Research Article

Mobile Interface CMS Method for Vibration Characteristics Prediction of Mistuned Bladed Disk with Different Coupling Degrees

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The prediction of vibration characteristics was studied in the mistuned bladed disk by the mobile interface prestressed component mode synthesis (CMS) superelement method. When the strongly, generally, and weakly coupling in the mistuned bladed disk, according to the results of the direct FEM method, the prediction accuracy of this method was verified and compared with the fixed-interface CMS method by using the relative error of dynamic frequency, vibration mode matching function, and dimensionless root mean square error of vibration amplitudes. It is pointed that for mistuned bladed disk in the strong coupling, the prediction accuracy of dynamic frequency and vibration amplitudes are higher by the mobile interface CMS method and the vibration modes are matched with the direct method. In weak coupling, the results of dynamic frequency and vibration modes predicted by the mobile interface CMS method and the fixed-interface CMS method are consistent with the direct method, but the vibration amplitudes' prediction error of the mobile interface CMS method is lower than that of the fixed-interface CMS method. In general coupling, the mobile interface CMS method has higher dynamic frequency prediction accuracy at low order, and the two methods have comparable dynamic frequency prediction accuracy at high order. The vibration modes predicted by the two methods are matched with the direct FEM method, and the prediction accuracy of vibration amplitude by the mobile interface CMS method is better than that of the fixed-interface CMS method. The results indicate that the mobile interface CMS method could more accurately predict vibration characteristics of the mistuned bladed disk with different coupling degrees and could be an effective measurement for studying the vibration characteristics of the mistuned bladed disk system.

1. Introduction

The bladed disk is an extremely important component in a gas turbine engine, and its vibration characteristics are directly related to the performance and safety of the engine. In the design stage, the finite element method is mainly used to predict the vibration characteristics of the bladed disk. Theoretically, the bladed disk is a harmonic structure; that is, it satisfies the condition of cyclic symmetry. Only the finite element model of the basic sector of the bladed disk is established, and the cyclic symmetry method can be used to predict its vibration characteristics. Literature [1] predicts the vibration characteristics of a gas turbine with a crowned harmonic blisk on the basis of the cyclic symmetry method. In practice, due to the influence of factors such as

mechanical manufacturing error, material parameter dispersion, wear in use, and chatter suppression, there are slight differences between the blades, which are mistuned [2]. The periodic mistuned disk still satisfies the condition of cyclic symmetry, and the cyclic symmetry method can be used to predict its vibration characteristics. Literature [3] analyzed the vibration localization of the periodic offset mistuned bladed disk using the cyclic symmetry method. However, the cyclic symmetry method will fail when any mistuned disk destroys the cyclic symmetry condition of the bladed disk structure. The overall model of the bladed disk must be established, and the direct method must be used to predict its vibration characteristics. Literature [4] calculated the dynamic frequency of a mistuned bladed disk under rated conditions on the basis of the direct method. Lee [5]

analyzed the forced vibration characteristics of the mistuned bladed disk using the direct method. Kan [6] investigated the effect of Coriolis force on the forced response of the mistuned bladed disk using the direct method. When the size of the bladed disk is large and the twisted shape of the blade is complex, in order to ensure the prediction accuracy, the number of units and nodes divided into the finite element model of the whole bladed disk is large, which makes the direct method very time-consuming and even impossible to complete the prediction. Therefore, it is necessary to adopt an efficient reduction method to predict the vibration characteristics of the mistuned bladed disk structure [7], which not only greatly reduces the number of degrees of freedom of the model but also satisfies the prediction accuracy requirements of the vibration characteristics.

The order reduction methods mainly include the static reduction method [8, 9], dynamic reduction method [10], and modal synthesis method. Modal synthesis methods include the fixed-interface method [11], free-interface method [12, 13], and modal synthesis superelement method [14]. Literature [15] developed and verified the component modal mistuned method. Literature [16] analyzed the static frequency of a gas turbine's mistuned bladed disk structure based on the modal synthesis superelement method. The papers [17, 18], respectively, proposed a reduction analysis method of the dynamic characteristics of the mistuned bladed disk, but there were not consider the influence of the centrifugal force of the rotational speed. The papers [19, 20] proposed a fixed-interface CMS method that combines a fixed-interface prestress and a free-interface CMS superelement method. Literature [21–25] carried out related researches on the vibration characteristics of mistuned bladed disks on the basis of the fixed-interface CMS method. Literature [26] identified the mistuned parameters of real blades using modal testing and finite element method, and the vibration characteristics of the real mistuned bladed disk structure were analyzed by the mobile interface prestressed CMS superelement method.

At present, the vibration characteristics of the mistuned bladed disk are mainly used in the fixed-interface CMS method. When the rotational speed prestresses analysis, an approximately fixed displacement constraint is applied to the base sector coupling surface of the bladed disk, which approximates the prediction accuracy of the vibration characteristics of the mistuned bladed disk. Based on the mobile interface prestress CMS superelement method that predicts the vibration characteristics of different coupling mistuned bladed disk structures, using the relative error of dynamic frequency, vibration mode matching function and dimensionless root mean square error of vibration amplitudes verify the prediction accuracy of the method, compared with the fixed-interface CMS method.

2. Establishment of Mistuned Blade Disk Model with Different Coupling Degrees

The coupling degree of the bladed disk is the characterization of the rigid relationship between the blade and the disk. When the rigidity of the blade and the disk is close, the

vibration of the blade can cause strong vibration of the disk, which is a strong coupling bladed disk. When the blade rigidity is not very small relative to the wheel disk, and the vibration of the blade and the wheel disk affect each other greatly, it is a general coupling bladed disk. When the blade rigidity is much smaller than the wheel disk, the vibration of the blade can only cause the weak vibration of the wheel disk, which is a weakly coupled bladed disk. In this paper, the basic sector and bladed disk system finite element models of strong coupling, weak coupling, and general coupling are established as shown in Figures 1(a)–1(c).

The bladed disk working speed is 11353 r/min. The number of blades of the strong coupled bladed disk is 12. The density of the bladed disk material is 4500 kg/m^3 , and Poisson's ratio is 0.3. The harmonic modulus of elasticity is $2.1 \times 10^{11} \text{ Pa}$. The number of blades of the generally coupled bladed disk is 38, the elastic modulus of the disk material is $1.15 \times 10^{11} \text{ Pa}$, Poisson's ratio is 0.3, and the density is 4640 kg/m^3 . The density of the blade material is 4380 kg/m^3 , Poisson's ratio is 0.3, and the harmonic modulus of elasticity is $1.135 \times 10^{11} \text{ Pa}$. The number of blades of the weakly coupled bladed disk is 8, Poisson's ratio of the bladed disk material is 0.3, the density is 7800 kg/m^3 , and the harmonic modulus of elasticity is $2.1 \times 10^{11} \text{ Pa}$. Using different simulation mistuned disks of the blade elastic modulus, Figures 2(a)–2(c) show the mistuned elastic modulus of each blade for three coupling degrees.

3. Mobile Interface CMS Method

In this paper, the vibration characteristics of the mistuned blisk system are predicted using the mobile interface prestressed CMS superelement method. The mobile interface CMS method consists of two parts: the mobile interface prestressed analysis and the free-interface CMS superelement analysis.

3.1. Prestressed Analysis of Mobile Interface. First, the bladed disk structure is assumed to be a harmonic system. Let the established bladed disk basic sector finite element model be the first bladed disk basic sector, and $\{v^1\}$ is the displacement vector of the first bladed disk basic sector. The displacement vector is divided into interface degrees of freedom $\{v_m^1\}$ and noninterface degrees of freedom $\{v_s^1\}$; there are

$$\{v^1\} = \begin{Bmatrix} v_m^1 \\ v_s^1 \end{Bmatrix}. \quad (1)$$

The constraint boundary, working speed load, and interface cyclic symmetry conditions are applied, and the displacement vector $\{v_m^1\}$ of the first basic sector interface degrees of freedom of the bladed disk system at the working speed is obtained using the cyclic symmetry analysis method.

The displacement vector of the i -th basic sector interface degree of freedom in the blade disk system is $\{v_m^i\}$, which can be expressed in the following form:

FIGURE 1: Finite element model of basic sector and bladed disk system with different coupling degrees. (a) Strong coupling. (b) Weak coupling. (c) General coupling.

FIGURE 2: Mistuned elastic modulus of blades of the bladed disk with different coupling degrees. (a) Strong coupling. (b) Weak coupling. (c) General coupling.

$$\{v_m^i\} = \{v_m^{i1} \dots v_m^{ik}\}^T, \quad (2)$$

where k represents the number of nodes on the interface; the displacement vector of the j -th node on the interface is

$$\{v_m^{ij}\} = \{\Delta x_m^{ij} \Delta y_m^{ij} \Delta z_m^{ij}\}, \quad 1 \leq j \leq k. \quad (3)$$

According to the cyclic symmetry condition, the displacement vector $\{v_m^{(i+1)j}\}$ of the j -th node on the interface of the $i+1$ -th basic sector in the bladed disk system can be calculated using the following formula:

$$\{v_m^{(i+1)j}\}^T = \begin{bmatrix} \cos\left(\pm \frac{360}{N}\right) & \sin\left(\pm \frac{360}{N}\right) & 0 \\ -\sin\left(\pm \frac{360}{N}\right) & \cos\left(\pm \frac{360}{N}\right) & 0 \\ 0 & 0 & 1 \end{bmatrix} \{v_m^{ij}\}^T. \quad (4)$$

Here, N is the number of blades, and the displacement vector of the interface degree of freedom of the $i+1$ -th basic sector in the bladed disk system is $\{v_m^{i+1}\}$, which can be calculated using the following form:

$$\{v_m^{i+1}\} = \{v_m^{(i+1)1} \dots v_m^{(i+1)k}\}^T \quad (5)$$

Since $\{v_m^1\}$ is known, we substitute it into equations (2)–(5) to get $\{v_m^i\}$, $2 \leq i \leq N$.

Secondly, the bladed disk structure is regarded as a mistuned system. The overall bladed disk is composed of N substructures, and each substructure is a basic sector. At a certain speed, the static equation of substructure i is

$$[K^i]\{u^i\} = \{F^i\}, \quad (6)$$

where $[K^i]$, $\{u^i\}$, and $\{F^i\}$ are, respectively, the stiffness matrix, displacement vector, and load vector generated by the rotational speed of substructure i . The displacement vector $\{u^i\}$ is divided into the following form:

$$\{u^i\} = \begin{Bmatrix} u_m^i \\ u_s^i \end{Bmatrix}. \quad (7)$$

Here, $\{u_m^i\}$ and $\{u_s^i\}$ are interface and noninterface degrees of freedom, respectively. Substitute equations (7) into (6) to get

$$\begin{bmatrix} K_{mm}^i & K_{ms}^i \\ K_{sm}^i & K_{ss}^i \end{bmatrix} \begin{Bmatrix} u_m^i \\ u_s^i \end{Bmatrix} = \begin{Bmatrix} F_m^i \\ F_s^i \end{Bmatrix}. \quad (8)$$

The mobile displacement constraint is applied to the degrees of freedom of the interface, i.e., $\{u_m^i\} = \{v_m^i\}$, and the prestressed analysis is carried out on the substructure. Equation (8) is simplified as

$$[K_{ss}^i]\{u_s^i\} = \{F_s^i\} - [K_{sm}^i]\{v_m^i\}. \quad (9)$$

Solve equation (9), and get the stiffness matrix $[K_l^i(\Omega)]$ and softening matrix $[K_r^i(\Omega)]$ related to the speed Ω . When the effects of stiffness and softening are considered at the same time, the stiffness matrix of substructure i is as follows:

$$[K_*^i] = [K^i] + [K_l^i(\Omega)] - [K_r^i(\Omega)]. \quad (10)$$

3.2. Free-Interface CMS Superelement Analysis. For an undamped structure, the dynamic equation of substructure i is

$$[M^i]\{\ddot{u}^i\} + [K_*^i]\{u^i\} = \begin{Bmatrix} F_m^i \\ F_s^i \end{Bmatrix}, \quad (11)$$

where $[M^i]$ is the mass matrix of the substructure, and $\{F_m^i\}$ and $\{F_s^i\}$ are the force vectors of the interface nodes of substructure i on other substructures and the external force vectors of noninterface nodes, respectively.

Substituting equations (7) into (11), when the structure vibrates freely, the force exerted on noninterface nodes is $\{F_s^i\} = \{0\}$, and equation (11) is simplified as

$$\begin{bmatrix} M_{mm}^i & M_{ms}^i \\ M_{sm}^i & M_{ss}^i \end{bmatrix} \begin{Bmatrix} \ddot{u}_m^i \\ \ddot{u}_s^i \end{Bmatrix} + \begin{bmatrix} K_{*mm}^i & K_{*ms}^i \\ K_{*sm}^i & K_{*ss}^i \end{bmatrix} \begin{Bmatrix} u_m^i \\ u_s^i \end{Bmatrix} = \begin{Bmatrix} F_m^i \\ 0 \end{Bmatrix}. \quad (12)$$

The mobile displacement constraints of the interface degrees of freedom are analyzed and released using the free-interface CMS superelement method, that is, delete the constraint $\{u_m^i\} = \{v_m^i\}$, and the undamped free vibration equation of the substructure i with the unconstrained interface is

$$[M^i]\{\ddot{u}^i\} + [K_*^i]\{u^i\} = \{0\}. \quad (13)$$

From equation (13), modal truncation is used to obtain the low-order modal set and the modal coordinate transformation matrix corresponding to the free-interface method. Taking the modal coordinate transformation matrix as the approximate modal of substructure i , substituting the modal coordinate transformation formula into equation (12), the dynamic equation of substructure i in modal coordinates is obtained as

$$[\hat{M}^i]\{\ddot{q}_D^i\} + [\hat{K}_*^i]\{q_D^i\} = \{\hat{F}^i\}, \quad i = 1, 2, \dots, N. \quad (14)$$

Here, $\{q_D^i\}$ is the modal coordinate vector, $[\hat{M}^i] = [T_D^i]^T [M^i] [T_D^i]$ is the mass matrix under the modal coordinate, $[\hat{K}_*^i] = [T_D^i]^T [K_*^i] [T_D^i]$ is the stiffness matrix under the modal coordinate, $\{\hat{F}^i\} = [T_D^i]^T \begin{Bmatrix} F_m^i \\ 0 \end{Bmatrix}$ is the load vector, and $[T_D^i]$ is the modal coordinate transformation matrix.

Combining the dynamic equations of N substructures forms the reduced dynamic equation of the overall mistuned bladed disk structure in modal coordinates:

$$\begin{bmatrix} [\hat{M}^1] & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & [\hat{M}^N] \end{bmatrix} \begin{Bmatrix} \ddot{q}_D^1 \\ \vdots \\ \ddot{q}_D^N \end{Bmatrix} + \begin{bmatrix} [\hat{K}_*^1] & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & [\hat{K}_*^N] \end{bmatrix} \begin{Bmatrix} q_D^1 \\ \vdots \\ q_D^N \end{Bmatrix} = \begin{bmatrix} [T_{Dm}^1]^T & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & [T_{Dm}^N]^T \end{bmatrix} \begin{Bmatrix} F_m^1 \\ \vdots \\ F_m^N \end{Bmatrix}. \quad (15)$$

The displacement coordination and force balance conditions are adopted to couple the degree of freedom of interface nodes of the substructure to obtain the generalized coordinate transformation matrix. The generalized

coordinate transformation formula is substituted into equation (15) to obtain the dynamic equation of the overall mistuned bladed disk structure under the generalized coordinates including the effects of rigidization and softening:

$$\begin{aligned}
[\bar{M}]\{\ddot{p}\} + [\bar{K}_*]\{p\} &= \{\bar{F}\}, \\
[\bar{M}] &= [a]^T \begin{bmatrix} \hat{M}^1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \hat{M}^N \end{bmatrix} [a], \\
[\bar{K}_*] &= [a]^T \begin{bmatrix} \hat{K}_*^1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \hat{K}_*^N \end{bmatrix} [a], \\
\{\bar{F}\} &= [a]^T \begin{bmatrix} [T_{Dm}^1]^T & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & [T_{Dm}^N]^T \end{bmatrix} \begin{Bmatrix} F_m^1 \\ \vdots \\ F_m^N \end{Bmatrix},
\end{aligned} \tag{16}$$

where $\{p\}$ is the reduced generalized coordinate vector, and $[a]$ is the generalized coordinate transformation matrix.

The free vibration equation for the reduction of the overall mistuned bladed disk is

$$[\bar{M}]\{\ddot{p}\} + [\bar{K}_*]\{p\} = \{0\}. \tag{17}$$

By solving equations (16) and (17), the dynamic frequency, the response, and mode shape under the reduced generalized coordinate $\{p\}$ of the overall mistuned bladed disk can be obtained, and the superelement can be used.

The generalized coordinate inverse transformation and the modal coordinate inverse transformation are used to obtain the response and mode shape of the overall mistuned bladed disk under the physical coordinate $\{u\}$ and complete the extended superelement.

4. Influence of Different Coupling Degrees on Prediction Accuracy of Vibration Characteristics

The vibration characteristics of the mistuned bladed disk system mainly include the vibration frequency, vibration shape, and amplitude. In this paper, the prediction result of the direct method is used as the theoretical value, and the relative error of dynamic frequency, vibration mode matching function, and dimensionless root mean square error of vibration amplitudes are introduced to verify the prediction accuracy of the mobile interface CMS method and compare it with the fixed-interface CMS method.

4.1. Prediction Accuracy Index

4.1.1. Relative Error of Dynamic Frequency. The dynamic frequency relative error prediction of the bladed disk system at a certain speed is

$$e_{ddfre}^l = \left| \frac{F_{cs}^l - F_{mfcms}^l}{F_{cs}^l} \right| \times 100\%. \tag{18}$$

Here, l is the mode order, F_{cs}^l is the l th-order dynamic frequency of the mistuned bladed disk system obtained by the direct method, and F_{mfcms}^l is the l th-order dynamic frequency obtained by the mobile interface CMS

superelement method or the fixed-interface CMS super-element method.

4.1.2. Vibration Mode Matching Function. The dimensionless amplitude vector of the l -th mode shape of the mistuned bladed disk predicted by the direct method is

$$\{X_{lz}\} = \{x_{lz}^1, x_{lz}^2, \dots, x_{lz}^k, \dots, x_{lz}^N\}. \tag{19}$$

The dimensionless amplitude vector of the l -th mode shape of the mistuned bladed disk predicted by mobile interface CMS method or fixed-interface CMS method is

$$\{X_l\} = \{x_l^1, x_l^2, \dots, x_l^k, \dots, x_l^N\}. \tag{20}$$

Taking the dimensionless amplitude vector element sequence predicted by the direct method as the template sequence, the matching degree function between sequence $\{X_l\}$ and template sequence $\{X_{lz}\}$ is as follows:

$$C_l(\tau) = \sum_{k=1}^N x_l^{k+\tau} x_{lz}^k, \quad \tau = 0, 1, 2, \dots, N-1. \tag{21}$$

When $N < k + \tau < 2N$, $x_l^{k+\tau} = x_l^{k+\tau-N}$.

The parameter τ is the independent variable of the mode matching degree function $C_l(\tau)$, assuming that the maximum value of $C_l(\tau)$ is $C_l(\tau_m)$, that is,

$$\max C_l(\tau) = C_l(\tau_m). \tag{22}$$

When $\tau_m = 0$, the mode shapes predicted by the mobile interface CMS method or the fixed-interface CMS method match those of the direct method. When $\tau_m \neq 0$, the mode shapes do not match.

4.1.3. Dimensionless Root Mean Square Error of Vibration Amplitudes. The dimensionless root mean square error of the vibration amplitude vector element of the l -th mode shape of the bladed disk structure at a certain speed is

$$\text{RMSE}_l = \sqrt{\frac{1}{N} \sum_{k=1}^N (x_l^k - x_{lz}^k)^2}, \tag{23}$$

where N is the number of blades, x_l^k is the dimensionless amplitude value of the k -th blade in the l -th mode obtained by mobile interface method or fixed-interface method, and x_{lz}^k is the dimensionless amplitude value of the k -th blade in the l -th mode obtained by the direct method.

4.2. Prediction Accuracy Verification and Comparative Analysis

4.2.1. Dynamic Frequency Relative Error Analysis. The dynamic frequency relative error of the strongly coupled mistuned blisk system obtained by the mobile interface CMS superelement method and the fixed-interface CMS super-element method is shown in Figure 3. From Figure 3, it can be seen that the dynamic frequency maximum relative error predicted by the fixed-interface method is 29.814%. As the

FIGURE 3: First-order bending dynamic frequency relative error of strongly coupling mistuned bladed disk.

order increases, the relative error shows a tendency to become smaller. The dynamic frequency maximum relative error predicted by the mobile interface method is 0.1466%. As the order increases, the relative error does not change significantly. For the strongly coupled mistuned bladed disk structure, the mobile interface method has higher accuracy in predicting the dynamic frequency, while the fixed-interface method has lower accuracy.

Table 1 shows the dynamic frequency of the weakly coupled mistuned bladed disk system predicted by the direct method, the mobile interface method, and the fixed-interface method at a certain speed. It can be seen from Table 1 that the dynamic frequency prediction results of the mobile interface method and the fixed-interface method are consistent with the direct method, and the dynamic frequency relative error is 0%. For the weakly coupled mistuned bladed disk system, both the mobile interface method and the fixed-interface method have higher dynamic frequency prediction accuracy.

Figure 4 shows the dynamic frequency relative error of the general coupling mistuned bladed disk structures obtained by the direct method, the mobile interface method, and the fixed-interface method. As can be seen from Figure 4, the dynamic frequency relative error of the general coupled mistuned bladed disk structure predicted by the mobile interface method is small, and it does not change significantly with the increase of order. However, the relative error of the general coupled mistuned bladed disk structure obtained by the fixed-interface method is larger at low order, especially when the order is no more than 5, and tends to decrease with the increase of the order. However, the relative error of the coupled mistuned bladed disk structure is smaller at high order. For general coupled mistuned bladed disk systems, the prediction accuracy of the mobile interface method is much better than that of the fixed-interface method when the order is lower, and the prediction accuracy of the two methods is similar when the order is higher.

4.2.2. Vibration Shape Matching Analysis. The mode shape matching function and parameters of the strongly coupled mistuned bladed disk predicted by the mobile interface method and the fixed-interface method are shown in Table 2. As can be seen from Table 2, when the mobile interface method is used to predict the 12th order mode of a bending family, when the mode matching degree parameter $\tau_m = 0$, the mode matching degree function $C_l(\tau)$ takes the maximum value, indicating that the mode shapes of the strongly coupled mistuned bladed disk predicted by the mobile interface method match the mode shapes of the direct method. When the fixed-interface method is used to predict the mode shape of the 5th order, when $\tau_m = 6$, the mode matching degree function $C_l(\tau)$ takes the maximum value. In the 10th and 11th modes, when $\tau_m = 1$, $C_l(\tau)$ takes the maximum. It is shown that the mode shapes of the 5th, 10th, and 11th order of the strongly coupled mistuned bladed disk predicted by the fixed-interface method do not match those predicted by the direct method.

Figures 5(a) and 5(b) are the vibration shape matching function of the weakly coupled mistuned bladed disk predicted by the mobile interface method and the fixed-interface method, respectively. It can be seen from Figures 5(a) and 5(b) that when the mobile interface method and the fixed-interface method are used, respectively, for a bending group of 8th order mode, when the mode matching degree parameter $\tau_m = 0$, the mode matching degree function $C_l(\tau)$ takes the maximum value. For the weakly coupled mistuned bladed disk structure, the mode shapes predicted by the mobile interface method and the fixed-interface method are matched with those predicted by the direct method.

The mode matching degree functions of the general coupled mistuned bladed disk predicted by the mobile interface method and the fixed-interface method are shown in Figures 6(a) and 6(b), respectively. It can be seen from Figures 6(a) and 6(b) that based on the prediction of the mobile interface method and the fixed-interface method, for the first 10 modes of a bend group, when $\tau_m = 0$, $C_l(\tau)$ takes the maximum value. For the general coupled mistuned bladed disk structure, the mode shape predicted by the above two methods matches the mode shape predicted by the direct method.

4.2.3. Dimensionless Vibration Amplitude Root Mean Square Error Analysis. Figures 7–9 show the variation curves of dimensionless amplitude root mean square error of strongly coupled, weakly coupled, and generally coupled mistuned bladed disks with the order of vibration modes, respectively. As can be seen from Figures 7–9, for different coupling degrees of mistuned bladed disks, the dimensionless amplitude root mean square errors predicted by the mobile interface method are all smaller than those predicted by the fixed-interface method.

4.2.4. Comparison of Analysis Accuracy and Analysis Time. The analysis accuracy of dynamic frequencies and analysis time of the three methods are compared. Since the general coupling model has the largest total number of degrees of

TABLE 1: Dynamic frequency of weakly coupling mistuned bladed disk at working speed.

Order	Dynamic frequency (Hz)		
	Direct method	Mobile interface method	Fixed-interface method
1	259.84	259.84	259.84
2	259.93	259.93	259.93
3	259.98	259.98	259.98
4	260.03	260.03	260.03
5	260.07	260.07	260.07
6	260.12	260.12	260.12
7	260.17	260.17	260.17
8	260.22	260.22	260.22

FIGURE 4: First-order bending dynamic frequency relative error of generally coupling mistuned bladed disk.

TABLE 2: Vibration mode matching degree of strongly coupling mistuned bladed disk.

Mode shape order	Mobile interface method		Fixed-interface method	
	$\max C_l(\tau)$	τ_m	$\max C_l(\tau)$	τ_m
1	1.5534	0	1.43309	0
2	1.83892	0	1.66608	0
3	1.83934	0	1.66948	0
4	2.5042	0	2.20679	0
5	2.61836	0	2.31116	6
6	3.53852	0	3.1394	0
7	3.53448	0	3.13897	0
8	5.09595	0	4.67008	0
9	5.16633	0	4.81133	0
10	6.13409	0	5.94246	1
11	6.15465	0	5.96189	1
12	6.39985	0	6.25401	0

freedom and the reduction rate of degrees of freedom, so only the analysis time of the general coupling model is compared in Table 3. As shown from Table 3, the analysis accuracy of dynamic frequencies of strong coupling bladed disk model based on the mobile interface method and the fixed-interface method is 99.8534% and 70.186%,

respectively; the analysis accuracy of dynamic frequencies of the general coupling bladed disk model is 99.9039% and 97.1675%, respectively, and the relative analysis time is 28.42% and 27.07%, respectively; the analysis accuracy of dynamic frequencies of the weak coupling bladed disk model is 100%. The analysis time of the mobile interface method is

FIGURE 5: Vibration mode matching function of weakly coupling mistuned bladed disk. (a) Mobile interface method. (b) Fixed-interface method.

FIGURE 6: Vibration mode matching function of generally coupling mistuned bladed disk. (a) Mobile interface method. (b) Fixed-interface method.

FIGURE 7: Root mean square error of dimensionless vibration amplitude of strongly coupling mistuned bladed disk.

FIGURE 8: Root mean square error of dimensionless vibration amplitude of weakly coupling mistuned bladed disk.

FIGURE 9: Root mean square error of dimensionless vibration amplitude of generally coupling mistuned bladed disk.

TABLE 3: The analysis accuracy of dynamic frequencies and analysis time of three methods.

Analysis method	Analysis accuracy of dynamic frequencies	Analysis time
Mobile interface prestressed CMS superelement method	Strong coupling: 99.8534% General coupling: 99.9039% Weak coupling: 100%	General coupling: 28.42%
Fixed-interface prestressed CMS superelement method	Strong coupling: 70.1860% General coupling: 97.1675% Weak coupling: 100%	General coupling: 27.07%
Direct FEM method	100%	General coupling: 100%

basically the same as that of the fixed-interface method, but the analysis accuracy is better than that of the fixed-interface method, especially for strongly coupling bladed disk.

5. Conclusions

Aiming at the shortcoming of the existing methods for predicting the vibration characteristics of bladed disk structures and other large and complex structures, a mobile interface prestressed CMS superelement method is proposed in this paper. Taking the prediction results of the direct method as the theoretical value, the relative error of dynamic frequency, the mode matching function, and the dimensionless root mean square error of vibration amplitude verify the prediction accuracy of the proposed method. The prediction accuracy of the proposed method is compared with that of the fixed-interface CMS method. The specific conclusions are as follows:

- (1) For the strongly coupled mistuned bladed disk structure, the relative error of dynamic frequency and the root mean square error of dimensionless amplitude predicted by the mobile interface method are small and the mode patterns are matched. The dynamic frequency relative error and the root mean square error of the dimensionless amplitude predicted by the fixed-interface method are large and the mode mismatch occurs.
- (2) For the weakly coupled mistuned bladed disk structures, the relative errors of dynamic frequencies predicted by the mobile interface method and fixed-interface method are both 0%, and the modes are matched, but the dimensionless root mean square error of amplitude predicted by mobile interface method is less than that by the fixed-interface method.
- (3) For the general coupled detuned bladed disk structures, the relative error of dynamic frequency predicted by the mobile interface method is small and basically unchanged, while the dynamic frequency relative error predicted by the fixed-interface method is large when the order is low, and the relative error of dynamic frequency decreases gradually with the increase of order. The mode shapes predicted by the two methods match those of the direct method, while the root mean square error of the dimensionless amplitude predicted by the mobile interface method is less than that by the fixed-interface method.

- (4) The mobile interface method is more accurate in predicting the vibration characteristics of the mistuned bladed disk structures with different coupling degrees. Compared with the direct FEM method, this method is less time-consuming.

Data Availability

The data used to support the findings of this paper are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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