Research Article

Modeling of Minimum and Maximum Void Ratios of Granular Soils

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Determining the maximum and minimum void ratios of granular soils is very important because it can be correlated with engineering behaviors such as soil permeability. Empirical relationships can be used for determining the void ratio, but they have many limitations related to the shape and the size distribution of the grains. Analytical methods improve the empirical relationships. In this paper, we present enhancements to the model of Chang et al. by combining the model of Youd et al., as Chang–Youd models, to make it more convenient and also to extend its usage to be suitable for determining the maximum void ratio. The Chang–Youd models are verified with experimental tests performed by the authors. Compared with the experimental results in the literature, the Chang–Youd models are also effective but more convenient and practical.

1. Introduction

The engineering properties of granular soils such as the density, strength, and permeability coefficient are directly affected by the grain size distribution and the shape of the grains, as well as the void ratio. The difference between the maximum and minimum void ratios is the indicator of the relative density of the soil which can be considered as the most important factor that reflects the strength state of the soil [1].

Mono-sized, binary-sized, multisized particles were investigated experimentally by An et al. [2], Fuggle et al. [3], McGeary [4], and Yu and Standish [5]. Also, there are some idealized models of single and two-sized spherical particles to determine the minimum and maximum void ratios [6], but those models do not represent the actual uniformity coefficient or angularity. Hence, some studies proposed empirical relationships to determine the void ratio such as [7] and [8]. Other studies investigated the minimum and maximum void ratio and correlated them with the soil properties such as the particles' diameter corresponding to

50 finer percent, D_{50} , and the soil angularity [9, 10]. There are also analytical models to determine the void ratio of granular soils, such as the ideal model suggested by Kezdi [11] which was found to overestimate the void ratio. A more accurate model was proposed by Aberg [12] and Chang et al. [13]. Recently, Chang et al. [14] presented an analytical model to determine the minimum void ratio, e_i^M , of granular soils composed of n -sized particles ($d_1, d_2, \dots, d_n$) and their volume fractions ($y_1, y_2, \dots, y_n$). They assumed that the minimum void ratio for packings containing particles of equal size of d_1 is e_1 , and the void ratio of packings containing particle size d_2 only is e_2 . In the same way, the void ratio for packings containing mono-sized particles ($d_3, d_4, d_5, \dots, d_n$) are e_i (i.e., $e_3, e_4, e_5, \dots, e_n$), respectively. The latter set of values will be used in estimation of the minimum void ratio of the whole mixture by taking a packing containing particles of single size d_i (dominant particle size) and considering this diameter as the base packing. The multisize particle packing is built up by adding the larger and the smaller particles to this base packing. The particles smaller than d_i will cause an increment in the solid volume and some

decrement in the volume of the voids combined with some value of increment in the voids' volume due to disturbance. Adding particles with sizes larger than d_i will cause increments in both the solid volume and the voids volume. Combining all of these factors together in a mathematical form, we get

$$e_i^M = \sum_{j=1}^n e_j y_j - \sum_{j=1}^{i-1} b_{ij} e_j y_j - \sum_{j=i+1}^n a_{ij} j (1 + e_j) y_j, \quad (1)$$

where y_j is the finer percent corresponding to particle diameter d_j , which can be calculated from the gradation curve, a_{ij} is the filling coefficient, b_{ij} is the embedment coefficient, $a_{ij} = f(d, p)$, $b_{ij} = f(d, s)$, and p and s are parameters determined from experimental tests. Chang et al. [14], using a power function to fit the experimental results, proposed the empirical relationship, equation (2), to determine the value of e_i :

$$e_i = \alpha d_i^\beta, \quad (2)$$

where α and β are two coefficients based on the fit curve of the experimental results. In Chang et al.'s [14] model, there are four parameters (p , s , α , and β) that should be determined, and hence, they required a large number of experimental tests, which makes that the model is not "readily useful in engineering practice for predicting minimum void ratio," as stated by Chang et al. [14].

In this paper, by combining the model of Chang et al. [14] and Youd [8], we proposed the Chang–Youd models to determine the minimum void ratios, which make the experimental tests required to determine the model parameters more convenient and practical. Also, we extended the applicability of equation (1) to be suitable for determining the maximum void ratio.

2. Enhanced Models to Determine the Minimum and Maximum Void Ratios

2.1. Enhanced Method to Calculate the Minimum Void Ratio. Four empirical parameters are required for the Chang et al. [14] method, namely, s , p , α , and β . Obtaining these parameters required tedious and time-consuming experimental tests, and for this reason, Chang et al. [14] stated that "Because of the number of experiments required for calibrating the model parameters, this model is not expected to be readily useful in engineering practice for predicting minimum void ratio."

The maximum and minimum void ratios depend on particle's shape, as R and C_u stated by Xiao et al. [15] and Youd [8]. Park and Santamarina [16], based on the study of Youd [8], proposed that the minimum void ratio e_i can be calculated using empirical formulas, equation (3), without loss of the model accuracy:

$$e_i = -0.012 + \frac{0.082}{R} + \frac{0.371}{C_u}, \quad (3)$$

where C_u is the coefficient of uniformity of the soil, as d_{60}/d_{10} , and R is the roundness of the particles ($R=0.14, 0.21,$

0.30, 0.41, 0.59, and 0.84 for very angular, angular, sub-angular, subrounded, rounded, and well-rounded particles, respectively). Using equation (3) here instead of equation (2) has many advantages. Firstly, the minimum void ratio can be obtained by the uniformity coefficient of the soil and roundness of the particles. Secondly, it can be assigned different values of particle roundness for each particle diameter d_i ; because for some granular mixtures, the roundness of the fine-grained particles is different from those of the larger ones, such as the soil tested by Xiao et al. [17] where the particles larger than 10 mm are rounded to subrounded ($R=0.41\sim0.59$) and the smaller particles are angular to subangular ($R=0.21\sim0.30$). Such a case of different roundness values can be easily taken into consideration in equation (3), while equation (2) gives average values of α and β for all the soil particles.

2.2. New Method to Calculate the Maximum Void Ratio. Following the same way of deriving equation (1), we found that it is also possible to determine the maximum void ratio from the model of Chang et al. [14] if we calculate the "maximum" void ratio for a packing containing monosized particles e_i (i.e., $e_1, e_2, \dots, e_n$), and this can be done by using equation (4) stated by Youd [8] to determine the maximum void ratio into equation (1):

$$e_i = 0.032 + \frac{0.154}{R} + \frac{0.522}{C_u}. \quad (4)$$

2.3. Flow Diagram of Chang–Youd Models. Combining Chang et al.'s [14] and Youd's [8] statements, we proposed Chang–Youd models, given in Figure 1. The soil gradation curve can be divided into n divisions. Particle size d_i ($i=1$ to n) is the average particle size for each division, which corresponds to a size ratio $d_i/d_{i+1} < 1.8$, stated by Chang et al. [14].

3. Verification of the Chang–Youd Models

3.1. Verification of the Enhanced Minimum Void Ratio Model. The data CB-Mix-No. 1 and CB-Mix-No. 2 of Youd [8] and G1-1 and G2-8 of experimental tests are used to describe the calculation process of the Chang–Youd model and Chang model. The gradation curve of CB-Mix-No. 1 (Figure 2) is divided into 2 divisions ($n=2$) and CB-Mix-No. 2 (Figure 2), G1-1 (Figure 3), and G2-8 (Figure 4) are divided into 4 divisions ($n=4$), respectively. The two sand mixtures' properties, minimum void ratios, and maximum void ratios were measured by the experiment. Roundness can be inscribed within the grain image [8] and are listed in Table 1. The data G1-1 and G2-8 of experimental tests performed by the authors here using 16 sand-gravel mixtures composed of subrounded grain ranged from 0.1 to 19 mm. Their grain size distribution curves are shown in Figures 3 and 4. Based on the study of Xiao et al. [17], where the particles are larger than 10 mm, $R=0.41\sim0.59$, and the particles are smaller than 10 mm, $R=0.21\sim0.30$, the minimum void ratio and the maximum void ratio of G1-1 and G2-8 were obtained

FIGURE 1: Flow diagram of Chang-Youd Models.

FIGURE 2: Grain size distribution curve of soils 1 and 2 tested by Youd [8].

FIGURE 3: Grain size distribution of soils G1-1 to G1-8.

according to ASTM D4254-16 [18], and the results are shown in Table 1.

The gradation curve of CB-Mix-No. 1 can be reduced to a simple model for binary packing, as CB-Mix-No. 2, G1-1, and G2-8 can be reduced to quaternary packing. Take the binary mixture, for example, let y_1 and y_2 be the solid weight fractions for large particle d_1 and small particle d_2 and C_{ui} be the coefficient of uniformity for each division, as shown in Table 1.

CB-Mix-No. 1 as binary packing, equation (1) leads to the following expressions:

(i) Large particle d_1 as dominant particle size:

$$e_1^M = e_1 y_1 + e_2 y_2 - a_{12} (1 + e_2) y_2. \quad (5)$$

(ii) Small particle d_2 as dominant particle size:

$$e_2^M = e_1 y_1 + e_2 y_2 - b_{21} e_1 y_1. \quad (6)$$

CB-Mix-No. 2, G1-1, and G2-8 as quaternary packing, equation (1) leads to the following expressions:

(iii) d_1 as dominant particle size:

$$e_1^M = e_1 y_1 + e_2 y_2 + e_3 y_3 + e_4 y_4 - a_{12} (1 + e_2) y_2 - a_{13} (1 + e_3) y_3 - a_{14} (1 + e_4) y_4. \quad (7)$$

(iv) d_2 as dominant particle size:

FIGURE 4: Grain size distribution of soils G2-1 to G2-8.

TABLE 1: Properties of sand mixes.

Soil ID	Divisions	Size range mm	Weight fractions γ_i	Roundness R_i	Coefficient of uniformity $C_{ui} = d_{60}/d_{10}$	Dominant $d_i = d_{50}$ mm	$e_{\min}$	$e_{\max}$
CB-Mix-No. 1	1	0.70–1.00	0.500	0.19	1.168	0.838	0.705	1.257
	2	0.50–0.70	0.500	0.19	1.192	0.596		
CB-Mix-No. 2	1	1.00–2.00	0.250	0.19	1.432	1.411	0.590	1.099
	2	0.70–1.00	0.250	0.19	1.180	0.833		
	3	0.50–0.70	0.250	0.19	1.186	0.588		
	4	0.25–0.50	0.250	0.19	1.380	0.348		
G1-1	1	15.72–18.92	0.149	0.59	1.100	17.317	0.242	0.541
	2	7.49–15.72	0.349	0.41	1.494	11.609		
	3	2.51–7.49	0.303	0.30	1.829	5.004		
	4	0.10–2.51	0.199	0.30	4.533	1.305		
G2-8	1	16.21–19.21	0.226	0.59	1.091	17.710	0.546	0.774
	2	12.86–16.21	0.218	0.59	1.127	14.538		
	3	10.05–12.86	0.238	0.41	1.136	11.458		
	4	7.49–10.05	0.318	0.30	1.165	8.775		

$$e_2^M = e_1 y_1 + e_2 y_2 + e_3 y_3 + e_4 y_4 - b_{21} e_1 y_1 - a_{23} (1 + e_3) y_3 - a_{24} (1 + e_4) y_4. \quad (8)$$

(v) d_3 as dominant particle size:

$$e_3^M = e_1 y_1 + e_2 y_2 + e_3 y_3 + e_4 y_4 - b_{31} e_1 y_1 - b_{32} e_2 y_2 - a_{34} (1 + e_4) y_4. \quad (9)$$

(vi) d_4 as dominant particle size:

$$e_4^M = e_1 y_1 + e_2 y_2 + e_3 y_3 + e_4 y_4 - b_{41} e_1 y_1 - b_{42} e_2 y_2 - b_{43} e_3 y_3. \quad (10)$$

Chang et al. [13] determined the filling coefficient a_{ij} and embedment coefficient b_{ij} from experimental results, which are fitted by a power function of the particle size ratio (d_j/d_i):

$$a_{ij} = \left(1 - \frac{d_j}{d_i}\right)^p, \quad (j > i), \quad (11)$$

$$b_{ij} = \left(1 - \frac{d_i}{d_j}\right)^s, \quad (j < i).$$

The p and s are shape parameters, which can be determined by “the residual test method”, and the overall range of $s = 2 \sim 6$, and the range of $p = 2 \sim 7$, stated by Chang et al. [13]. The optimum values $p = 2.6$ and $s = 2.45$ has the lowest residual for the data CB-Mix-No. 1, and $p = 3.8$ and $s = 1.95$ for CB-Mix-No. 2 of Youd [8], G1-1, and G2-8. The result of filling coefficient a_{ij} and embedment coefficient b_{ij} is listed in Table 2.

Chang model and Chang-Youd model predictions are conducted to obtain the minimum void ratios of CB-Mix-No. 1, CB-Mix-No. 2, G1-1, and G2-8, which are compared with the actual values are listed in Table 3. In Chang model, the

TABLE 2: Filling coefficient (a_{ij}) and embedment coefficient (b_{ij}).

Soil ID	Division	d_i	a_{ij}			b_{ij}			p	s
		mm	$j=2$	$j=3$	$j=4$	$j=1$	$j=2$	$j=3$		
CB-Mix-No. 1	$i=1$	0.838	0.048	—	—	—	—	—	2.60	2.45
	$i=2$	0.596	—	—	—	0.058	—	—		
CB-Mix-No. 2	$i=1$	1.411	0.034	0.129	0.341	—	—	—	3.80	1.95
	$i=2$	0.833	—	0.010	0.128	0.175	—	—		
	$i=3$	0.588	—	—	0.033	0.350	0.092	—		
	$i=4$	0.348	—	—	—	0.576	0.348	0.174		
G1-1	$i=1$	17.317	0.014	0.269	0.740	—	—	—	3.80	1.95
	$i=2$	11.609	—	0.114	0.632	0.115	—	—		
	$i=3$	5.004	—	—	0.313	0.514	0.333	—		
	$i=4$	1.305	—	—	—	0.858	0.793	0.555		
G2-8	$i=1$	17.710	0.001	0.018	0.072	—	—	—	3.80	1.95
	$i=2$	14.538	—	0.003	0.028	0.035	—	—		
	$i=3$	11.458	—	—	0.004	0.131	0.049	—		
	$i=4$	8.775	—	—	—	0.263	0.165	0.059		

TABLE 3: Minimum and maximum void ratios for CB-Mix-No. 1 (CB-1), CB-Mix-No. 2 (CB-2), G1-1, and G2-8, obtained from the Chang model and Chang-Youd models' prediction and test.

Soil ID	n	e_i		e_i^M		Chang model $e_{\min}$	Chang-Youd model $e_{\min}$	Test $e_{\min}$	Chang-Youd model			Test
		Chang model	Chang-Youd model	Chang model	Chang-Youd model				e_i	e_i^M	$e_{\max}$	
CB-1	1	0.707	0.737	0.701	0.692	0.701	0.713	0.705	1.289	1.230	1.248	1.257
	2	0.716	0.731	0.675	0.713				1.280	1.248		
CB-2	1	0.692	0.679	0.495	0.494	0.622	0.620	0.59	1.207	0.967	1.119	1.099
	2	0.707	0.734	0.622	0.620				1.285	1.119		
	3	0.717	0.732	0.621	0.618				1.283	1.095		
	4	0.732	0.688	0.519	0.515				1.221	0.907		
G1-1	1	0.832	0.464	0.139	0.106	0.214	0.260	0.242	0.768	0.359	0.511	0.541
	2	0.558	0.436	0.213	0.203				0.757	0.476		
	3	0.241	0.464	0.209	0.260				0.831	0.511		
	4	0.063	0.343	0.103	0.172				0.660	0.315		
G2-8	1	0.781	0.467	0.616	0.469	0.636	0.494	0.546	0.772	0.807	0.836	0.774
	2	0.710	0.456	0.638	0.493				0.756	0.836		
	3	0.632	0.515	0.626	0.491				0.867	0.828		
	4	0.555	0.580	0.578	0.460				0.994	0.776		

coefficients $\alpha=0.7016$ and $\beta=-0.04$ of equation (2) for CB-Mix-No. 1 and CB-Mix-No. 2, which is stated from Chang et al. [14]. In the Chang model, using equation (2) to fit the G1-1~G1-8 and G2-1~G2-8 experimental data, and the fit curve is shown in Figure 5. According to the fit curve coefficients, $\alpha=0.0483$ and $\beta=0.9981$ for G1-1, and $\alpha=0.1939$ and $\beta=0.4852$ for G2-8.

Through the discrepancy between predicted and measured minimum void ratios in Table 3, both models can well predict the minimum void ratio.

The Chang-Youd model of the minimum void ratio was verified with a data set of 45 soil samples obtained from the literature; 8 samples were obtained from Youd [8], (same samples were also employed by Chang et al. [14]), 6 samples were obtained from Indraratna et. al [19], 31 samples were obtained from Fragaszy and Sneider

[20], and 16 samples from the experimental tests performed by the authors here using 16 sand-gravel mixtures composed of subrounded grain that ranged from 0.1 to 19 mm, and their grains' size distribution curves are shown in Figures 3 and 4. Same samples are calculated by the Chang model too. The data sets involve different grain sizes ranging from silt to gravel; they also involve different particle shapes ranging from angular to rounded particles. The soil properties, as well as the grain size distribution curves, are shown in Figures 2–4 and 6 and Table 4.

Equation (3), suggested here, as well as the other model relations are included in a computer program written by the authors to compute the minimum void ratio. The results of the predicted and the actual minimum void ratios are shown in Figure 7, which are located very close to the equality line.

FIGURE 5: Experimental data G1-1~G1-8 and G2-1~G2-8 fitted by equation (2).

FIGURE 6: Continued.

(e)

(f)

(g)

(h)

FIGURE 6: Grain size distribution curves of soils tested by Fragaszy and Sneider [20]. (a) Soil 1. (b) Soil 2. (c) Soil 3. (d) Soil 4. (e) Soil 5. (f) Soil 6. (g) Soil 7. (h) Soil 8.

Accordingly, we recommend using equation (3) as another alternative to equation (2) to make the model more useful in the engineering practice.

3.2. Verification of the Maximum Void Ratio Model. The fitted curve of filling coefficient (a_{ij}) and embedment coefficient (b_{ij}) for the maximum void ratio has the similar trend with that for minimum void ratios [21]. Using the Chang–Youd model, equation (4) replaced equation (2) to obtain the maximum void

ratio for CB-Mix-No. 1, CB-Mix-No. 2, G1-1, and G2-8 and are listed in Table 3. The discrepancy between the predicted and tested maximum void ratio is less than 8%, as shown in Table 3.

The newly suggested model to calculate the maximum void ratio of granular soils was verified with 30 experimental tests, and 8 samples were performed by Youd [8], and 6 samples were performed by Indraratna et al. [19], and the model was also verified with experimental tests performed by the authors here using 16 sand-gravel mixtures composed of subrounded grain that ranged from 0.1 to 19 mm, and

TABLE 4: Soil properties of the soil samples.

No.	Reference	Soil ID	$e_{\min}$	$e_{\max}$
1	Indraratna et al. [19]	G1	0.615	0.825
2		G2	0.567	0.785
3		G3	0.535	0.775
4		G4	0.484	0.747
5		G5	0.471	0.743
6		G6	0.465	0.730
7	Youd [8]	MOL-Mix-No. 1	0.458	0.799
8		MOL-Mix-No. 2	0.370	0.688
9		MOL-Mix-No. 3	0.300	0.577
10		MOL-Mix-No. 4	0.271	0.491
11		CB-Mix-No. 1	0.705	1.257
12		CB-Mix-No. 2	0.590	1.099
13		CB-Mix-No. 3	0.480	0.993
14		CB-Mix-No. 4	0.439	0.800
15	Fragaszy and Sneider [20]	Soil 1, $G = 40\%$	0.315	—
16		Soil 1, $G = 70\%$	0.351	—
17		Soil 1, $G = 85\%$	0.453	—
18		Soil 2, $G = 20\%$	0.309	—
19		Soil 2, $G = 40\%$	0.260	—
20		Soil 2, $G = 70\%$	0.220	—
21		Soil 2, $G = 85\%$	0.329	—
22		Soil 3, $G = 20\%$	0.303	—
23		Soil 3, $G = 40\%$	0.275	—
24		Soil 3, $G = 70\%$	0.297	—
25		Soil 3, $G = 85\%$	0.492	—
26		Soil 4, $G = 40\%$	0.280	—
27		Soil 4, $G = 70\%$	0.226	—
28		Soil 4, $G = 85\%$	0.319	—
29		Soil 5, $G = 20\%$	0.315	—
30		Soil 5, $G = 40\%$	0.280	—
31		Soil 5, $G = 70\%$	0.323	-
32		Soil 5, $G = 85\%$	0.437	—
33		Soil 6, $G = 20\%$	0.223	—
34		Soil 6, $G = 50\%$	0.217	—
35		Soil 6, $G = 70\%$	0.289	—
36		Soil 6, $G = 85\%$	0.345	—
37		Soil 7, $G = 20\%$	0.289	—
38		Soil 7, $G = 50\%$	0.278	—
39		Soil 7, $G = 70\%$	0.340	—
40		Soil 7, $G = 85\%$	0.504	—
41		Soil 8, $G = 20\%$	0.200	—
42		Soil 8, $G = 40\%$	0.227	—
43		Soil 8, $G = 50\%$	0.247	—
44		Soil 8, $G = 70\%$	0.348	—
45		Soil 8, $G = 85\%$	0.516	—
46	This study	G1-1	0.242	0.541
47		G1-2	0.254	0.568
48		G1-3	0.268	0.570
49		G1-4	0.345	0.589
50		G1-5	0.376	0.630
51		G1-6	0.398	0.669
52		G1-7	0.431	0.673
53		G1-8	0.483	0.738
54		G2-1	0.296	0.631
55		G2-2	0.285	0.645
56		G2-3	0.323	0.643
57		G2-4	0.357	0.666
58		G2-5	0.401	0.672
59		G2-6	0.437	0.699
60		G2-7	0.488	0.726
61		G2-8	0.546	0.774

FIGURE 7: Actual and predicted values of the minimum void ratio.

FIGURE 8: The actual and the predicted values of the maximum void ratio.

their grains' size distribution curves are shown in Figures 3 and 4. The maximum void ratios of these mixtures were obtained according to ASTM D4254-16 [18], and the results are shown in Table 4.

The values of other samples' maximum void ratios were computed. The results of the predicted and the actual void ratios are shown in Figure 8, and as can be shown, there is a very good agreement between the predicted and the actual values, and all the data points are located around the equality line.

The results of a large body of experimental tests (altogether 91 samples) confirm that Chang-Youd models presented here can be used to determine the maximum as well as the minimum void ratio. Also, it requires fewer experimental tests to determine the model parameters.

4. Summary and Conclusion

There are many methods to determine the maximum and minimum void ratios of the granular soils, including empirical and analytical methods. Recently, Chang et al. [14] presented an analytical model to determine the minimum void ratio of the granular soils. Although that Chang et al.'s [14] model gives a good agreement between the predicted and the actual values of the minimum void ratios, it requires many experimental tests to determine the values of the four model parameters, namely, p , s , α , and β which make the model to be not readily usable in the engineering practice. In this paper, we enhanced the Chang et al. [14] model by combining Chang et al.'s [14] and Youd's [8] model, presented as the Chang-Youd model. The minimum void ratio can be obtained by the uniformity coefficient of the soil and roundness of the particles using the Chang-Youd model, which makes the experimental tests required to determine the model parameters more convenient and practical. Also, it is proposed to extend the Chang-Youd model usability to be suitable for determining not only the minimum void ratio but also the maximum void ratio, while the Chang et al. [14] model cannot predict the maximum void ratio. A large number of experimental tests performed by the authors here as well as experimental data available in the literatures were used to verify the Chang-Youd models, and the results show that there is a good agreement between the predicted and the estimated values of the void ratios.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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