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Research Article

An Analytical View of Fractional-Order Fisher's Type Equations within Caputo Operator

Nehad Ali Shah [1,2]

[1] *Informetrics Research Group, Ton Duc Thang University, Ho Chi Minh City, Vietnam*
[2] *Faculty of Mathematics & Statistics, Ton Duc Thang University, Ho Chi Minh City, Vietnam*

Correspondence should be addressed to Nehad Ali Shah; nehadali199@yahoo.com

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The present research article is related to the analytical investigation of some nonlinear fractional-order Fisher's equations. The homotopy perturbation technique and Shehu transformation are implemented to discuss the fractional view analysis of Fisher's equations. For a better understanding of the proposed procedure, some examples related to Fisher's equations are presented. The identical behavior of the derived and actual solutions is observed. The solutions at different fractional are calculated, which describe some useful dynamics of the given problems. The proposed technique can be modified to study the fractional view analysis of other problems in various areas of applied sciences.

1. Introduction

In mathematical science, the construction of exact and explicit solutions to nonlinear fractional-order partial differential equations (PDEs) is very significant and is one of the most exciting and especially active fields of study. It is well recognized that it is possible to divide all nonlinear PDEs into two parts: the nonintegrable ones and the integrable partial differential equations. There is an infinite number of exact solutions to the first form, i.e., the integrable equations. The most well-known problems among them are the sine-Gordon equation, Korteweg-de Vries equation, Boussinesq equations, Kawahara type equations, and nonlinear Schrodinger equation and the list can be expanded with other fundamental integrable problems, but it is not our purpose to give all the lists [1–5]. Nonlinear PDEs are considered to be in the class of nonintegrable partial differential equations with certain precise solutions or without precise solutions and will need special care to achieve their solutions because of the shape of the nonlinear differential equation and the pole of its solution. The Fitzhugh-Nagumo equation, Fisher equation, Burger-Huxley equation, and Ginzburg-Landau equation can be mentioned as the well-known nonintegrable PDEs among them all [6–13].

Over the last few decades, considerable progress has been made in developing methods for obtaining precise solutions to nonlinear equations, but the progress accomplished is insufficient. Since, from our point of view, there is no single optimal way to achieve correct solutions to nonlinear differential equations of all forms. Based on the researchers' expertise and the sympathy for the method used, each method has its benefits and shortcomings. Also, all these techniques can be seen to be problem-dependent, namely, that certain techniques perform well on some concerns, but others do not. Therefore, it is very important to apply certain well-known methods to nonlinear partial differential equations in the literature that are not solved with that method to look for potential new exact solutions or to check current solutions with different approaches [14–17].

Fisher-Kolmogorov-Petrovsky-Piscounov (Fisher-KPP) equation was first introduced by Fisher [18] and was later renamed Fisher equation. FEs have numerous applications in the fields of engineering and science [19–22]. The researchers investigated some important generalizations of this equation [23–25]. Numerous reaction-diffusion equations have wavefronts that show a vital part in explaining chemical, physical, and biological phenomena [26, 27]. The reaction-diffusion systems can explain how changes in the concentration of one or more chemicals occur. One is the local chemical reactions that transform the substances into each other and the other is the diffusion, which allows the substances to spread through the air.

The simplest equation for reaction-diffusion in one spatial dimension,

$$\psi_{\mathfrak{I}} = P\psi_{\mu\mu} + Q(\psi), \tag{1}$$

where $\psi(\mu, \mathfrak{I})$ shows single material concentration, P represents diffusion coefficients, and Q represents all local reactions. If $R(\psi) = \psi(1-\psi)$, we get FE which is used to define the biological populations dispersion. The Fisher-KPP advection equation is used to define population dynamics in advective environments [28]. The partial differential equation proposed by Fisher is nonlinear as

$$\psi_{\mathfrak{I}} = D\psi_{\mu\mu} + \psi(1-\psi). \tag{2}$$

Fisher proposed equation (2) as a model for gene selection, with ψ denoting the population density. The same equation also arises in the autocatalytic chemical reactions, nuclear reactor theory, flame propagation, neurophysiology, and Brownian motion process. The Fisher equation is considered to be an important equation because of its vast number of applications in the field of engineering.

The homotopy perturbation technique was developed by He [29, 30] in 1998. HPM provides the solution as a sum of the sequence having an infinite sum that converges rapidly to the exact results. HPM can be used to solve PDEs of higher dimensions and nonlinearity effectively.

In the present research article, effective utilization of the new developed technique, the homotopy perturbation method and Shehu transform, has been implemented to solve fractional FEs. The suggested technique is very effective for the solutions of other fractional PDEs because its required small computational work and higher degree accuracy. Moreover, the obtained results are in close resemblance with the actual solution of all fractional FEs.

2. Preliminaries

2.1. Definition. The fractional-order Riemann–Liouville integral is define by [31, 32]

$$I_0^{\delta} h(\tau) = \frac{1}{\Gamma(\alpha)} \int_0^{\eta} (\eta - s)^{\alpha-1} h(s) \mathrm{d}s. \tag{3}$$

2.2. Definition. The fractional-order Caputo's derivative of $h(\eta)$ is given as [31, 32]

$$\begin{aligned} D_{\eta}^{\alpha} h(\eta) &= I^{n-\alpha} f^{n}, \quad n-1 < \alpha < n, n \in \mathbb{N}, \\ &\frac{d^n}{d\eta^n} h(\eta), \quad \alpha = n, n \in \mathbb{N}. \end{aligned} \tag{4}$$

2.3. Definition. The integral of Shehu transformation is new and similar to other integral transformation which is described for exponential order functions. In set A, we take a function which is described by [33–35]

$$A = \left\{ \nu(\eta) \colon \exists, \rho_1, \rho_2 > 0, |\nu(\eta)| < M e^{(|\eta|/\rho_i)}, \quad \text{if } \eta \in [0, \infty). \right. \tag{5}$$

The Shehu transformation which is defined by $S(\cdot)$ for a function $\nu(\eta)$ is given as

$$S\{\nu(\eta)\} = V(s, \mu) = \int_0^{\infty} e^{(-s\eta/\mu)} \nu(\eta) \mathrm{d}\eta, \quad \eta > 0, s > 0. \tag{6}$$

The Shehu transformation of a function $\nu(\eta)$ is $V(s, \mu)$, and then $\nu(\eta)$ is known as the inverse of $V(s, \mu)$ which is define as

$$S^{-1}\{V(s, \mu)\} = \nu(\eta), \quad \text{for } \eta \geq 0, S^{-1} \text{ is inverse Shehu transformation.} \tag{7}$$

2.4. Definition. The Shehu transformation for nth derivatives is defined as [33–35]

$$S\left\{\nu^{(n)}(\eta)\right\} = \frac{s^n}{u^n} V(s, u) - \sum_{k=0}^{n-1} \left(\frac{s}{u}\right)^{n-k-1} \nu^{(k)}(0). \tag{8}$$

2.5. Definition. The fractional-order derivatives of Shehu transformation are given as [33–35]

$$S\left\{\nu^{(\alpha)}(\eta)\right\} = \frac{s^{\alpha}}{u^{\alpha}} V(s, u) - \sum_{k=0}^{n-1} \left(\frac{s}{u}\right)^{\alpha-k-1} \nu^{(k)}(0), \quad 0 < \beta \leq n. \tag{9}$$

2.6. Definition. The Mittag-Leffler function of $E_{\alpha}(z)$ for $\alpha > 0$ is given as

$$E_{\alpha}(z) = \sum_{m=0}^{\infty} \frac{z^m}{\Gamma(\alpha m + 1)}, \quad \alpha > 0, z \in \mathbb{C}. \tag{10}$$

3. Homotopy Perturbation Transform Method

To explain the fundamental ideas of this method, we get the following equation:

$$\begin{aligned} D_{\mathfrak{I}}^{\alpha} \psi(\mu, \mathfrak{I}) + M\psi(\mu, \mathfrak{I}) + N\psi(\mu, \mathfrak{I}) &= h(\mu, \mathfrak{I}), \quad \mathfrak{I} > 0, 0 < \alpha \leq 1, \\ \psi(\mu, 0) &= g(\mu), \quad \mu \in \mathfrak{R}, \end{aligned} \tag{11}$$

where $D_{\mathfrak{I}}^{\alpha} = (\partial^{\alpha}/\partial \mathfrak{I}^{\alpha})$ is Caputo's derivative, M, N is the linear and nonlinear operator in μ, and $h(\mu, \mathfrak{I})$ is the source function.

By taking Shehu transformation, we can write (11) as

$$S\left[D_{\mathfrak{I}}^{\alpha}\psi(\mu,\mathfrak{I})+M\psi(\mu,\mathfrak{I})+N\psi(\mu,\mathfrak{I})\right]=S[h(\mu,\mathfrak{I})],$$

$$R(\mu,s,u)=\frac{g(\mu)}{s}+\frac{u^{\alpha}}{s^{\alpha}}S[h(\mu,\mathfrak{I})]-\frac{u^{\alpha}}{s^{\alpha}}S[M\psi(\mu,\mathfrak{I})+N\psi(\mu,\mathfrak{I})]. \tag{12}$$

Now, using inverse Shehu transformation, we get

$$\psi(\mu,\mathfrak{I})=F(\mu,\mathfrak{I})-S^{-1}\left[\frac{u^{\alpha}}{s^{\alpha}}S\{M\psi(\mu,\mathfrak{I})+N\psi(\mu,\mathfrak{I})\}\right], \tag{13}$$

where

$$\begin{aligned}F(\mu,\mathfrak{I})&=S^{-1}\left[\frac{g(\mu)}{s}+\frac{u^{\alpha}}{s^{\alpha}}S[h(\mu,\mathfrak{I})]\right]\\&=g(\mu)+S^{-1}\left[\frac{u^{\alpha}}{s^{\alpha}}S[h(\mu,\mathfrak{I})]\right].\end{aligned} \tag{14}$$

Now, if ρ is the parameter perturbation, we can write as

$$\psi(\mu,\mathfrak{I})=\sum_{k=0}^{\infty}\rho^{k}\psi_{k}(\mu,\mathfrak{I}), \tag{15}$$

where ρ is the perturbation parameter and $\rho\in[0,1]$.

The nonlinear term can be decomposed as

$$N\psi(\mu,\mathfrak{I})=\sum_{k=0}^{\infty}\rho^{k}H_{n}(\psi), \tag{16}$$

where H_n are He's polynomials of the form $\psi_0,\psi_1,\psi_2,\ldots,\psi_n$, and can be determined as

$$H_{n}(\psi_0,\psi_1,\ldots,\psi_n)=\frac{1}{n!}\frac{\partial^{n}}{\partial p^{n}}\left[N\left(\sum_{k=0}^{\infty}\rho^{k}\psi_{k}\right)\right]_{\rho=0}. \tag{17}$$

Using relations (15) and (16) in (2) and constructing the homotopy, we get

$$\sum_{k=0}^{\infty}\rho^{k}\psi_{k}(\mu,\mathfrak{I})=F(\mu,\mathfrak{I})-\rho\times\left[S^{-1}\left\{\frac{u^{\alpha}}{s^{\alpha}}S\left\{M\sum_{k=0}^{\infty}\rho^{k}\psi_{k}(\mu,\mathfrak{I})+\sum_{k=0}^{\infty}\rho^{k}H_{k}(\psi)\right\}\right\}\right]. \tag{18}$$

On comparing coefficient of ρ on both sides, we obtain

$$\begin{aligned}&\rho^{0}\colon\ \psi_{0}(\mu,\mathfrak{I})=F(\mu,\mathfrak{I}),\\&\rho^{1}\colon\ \psi_{1}(\mu,\mathfrak{I})=S^{-1}\left[\frac{u^{\alpha}}{s^{\alpha}}S\{M\psi_{0}(\mu,\mathfrak{I})+H_{0}(\psi)\}\right],\\&\rho^{2}\colon\ \psi_{2}(\mu,\mathfrak{I})=S^{-1}\left[\frac{u^{\alpha}}{s^{\alpha}}S\{M\psi_{1}(\mu,\mathfrak{I})+H_{1}(\psi)\}\right],\\&\qquad\vdots\\&\rho^{k}\colon\ \psi_{n}(\mu,\mathfrak{I})=S^{-1}\left[\frac{u^{\alpha}}{s^{\alpha}}S\{M\psi_{k-1}(\mu,\mathfrak{I})+H_{k-1}(\psi)\}\right],\\&\qquad\qquad k>0,k\in N.\end{aligned} \tag{19}$$

The component $\psi_k(\mu,\mathfrak{I})$ can be calculated easily, which leads us to the convergent series rapidly. By taking $\rho\longrightarrow 1$, we obtain

$$\psi(\mu,\mathfrak{I})=\lim_{M\longrightarrow\infty}\sum_{k=1}^{M}\psi_{k}(\mu,\mathfrak{I}). \tag{20}$$

The obtained result is in the form of series and easily converges to exact solution of the problem.

4. Test Problems

To show the validity of the suggested technique, the following test problems are solved.

4.1. Example. Consider the fractional-order Fisher equation is given by

$$D_{\mathfrak{I}}^{\alpha}\psi=\psi_{\mu\mu}+\psi(1-\psi),\quad 0<\alpha\leq 1, \tag{21}$$

with initial condition

$$\psi(\mu,0)=\beta. \tag{22}$$

Applying Shehu transform to (21), we have

$$\begin{aligned}&\frac{s^{\alpha}}{u^{\alpha}}S[\psi(\mu,\mathfrak{I})]-\frac{s^{\alpha-1}}{u^{\alpha}}\psi(\mu,0)=S\big(\psi_{\mu\mu}+\psi(1-\psi)\big).\\&S[\psi(\mu,\mathfrak{I})]=\frac{\beta}{s}+\frac{u^{\alpha}}{s^{\alpha}}\left[S\big(\psi_{\mu\mu}+\psi(1-\psi)\big)\right].\end{aligned} \tag{23}$$

Using inverse Shehu transformation, we get

$$\psi(\mu,\mathfrak{I})=\beta+S^{-1}\left[\frac{u^{\alpha}}{s^{\alpha}}\left\{S\big(\psi_{\mu\mu}+\psi(1-\psi)\big)\right\}\right]. \tag{24}$$

Applying the abovementioned homotopy perturbation technique as in (18), we get

$$\sum_{k=0}^{\infty}\rho^{k}\psi_{k}(\mu,\mathfrak{I})=\beta+\rho\left[S^{-1}\left\{\frac{u^{\alpha}}{s^{\alpha}}S\left(\left(\sum_{k=0}^{\infty}\rho^{k}\psi_{k}(\mu,\mathfrak{I})\right)_{\mu\mu}+\sum_{k=0}^{\infty}\rho^{k}\psi_{k}(\mu,\mathfrak{I})\left(1-\sum_{k=0}^{\infty}\rho^{k}\psi_{k}(\mu,\mathfrak{I})\right)\right)\right\}\right]. \tag{25}$$

Comparing the coefficient of power ρ, we get

$$\begin{aligned}
\rho^0&: \psi_0(\mu,\mathfrak{T}) = \beta,\\
\rho^1&: \psi_1(\mu,\mathfrak{T}) = S^{-1}\left[\frac{u^\alpha}{s^\alpha}S\left\{\psi_{0\mu\mu}+\psi_0-\psi_0^2\right\}\right] = \beta(1-\beta)\frac{\mathfrak{T}^\alpha}{\Gamma(\alpha+1)},\\
\rho^2&: \psi_2(\mu,\mathfrak{T}) = S^{-1}\left[\frac{u^\alpha}{s^\alpha}S\left\{\psi_{1\mu\mu}+\psi_1-2\psi_0\psi_1\right\}\right] = \beta(1-\beta)(1-2\beta)\frac{\mathfrak{T}^{2\alpha}}{\Gamma(2\alpha+1)},\\
\rho^3&: \psi_3(\mu,\mathfrak{T}) = S^{-1}\left[\frac{u^\alpha}{s^\alpha}S\left\{\psi_{2\mu\mu}+\psi_2-\psi_1^2-2\psi_0\psi_2\right\}\right] = \left(\beta-5\beta^2+8\beta^3-4\beta^4\right)\frac{\mathfrak{T}^{3\alpha}}{\Gamma(3\alpha+1)}\\
&\quad -\left(\beta^2-2\beta^3+\beta^4\right)\frac{\Gamma(2\alpha+1)}{\Gamma(\alpha+1)^2}\frac{\mathfrak{T}^{3\alpha}}{\Gamma(3\alpha+1)}.\\
&\vdots
\end{aligned}\tag{26}$$

Now, by taking $\rho \longrightarrow 1$, we obtain convergent series form solution as

$$\begin{aligned}
\psi(\mu,\mathfrak{T}) &= \psi_0+\psi_1+\psi_2+\psi_3+\cdots,\\
\psi(\mu,\mathfrak{T}) &= \beta+\beta(1-\beta)\frac{\mathfrak{T}^\alpha}{\Gamma(\alpha+1)}+\beta(1-\beta)(1-2\beta)\\
&\quad \frac{\mathfrak{T}^{2\alpha}}{\Gamma(2\alpha+1)}+\left(\beta-5\beta^2+8\beta^3-4\beta^4\right)\frac{\mathfrak{T}^{3\alpha}}{\Gamma(3\alpha+1)}\\
&\quad -\left(\beta^2-2\beta^3+\beta^4\right)\left(\frac{\Gamma(2\alpha+1)}{\Gamma(\alpha+1)^2}\right)\frac{\mathfrak{T}^{3\alpha}}{\Gamma(3\alpha+1)}+\cdots.
\end{aligned}\tag{27}$$

Putting $\alpha=1$, we get the same solution,

$$\psi(\mu,\mathfrak{T}) = \frac{\beta\exp^\mu}{1-\beta+\beta\exp^{\mathfrak{T}}}.\tag{28}$$

Figure 1 compares the exact solution and approximate solution for the nonlinear fractional-order Fisher equation at $\alpha=1$. Figure 2 represents the graph of 2D of exact and analytical solutions and the second graph in Figure 2 shows the different fractional-order graphs of α.

4.2. Example. Consider the fractional-order Fisher equation is given by

$$D_{\mathfrak{T}}^\alpha\psi = \psi_{\mu\mu}+6\psi(1-\psi),\quad 0<\alpha\le 1,\tag{29}$$

with initial conditions

$$\psi(\mu,0) = \frac{1}{(1+\exp^\mu)^2}.\tag{30}$$

Applying Shehu transform of (29), we have

$$\begin{aligned}
&\frac{s^\alpha}{u^\alpha}S[\psi(\mu,\mathfrak{T})]-\frac{s^{\alpha-1}}{u^\alpha}\psi(\mu,0) = S\left(\psi_{\mu\mu}+6\psi(1-\psi)\right).\\
&S[\psi(\mu,\mathfrak{T})] = \frac{1}{s}\frac{1}{(1+\exp^\mu)^2}+\frac{u^\alpha}{s^\alpha}\\
&\quad\cdot\left[S\left(\psi_{\mu\mu}+6\psi(1-\psi)\right)\right].
\end{aligned}\tag{31}$$

Using inverse Shehu transformation, we get

$$\psi(\mu,\mathfrak{T}) = \frac{1}{(1+\exp^\mu)^2}+S^{-1}\left[\frac{u^\alpha}{s^\alpha}\left\{S\left(\psi_{\mu\mu}+6\psi(1-\psi)\right)\right\}\right].\tag{32}$$

Applying the abovementioned homotopy perturbation technique as in (18), we get

$$\sum_{k=0}^\infty\rho^k\psi_k(\mu,\mathfrak{T}) = \frac{1}{(1+\exp^\mu)^2}+\rho\left[S^{-1}\left\{\frac{u^\alpha}{s^\alpha}S\left(\left(\sum_{k=0}^\infty\rho^k\psi_k(\mu,\mathfrak{T})\right)_{\mu\mu}+6\sum_{k=0}^\infty\rho^k\psi_k(\mu,\mathfrak{T})\left(1-\sum_{k=0}^\infty\rho^k\psi_k(\mu,\mathfrak{T})\right)\right)\right\}\right].\tag{33}$$

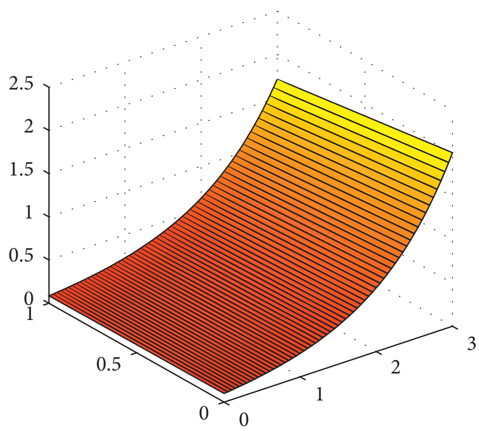

Figure 1: The graphs of exact and HPTM solutions for equation (21) at $\alpha = 1$.

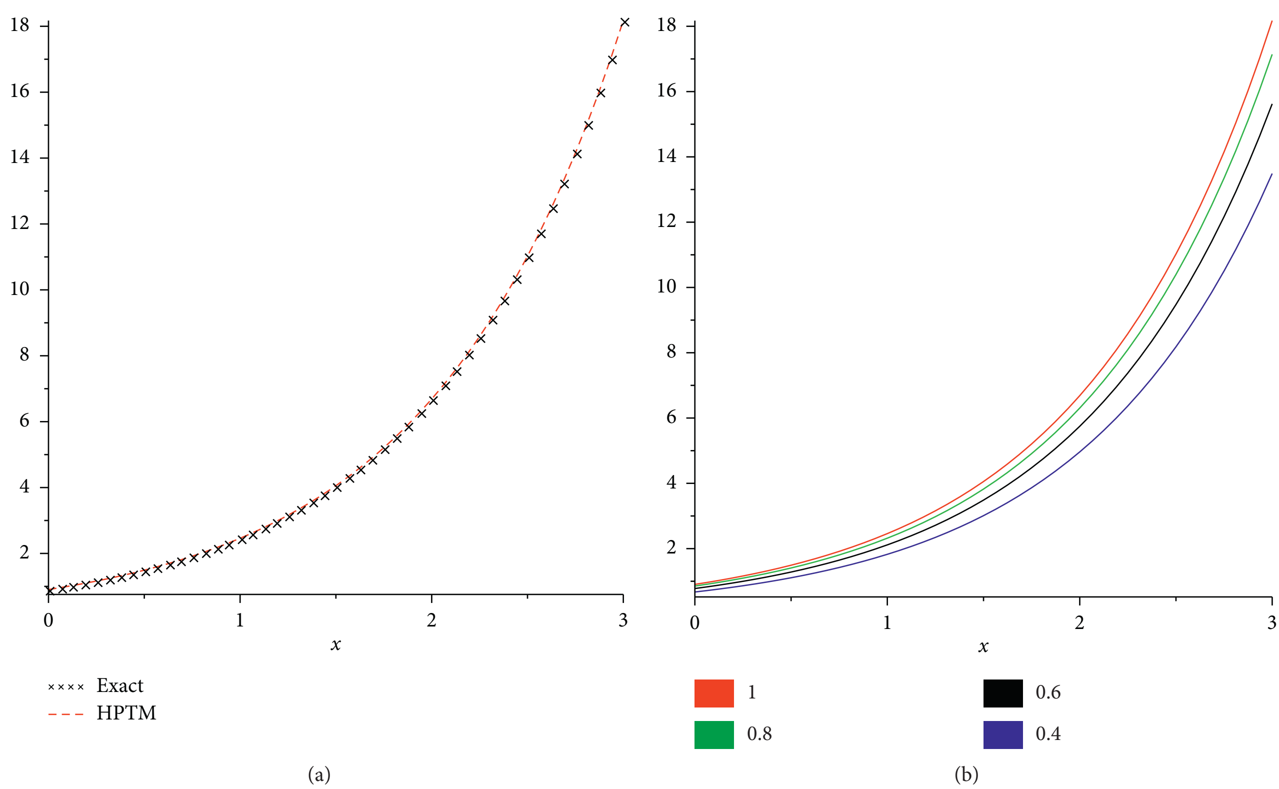

Figure 2: The graphs of exact and HPTM solutions and different fractional-order α of example 1.

Comparing the coefficient of power ρ, we get

$$\begin{aligned}
\rho^0&: \psi_0(\mu,\mathfrak{I}) = \frac{1}{(1+\exp^{\mu})^2},\\
\rho^1&: \psi_1(\mu,\mathfrak{I}) = S^{-1}\left[\frac{u^\alpha}{s^\alpha}S\left\{\psi_{0\mu\mu}+6\psi_0-6\psi_0^2\right\}\right] = 10\frac{\exp^{\mu}}{(1+\exp^{\mu})^3}\frac{\mathfrak{I}^\alpha}{\Gamma(\alpha+1)},\\
\rho^2&: \psi_2(\mu,\mathfrak{I}) = S^{-1}\left[\frac{u^\alpha}{s^\alpha}S\left\{\psi_{1\mu\mu}+6\psi_1-12\psi_0\psi_1\right\}\right] = 50\frac{\exp^{\mu}(-1+2\exp^{\mu})}{(1+\exp^{\mu})^4}\frac{\mathfrak{I}^{2\alpha}}{\Gamma(2\alpha+1)},\\
\rho^3&: \psi_3(\mu,\mathfrak{I}) = S^{-1}\left[\frac{u^\alpha}{s^\alpha}S\left\{\psi_{2\mu\mu}+6\psi_2-6\psi_1^2-12\psi_0\psi_2\right\}\right] = 50\exp^{\mu}(5-6\exp^{\mu}\\
&\quad -15\exp^{2\mu},n+20\exp^{3\mu}-12\exp^{\mu}\frac{\Gamma(2\alpha+1)}{(\Gamma(\alpha+1))^2}\bigg)\frac{\mathfrak{I}^{3\alpha}}{(1+\exp^{\mu})^6\,\Gamma(3\alpha+1)}.\\
&\vdots
\end{aligned} \tag{34}$$

Now, by taking $\rho \longrightarrow 1$, we obtain convergent series form solution as

$$\begin{aligned}
\psi(\mu,\mathfrak{I}) &= \psi_0+\psi_1+\psi_2+\psi_3+\cdots\\
&= \frac{1}{(1+\exp^{\mu})^2}+10\frac{\exp^{\mu}}{(1+\exp^{\mu})^3}\frac{\mathfrak{I}^\alpha}{\Gamma(\alpha+1)}+50\frac{\exp^{\mu}(-1+2\exp^{\mu})}{(1+\exp^{\mu})^4}\frac{\mathfrak{I}^{2\alpha}}{\Gamma(2\alpha+1)}\\
&\quad +50\exp^{\mu}\left(5-6\exp^{\mu}-15\exp^{2\mu}+20\exp^{3\mu}-12\exp^{\mu}\frac{\Gamma(2\alpha+1)}{(\Gamma(\alpha+1))^2}\right)\frac{\mathfrak{I}^{3\alpha}}{(1+\exp^{\mu})^6\,\Gamma(3\alpha+1)}+\cdots.
\end{aligned} \tag{35}$$

Putting $\alpha = 1$, we get the same solution

$$\psi(\mu,\mathfrak{I}) = \frac{1}{\left(1-\exp^{\mu-5\mathfrak{I}}\right)^2}. \tag{36}$$

Figure 3 compares the exact solution and approximate solution for the nonlinear fractional-order Fisher equation at $\alpha = 1$. Figure 3 represents the graph of 2D of exact and analytical solutions and the second graph in Figure 3 shows the different fractional-order graphs of α of example 2.

4.3. Example. Consider the fractional-order Fisher equation is given by

$$D_{\mathfrak{I}}^{\alpha}\psi = \psi_{\mu\mu}+\psi\left(1-\psi^6\right), \quad 0<\alpha\le 1, \tag{37}$$

with initial conditions

$$\psi(\mu,0) = \frac{1}{\left(1+\exp^{(3/2)\mu}\right)^{(1/3)}}. \tag{38}$$

Applying Shehu transform of (37), we have

$$\begin{aligned}
&\frac{s^\alpha}{u^\alpha}S[\psi(\mu,\mathfrak{I})]-\frac{s^{\alpha-1}}{u^\alpha}\psi(\mu,0) = S\left(\psi_{\mu\mu}+\psi\left(1-\psi^6\right)\right),\\
&S[\psi(\mu,\mathfrak{I})] = \frac{1}{s}\frac{1}{\left(1+\exp^{(3/2)\mu}\right)^{(1/3)}}+\frac{u^\alpha}{s^\alpha}\\
&\quad\cdot\left[S\left(\psi_{\mu\mu}+\psi\left(1-\psi^6\right)\right)\right].
\end{aligned} \tag{39}$$

Using inverse Shehu transformation, we get

$$\begin{aligned}
\psi(\mu,\mathfrak{I}) &= \frac{1}{\left(1+\exp^{(3/2)\mu}\right)^{(1/3)}}+S^{-1}\\
&\quad\cdot\left[\frac{u^\alpha}{s^\alpha}\left\{S\left(\psi_{\mu\mu}+\psi\left(1-\psi^6\right)\right)\right\}\right].
\end{aligned} \tag{40}$$

Applying the abovementioned homotopy perturbation technique as in (18), we get

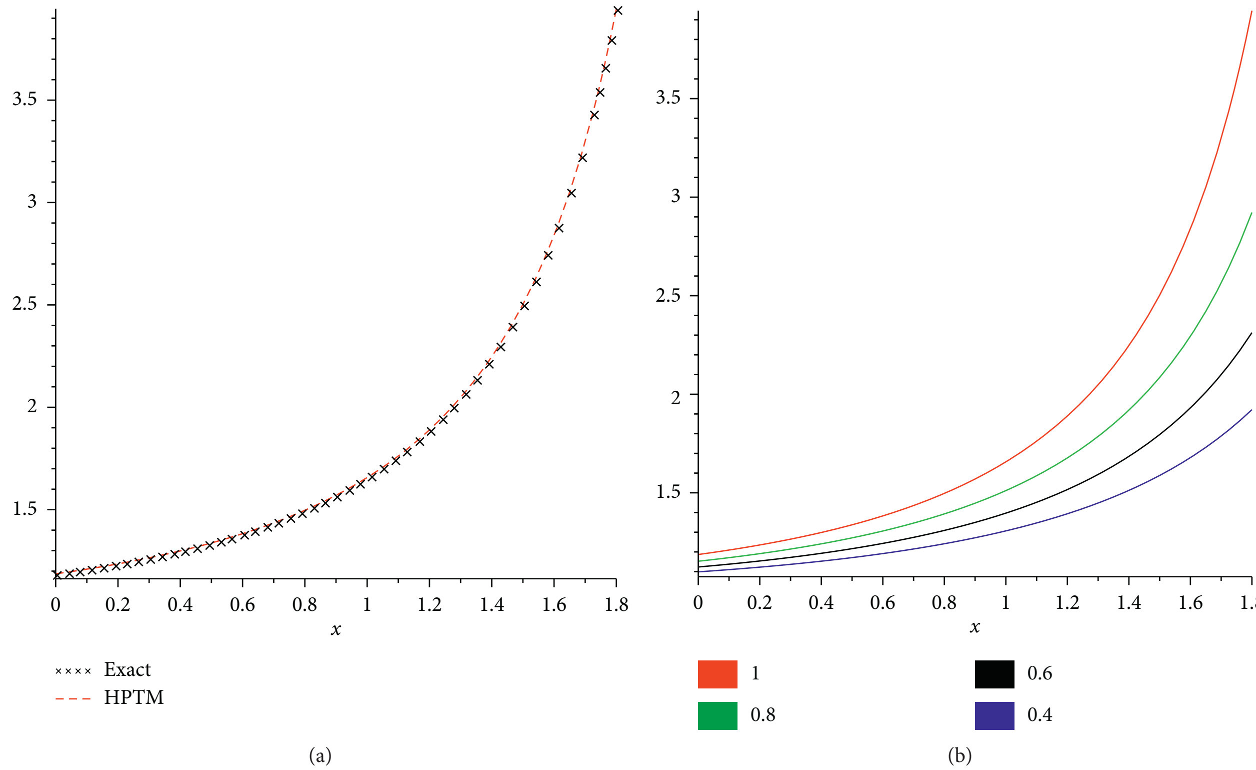

FIGURE 3: The graphs of exact and HPTM solutions and different fractional-order α for equation (29).

$$\sum_{k=0}^{\infty}\rho^{k}\psi_{k}(\mu,\mathfrak{I})=\frac{1}{\left(1+\ \exp^{(3/2)\mu}\right)^{(1/3)}}+\rho\left[S^{-1}\left\{\frac{u^{\alpha}}{s^{\alpha}}S\left(\left(\sum_{k=0}^{\infty}\rho^{k}\psi_{k}(\mu,\mathfrak{I})\right)_{\mu\mu}+\sum_{k=0}^{\infty}\rho^{k}\psi_{k}(\mu,\mathfrak{I})\left(1-\left(\sum_{k=0}^{\infty}\rho^{k}\psi_{k}(\mu,\mathfrak{I})\right)^{6}\right)\right)\right\}\right]. \tag{41}$$

Comparing the coefficient of the same power of ρ, we get

$$\begin{aligned}
&\rho^{0}:\ \psi_{0}(\mu,\mathfrak{I})=\frac{1}{\left(1+\ \exp^{(3/2)\mu}\right)^{(1/3)}},\\
&\rho^{1}:\ \psi_{1}(\mu,\mathfrak{I})=S^{-1}\left[\frac{u^{\alpha}}{s^{\alpha}}S\left\{\psi_{0\mu\mu}+\psi_{0}-\psi_{0}^{7}\right\}\right]=\frac{5\ \exp^{(3/2)\mu}}{4\left(1+\ \exp^{(3/2)\mu}\right)^{(4/3)}}\frac{\mathfrak{I}^{\alpha}}{\Gamma(\alpha+1)},\\
&\rho^{2}:\ \psi_{2}(\mu,\mathfrak{I})=S^{-1}\left[\frac{u^{\alpha}}{s^{\alpha}}S\left\{\psi_{1\mu\mu}+\psi_{1}-7\psi_{0}^{6}\psi_{1}\right\}\right]=\frac{25\ \exp^{(3/2)\mu}\left(\exp^{(3/2)\mu}-3\right)}{16\left(1+\ \exp^{(3/2)\mu}\right)^{(7/3)}}\frac{\mathfrak{I}^{2\alpha}}{\Gamma(2\alpha+1)},\\
&\quad\vdots
\end{aligned} \tag{42}$$

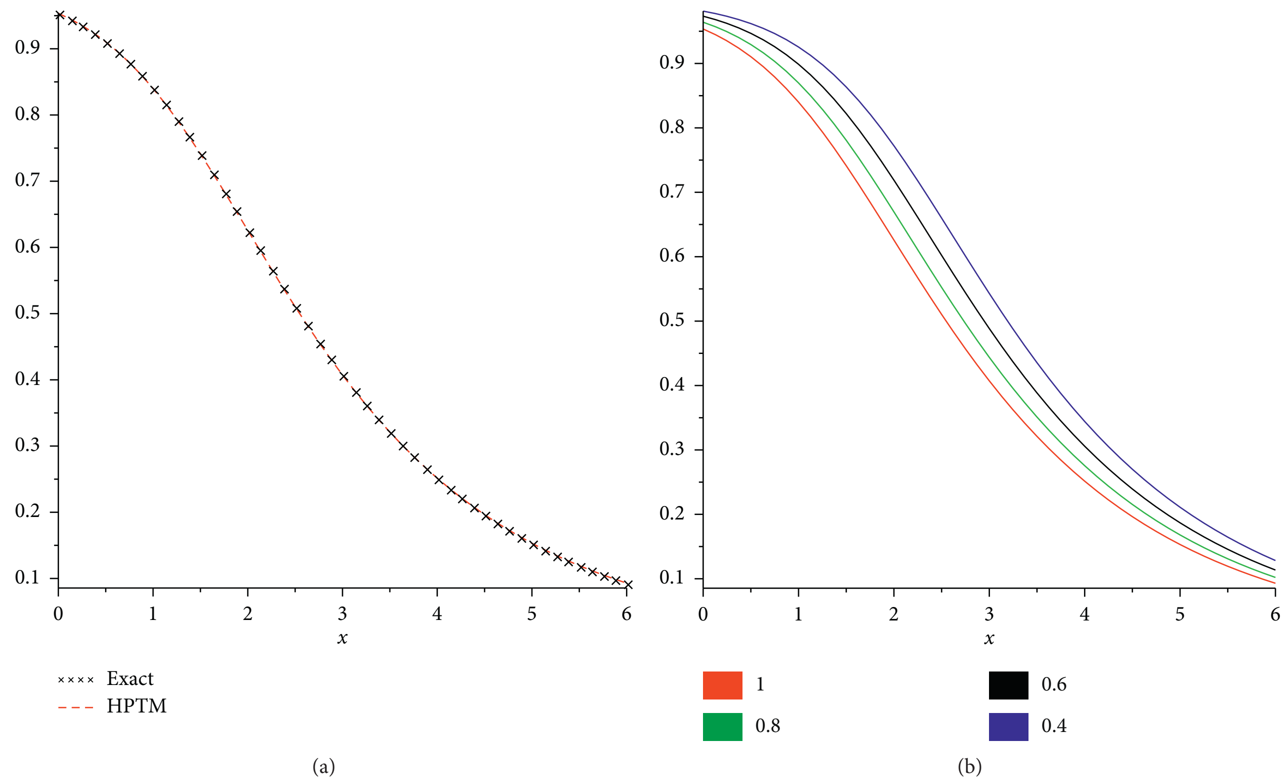

Figure 4: The graphs of exact and HPTM solutions and different fractional-order α for equation (37).

Now, by taking $\rho \longrightarrow 1$, we obtain convergent series form solution as

$$\psi(\mu, \mathfrak{I}) = \psi_0 + \psi_1 + \psi_2 + \psi_3 + \cdots.$$

$$\psi(\mu, \mathfrak{I}) = \frac{1}{\left(1 + \exp^{(3/2)\mu}\right)^{1/3}} + \frac{5 \exp^{(3/2)\mu}}{4\left(1 + \exp^{(3/2)\mu}\right)^{4/3}} \frac{\mathfrak{I}^{\alpha}}{\Gamma(\alpha + 1)} + \frac{25 \exp^{(3/2)\mu}\left(\exp^{(3/2)\mu} - 3\right)}{16\left(1 + \exp^{(3/2)\mu}\right)^{(7/3)}} \frac{\mathfrak{I}^{2\alpha}}{\Gamma(2\alpha + 1)} + \cdots. \tag{43}$$

Putting $\alpha = 1$, we get the same solution

$$\psi(\mu, \mathfrak{I}) = \left\{\frac{1}{2}\tanh\left(\frac{15}{8}\mathfrak{I} - \frac{3}{4}\mu\right) + \frac{1}{2}\right\}^{(1/3)}. \tag{44}$$

Figure 4 compares the exact solution and approximate solution for the nonlinear fractional-order Fisher equation at $\alpha = 1$. Figure 4 represents the graph of 2D of exact and analytical solutions and the second graph in Figure 4 shows the different fractional-order graphs of α.

5. Conclusion

In this paper, some computational works have been done to analyze Fisher's equations' fractional view analysis. For this purpose, the Shehu transformation is mixed with the homotopy perturbation method and derived a useful hybrid technique to handle the solution. The graphical representation of the solution of some illustrative examples is shown to be in closed contact. The fractional problem solution is convergent toward the integer-order solutions. Moreover, the accuracy of the proposed method is high and required less number of calculations. The suggested method can solve other fractional-order problems because of its simple and straight forward implementation.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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