

Research Article

Interval Type-2 Fuzzy Standardized Cumulative Sum Control Charts in Production of Fertilizers

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Statistical process control is a method used for controlling processes in which causes of variations and correction actions can be observed. Control chart is one of the powerful tools of statistical process control that are used to control nonconforming products. Previous literature suggests that fuzzy charts are more sensitive than conventional control charts, and hence, they provide better quality and conformance of products. Nevertheless, some of the data used are more suitable to be presented in interval type-2 fuzzy numbers compared to type-1 fuzzy numbers as interval type-2 fuzzy numbers have more ability to capture uncertain and vague information. In this paper, we develop an interval type-2 fuzzy standardized cumulative sum (IT2F-SCUSUM) control chart and apply it to data of fertilizer production. This new approach combines the advantages of interval type-2 fuzzy numbers and standardized sample means which can control the variability. Twenty samples with a sample size of six were examined for testing the conformance. The proposed IT2F-SCUSUM control chart unveils that 15 samples are “out of control.” The results are also compared to the conventional CUSUM chart and type-1 fuzzy CUSUM chart. The conventional chart shows that 13 samples are “out of control.” In contrast, the type-1 fuzzy CUSUM chart shows that the process is “out of control” for 14 samples. In the analysis of average run length, the proposed IT2F-SCUSUM chart outperforms the other two CUSUM charts. Thus, we can conclude that the IT2F-SCUSUM chart is more sensitive and takes lesser number of observations to identify the shift in the process. The analyses suggest that the IT2F-SCUSUM chart is a promising tool in examining conformance of the quality of the fertilizer production.

1. Introduction

Over the past few decades, quality is one of the most imperative consumer decision factors in selecting the faultless products and services [1]. It had been evolutionary developed since 1900 through various improvements in the quality of the products [2]. Some of the definitions in the quality terms are discussed as a viewpoint as a need for the technical community in the various organization such as European Organization for Quality Control and the American Society for Quality Control [3]. Statistical process

control (SPC) is one of the techniques used for controlling processes to distinguish causes of variation and signal the need for corrective actions [2]. Walter Shewhart, Bell Telephone Laboratories, USA, in 1924 developed SPC methods for the improvement of manufacturing quality, and these methods were incorporated into a management philosophy [1]. In achieving process stability of the products and services, SPC is very useful to be applied in which variability of products can be reduced. The process is said to be in statistical control if disturbances or special causes of variation are eliminated.

In SPC, the most important tool that is useful in the process of monitoring technique is control chart. Control charts help to distinguish the products, that is, less or more than the control limits. The use of classical control chart that was proposed by Walter Andrew Shewhart in 1920s provides a graphical depiction and record of data series. It is suitable to analyse the variation in the process when the data are known precisely and exactly. The most crucial thing is the possibility of loss of information, and this situation has merely happened in the qualitative data [4]. However, it might not be possible to determine the data clearly especially when analysing vague and qualitative data. The classical control charts may not be applicable since they require certain information where human subjectivity plays an important role in defining the quality characteristics. Hence, fuzzy control charts were proposed, and it is believed that these types of control charts will provide a systematic base to deal with the scenario which is ambiguous or not well defined. Fuzzy charts are useful when the data are uncertain or vague, and the process is incomplete or includes human subjectivity [5]. The vagueness can be handled by transforming incomplete or nonprecise quantities to their representative values for control chart decisions as “in control” or “out of control” [6]. In fact, fuzzy set theory is a perfect means for modelling uncertainty (or imprecision) arising from mental phenomena which is neither random nor stochastic [7].

Nowadays, type-1 fuzzy set is widely used in the manufacturing and agricultural area. Mojtaba Zabihinpour et al. [8] constructed the fuzzy and s control charts with an unbiased estimation of standard deviation to monitor quality characteristics. The study on noodle production food industry proved that the proposed technique improves the detection of abnormal shift in process mean. Recently, Sabahno et al. [9] investigated the adaptive \bar{X} and R fuzzy control chart that allows all the chart parameters to adapt based on the process state in the sample. In a nutshell, they found that their adaptive scheme was able to detect the process shift faster than the classical one. Nevertheless, some of the data used in our daily lives can only be used for the type-2 fuzzy control chart and cannot be expressed by other fuzzy charts. This indicates that if the data are not suitable to be used in the type-1 fuzzy control chart or conventional chart, it might affect the number of defective products produced by the company. This means the company will have a high tendency to sell the defective products to customers. Any mistakes in intervention of the process, delay of alarming excessive product defects, and scraps or reworks of final products will result in an increase of production costs. In manufacturing industries, they need to reduce the percentage of nonconformities in order to reduce the costs and to fulfill consumer satisfaction. Therefore, they need to find the best method for getting the best result of the product's quality. The type-2 fuzzy control chart needs to be used if the data are suitable to its analysis. In fact, the efficiency of the type-2 fuzzy control chart is more than the analysing of crisp data, but it also gives essential alerts by means of linguistic terms.

Consequently, the type-2 fuzzy chart is much capable to detect the meaning of process shifts and hence it would help

managers to establish a predictable and consistent level of quality of the product of the company. Erginel et al. [10] examined the fraction nonconforming products by using the interval type-2 fuzzy control chart. Other than that, Şentürk and Antucheviciene [11] analysed the type-2 fuzzy nonconformities control charts. Type-2 fuzzy control charts came into consideration when the researcher wanted to investigate the imprecision of membership functions in three dimensions. The classical c control charts were not suitable to be used when the data were collected as the type-2 fuzzy numbers. Hence, they applied the interval type-2 fuzzy charts to reduce the vagueness and uncertainty of the observation data. Teksen and Anagün [4] explored type-2 fuzzy charts using likelihood and defuzzification methods. The different methods for analysing interval type-2 fuzzy \bar{X} and R charts are defuzzification, distance, ranking, and likelihood methods. Control charts are used to compare with the crisp number to choose the best method. Other than that, Kaya and Turgut [12] analysed the type-2 fuzzy variables control chart and applied it on a real case application from electronic industry. They concluded that type-2 fuzzy control charts can evaluate the process in more sensitive and precise way. Thus far, however, there has been little discussion about interval type-2 fuzzy standardized cumulative sum (IT2F-SCUSUM) control charts. Generally, the conventional control chart is often used as an alternative to cumulative sum (CUSUM) in diagnostics aspects of bringing an uncontrollable product to be “in control.” Nevertheless, the conventional chart, also known as Shewhart chart, is quite insensitive to small process shifts which means assignable causes do not result in large process disturbance. Therefore, CUSUM is the best chart to be used in detecting small process shifts in monitoring analysis. The CUSUM chart, also known as time-weighted control chart, is used for controlling cumulative sum of quality characteristics measurement. It helps in detecting small shifts in a process which is less than 1.5σ [1].

This study wishes to develop the IT2F-SCUSUM control charts as a new approach in quality control. This study also compares the IT2F-SCUSUM control chart with conventional standardized cumulative sum (SCUSUM) and type-1 fuzzy standardized cumulative sum (T1F-SCUSUM) control charts. The comparative analysis will be conducted to determine which control chart is the most sensitive as it would help the manufacturers to reduce the percentage of nonconformists, thereby reducing the manufacturing costs in their company. The proposed IT2F-SCUSUM control chart is a maiden study in CUSUM control charts. This paper is organised as follows. Section 2 provides the literature review of CUSUM charts. The theoretical structure of the proposed method is explained in Section 3, while Section 4 covers the application of the IT2F-SCUSUM control chart using fertilizer production data. The comparative analysis results are given in Section 5, whereas Section 6 concludes the study.

2. Literature Review

Over the past few decades, CUSUM charts have been widely used for monitoring process stability and capability in identifying small shifts in the process. It was first proposed

by Page [13] and is much better than conventional charts in perceiving small process shifts, and hence, it is a good chart to be used in analysing the data [1]. This section provides a review of past research that is related to CUSUM.

Very recently, Wang et al. [14] constructed a convolution model for oil leakage detection in electrohydraulic railway point systems in Beijing, China. The electrohydraulic railway point system is widely adopted due to its high efficiency and long service life. Nevertheless, its operation highly relies on a sufficient volume of oil which sometimes leads to leakage of oil and hence causes a failure to the electrohydraulic railway point system. In a nutshell, they conclude that the CUSUM chart is sensitive in detecting the small mean shift in the process.

Chen et al. [15] used multiple estimators to estimate the mean and variance of the population by using samples in phase I of their experiment. When the testing process in phase II is out of control, they analysed the influence of each estimator combination on the CUSUM chart based on running length distribution of control charts. They comprehensively analysed the average, standard deviation, and percentile of the control chart running length in four different environments in order to find the parameter combination that optimizes the control chart performance. The research results show that the CUSUM control chart based on $X\text{-}\sigma\text{IQR}$ and $WH\text{-}\sigma\text{IQR}$ estimators performs best in a polluted environment.

Yu and Cheng [16] studied the CUSUM charts in psychometric research to detect aberrant responses in a response sequence such as test speediness, inattentiveness, or cheating. They compared the CUSUM chart and change-point analysis (CPA) in detecting the test speediness. Simulation studies show that the performances of the statistics are affected by the underlying data generating model, the severity of the speediness, and the length of the test. In a nutshell, they conclude that CUSUM analysis shows better performance in a wide range of process means compared to the CPA method.

Xue and Qiu [17] developed the multivariate statistical process control (MSPC) based on some nonparametric distribution. They concluded that the CUSUM chart can accommodate stationary serial data correlation and perform well in different large process shifts. The CUSUM chart method was used by [18] for continuous monitoring of antifouling (AF) treatment. As a result, it showed that the CUSUM chart is the best tool to deal with reducing the operation and maintenance costs.

Boullousa-Falces et al. [19] examined the validation of CUSUM chart for biofouling detection in heat exchangers. The CUSUM chart is very efficient in the early detection of slow and progressive changes within the process. They reported that CUSUM graphs demonstrated a greater capability to detect changes in the biological adherence process. Lawson [20] investigated monitoring a process mean by collecting one observation in every 12 minutes rather than a subgroup of five every 60 minutes. The results showed that the average time to signal (ATS) of both CUSUM and EWMA charts is substantially shorter than the ATS for the previous method, R chart. Volodarsky and Pototskiy [21]

compared the CUSUM charts with the overlay of the V-mask and the conventional charts of the mean values to the setting level offset. As a result, they found that CUSUM charts more sensitive to small displacements of the process setting level.

In Spain, Fortea-Sanchis and Escrig-Sos [22] applied CUSUM charts in monitoring clinical-care processes, a new aspect in clinical research. This study is really useful for studying learning curves which could not be observed with other methods. In medical research, Fortea-Sanchis et al. [23] studied the quality of nodal analysis in colon cancer and used a population registry cancer database to estimate the optimal number of lymph nodes for adequate prognostic analysis using CUSUM analysis.

A standardized CUSUM chart had been implemented by Ramasamy [24] to three different types of sample size which are variable sample size (VSS), fixed sample size (FSS), and Markov-dependent sample size (MDSS). He investigated the effect of the three types of sample size on the conventional control chart in monitoring small shifts in the process mean. He concluded that the standardized CUSUM chart with MDSS is the most vulnerable chart compared to other chart.

Over the past few years, fuzzy charts are being widely used in statistical process control research.

Al-Refaie et al. [25] analysed the CUSUM chart and EWMA chart in a manufacturing process using the triangular membership function. A set of three real case studies had been implemented to illustrate the proposed method which includes piston inside diameter, caps' angel, and tablet weight. The α -cut values show that the proposed CUSUM chart and EWMA chart efficiently help in monitoring the fuzzy observation in the process means. In a nutshell, the researcher revealed that the proposed charts have better detection ability in monitoring the quality characteristics of fuzzy observations and can be applied to other business applications.

Erginel and Şentürk [26] developed fuzzy EWMA and CUSUM control charts, and they reported that both conventional charts are not able to obtain the uncertainty in the case of incomplete data. Ghobadi et al. [27] constructed a fuzzy multivariate cumulative sum (CUSUM) control chart through a numerical comparison via a simulation study on the basis of the average run length (ARL). They concluded that the fuzzy multivariate cumulative sum (CUSUM) control chart performed better in detecting small- and medium-sized shifts in the process.

However, some of the data used in the manufacturing analysis not only can be expressed by type-1 fuzzy sets but are also more appropriate to be used in type-2 fuzzy sets. If there is no uncertainty, then a type-2 fuzzy set reduces to a type-1 fuzzy set, which is analogous to probability reducing to determinism when unpredictability vanishes. However, no study had been conducted on the type-2 fuzzy CUSUM control chart. Table 1 shows the summary of previous studies on the CUSUM control chart.

Based on the review in Table 1, we can conclude that majority of previous research focused on the conventional CUSUM chart and only two research studies had studied the T1F-CUSUM control charts. In fact, only one study towards the SCUSUM chart had been conducted. Therefore, this

TABLE 1: Summary of selected studies on the CUSUM control chart.

Existing literature	Application area	Findings	Type of CUSUM	Type of number used in analysis
Wang et al. [14]	Transportation sector (electrohydraulic railway point systems)	CUSUM control chart is constructed to monitor the residual signal since it is sensitive to the small mean shift	Conventional chart	Crisp numbers
Chen et al. [15]	Polluted environment	CUSUM control chart based on estimators performs best in a polluted environment	Conventional chart	Crisp numbers
Yu and Cheng [16]	Medical sector (psychometric research)	CUSUM analysis shows better performance in a wide range of conditions compared to change-point analysis (CPA) method	Conventional chart	Crisp numbers
Xue and Qiu [17]	Manufacturing sector	CUSUM chart accommodates stationary serial data correlation properly and it performs well in different cases	Conventional chart	Crisp numbers
Boullosa-Falces et al. [18]	Manufacturing sector (antifouling (AF) treatment of tubular heat exchangers)	CUSUM chart is the best tool for reducing the operation and maintenance costs	Conventional chart	Crisp numbers
Boullosa-Falces et al. [19]	Manufacturing sector (biofouling detection in heat exchangers)	CUSUM chart is simple and economical to be used	Conventional chart	Crisp numbers
Lawson [20]	Manufacturing sector	The average time to signal both CUSUM and EWMA charts are substantially better than the previous method, R chart	Conventional chart	Crisp numbers
Volodarsky and Pototskiy [21]	Manufacturing sector	CUSUM chart is more sensitive to small displacements	Conventional chart	Crisp numbers
Fortea-Sanchis and Escrig-Sos [22]	Medical sector (clinical-care processes)	Useful for assessing the quality-of-care outcomes by using learning curves	Conventional chart	Crisp numbers
Fortea-Sanchis et al. [23]	Medical sector	CUSUM has more appropriate cutoff point for diagnosing a high-quality prognosis in colon cancer patients	Conventional chart	Crisp numbers
Ramasamy [24]	Manufacturing sector (piston ring)	Standardized CUSUM chart is the most economical chart in detecting the small shift in the process	Conventional standardized chart	Crisp numbers
Al-Refaie et al. [25]	Manufacturing sector (piston ring, cap's angel, and tablet weight)	The proposed charts have better detection ability in monitoring the quality characteristics of fuzzy observations and can be applied to the other business applications	Type-1 fuzzy chart	Triangular fuzzy numbers
Erginel and Şentürk [26]	Manufacturing sector	Conventional EWMA and CUSUM control charts are not able to obtain the uncertainty of incomplete data	Type-1 fuzzy chart	Trapezoidal fuzzy numbers
Ghobadi et al. [27]	Manufacturing sector	The developed multivariate control chart shows better performance in detecting small- and medium-sized shifts in the process	Type-1 fuzzy chart	Trapezoidal fuzzy numbers

research differs from other studies by improving the fuzzy control chart for type-2 CUSUM control chart in agricultural sector. Different from most of the past studies where the CUSUM control chart was proposed, this study focuses on standardized CUSUM control charts. This study proposes a new standardized CUSUM control chart where interval type-2 fuzzy numbers are embedded in production data. This is the first identifiable study where the IT2F-SCUSUM control chart is proposed.

3. Proposed IT2F-SCUSUM Control Charts

In order to achieve the aforementioned objectives, this section revisits a brief conventional CUSUM control chart and type 1 fuzzy CUSUM control chart. More importantly,

this section proposes IT2F-SCUSUM control charts. CUSUM control charts are well recognized as a potentially advanced process monitoring tools because of their sensitivity against small and moderate shifts [28]. CUSUM is used to examine the process mean as it can easily detect even a small shift in the procedure.

In type-1 fuzzy control charts, the trapezoidal fuzzy numbers need to transform into crisp numbers. This transformation is called as defuzzification method. Four ways of representative (scalar) values for the fuzzy sets that transform fuzzy sets into crisp values are fuzzy mode, α -level fuzzy midrange, fuzzy median, and fuzzy average. Therefore, in this study, we will use the fuzzy midrange transformation method for the process of defuzzification of the data. Fuzzy midrange is the midpoint of the ends of the α -level cuts,

denoted as A^α , which is a nonfuzzy set that comprises all elements whose membership is greater than or equal to α -cuts [29]. There is no theoretical basis supporting any one specifically, and the selection between them should be mainly based on the ease of computation or preference of the user [11]. Therefore, in their study, they analysed the data using the α -level fuzzy midrange method because it is simpler to be used.

In this paper, we will analyse the data using conventional SCUSUM charts and T1F-SCUSUM control charts prior to proceeding with the proposed IT2F-SCUSUM control charts. In type-2 fuzzy sets, the membership functions are three dimensional which has a new third dimension that provides additional degrees of freedom that make it possible to directly model uncertainties [30]. An example of type-2 fuzzy sets is interval type-2 fuzzy sets. It is the most preferred type-2 fuzzy sets in scientific publications because computations with interval type-2 fuzzy sets are rather simple and manageable [31]. Therefore, the definitions of type-2 fuzzy sets, interval type-2 fuzzy sets, and some of interval type-2 arithmetic operations for two trapezoidal interval type-2 fuzzy sets are explained as follows.

Definition 1. A type-2 fuzzy set, denoted by \tilde{A} , is characterized by a type-2 membership function $\mu_{\tilde{A}}$ and presented as follows [31]:

$$\begin{aligned} \tilde{A} &= \{(x, u), \mu_{\tilde{A}}(x, u) | x \in X, u \in [0, 1]\}, \\ I_x &= \{u \in [0, 1] | (\mu_{\tilde{A}}(x, u) > 0)\}. \end{aligned} \quad (1)$$

An interval type-2 fuzzy set is a type-2 fuzzy set and can be expressed as follows:

$$I_x = \{u \in [0, 1] | (\mu_{\tilde{A}}(x, u) > 1)\}. \quad (2)$$

Therefore, interval type-2 trapezoidal fuzzy sets are called as closed interval type-2 fuzzy sets if I_x is the closed interval for every $x \in X$.

Definition 2. The upper membership function and the lower membership function of interval type-2 fuzzy sets are given in Figure 1 [32].

A trapezoidal interval type-2 fuzzy set is as follows:

$$\tilde{A}_i = (\tilde{A}_i^U, \tilde{A}_i^L) = \left((a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U; H_1(A_i^U), H_2(A_i^U)), (a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L; H_1(A_i^L), H_2(A_i^L)) \right), \quad (3)$$

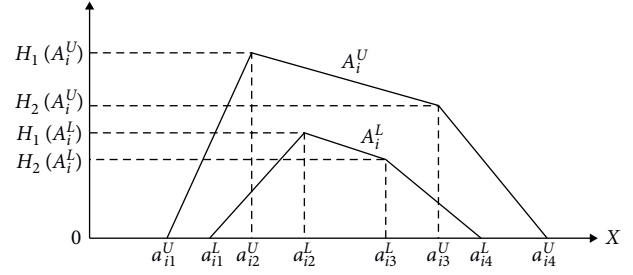


FIGURE 1: The membership functions of interval type-2 fuzzy set \tilde{A} .

where A_i^U and A_i^L denote type-1 fuzzy sets; $a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U$, $a_{i1}^L, a_{i2}^L, a_{i3}^L$, and a_{i4}^L are the reference points of the interval type-2 fuzzy \tilde{A}_i ; $H_j(A_i^U)$ signifies the membership value of the element $a_{i(j+1)}^U$ in the upper trapezoidal membership function A_i^U , $1 \leq j \leq 2$; and $H_j(A_i^L)$ indicates the membership value of the element $a_{i(j+1)}^L$ in the lower trapezoidal membership function A_i^L :

$$\begin{aligned} 1 &\leq j \leq 2, \\ H_1(A_i^U), H_2(A_i^U), H_1(A_i^L), H_2(A_i^L) &\in [0, 1], \\ 1 &\leq i \leq n. \end{aligned} \quad (4)$$

Let \tilde{A}_1 and \tilde{A}_2 be the two trapezoidal interval type-2 fuzzy sets:

$$\begin{aligned} \tilde{A}_1 &= (A_1^U, A_1^L) = \left((a_{11}^U, a_{12}^U, a_{13}^U, a_{14}^U; H_1(A_1^U), H_2(A_1^U)), (a_{11}^L, a_{12}^L, a_{13}^L, a_{14}^L; H_1(A_1^L), H_2(A_1^L)) \right), \\ \tilde{A}_2 &= (A_2^U, A_2^L) = \left((a_{21}^U, a_{22}^U, a_{23}^U, a_{24}^U; H_1(A_2^U), H_2(A_2^U)), (a_{21}^L, a_{22}^L, a_{23}^L, a_{24}^L; H_1(A_2^L), H_2(A_2^L)) \right). \end{aligned} \quad (5)$$

Then, the arithmetic operations for the two trapezoidal interval type-2 fuzzy sets are given as follows [32]:

Addition operation:

$$\begin{aligned} \tilde{A}_1 \oplus \tilde{A}_2 &= (A_1^U, A_1^L) \oplus (A_2^U, A_2^L) \\ &= \left((a_{11}^U + a_{21}^U, a_{12}^U + a_{22}^U, a_{13}^U + a_{23}^U, a_{14}^U + a_{24}^U; \min(H_1(A_1^U), H_1(A_2^U)), \min(H_2(A_1^U), H_2(A_2^U))), (a_{11}^L + a_{21}^L, a_{12}^L + a_{22}^L, a_{13}^L + a_{23}^L, a_{14}^L + a_{24}^L; \min(H_1(A_1^L), H_1(A_2^L)), \min(H_2(A_1^L), H_2(A_2^L))) \right). \end{aligned} \quad (6)$$

Subtraction operation:

$$\begin{aligned}\tilde{A}_1 \ominus \tilde{A}_2 &= (A_1^U, A_1^L) \ominus (A_2^U, A_2^L) \\ &= \left((a_{11}^U - a_{24}^U, a_{12}^U - a_{23}^U, a_{13}^U - a_{22}^U, a_{14}^U - a_{21}^U); \min(H_1(A_1^U); H_1(A_2^U)), \min(H_2(A_1^U); H_2(A_2^U)), \right. \\ &\quad \left. (a_{11}^L - a_{24}^L, a_{12}^L - a_{23}^L, a_{13}^L - a_{22}^L, a_{14}^L - a_{21}^L); \min(H_1(A_1^L); H_1(A_2^L)), \min(H_2(A_1^L); H_2(A_2^L)) \right). \quad (7)\end{aligned}$$

Multiplication operation:

$$\begin{aligned}\tilde{A}_1 \otimes \tilde{A}_2 &= (A_1^U, A_1^L) \otimes (A_2^U, A_2^L) \\ &= \left((a_{11}^U \times a_{21}^U, a_{12}^U \times a_{22}^U, a_{13}^U \times a_{23}^U, a_{14}^U \times a_{24}^U); \min(H_1(A_1^U); H_1(A_2^U)), \min(H_2(A_1^U); H_2(A_2^U)), \right. \\ &\quad \left. (a_{11}^L \times a_{21}^L, a_{12}^L \times a_{22}^L, a_{13}^L \times a_{23}^L, a_{14}^L \times a_{24}^L); \min(H_1(A_1^L); H_1(A_2^L)), \min(H_2(A_1^L); H_2(A_2^L)) \right). \quad (8)\end{aligned}$$

Arithmetic operations with crisp value k :

$$\begin{aligned}k \times \tilde{A}_1 &= \left((k \times a_{11}^U, k \times a_{12}^U, k \times a_{13}^U, k \times a_{14}^U); H_1(A_1^U), H_2(A_1^U), \right. \\ &\quad \left. (k \times a_{11}^L, k \times a_{12}^L, k \times a_{13}^L, k \times a_{14}^L); H_1(A_1^L), H_2(A_1^L), \right) \\ \frac{\tilde{A}_1}{k} &= \left(\left(\frac{1}{k} \times a_{11}^U, \frac{1}{k} \times a_{12}^U, \frac{1}{k} \times a_{13}^U, \frac{1}{k} \times a_{14}^U \right); H_1(A_1^U), H_2(A_1^U), \right. \\ &\quad \left. \left(\frac{1}{k} \times a_{11}^L, \frac{1}{k} \times a_{12}^L, \frac{1}{k} \times a_{13}^L, \frac{1}{k} \times a_{14}^L \right); H_1(A_1^L), H_2(A_1^L), \right) \quad (9)\end{aligned}$$

where $k > 0$.

There are some differences between type-1 fuzzy numbers and type-2 fuzzy numbers. Table 2 summarises the differences between them.

Based on Table 2, the type-1 fuzzy sets use a single membership function of data, whereas type-2 fuzzy sets use upper and lower membership functions. Next, in defuzzification techniques, there are four methods that can be used by using type-1 fuzzy sets, while in type-2 fuzzy sets, there are five methods that can be used. Besides that, the graph for type-1 fuzzy sets is in single trapezoid but the graph for type-2 fuzzy sets is in two trapezoids with three-dimensional sets.

3.1. IT2F-SCUSUM Control Chart. In the literature, the fuzzy approach to the CUSUM control chart was first introduced by Ghobadi et al. [27]. They developed the fuzzy CUSUM control chart by means of the fuzzy set theory

through a numerical example. Since then, there has been no study that combines type-2 fuzzy numbers with CUSUM control charts. In this section, we develop an IT2F-SCUSUM control chart where interval type-2 fuzzy numbers and standard sample means are combined.

The CUSUM chart is plot by the following quantity [1]:

$$C_i = \sum_{j=1}^i (\bar{x}_j - \mu_0), \quad (10)$$

where C_i is known as cumulative sum up to and including the i th sample, while μ_0 is the estimate of the in-control mean and \bar{x}_j is the mean of the j th sample. In this study, the number of defects was defined as trapezoidal number (a, b, c , and d). However, if $b = c$, the number of traps was converted into a triangular fuzzy number.

The interval type-2 fuzzy control chart uses fuzzy membership functions that have the grades themselves while constructing the limits of the control chart. The fuzzy sample mean is expressed as follows [34]:

TABLE 2: Differences between trapezoidal type-1 fuzzy numbers and type-2 fuzzy numbers.

	Type-1 fuzzy numbers [33]	Type-2 fuzzy numbers [32]
Fuzzy numbers	$A_i = (a_i, b_i, c_i, d_i)$	$(\tilde{A}_i^U, \tilde{A}_i^L) = \left((a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U; H_1(A_i^U), H_2(A_i^U)), (a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L; H_1(A_i^L), H_2(A_i^L)) \right)$
Defuzzification	(i) Fuzzy mode (ii) Fuzzy midrange (iii) Fuzzy median (iv) Fuzzy average	(i) Centroid method (ii) Indices method (iii) Ranking method (iv) Distance method (v) Likelihood method
Graph		

$$\begin{aligned} \tilde{A}_1 &= (A_1^U, A_1^L) = \left[\left(\overline{a_{i1}^U}, \overline{a_{i2}^U}, \overline{a_{i3}^U}, \overline{a_{i4}^U}; H_1(A_1^U), H_2(A_1^U) \right), \right. \\ &\quad \left. \left(\overline{a_{i1}^L}, \overline{a_{i2}^L}, \overline{a_{i3}^L}, \overline{a_{i4}^L}; H_1(A_1^L), H_2(A_1^L) \right) \right] \\ &= \left(\left(\frac{\sum_{i=1}^m \overline{a_{i1}^U}}{m}, \frac{\sum_{i=1}^m \overline{a_{i2}^U}}{m}, \frac{\sum_{i=1}^m \overline{a_{i3}^U}}{m}, \frac{\sum_{i=1}^m \overline{a_{i4}^U}}{m}; \min(H_1(A_1^U), H_2(A_1^U)) \right), \right. \\ &\quad \left. \left(\frac{\sum_{i=1}^m \overline{a_{i1}^L}}{m}, \frac{\sum_{i=1}^m \overline{a_{i2}^L}}{m}, \frac{\sum_{i=1}^m \overline{a_{i3}^L}}{m}, \frac{\sum_{i=1}^m \overline{a_{i4}^L}}{m}; \min(H_1(A_1^L), H_2(A_1^L)) \right) \right). \end{aligned} \quad (11)$$

Nevertheless, some researchers prefer to standardize the variable \bar{X}_i in equation (10) before performing the calculations because many CUSUM charts can have the same

value parameters. By standardizing the CUSUM chart, it can have the same values of κ and H ; hence, it leads naturally to a CUSUM for controlling variability by using

$$\begin{pmatrix} (Z_a^U, Z_b^U, Z_c^U, Z_d^U; H_1(A_i^U), H_2(A_i^U)) \\ (Z_a^L, Z_b^L, Z_c^L, Z_d^L; H_1(A_i^L), H_2(A_i^L)) \end{pmatrix} = \begin{pmatrix} \left(\frac{\overline{X}_a^U, \overline{X}_b^U, \overline{X}_c^U, \overline{X}_d^U}{\overline{X}_a^L, \overline{X}_b^L, \overline{X}_c^L, \overline{X}_d^L} \right) - \left(\frac{\mu_0^U}{\mu_0^L} \right) \\ \left(\frac{S_a^U, S_b^U, S_c^U, S_d^U}{S_a^L, S_b^L, S_c^L, S_d^L} \right) \end{pmatrix}, \quad \text{for } i = 1, 2, 3, \dots, n, \quad (12)$$

where

$$\begin{pmatrix} S_a^U, S_b^U, S_c^U, S_d^U \\ S_a^L, S_b^L, S_c^L, S_d^L \end{pmatrix} = \sqrt{\frac{\sum_{i=1}^n \left[\begin{pmatrix} X_a^U, X_b^U, X_c^U, X_d^U \\ X_a^L, X_b^L, X_c^L, X_d^L \end{pmatrix}_{ij} - \begin{pmatrix} \overline{X}_a^U, \overline{X}_b^U, \overline{X}_c^U, \overline{X}_d^U \\ \overline{X}_a^L, \overline{X}_b^L, \overline{X}_c^L, \overline{X}_d^L \end{pmatrix}_j \right]^2}{n-1}}, \quad (13)$$

an unbiased standard deviation for the i th sample. It can be seen that all sample means and standard deviations are presented in interval type-2 fuzzy numbers.

In representing the CUSUMs, there are two ways that can be used: the tabular or algorithmic form and the V-mask form of the CUSUM. The tabular CUSUM is more desirable

to be used in monitoring the process mean compared to V-mask because it is hard for practitioners to make an interpretation from the analysis [1]. Then, the tabular

standardized CUSUM works by accumulating deviations as follows [29]:

$$(C_{ai}^+, C_{bi}^+, C_{ci}^+, C_{di}^+) = (\max[0, Z_{ai} - \kappa + C_{ai-1}^+], \max[0, Z_{bi} - \kappa + C_{bi-1}^+], \max[0, Z_{ci} - \kappa + C_{ci-1}^+], \max[0, Z_{di} - \kappa + C_{di-1}^+]), \quad (14)$$

$$(C_{ai}^-, C_{bi}^-, C_{ci}^-, C_{di}^-) = (\max[0, Z_{ai} - \kappa + C_{ai-1}^-], \max[0, Z_{bi} - \kappa + C_{bi-1}^-], \max[0, Z_{ci} - \kappa + C_{ci-1}^-], \max[0, Z_{di} - \kappa + C_{di-1}^-]), \quad (15)$$

where κ is the reference value and C_i^+ and C_i^- are one-sided upper and lower SCUSUMs, respectively. Besides, $(C_{ai}^+, C_{bi}^+, C_{ci}^+, C_{di}^+)$ and $(C_{ai}^-, C_{bi}^-, C_{ci}^-, C_{di}^-)$ accumulate deviations from the target value that is greater than κ with both quantities reset to zero on becoming negative. This means the process is considered as “out of control” if either C_i^+ or C_i^- exceeds the decision interval H , based on N^+ and N^- . H is the recommended value of the decision interval as five times of the process standard deviation σ [1]. Based on this procedure, $H = h\sigma$ and $\kappa = k\sigma$, where σ is the standard deviation of the sample variable used in the CUSUM chart. In this procedure, $\kappa = 0.5$ and $H = 5$ are taken as decision parameters for optimum level [29]. In fact, Montgomery [1] and Ramasamy [35] also said that using the parameter of $H = 5$ and $\kappa = 0.5$ will provide a good run length value compared to a shift of about 1σ in the process mean.

However, some CUSUM charts can have the same values of κ and H , and these parameters are not scale dependent as they do not depend on S . Therefore, the standardized CUSUM chart is the best alternative as it leads naturally to a CUSUM for controlling variability. The head start or fast initial response (FIR) essentially sets the starting values C_i^+ and C_i^- to nonzero values, typically $H/2$ for effective detection of any shift in the mean.

3.2. Defuzzification Method for IT2F-SCUSUM Control Chart. Defuzzification provides the best representation value of interval type-2 fuzzy sets as it finds only one value for each fuzzy set. This means the output of the defuzzification is a crisp value. Interval type-2 fuzzy control charts generated by these methods are similar to control charts; hence, defuzzification is able to evaluate the process as “in control” and “out of control” in the same way as the classical method [36].

There are various methods that can be used in analysing the defuzzification process in type-2 fuzzy control charts such as centroid method [30], indices method [37], and best nonfuzzy performance (BNP) method [38]. Consequently, Ercan and Anagun [34] made a comparison between four methods used in analysing the interval type-2 fuzzy sets. The methods are Kahraman et al.’s defuzzification method [39], Qin and Liu’s ranking method [40], Chen’s distance method [41], and Chen and Lee’s likelihood method [42]. As a result, the researchers concluded that all the methods show a similar result in terms of “in control” or “out of control” situation. Therefore, in this research, we use Kahraman et al.’s defuzzification method [39] as it is much simpler and more flexible in evaluating the process. Nevertheless, Kahraman et al. [39] modified the BNP method for application with trapezoidal type-2 fuzzy sets as follows:

$$\text{DIT2}_{\text{Trap}(i)}^U = \frac{(\bar{u}_{a_4^U} - \bar{u}_{a_1^U}) + (H_2(A_1^U)\bar{u}_{a_2^U} - \bar{u}_{a_1^U}) + (H_1(A_1^U)\bar{u}_{a_3^U} - \bar{u}_{a_1^U})}{4} + \bar{u}_{a_1^U}, \quad (16)$$

$$\text{DIT2}_{\text{Trap}(i)}^L = \frac{(\bar{u}_{a_4^L} - \bar{u}_{a_1^L}) + (H_2(A_1^L)\bar{u}_{a_2^L} - \bar{u}_{a_1^L}) + (H_1(A_1^L)\bar{u}_{a_3^L} - \bar{u}_{a_1^L})}{4} + \bar{u}_{a_1^L}, \quad (17)$$

$$\text{DIT2}_{\text{Trap}(i)} = \frac{\text{DIT2}_{\text{Trap}(i)}^U + \text{DIT2}_{\text{Trap}(i)}^L}{2}, \quad (18)$$

where $H_1(A_1^U)$ and $H_2(A_1^U)$ are the maximum membership degree of the upper membership functions, while a_{i4}^U and a_{i1}^U are the largest and the least possible values of the upper membership function. a_{i2}^U and a_{i3}^U are the second and third parameters of the upper membership functions. a_{i4}^L and a_{i1}^L are the largest and the least possible values of the lower membership function, while a_{i2}^L and a_{i3}^L are the second and third parameters of the lower membership functions. On the other hand, $\text{DIT2}_{\text{Trap}(i)}$ is the defuzzification value of each

data of interval type-2 fuzzy number on standardized cumulative sum per unit.

3.3. Performance of Control Chart. The control chart’s performance will be analysed by using the average run length (ARL). ARL is the average number of points that must be plotted before a point indicates an “out-of control” condition [1]. It is the expectation of the time before the control

chart gives a false alarm that an “in control” process has gone “out of control” [43]. This signifies the control chart is defined as the most effective chart based on the least value of ARL in the process shifts. Previously, Roberts [44] developed monographs of ARLs for normally distributed observations, while Robinson and Ho [45] used a numeric procedure to determine the ARL. However, Crowder [46] calculates the ARL using a computer program and Lucas and Saccucci [47] presented table and graph of ARL values for different values of L and λ for the EWMA chart. In fact, they evaluated the run length properties as a continuous Markov chain. Markov chain is the best analysis that can be applied as it is fast and accurate to compute ARLs. Hence, in this study, simulation is carried out to calculate the ARL values using Sigma XL software based on Markov chain rule. The evaluation criteria of the ARL approximation are given as follows:

$$ARL = \frac{\exp(-2\Delta b) + 2\Delta b - 1}{2\Delta^2}. \quad (19)$$

For $\Delta \neq 0$, $\Delta = -\delta^* - k$ for the upper one-sided CUSUM C_i^+ , $\Delta = -\delta^* - k$ for the lower one-sided CUSUM C_i^- , $b = h + 1.166$, and $\delta^* = ((\mu_1 - \mu_0)/\sigma)$. If $\Delta = 0$, one can use $ARL = b^2$. On the other hand, the quantity δ^* denotes the shift in the mean, which is in the units of σ , for which the ARL is to be calculated. Hence, if $\delta^* = 0$, use equation (19) to calculate the ARL_0 , while if $\delta^* \neq 0$, the value of ARL_1 is calculated based on shift of size δ^* . The ARL^+ and ARL^- are calculated based on the following formula:

$$\frac{1}{ARL} = \frac{1}{ARL^+} + \frac{1}{ARL^-}. \quad (20)$$

The developed IT2F-SCUSUM is a complete version of quality control where the chart is being examined based on the small shift in the process and proved by the performance of the average run length. There are two types of ARLs: the “in-control” state, ARL_0 , and the “out-of-control” state, ARL_1 . The smaller value of ARL indicates that it is better in detecting the small shift of the analysis [48, 49]. Figure 2 shows the summary of flowchart on analysis of the IT2F-SCUSUM control chart.

The proposed work is subjected to a comparative analysis where the performance of IT2F-SCUSUM chart is being compared with the performance of T1F-SCUSUM chart and conventional SCUSUM chart to find out the best method for analysing the defects.

4. Case Study: Application to Fertilizer Production

The IT2F-SCUSUM control chart is applied to fertilizer production in an agricultural system. Fertilizers provide macro- and micronutrients to the plants. It is also a source of food for plants and soil to enable the plant growth. There must be a perfect balance of sun, water, and food for plants to grow successfully. In fact, the production of plant might be affected if the number of fertilizers used in a plant is not sufficient. For example, the essential nutrients needed by

plants are macronutrients which consist of the elements nitrogen, phosphorus, and potassium to keep the plants well nourished. There are two types of fertilizers which are organic and chemical. Organic fertilizers mean the production of fertilizers is based on natural products that are free from additives or chemical substances, for instance, leaf mould and cow manure. In contrast, chemical fertilizers typically contain some additives or nonorganic fillers. However, chemical fertilizers can give good improvement to the plants just in days, are easy to handle, and are not expensive.

Twenty samples of chemical fertilizers were collected every ten minutes for an hour from an agriculture and rural development company in Malaysia. The weights of the fertilizers are in grams with a sample size of six. Defective fertilizers might result in high toxic chemicals that may affect the soil pH. The company used two types of machines which are packaging machine to pack the soil and granulation machine to granulate the soil in mixing the materials and suitable speed in making the fertilizers. However, some uncertainty and vagueness can occur due to the operators' judgement or mechanical errors in handling the fertilizers as human cognitive decisions play an important role; hence, an IT2F-SCUSUM control chart modelled by membership functions is the inevitable tool for these uncertainties.

4.1. Implementation. The data are modelled as IT2F-SCUSUM fuzzy numbers using trapezoidal membership functions. The IT2F-SCUSUM control chart used the fuzzy membership functions that have the grades themselves while constructing the limits of the control chart.

The data of fertilizer production are shown in Table 3. Then, the data are fuzzified to interval type-2 fuzzy numbers as follows.

Values for upper interval type-2 fuzzy sets $(a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U)$ are defined as changes of $(a - \Delta, a, a + \Delta, a + 2\Delta)$. In this study, $\Delta = 0.1$ is chosen to show the flexibility of fuzzy numbers. An example of fuzzification for first sample is illustrated as follows:

$$\begin{aligned} a_{1a}^U &= 15.8 - 0.1 = 15.7, \\ a_{1b}^U &= 15.8, \\ a_{1c}^U &= 15.8 + 0.1 = 15.9, \\ a_{1d}^U &= 15.8 + 0.2 = 16. \end{aligned} \quad (21)$$

Next, values of lower interval type-2 fuzzy sets $(a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L)$ are the changes of $\Delta + 0.2$ to differentiate between A^U and A^L of interval type-2:

$$\begin{aligned} a_{1a}^L &= 15.7 + 0.1 = 15.8, \\ a_{1b}^L &= 15.8 + 0.1 = 15.9, \\ a_{1c}^L &= 15.9 + 0.1 = 16, \\ a_{1d}^L &= 16 + 0.1 = 16.1. \end{aligned} \quad (22)$$

All the fuzzified data results are shown in Tables 4 and 5. Table 4 shows the upper interval type-2 fuzzy number, and Table 5 shows the lower interval type-2 fuzzy number.

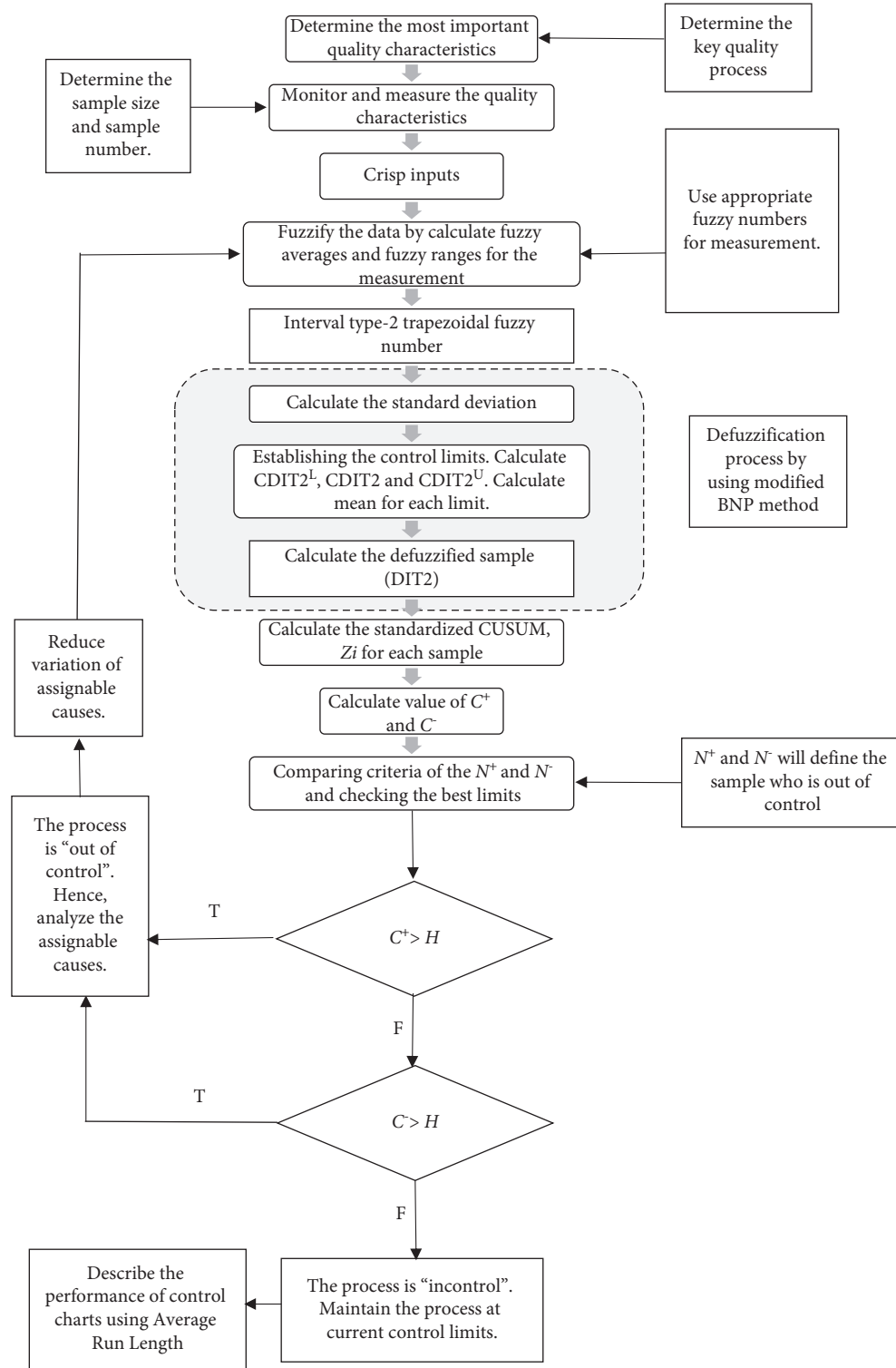


FIGURE 2: Flowchart of analysis of the IT2F-SCUSUM control chart.

TABLE 3: Data of 20 fertilizers' production in grams.

10 min	20 min	30 min	40 min	50 min	60 min
15.8	16.3	16.2	16.1	16.6	16.4
16.3	15.9	15.9	16.2	16.4	16.2
16.1	16.2	16.5	16.4	16.3	16.1
16.3	16.2	15.9	16.4	16.2	16
16.1	16.3	16.4	16.3	16	15.8
16.1	15.8	16.7	16.6	16.4	16.2
16.1	16.3	16.5	16.1	16.5	16.3
16.2	16.1	16.2	16.1	16.3	16.1
16.3	16.4	16.4	16.1	16.5	16.3
15.3	15.4	15.5	15.3	15.2	15.3
16.2	16.6	15.9	16.1	16.4	16.2
14.9	15.1	15.2	15.1	15.4	15.5
16.4	16.3	16.6	16.2	16.2	16
16.5	16.5	16.2	16.1	16.4	16.2
15.2	15.5	15.5	15.7	15.8	15.9
16	16.4	16.3	16.1	16.2	16
16.4	16	16.4	16.1	16.2	16
16	16.2	16.4	16.5	16.1	15.9
16.4	16.2	16.3	16.2	16.4	16.2
16.4	16.4	16.5	16	15.8	15.6

Then, the data from Tables 4 and 5 are calculated using equation (11), and the results are presented in Tables 6 and 7.

The calculation for the first sample of X_a from Table 6 is the mean of the first row of X_a in Table 4. Parts of the computations are shown as follows:

$$X_a = \frac{(15.7 + 16.2 + 16.1 + 16 + 16.5 + 16.3)}{6} = 16.1333. \quad (23)$$

For the second sample of X_b from Table 6, the calculation is as follows:

$$X_b = \frac{(15.8 + 16.3 + 16.2 + 16.1 + 16.6 + 16.4)}{6} = 16.2333. \quad (24)$$

For the third sample of X_c , the calculation is as follows:

$$X_c = \frac{(15.9 + 16.4 + 16.3 + 16.2 + 16.7 + 16.5)}{6} = 16.3333. \quad (25)$$

For the fourth sample of X_d , the calculation is as follows:

$$X_d = \frac{(16 + 16.5 + 16.4 + 16.3 + 16.8 + 16.6)}{6} = 16.4333. \quad (26)$$

Table 6 shows the upper interval type-2 fuzzy number for \bar{X} control chart, and Table 7 shows the lower interval type-2 fuzzy \bar{X} chart.

The calculation for S_1 standard deviation is as follows:

$$S_{1a} = \sqrt{\frac{\sum_{i=1}^n [(15.7, 16.2, 16.1, 16, 16.5, 16.3) - 16.1333]^2}{6 - 1}} = 0.2733. \quad (27)$$

Similar calculations are implemented for other values of standard deviations.

Then, standardized CUSUM and Z_i are calculated by using equation (12) for each of the sample, and the results are presented in Tables 8 and 9. In this study, we will use a tabular CUSUM with $\kappa = 0.5$ (because the shift size is 1σ and $\sigma = 1$), $\mu_0 = 16$, and $H = 5$ (as the recommended value of the decision interval is $H = 5\sigma = 5$). The calculation for the first sample of Z_a from Table 8 is as follows:

$$Z_{1a} = \frac{(16.1333 - 16)}{0.2733} = 0.4880. \quad (28)$$

Next, for the first sample of Z_b , the calculation is as follows:

$$Z_{1b} = \frac{(16.2333 - 16)}{0.2733} = 0.8539. \quad (29)$$

For the first sample of Z_c , the calculation is as follows:

$$Z_{1c} = \frac{(16.3333 - 16)}{0.2733} = 1.2199. \quad (30)$$

Afterwards, the calculation for Z_{1d} is as follows:

$$Z_{1d} = \frac{(16.4333 - 16)}{0.2733} = 1.5858. \quad (31)$$

Table 8 shows the upper IT2F-SCUSUM control chart, and Table 9 shows the lower IT2F-SCUSUM control chart.

Next, each sample of the standardized CUSUM control chart has been defuzzified based on the methods suggested by [39] for the evaluation of the process control using equations (16)–(18) as follows. The results for all samples are given in Table 10.

For first sample of $DIT2_{Upper}$, $DIT2_{Lower}$, and $DIT2$, the calculations are as follows:

TABLE 4: Upper IT2F-SCUSUM number for 20 subgroups.

	X_a						X_b						X_c						X_d					
	10	20	30	40	50	60	10	20	30	40	50	60	10	20	30	40	50	60	10	20	30	40	50	60
10	15.7	16.2	16.1	16	16.5	16.3	15.8	16.3	16.2	16.1	16.6	16.4	15.9	16.4	16.3	16.2	16.7	16.5	16	16.5	16.4	16.3	16.8	16.6
15.7	16.2	15.8	15.8	16.1	16.3	16.1	16.3	15.9	15.9	16.2	16.4	16.2	16.4	16	16	16.3	16.5	16.3	16.5	16.1	16.1	16.4	16.6	16.4
16	16.1	16.4	16.3	16.2	16	16.1	16.2	16.5	16.4	16.3	16.1	16.2	16.3	16.3	16.6	16.5	16.4	16.2	16.3	16.4	16.7	16.6	16.5	16.3
16.2	16.2	16.1	15.8	16.3	16.1	15.9	16.3	16.2	15.9	16.4	16.2	16	16.4	16.3	16	16.5	16.3	16.1	16.5	16.4	16.1	16.6	16.4	16.2
16	16	16.2	16.3	16.2	15.9	15.7	16.1	16.3	16.4	16.3	16	15.8	16.2	16.4	16.5	16.4	16.1	15.9	16.3	16.5	16.6	16.5	16.2	16
16	16	15.7	16.6	16.5	16.3	16.1	16.1	15.8	16.7	16.6	16.4	16.2	16.2	15.9	16.8	16.7	16.5	16.3	16.3	16	16.9	16.8	16.6	16.4
16	16	16.2	16.4	16	16.4	16.2	16.1	16.3	16.5	16.1	16.5	16.3	16.2	16.4	16.6	16.2	16.6	16.4	16.3	16.5	16.7	16.3	16.7	16.5
16.1	16.1	16	16.1	16	16.2	16	16.2	16.1	16.2	16.1	16.3	16.1	16.3	16.2	16.3	16.2	16.4	16.2	16.4	16.3	16.4	16.3	16.5	16.3
16.2	16.2	16.3	16.3	16	16.4	16.2	16.3	16.4	16.4	16.1	16.5	16.3	16.4	16.5	16.5	16.2	16.6	16.4	16.5	16.6	16.6	16.3	16.7	16.5
15.2	15.3	15.4	15.2	15.1	15.2	15.3	15.3	15.4	15.5	15.3	15.2	15.3	15.4	15.5	15.6	15.4	15.3	15.4	15.5	15.6	15.7	15.5	15.4	15.5
16.1	16.5	15.8	16	16	16.3	16.1	16.2	16.6	15.9	16.1	16.4	16.2	16.3	16.7	16	16.2	16.5	16.3	16.4	16.8	16.1	16.3	16.6	16.4
14.8	15	15.1	15	15	15.3	15.4	14.9	15.1	15.2	15.1	15.4	15.5	15	15.2	15.3	15.2	15.5	15.6	15.1	15.3	15.4	15.3	15.6	15.7
16.3	16.2	16.5	16.1	16.1	16.1	15.9	16.4	16.3	16.6	16.2	16.2	16	16.5	16.4	16.7	16.3	16.3	16.1	16.6	16.5	16.8	16.4	16.4	16.2
16.4	16.4	16.1	16	16	16.3	16.1	16.5	16.5	16.2	16.1	16.4	16.2	16.6	16.6	16.6	16.3	16.2	16.5	16.7	16.7	16.4	16.3	16.6	16.4
15.1	15.4	15.4	15.6	15.6	15.7	15.8	15.2	15.5	15.5	15.7	15.8	15.9	15.3	15.6	15.6	15.8	15.9	16	15.4	15.7	15.7	15.9	16	16.1
15.9	16.3	16.2	16	16	16.1	15.9	16	16.4	16.3	16.1	16.2	16	16.1	16.5	16.4	16.2	16.3	16.1	16.2	16.6	16.5	16.3	16.4	16.2
16.3	15.9	16.3	16	16	16.1	15.9	16.4	16	16.4	16.1	16.2	16	16.5	16.1	16.5	16.2	16.3	16.1	16.6	16.2	16.6	16.3	16.4	16.2
15.9	16.1	16.3	16.4	16	16	15.8	16	16.2	16.4	16.5	16.1	15.9	16.1	16.3	16.5	16.6	16.2	16	16.2	16.4	16.6	16.7	16.3	16.1
16.3	16.1	16.2	16.1	16.2	16.3	16.1	16.4	16.2	16.3	16.2	16.4	16.2	16.5	16.3	16.4	16.3	16.5	16.3	16.6	16.4	16.5	16.4	16.6	16.4
16.3	16.3	16.4	15.9	15.7	15.5	16.4	16.4	16.4	16.5	16	15.8	15.6	16.5	16.5	16.6	16.1	15.9	15.7	16.6	16.6	16.7	16.2	16	15.8

TABLE 5: Lower IT2F-SCUSUM number for 20 subgroups.

X_a						X_b						X_c						X_d					
10	20	30	40	50	60	10	20	30	40	50	60	10	20	30	40	50	60	10	20	30	40	50	60
15.8	16.3	16.2	16.1	16.6	16.4	15.9	16.4	16.3	16.2	16.7	16.5	16	16.5	16.4	16.3	16.8	16.6	16.1	16.6	16.5	16.4	16.9	16.7
16.3	15.9	15.9	16.2	16.4	16.2	16.4	16	16	16.3	16.5	16.3	16.5	16.1	16.1	16.4	16.6	16.4	16.6	16.2	16.2	16.5	16.7	16.5
16.1	16.2	16.5	16.4	16.3	16.1	16.2	16.3	16.6	16.5	16.4	16.2	16.3	16.4	16.7	16.6	16.5	16.3	16.4	16.5	16.8	16.7	16.6	16.4
16.3	16.2	15.9	16.4	16.2	16	16.4	16.3	16	16.5	16.3	16.1	16.5	16.4	16.1	16.6	16.4	16.2	16.6	16.5	16.2	16.7	16.5	16.3
16.1	16.3	16.4	16.3	16	15.8	16.2	16.4	16.5	16.4	16.1	15.9	16.3	16.5	16.6	16.5	16.2	16	16.4	16.6	16.7	16.6	16.3	16.1
16.1	15.8	16.7	16.6	16.4	16.2	16.2	15.9	16.8	16.7	16.5	16.3	16.3	16	16.9	16.8	16.6	16.4	16.4	16.1	17	16.9	16.7	16.5
16.1	16.3	16.5	16.1	16.5	16.3	16.2	16.4	16.6	16.2	16.6	16.4	16.3	16.5	16.7	16.3	16.7	16.5	16.4	16.6	16.8	16.4	16.8	16.6
16.2	16.1	16.2	16.1	16.3	16.1	16.3	16.2	16.3	16.2	16.4	16.2	16.4	16.3	16.4	16.3	16.5	16.3	16.5	16.4	16.5	16.4	16.6	16.4
16.3	16.4	16.4	16.1	16.5	16.3	16.4	16.5	16.5	16.2	16.6	16.4	16.5	16.6	16.6	16.3	16.7	16.5	16.6	16.7	16.7	16.4	16.8	16.6
15.3	15.4	15.5	15.3	15.2	15.3	15.4	15.5	15.6	15.4	15.3	15.4	15.5	15.6	15.7	15.5	15.4	15.5	15.6	15.7	15.8	15.6	15.5	15.6
16.2	16.6	15.9	16.1	16.4	16.2	16.3	16.7	16	16.2	16.5	16.3	16.4	16.8	16.1	16.3	16.6	16.4	16.5	16.9	16.2	16.4	16.7	16.5
14.9	15.1	15.2	15.1	15.4	15.5	15	15.2	15.3	15.2	15.5	15.6	15.1	15.3	15.4	15.3	15.6	15.7	15.2	15.4	15.5	15.4	15.7	15.8
16.4	16.3	16.6	16.2	16.2	16	16.5	16.4	16.7	16.3	16.3	16.1	16.6	16.5	16.8	16.4	16.4	16.2	16.7	16.6	16.9	16.5	16.5	16.3
16.5	16.5	16.2	16.1	16.4	16.2	16.6	16.6	16.3	16.2	16.5	16.3	16.7	16.7	16.4	16.3	16.6	16.4	16.8	16.8	16.5	16.4	16.7	16.5
15.2	15.5	15.5	15.7	15.8	15.9	15.3	15.6	15.6	15.8	15.9	16	15.4	15.7	15.7	15.9	16	16.1	15.5	15.8	15.8	16	16.1	16.2
16	16.4	16.3	16.1	16.2	16	16.1	16.5	16.4	16.2	16.3	16.1	16.2	16.6	16.5	16.3	16.4	16.2	16.3	16.7	16.6	16.4	16.5	16.3
16.4	16	16.4	16.1	16.2	16	16.5	16.1	16.5	16.2	16.3	16.1	16.6	16.2	16.6	16.3	16.4	16.2	16.7	16.3	16.7	16.4	16.5	16.3
16	16.2	16.4	16.5	16.1	15.9	16.1	16.3	16.5	16.6	16.2	16	16.2	16.4	16.6	16.7	16.3	16.1	16.3	16.5	16.7	16.8	16.4	16.2
16.4	16.2	16.3	16.2	16.4	16.2	16.5	16.3	16.4	16.3	16.5	16.3	16.6	16.4	16.5	16.4	16.6	16.4	16.7	16.5	16.6	16.5	16.7	16.5
16.4	16.4	16.5	16	15.8	15.6	16.5	16.5	16.6	16.1	15.9	15.7	16.6	16.6	16.7	16.2	16	15.8	16.7	16.7	16.8	16.3	16.1	15.9

TABLE 6: Upper interval type-2 fuzzy \bar{X} of 20 subgroups.

a	b	c	d	H_1	H_2
16.1333	16.2333	16.3333	16.4333	1	1
16.0500	16.1500	16.2500	16.3500	1	1
16.1667	16.2667	16.3667	16.4667	1	1
16.0667	16.1667	16.2667	16.3667	1	1
16.0500	16.1500	16.2500	16.3500	1	1
16.2000	16.3000	16.4000	16.5000	1	1
16.2000	16.3000	16.4000	16.5000	1	1
16.0667	16.1667	16.2667	16.3667	1	1
16.2333	16.3333	16.4333	16.5333	1	1
15.2333	15.3333	15.4333	15.5333	1	1
16.1333	16.2333	16.3333	16.4333	1	1
15.1000	15.2000	15.3000	15.4000	1	1
16.1833	16.2833	16.3833	16.4833	1	1
16.2167	16.3167	16.4167	16.5167	1	1
15.5000	15.6000	15.7000	15.8000	1	1
16.0667	16.1667	16.2667	16.3667	1	1
16.0833	16.1833	16.2833	16.3833	1	1
16.0833	16.1833	16.2833	16.3833	1	1
16.1833	16.2833	16.3833	16.4833	1	1
16.0167	16.1167	16.2167	16.3167	1	1

TABLE 7: Lower interval type-2 fuzzy \bar{X} of 20 subgroups

a	b	c	d	H_1	H_2
16.2333	16.3333	16.4333	16.5333	0.6	0.5
16.1500	16.2500	16.3500	16.4500	0.7	0.6
16.2667	16.3667	16.4667	16.5667	0.7	0.6
16.1667	16.2667	16.3667	16.4667	0.6	0.5
16.1500	16.2500	16.3500	16.4500	0.6	0.6
16.3000	16.4000	16.5000	16.6000	0.6	0.5
16.3000	16.4000	16.5000	16.6000	0.5	0.7
16.1667	16.2667	16.3667	16.4667	0.7	0.5
16.3333	16.4333	16.5333	16.6333	0.8	0.6
15.3333	15.4333	15.5333	15.6333	0.7	0.4
16.2333	16.3333	16.4333	16.5333	0.6	0.6

TABLE 7: Continued.

a	b	c	d	H_1	H_2
15.2000	15.3000	15.4000	15.5000	0.7	0.5
16.2833	16.3833	16.4833	16.5833	0.6	0.6
16.3167	16.4167	16.5167	16.6167	0.8	0.6
15.6000	15.7000	15.8000	15.9000	0.6	0.8
16.1667	16.2667	16.3667	16.4667	0.7	0.6
16.1833	16.2833	16.3833	16.4833	0.6	0.4
16.1833	16.2833	16.3833	16.4833	0.6	0.6
16.2833	16.3833	16.4833	16.5833	0.8	0.4
16.1167	16.2167	16.3167	16.4167	0.7	0.6

TABLE 8: Upper IT2F-SCUSUM of 20 subgroups.

Z_a	Z_b	Z_c	Z_d	H_1	H_2
0.4880	0.8539	1.2199	1.5858	1	1
0.2411	0.7234	1.2056	1.6878	1	1
1.0206	1.6330	2.2454	2.8577	1	1
0.3581	0.8951	1.4322	1.9693	1	1
0.2214	0.6642	1.1070	1.5498	1	1
0.5976	0.8964	1.1952	1.4940	1	1
1.1180	1.6771	2.2361	2.7951	1	1
0.8165	2.0412	3.2660	4.4907	1	1
1.7078	2.4398	3.1717	3.9036	1	1
-7.4232	-6.4550	-5.4867	-4.5185	1	1
0.5505	0.9633	1.3762	1.7891	1	1
-4.1079	-3.6515	-3.1950	-2.7386	1	1
0.8981	1.3880	1.8779	2.3678	1	1
1.2579	1.8385	2.4191	2.9997	1	1
-1.9764	-1.5811	-1.1859	-0.7906	1	1
0.4082	1.0206	1.6330	2.2454	1	1
0.4542	0.9992	1.5442	2.0892	1	1
0.3597	0.7914	1.2231	1.6547	1	1
1.8647	2.8818	3.8989	4.9160	1	1
0.0449	0.3144	0.5840	0.8535	1	1

TABLE 9: Lower IT2F-SCUSUM of 20 subgroups.

Z_a	Z_b	Z_c	Z_d	H_1	H_2
1.2199	1.5858	1.9518	2.3178	0.6	0.7
1.2056	1.6878	2.1701	2.6523	0.7	0.6
2.2454	2.8577	3.4701	4.0825	0.7	0.6
1.4322	1.9693	2.5064	3.0435	0.6	0.7
1.1070	1.5498	1.9926	2.4354	0.6	0.6
1.1952	1.4940	1.7928	2.0917	0.6	0.7
2.2361	2.7951	3.3541	3.9131	0.8	0.7
3.2660	4.4907	5.7155	6.9402	0.7	0.8
3.1717	3.9036	4.6355	5.3675	0.8	0.6
-5.4867	-4.5185	-3.5502	-2.5820	0.7	0.9
1.3762	1.7891	2.2019	2.6148	0.6	0.6
-3.1950	-2.7386	-2.2822	-1.8257	0.7	0.8
1.8779	2.3678	2.8577	3.3476	0.6	0.6
2.4191	2.9997	3.5803	4.1609	0.8	0.6
-1.1859	-0.7906	-0.3953	0.0000	0.6	0.8
1.6330	2.2454	2.8577	3.4701	0.7	0.6
1.5442	2.0892	2.6342	3.1792	0.6	0.9
1.2231	1.6547	2.0864	2.5181	0.6	0.6
3.8989	4.9160	5.9331	6.9502	0.8	0.9
0.5840	0.8535	1.1230	1.3925	0.7	0.6

TABLE 10: IT2F-SCUSUM control charts for 20 subgroups.

No.	DIT2 _{Upper}	DIT2 _{Lower}	DIT2	C ⁺	N ⁺	C ⁻	N ⁻
1	1.0369	1.4547	1.2458	0.746	1	0.000	0
2	0.9645	1.5974	1.2810	1.527	2	0.000	0
3	1.9392	2.6179	2.2785	3.305	3	0.000	0
4	1.1637	1.8395	1.5016	4.307	4	0.000	0
5	0.8856	1.4170	1.1513	4.958	5	0.000	0
6	1.0458	1.3521	1.1990	5.657	6	0.000	0
7	1.9566	2.6973	2.3269	7.484	7	0.000	0
8	2.6536	4.4499	3.5518	10.536	8	0.000	0
9	2.8057	3.6474	3.2266	13.262	9	0.000	0
10	-5.9708	-3.6551	-4.8130	7.949	10	4.313	1
11	1.1698	1.5964	1.3831	8.832	11	2.430	0
12	-3.4233	-2.2023	-2.8128	5.520	12	4.743	1
13	1.6330	2.0902	1.8616	6.881	13	2.381	0
14	2.1288	2.8110	2.4699	8.851	14	0.000	0
15	-1.3835	-0.5139	-0.9487	7.403	15	0.449	1
16	1.3268	2.1127	1.7197	8.622	16	0.000	2
17	1.2717	2.0460	1.6589	9.781	17	0.000	0
18	1.0072	1.4964	1.2518	10.533	18	0.000	0
19	3.3903	5.0050	4.1976	14.231	19	0.000	0
20	0.4492	0.8187	0.6339	14.365	20	0.000	0

TABLE 11: Result for the conventional SCUSUM control chart and T1F-SCUSUM control chart.

No.	Conventional SCUSUM chart					T1F-SCUSUM chart				
	Z _i	C ⁺	N ⁺	C ⁻	N ⁻	Z _i	C ⁺	N ⁺	C ⁻	N ⁻
1	0.8539	0.3539	1	0.0000	0	1.0369	0.5369	1	0.0000	0
2	0.7234	0.5773	2	0.0000	0	0.9645	1.0014	2	0.0000	0
3	1.6330	1.7103	3	0.0000	0	1.9392	2.4406	3	0.0000	0
4	0.8951	2.1054	4	0.0000	0	1.1637	3.1042	4	0.0000	0
5	0.6642	2.2696	5	0.0000	0	0.8856	3.4899	5	0.0000	0
6	0.8964	2.6660	6	0.0000	0	1.0458	4.0357	6	0.0000	0
7	1.6771	3.8431	7	0.0000	0	1.9566	5.4922	7	0.0000	0
8	2.0412	5.3843	8	0.0000	0	2.6536	7.6459	8	0.0000	0
9	2.4398	7.3241	9	0.0000	0	2.8057	9.9516	9	0.0000	0
10	-6.4550	0.3691	10	6.9550	1	-5.9708	3.4807	10	6.4708	1
11	0.9633	0.8325	11	6.4916	2	1.1698	4.1505	11	5.8011	2
12	-3.6515	0.0000	0	10.6431	3	-3.4233	0.2272	12	9.7243	3
13	1.3880	0.8880	1	9.7551	4	1.6330	1.3602	13	8.5913	4
14	1.8385	2.2266	2	8.4166	5	2.1288	2.9890	14	6.9625	5
15	-1.5811	0.1454	3	10.4977	6	-1.3835	1.1055	15	8.8460	6
16	1.0206	0.6660	4	9.9771	7	1.3268	1.9323	16	8.0192	7
17	0.9992	1.1652	5	9.4779	8	1.2717	2.7040	17	7.2476	8
18	0.7914	1.4566	6	9.1865	9	1.0072	3.2112	18	6.7403	9
19	2.8818	3.8384	7	6.8047	10	3.3903	6.1016	19	3.8500	10
20	0.3144	3.6528	8	6.9903	11	0.4492	6.0508	20	3.9008	11

$$\text{DIT2}_{\text{Trap}(1)}^U = \frac{(1.5858 - 0.4880) + (1 \times 0.8539 - 0.4880) + (1 \times 1.2199 - 0.4880)}{4} + 0.4880 = 1.0369,$$

$$\text{DIT2}_{\text{Trap}(1)}^L = \frac{(2.3178 - 1.2199) + (0.7 \times 1.5858 - 1.2199) + (0.6 \times 1.9518 - 1.2199)}{4} + 1.2199 = 1.4547,$$

$$\text{DIT2}_{\text{Trap}(1)} = \frac{1.0369 + 1.4547}{2} = 1.2458.$$

(32)

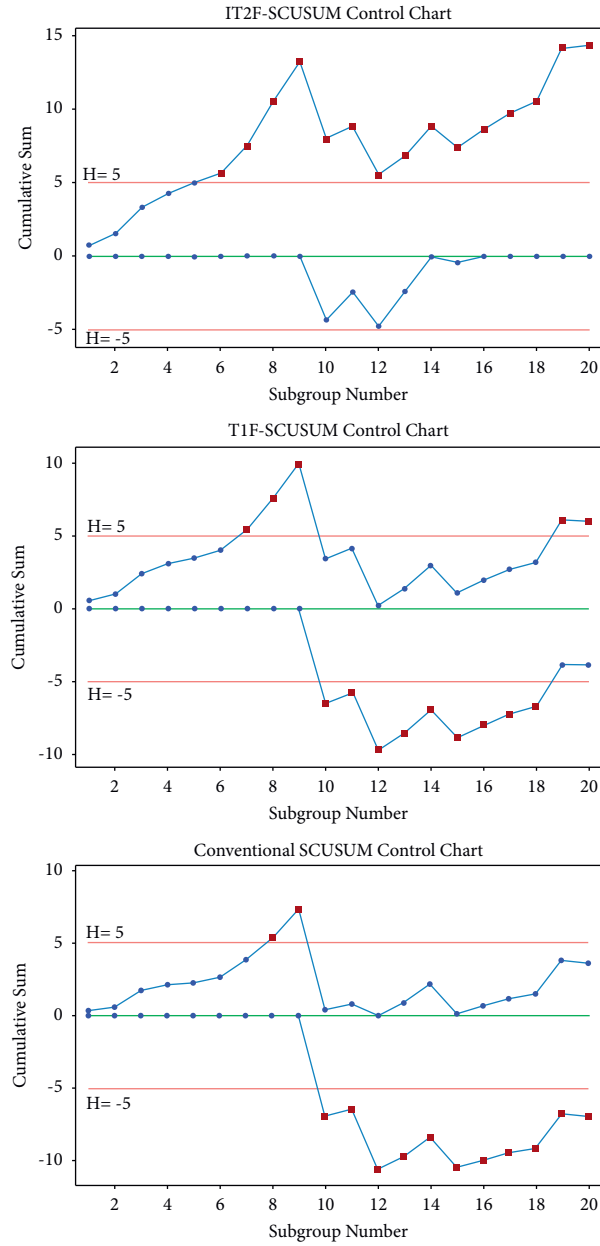


FIGURE 3: Control charts of conventional SCUSUM, T1F-SCUSUM, and IT2F-SCUSUM charts.

TABLE 12: Average run length (ARL) for conventional SCUSUM, T1F-SCUSUM, and IT2F-SCUSUM charts.

Shift in mean (multiple of sigma)	IT2F-SCUSUM 2	T1F-SCUSUM	Conventional SCUSUM
0	301.69	302.69	310.44
0.25	107.40	107.62	109.30
0.5	32.91	32.94	33.23
0.75	15.43	15.44	15.53
1	9.52	9.53	9.58
1.25	6.82	6.83	6.86
1.5	5.32	5.32	5.35
1.75	4.37	4.37	4.39
2	3.72	3.73	3.74
2.25	3.25	3.26	3.27
2.5	2.90	2.90	2.91
2.75	2.63	2.63	2.64
3	2.41	2.41	2.42
3.5	2.10	2.10	2.11
4	1.90	1.90	1.91
4.5	1.72	1.73	1.73
5	1.53	1.53	1.54

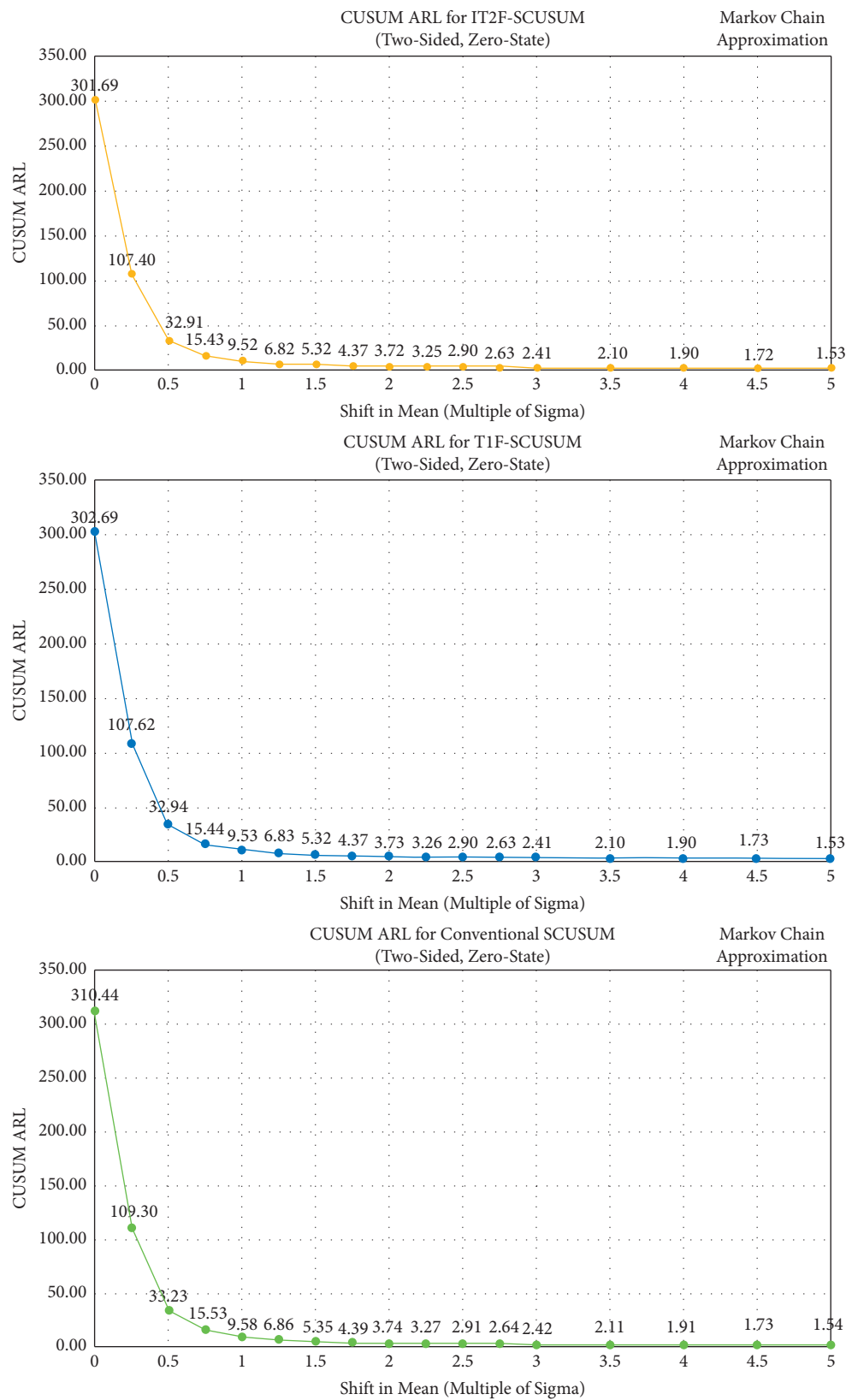


FIGURE 4: Graph of ARL on SCUSUM, T1F-SCUSUM, and IT2F-SCUSUM charts.

Then, we calculate C_i^+ and C_i^- by using equations (14) and (15). The initial value is $C^+(0) = 0 = C^-(0)$. For the first sample, $C^+ = C_1^+ = \max[0, 1.2458 - 0.5 + 0] = 0.746$ and $C^- = C_1^- = \max[0, -0.5 - 1.2458 + 0] = 0$.

For the second sample, $C^+ = C_2^+ = \max[0, 1.2810 - 0.5 + 0.746] = 1.527$ and $C^- = C_1^- = \max[0, -0.5 - 1.2810 + 0] = 0$.

Based on the result of N^+ , we start to count from 1 since the result of C_1^+ is 0.746. Then, C_6^+ shows the result is 5.657 which is greater than the limits of $H = 5$. Therefore, we can conclude that the process is out of control from sample 6 until sample 20 since the value of N^+ is greater than $H = 5$. In this study, $H = 5$ is taken as decision parameter for optimal level as recommended by [1, 35].

5. Comparative Analysis

To see the stability of the three types of control charts, this section provides a comparative result of the analysis. Table 11 shows the result for the conventional SCUSUM control chart and T1F-SCUSUM control chart.

Figure 3 presents the control charts using conventional SCUSUM, T1F-SCUSUM, and the proposed IT2F-SCUSUM.

Based on the conventional SCUSUM chart in Table 11 and Figure 3, we can see that, samples 8–20 are “out of control” since they exceed the control limits which means 13 points are uncontrolled. However, for the T1F-SCUSUM control chart, 14 samples are “out of control,” and 15 samples are “out of control” for the IT2F-SCUSUM control chart. Hence, this shows that the IT2F-SCUSUM control chart is more sensitive than the conventional SCUSUM chart and T1F-SCUSUM control chart since it captures the least number of samples compared to other charts.

Next, ARL with different values of shift is used to evaluate the control chart's performance. ARL is the average number of samples taken before any signal of “out of control” condition is detected in the control chart. It was determined using Sigma XL, and the outcomes are provided in Table 12.

From Table 12, we can see that the “in-control” ARL of the IT2F-SCUSUM chart is 301.69, which is lower than the “in-control” ARL of the conventional SCUSUM chart ($ARL_0 = 310.44$) and T1F-SCUSUM chart ($ARL_0 = 302.69$). This concludes that if the process is in control, we expect to get a signal every 301 samples on average which is faster than the other two types of charts. In fact, the “out-of-control” ARL for the IT2F-SCUSUM chart is 9.52, also lower than the out-of-control ARL of the conventional SCUSUM chart and T1F-SCUSUM chart. This indicates that the IT2F-SCUSUM chart control chart is quicker in indicating small shifts in the process compared to other charts. It also proves that when the magnitude of the shifts increases, the power of control charts is then augmented. In fact, it shows that the control chart's performance is better when fuzzy numbers are being implemented.

Figure 4 is presented to visualise the three ARLs that are obtained using Markov chain approximation where the shifts in the control process can be observed.

From Table 12 and Figure 4, we can see that the IT2F-SCUSUM chart has lower value of ARL compared to the conventional SCUSUM chart and T1F-SCUSUM chart. This indicates that the IT2F-SCUSUM chart control chart is quicker in indicating small shifts in the process compared to other charts. It shows that the control chart's performance is better when fuzzy numbers are being implemented.

6. Conclusions

Fuzzy control charts have been widely used in sociological, medical, engineering, economics, service, and management research. The fuzzy set theory has the capability of systematic dealing with fuzzy data. Most previous studies designed fuzzy control chart for linguistic data and fuzzy numbers since fuzzy logic helps in explaining vague and imprecise data. Traditionally, the conventional control chart is used to identify the process shift with real-value data. The efficiency of the IT2F-SCUSUM charts is more than analysing of crisp data, but it also gives essential alerts by means of flexibility of interval type-2 fuzzy numbers. The CUSUM control chart through a numerical comparison via a simulation study shows better performance in detecting small- and medium-sized shifts in the process. In fact, it is the best chart for detecting small process shifts which are less than 1.5σ . Besides, CUSUM charts are used for controlling cumulative sum of quality characteristics measurement in monitoring analysis.

The contributions of this paper are threefold: (1) we proposed to use IT2F-SCUSUM sets since this method can model higher levels of uncertainty compared to T1F-SCUSUM sets. Pertaining to the SCUSUM charts, there are some studies on ordinary fuzzy control charts, yet there is no study on IT2F-SCUSUM charts so far. Indeed, modelling the fuzzy control charts using the IT2F-SCUSUM sets may contribute to more accuracy in monitoring the process as the membership functions of the data are already imprecise. (2) Then, we compare the new method with the conventional SCUSUM chart and T1F-SCUSUM control chart. This comparative analysis is not only to gain insights about the performance of the three charts but also to help researchers make decisions either in reducing or eliminating the need for inspection in the products. (3) Last but not the least, we computed the ARL for each chart to evaluate the exact probabilities of false alarm in designing the best charts among them. The simulation study shows better performance in detecting small- and medium-sized shifts in the process which are less than 1.5σ .

Based on the case study results, we can conclude that the IT2F-SCUSUM control chart is more sensitive to examine the variations of the fertilizer production characteristics compared to the T1F-SCUSUM control chart and conventional SCUSUM since the IT2F-SCUSUM control chart found 15 defects, but the T1F-SCUSUM control chart found 14 defects and conventional SCUSUM found 13 defects in fertilizers. Hence, this means the defect should be removed, and if the company includes the defects, the fertilizers will be low in quality and it might affect the plantations of the consumers. The company should consider the quality of the product to increase the level of customer satisfaction.

In a nutshell, we can conclude that the IT2F-SCUSUM control chart is more sensitive to monitor the variations of the fertilizer production compared to the T1F-SCUSUM control chart and conventional SCUSUM chart. Further research can investigate the IT2F-SCUSUM control chart for monitoring defects using a variable sample size and fuzzy theory control charts using hesitant fuzzy theory, intuitionistic fuzzy theory, or neutrosophic fuzzy theory. Additionally, future research can use high number of samples to see more variabilities in the analysis. Other than that, the stated proposal may be extended in the future by using the CUSUM structure proposed by Faisal et al. [50], where a link relative variable transformation could be introduced to IT2F-CUSUM.

Data Availability

The data used to support the findings of this study are included within the article.

Disclosure

Publication of this research is part of the requirement for graduation from University Malaysia Terengganu.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors' Contributions

All authors contributed equally to the writing of this manuscript and read and approved the final manuscript.

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