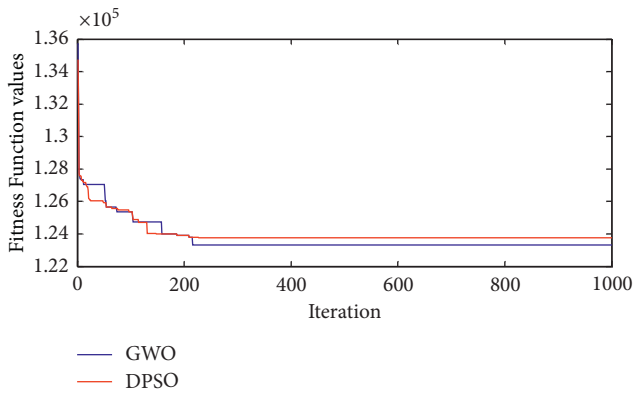

FIGURE 9: Convergence case of Test System 3 for both techniques.

5. Conclusion

In this paper, the DPSO and GWO have been applied successfully to model and solve the multiarea economic dispatch in three different test cases. First, the MAED problem with nonlinear cost function is solved on a single-area test system consisting of 3 thermal generators. Second, DPSO and GWO are applied on two-area test systems with four thermal units with single tie lines. These are, after that, employed on a multiarea test system consisting of 40 thermal units and six tie lines. Optimum demand sharing of power generating units is evaluated using DPSO and GWO optimization techniques. The simulation results reveal that GWO techniques produce qualitative cost solutions without any constraint violation. A significant improvement in the cost results has been obtained compared to other optimization techniques discussed in the literature. In the future, this work can be extended to work in deregulated, stochastic, and contingent environments. The losses can also be calculated using the B-coefficients of thermal generators in future research work. Ramp rate is also an important constraint that makes power system problems more realistic and can also be considered in future research work.

Nomenclatures

a_{ij}, b_{ij}, and c_{ij}:	The cost coefficients of the jth generator in area i (Rs/hr), (Rs/hr MW^{-1}), and (Rs/hr MW^{-2})
C_1 and C_2:	Acceleration coefficients for the best and social experience of PSO
C_{1b} and C_{1p}:	Acceleration coefficients for best and preceding experience
e_{ij} and f_{ij}:	The valve point effect coefficients of the jth generator in area i (Rs/hr, MW^{-1})
$g$best^t:	The best particle during tth iteration
$g$rms^t:	Root mean the square experience of the swarm during tth iteration
itr:	Current iteration count
itr$_{\max}$:	Maximum iteration count
P_{Gij}:	The real power output of the jth generator in area i (MW)
$P_{G_{ij}}^{\min}/P_{G_{ij}}^{\max}$:	Minimum/maximum generation limits of jth generator in area i (MW)
$P_{Tim}^{\min}/P_{Tim}^{\max}$:	Minimum/maximum tie line power limit from area i to area m (MW)
preceding$_n$:	Preceding position of nth particle achieved based on its just previous experience
P_{Tim}:	Tie line real power flow from area i to area m (MW) rand$_1$() and rand$_2$() random numbers in [0, 1]
V_n^t:The velocity of nth particle at tth iteration$W$:	The velocity of nth particle at tth iteration$W$:Inertia weight
PSO:	Particle swarm optimization
GWO:	Grey wolf optimizer
RCGA:	Real codec genetic algorithm
ABCO:	Artificial bee colony optimization
MAED:	Multiarea economic dispatch
P_{Gi}:	Total real power generation in area i (MW)
k:The ratio of dynamic cognitive and social acceleration coefficients:	The ratio of dynamic cognitive and social acceleration coefficientsk_w: The ratio of maximum and minimum bound of the inertia weight
M:	Number of areas
N_{Gi}:	Number of generating units in the system in area i
P_{Di}:	The total real power demand of area i (MW)
Pbest$_n$:	The best position of nth particle achieved based on its own experience
PD:	The total actual power demand of the system (MW)
$W_{\min}/W_{\max}$:	Minimum/maximum value of inertia weight
Δt:	Time step (s)
ζ_1 and ζ_2:	Exponential constriction functions
η:	The ratio of the current and maximum iteration count
η_i:	The value of g at which cognitive and social behavior equalizes
μ:	Constant
μ_1 and μ_2:	Coefficients of exponent terms
PSO-TVAC:	PSO time-varying acceleration coefficients

DE:	Differential evolution
EP:	Evolutionary programming
DPSO:	Dynamically controlled particle swarm optimization
ED:	Economic dispatch.

Data Availability

The data used in this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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