Research Article

Multifuzzy Cubic Sets and Their Correlation Coefficients for Multicriteria Group Decision-Making

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The notion of multifuzzy sets (MFSs) or multi-interval-valued fuzzy sets (MIVFSs) provides a new method to represent some problems with a sequence of the different and/or same fuzzy/interval-valued fuzzy membership values of an element to the set. Then, a fuzzy cubic set (FCS) consists of a certain part (a fuzzy value) and an uncertain part (an interval-valued fuzzy value) but cannot represent hybrid information of both MFS and MIVFS. To adequately depict the opinion of several experts/decision-makers by using a union/sequence of the different and/or same fuzzy cubic values for an object assessed in group decision-making (GDM) problems, this paper proposes a multifuzzy cubic set (MFCS) notion as the conceptual extension of FCS to express the hybrid information of both MFS and MIVFS in the fuzzy setting of both uncertainty and certainty. Then, we propose three correlation coefficients of MFCSs and then introduce correlation coefficients of MFSs and MIVFSs as special cases of the three correlation coefficients of MFCSs. Further, the multicriteria GDM methods using three weighted correlation coefficients of MFCSs are developed under the environment of MFCSs, which contains the MFS and MIVFS GDM methods. Lastly, these multicriteria GDM methods are applied in an illustrative example on the selection problem of equipment suppliers; then their decision results and comparative analysis indicate that the developed GDM methods are more practicable and effective and reflect that either different correlation coefficients or different information expressions can also impact on the ranking of alternatives. Therefore, this study indicates the main contribution of the multifuzzy cubic information expression, correlation coefficients, and GDM methods in the multifuzzy setting of both uncertainty and certainty.

1. Introduction

Fuzzy set (FS) [1] is usually depicted by a membership degree in the interval $[0, 1]$, where only indicate one occurrence of each element. Hence, FSs have been widely applied in various areas [2–10] regarding vague and incomplete real problems. As the extension of FSs, Yager [11] and Sebastian and Ramakrishnan [12] presented a fuzzy multiset/bag/multifuzzy set (MFS), which provides a better method to represent the numbers of copies of an element to the set by the different and/or same membership values. Thus, fuzzy multisets/MFSs have been utilized for various applications [13–17]. By extending the fuzzy multiset/MFS to the interval-valued fuzzy set (IVFS) [18], Kreinovich and Sriboonchitta [19] defined the notion of a multi-interval-valued fuzzy set (MIVFS), where each element can be

repeated more than once with the different and/or same interval-valued membership values to adequately describe a union/sequence of interval-valued fuzzy values (IVFVs).

Then, there exist the certainty and uncertainty of human judgments regarding complicated real-world problems. Hence, Jun et al. [20] proposed a notion of a fuzzy cubic set (FCS), which can depict the hybrid fuzzy information of the partial certainty (fuzzy value) and partial uncertainty (IVFV). Afterward, some researchers [21–24] also introduced some theory and applications of FCSs since FCS is a very useful tool for expressing potentially uncertain and certainty information provided by the decision-makers. Based on the hybrid notion of FCS and a hesitant FS [25], some researchers proposed hesitant FCSs and their decision-making methods [26, 27] due to the hesitancy of human judgments in the setting of FCSs; then some researchers

presented a cubic hesitant fuzzy set (CHFS) by means of a hybrid form of both an IVFV and several hesitant fuzzy values and after that developed similarity measures of CHFSs for medical diagnosis/assessments [28, 29] and decision-making [30]. More recently, Fu and Ye [31] proposed the notion of cubic hesitant neutrosophic numbers (CHNNs) by using a hybrid form of both an IVFV and several neutrosophic numbers and then developed the indeterminate parameter-based similarity measure of CHNNs for the risk grade assessment of prostate cancer patients. Next, Ye et al. [32] further proposed a notion of fuzzy credibility cubic numbers (FCCNs) by a hybrid form of both a cubic credibility degree and a fuzzy cubic value (FCV) and then developed two weighted aggregation operators of FCCNs and used them in the decision-making problem of slope design schemes so as to enhance the credibility degree/level of fuzzy decision-making problems. Based on the mixed form of an intuitionistic fuzzy set (IFS) and an interval-valued IFS (IVIFS), some researchers [33–35] presented cubic intuitionistic fuzzy sets (CIFSSs) and their applications in decision-making problems. Although extension forms of various fuzzy information, such as Pythagorean FSs [36], interval-valued Pythagorean FSs [37], and neutrosophic sets [38], were introduced in recent years and applied in GDM problems, they lack corresponding cubic information expression forms.

However, the hesitant FS concept can only express a set of several different cubic membership values of each element to the set in the hesitant environment, but it cannot reflect the number of copies of an element to the set by a union/sequence of the different and/or same membership values so as to lose some useful information in the group decision-making (GDM) process. Generally, the aforementioned various fuzzy cubic concepts cannot also reflect the number of copies of an element to the set by a union/sequence of the different and/or same FCVs. Then, MFS or MIVFS can reflect a union of the different and/or same fuzzy values or IVFVs of an element to the set, but MFS or MIVFS cannot express the mixed information of both MFS and MIVFS. To suitably describe the union/sequence of the different and/or same FCVs, this study presents the concept of MFCS based on the hybrid information of MFS and MIVFS. The MFCS notion can provide a new expression form to overcome the insufficiency of the hesitant FCS and to extend the notion of FCS. Thus, MFCS can describe the opinions that the several experts or decision-makers propose by using a union/sequence of the different and/or same FCVs in GDM problems. Next, we propose three correlation coefficients of MFCSs and then introduce the correlation coefficients of MFSs and MIVFSs as special cases of the three correlation coefficients of MFCSs. GDM methods using the weighted correlation coefficients of MFCSs (including weighted correlation coefficients of MFSs and MIVFSs) are developed and used in an illustrative example on the selection problem of equipment suppliers. Finally, the comparative analysis of the developed GDM methods and the related methods are investigated to illustrate the practicability and suitability of the developed GDM methods in the setting MFCSs.

In the original study, the main contributions are summarized in the following:

- (i) The proposed MFCS can solve the hybrid information expression problem of both MFS and MIVFS and provide an adequate expression form to depict the opinion of several experts/decision-makers by using a union/sequence of several different and/or the same FCVs in multicriteria GDM problems
- (ii) The proposed correlation coefficients of MFCSs can provide effective mathematical models for the multicriteria GDM methods in the environments of MFCSs, MFSs, and MIVFSs
- (iii) The developed multifuzzy cubic GDM methods can solve GDM problems under the environment of MFCSs and show their superiority and usability in the setting of MFCSs

For this study, this paper is composed of the following structures. Section 2 introduces some notions of MFSs, MIVFSs, and FCSs. In Section 3, we present the concept of MFCS based on the hybrid information of MFS and MIVFS and define the relations of multifuzzy cubic values (MFCVs). Section 4 proposes three correlation coefficients of MFCSs and then introduces the correlation coefficients of MFSs and MIVFSs as special cases of the three correlation coefficients of MFCSs. Section 5 develops multicriteria GDM methods using the proposed weighted correlation coefficients of MFCSs in the MFCS setting. In Section 6, the developed GDM methods are used for an illustrative example on the selection problem of equipment suppliers, and then the comparative analysis of the developed multicriteria GDM methods and the related methods are investigated to illustrate the practicability and suitability of the developed GDM methods in the setting of MFCSs. Conclusions and further work are included in Section 7.

2. Some Notions of MFSs, MIVFSs, and FCSs

Set $U = \{u_1, u_2, \dots, u_n\}$ as a universe set. Then, a MFS S on U is defined as the following form [11, 12]:

$$S = \left\{ \langle u_j, M_S(u_j) \rangle \mid u_j \in U \right\}, \quad (1)$$

where $M_S(u_j)$ is the membership sequence denoted by $(m_{S1}(u_j), m_{S2}(u_j), \dots, m_{Sq(j)}(u_j))$ for $m_{Sk}(u_j) \in [0, 1]$ ($k = 1, 2, \dots, q(j); j = 1, 2, \dots, n$) and $u_j \in U$, which is named a MFS/FMS. In the MFS S , each element u_j in U may occur more than once with the different and/or same membership values.

To depict the fuzzy uncertain information in real-world problems, Kreinovich and Sriboonchitta [19] put forward a MIVFS T on a universe set $U = \{u_1, u_2, \dots, u_n\}$ and defined the following form:

$$T = \left\{ \langle u_j, M_T(u_j) \rangle \mid u_j \in U \right\}, \quad (2)$$

where $M_T(u_j)$ is a union/sequence of the interval-valued membership values defined by $(m_{T1}(u_j), m_{T2}(u_j), \dots, m_{Tq(j)}(u_j))$ for $m_{Tk}(u_j) \subseteq [0, 1]$ ($k = 1, 2, \dots, q(j); j = 1, 2, \dots, n$) and

$u_j \in U$, which is named a MIVFS. In the MIVFS T , each element u_j in U may occur more than once with the different and/or same interval-valued membership values.

To represent the fuzzy hybrid information of the uncertainty and the certainty in real-world problems, Jun et al. [20] put forward a FCS on a universe set $U = \{u_1, u_2, \dots, u_n\}$, which is denoted by

$$C = \{ \langle u_j, A_C(u_j), \alpha_C(u_j) \rangle \mid u_j \in U \}, \quad (3)$$

where $A_C(u_j): U \rightarrow [0, 1]$ and $\alpha_C(u_j): U \rightarrow [0, 1]$ are an IVFV and a fuzzy value, respectively, such that $A_C(u_j) = [a(u_j), b(u_j)] \subseteq [0, 1]$ and $\alpha_C(u_j) \in [0, 1]$ for $u_j \in U$ ($j = 1, 2, \dots, n$). In the FCS C , each element u_j in U only occurs once with the hybrid information of both an interval-valued membership value and a single-valued membership value.

For the simplified expression, each element $\langle u_j, A_C(u_j), \alpha_C(u_j) \rangle$ ($j = 1, 2, \dots, n$) in C is simply denoted as $c_j = \langle [a_j, b_j], \alpha_j \rangle$ and named FCV. If $a_j \leq \alpha_j \leq b_j$, then $c_j = \langle [a_j, b_j], \alpha_j \rangle$ is named an internal FCV; if $\alpha_j \notin [a_j, b_j]$, then $c_j = \langle [a_j, b_j], \alpha_j \rangle$ is named an external FCV.

For any two FCVs $c_{1j} = \langle [a_{1j}, b_{1j}], \alpha_{1j} \rangle$ and $c_{2j} = \langle [a_{2j}, b_{2j}], \alpha_{2j} \rangle$, their relations are introduced as follows [20]:

- (1) $c_{1j} \supseteq c_{2j} \Leftrightarrow [a_{1j}, b_{1j}] \supseteq [a_{2j}, b_{2j}]$ and $\alpha_{1j} \geq \alpha_{2j}$
- (2) $c_{1j} = c_{2j} \Leftrightarrow c_{1j} \supseteq c_{2j}$ and $c_{2j} \supseteq c_{1j}$, namely, $[a_{1j}, b_{1j}] = [a_{2j}, b_{2j}]$ and $\alpha_{1j} = \alpha_{2j}$
- (3) $c_{1j} \cup c_{2j} = \langle [a_{1j} \vee a_{2j}, b_{1j} \vee b_{2j}], \alpha_{1j} \vee \alpha_{2j} \rangle$
- (4) $c_{1j} \cap c_{2j} = \langle [a_{1j} \wedge a_{2j}, b_{1j} \wedge b_{2j}], \alpha_{1j} \wedge \alpha_{2j} \rangle$
- (5) $c_{1j}^c = \langle [1 - b_{1j}, 1 - a_{1j}], 1 - \alpha_{1j} \rangle$ (complement of c_{1j})

3. Multifuzzy Cubic Sets (MFCSSs)

Based on the hybrid concept of MFS and MIVFS, this section presents the concept of MFCS as an extension of the FCS notion and defines the relations of MFCVs.

Definition 1. Set $U = \{u_1, u_2, \dots, u_n\}$ as a universe set. A MFCS M over U is defined by the following mathematical expression:

$$M = \left\{ \left\langle \begin{array}{l} u_j, ([a_{M1}(u_j), b_{M1}(u_j)], \alpha_{M1}(u_j)), \\ [a_{M2}(u_j), b_{M2}(u_j)], \alpha_{M2}(u_j), \dots, \\ ([a_{Mq(j)}(u_j), b_{Mq(j)}(u_j)], \alpha_{Mq(j)}(u_j)) \end{array} \right\rangle \mid u_j \in U \right\}, \quad (4)$$

where $([a_{M1}(u_j), b_{M1}(u_j)], \alpha_{M1}(u_j)), ([a_{M2}(u_j), b_{M2}(u_j)], \alpha_{M2}(u_j)), \dots, ([a_{Mq(j)}(u_j), b_{Mq(j)}(u_j)], \alpha_{Mq(j)}(u_j))$ is a membership sequence of $q(j)$ copies of an element u_j to the set M , such that IVFVs $[a_{Mk}(u_j), b_{Mk}(u_j)] \subseteq [0, 1]$ and fuzzy values $\alpha_{Mk}(u_j) \in [0, 1]$ ($j = 1, 2, \dots, n$; $k = 1, 2, \dots, q(j)$) for $u_j \in U$. In the MFCS M , each element u_j may occur more than once with the different and/or same FCVs.

For example, there is a MFCS $M_1 = \{ \langle u_1, ([0.7, 0.9], 0.8), ([0.7, 0.9], 0.8), ([0.5, 0.7], 0.6) \rangle, \langle u_2, ([0.7, 0.8], 0.7), ([0.7, 0.8], 0.7), ([0.6, 0.7], 0.6) \rangle \}$ in the universe set $U = \{u_1, u_2\}$.

In particular, when $q(j) = 1$ ($j = 1, 2, \dots, n$) in M , the MFCS M is reduced to FCS. If $\alpha_{jk} \in [a_{jk}, b_{jk}]$ ($j = 1, 2, \dots, n$; $k = 1, 2, \dots, q(j)$), M is named the internal MFCS; and if $\alpha_{jk} \notin [a_{jk}, b_{jk}]$ ($j = 1, 2, \dots, n$; $k = 1, 2, \dots, q(j)$), M is named the external MFCS.

Each MFCV $\langle u_j, ([a_{M1}(u_j), b_{M1}(u_j)], \alpha_{M1}(u_j)), ([a_{M2}(u_j), b_{M2}(u_j)], \alpha_{M2}(u_j)), \dots, ([a_{Mq(j)}(u_j), b_{Mq(j)}(u_j)], \alpha_{Mq(j)}(u_j)) \rangle$ ($j = 1, 2, \dots, n$) in M is simply denoted as $m_j = \langle ([a_{j1}, b_{j1}], \alpha_{j1}), ([a_{j2}, b_{j2}], \alpha_{j2}), \dots, ([a_{jq(j)}, b_{jq(j)}], \alpha_{jq(j)}) \rangle$.

Definition 2. Set any two MFCVs as $m_{1j} = \langle ([a_{1j1}, b_{1j1}], \alpha_{1j1}), ([a_{1j2}, b_{1j2}], \alpha_{1j2}), \dots, ([a_{1jq(j)}, b_{1jq(j)}], \alpha_{1jq(j)}) \rangle$ and $m_{2j} = \langle ([a_{2j1}, b_{2j1}], \alpha_{2j1}), ([a_{2j2}, b_{2j2}], \alpha_{2j2}), \dots, ([a_{2jq(j)}, b_{2jq(j)}], \alpha_{2jq(j)}) \rangle$ ($j = 1, 2, \dots, n$). Then, their relations are defined as follows:

- (1) $m_{1j} \subseteq m_{2j} \Leftrightarrow [a_{1jk}, b_{1jk}] \subseteq [a_{2jk}, b_{2jk}]$ and $\alpha_{1jk} \leq \alpha_{2jk}$ ($k = 1, 2, \dots, q(j)$);

- (2) $m_{1j} = m_{2j} \Leftrightarrow m_{1j} \subseteq m_{2j}$ and $m_{2j} \subseteq m_{1j}$, namely, $[a_{1jk}, b_{1jk}] = [a_{2jk}, b_{2jk}]$ and $\alpha_{1jk} = \alpha_{2jk}$ ($k = 1, 2, \dots, q(j)$)
- (3) $m_{1j} \cup m_{2j} = \langle ([a_{1j1} \vee a_{2j1}, b_{1j1} \vee b_{2j1}], \alpha_{1j1} \vee \alpha_{2j1}), ([a_{1j2} \vee a_{2j2}, b_{1j2} \vee b_{2j2}], \alpha_{1j2} \vee \alpha_{2j2}), \dots, ([a_{1jq(j)} \vee a_{2jq(j)}, b_{1jq(j)} \vee b_{2jq(j)}], \alpha_{1jq(j)} \vee \alpha_{2jq(j)}) \rangle$
- (4) $m_{1j} \cap m_{2j} = \langle ([a_{1j1} \wedge a_{2j1}, b_{1j1} \wedge b_{2j1}], \alpha_{1j1} \wedge \alpha_{2j1}), ([a_{1j2} \wedge a_{2j2}, b_{1j2} \wedge b_{2j2}], \alpha_{1j2} \wedge \alpha_{2j2}), \dots, ([a_{1jq(j)} \wedge a_{2jq(j)}, b_{1jq(j)} \wedge b_{2jq(j)}], \alpha_{1jq(j)} \wedge \alpha_{2jq(j)}) \rangle$
- (5) $m_{1j}^c = \langle ([1 - b_{1j1}, 1 - a_{1j1}], 1 - \alpha_{1j1}), ([1 - b_{1j2}, 1 - a_{1j2}], 1 - \alpha_{1j2}), \dots, ([1 - b_{1jq(j)}, 1 - a_{1jq(j)}], 1 - \alpha_{1jq(j)}) \rangle$ (complement of m_{1j})

4. Correlation Coefficients of MFCSSs

Three correlation coefficients of MFCSSs and their special cases are proposed in this section under environments of MFCSSs, MFSs, and MIVFSs.

Definition 3. Set $M_1 = \{m_{11}, m_{12}, \dots, m_{1n}\}$ and $M_2 = \{m_{21}, m_{22}, \dots, m_{2n}\}$ as two MFCSSs, where $m_{1j} = \langle ([a_{1j1}, b_{1j1}], \alpha_{1j1}), ([a_{1j2}, b_{1j2}], \alpha_{1j2}), \dots, ([a_{1jq(j)}, b_{1jq(j)}], \alpha_{1jq(j)}) \rangle$ and $m_{2j} = \langle ([a_{2j1}, b_{2j1}], \alpha_{2j1}), ([a_{2j2}, b_{2j2}], \alpha_{2j2}), \dots, ([a_{2jq(j)}, b_{2jq(j)}], \alpha_{2jq(j)}) \rangle$ ($j = 1, 2, \dots, n$) are MFCVs. Then, three correlation coefficients between M_1 and M_2 are proposed as follows:

$$\begin{aligned}
C_1(M_1, M_2) &= \frac{N(M_1, M_2)}{\sqrt{N(M_1, M_1)}\sqrt{N(M_2, M_2)}} = \frac{\sum_{j=1}^n \sum_{k=1}^{q(j)} (a_{1jk}a_{2jk} + b_{1jk}b_{2jk} + \alpha_{1jk}\alpha_{2jk})}{\sqrt{\sum_{j=1}^n \sum_{k=1}^{q(j)} (a_{1jk}^2 + b_{1jk}^2 + \alpha_{1jk}^2)}\sqrt{\sum_{j=1}^n \sum_{k=1}^{q(j)} (a_{2jk}^2 + b_{2jk}^2 + \alpha_{2jk}^2)}}, \\
C_2(M_1, M_2) &= \frac{N(M_1, M_2)}{\max(N(M_1, M_1), N(M_2, M_2))} = \frac{\sum_{j=1}^n \sum_{k=1}^{q(j)} (a_{1jk}a_{2jk} + b_{1jk}b_{2jk} + \alpha_{1jk}\alpha_{2jk})}{\max(\sum_{j=1}^n \sum_{k=1}^{q(j)} (a_{1jk}^2 + b_{1jk}^2 + \alpha_{1jk}^2), \sum_{j=1}^n \sum_{k=1}^{q(j)} (a_{2jk}^2 + b_{2jk}^2 + \alpha_{2jk}^2))}, \quad (5) \\
C_3(M_1, M_2) &= \frac{\sum_{j=1}^n (m_{1j} \wedge m_{2j})}{\sum_{j=1}^n (m_{1j} \vee m_{2j})} = \frac{\sum_{j=1}^n \sum_{k=1}^{q(j)} (a_{1jk} \wedge a_{2jk} + b_{1jk} \wedge b_{2jk} + \alpha_{1jk} \wedge \alpha_{2jk})}{\sum_{j=1}^n \sum_{k=1}^{q(j)} (a_{1jk} \vee a_{2jk} + b_{1jk} \vee b_{2jk} + \alpha_{1jk} \vee \alpha_{2jk})},
\end{aligned}$$

where $N(M_1, M_2) = \sum_{j=1}^n \sum_{k=1}^{q(j)} (a_{1jk}a_{2jk} + b_{1jk}b_{2jk} + \alpha_{1jk}\alpha_{2jk})$ is a correlation between M_1 and M_2 , $N(M_1, M_1) = \sum_{j=1}^n \sum_{k=1}^{q(j)} (a_{1jk}^2 + b_{1jk}^2 + \alpha_{1jk}^2)$ and $N(M_2, M_2) = \sum_{j=1}^n \sum_{k=1}^{q(j)} (a_{2jk}^2 + b_{2jk}^2 + \alpha_{2jk}^2)$ are information energy of M_1 and M_2 ; then, the symbols “ $\vee$ ” and “ $\wedge$ ” are the maximum and minimum operations.

Regarding the properties of the correlation coefficient [17], the correlation coefficients $C_p(M_1, M_2)$ ($p = 1, 2, 3$) also indicate the following proposition.

Proposition 1. *The correlation coefficients $C_p(M_1, M_2)$ ($p = 1, 2, 3$) include the following properties:*

$$(P1) \quad C_p(M_1, M_2) = 1 \text{ for } M_1 = M_2$$

$$(P2) \quad C_p(M_1, M_2) = C_p(M_2, M_1)$$

$$(P3) \quad C_p(M_1, M_2) \in [0, 1]$$

Proof. It is obvious that the verification of the properties (P1) and (P2) in Proposition 1 is straightforward. Hence, one only verifies the property (P3).

Based on the Cauchy-Schwarz inequality $(\sum_{j=1}^n y_j^2) \times (\sum_{j=1}^n z_j^2) \geq (\sum_{j=1}^n y_j z_j)^2$ for any positive real numbers y_j and z_j ($j = 1, 2, \dots, n$), there is the inequality $\sqrt{\sum_{j=1}^n y_j^2} \times \sqrt{\sum_{j=1}^n z_j^2} \geq \sum_{j=1}^n y_j z_j$. Hence, $C_1(M_1, M_2) \in [0, 1]$ can hold based on the inequality. Then, it is obvious that there exist the following inequalities:

$$\begin{aligned}
\max\left(\sum_{j=1}^n \sum_{k=1}^{q(j)} (a_{1jk}^2 + b_{1jk}^2 + \alpha_{1jk}^2), \sum_{j=1}^n \sum_{k=1}^{q(j)} (a_{2jk}^2 + b_{2jk}^2 + \alpha_{2jk}^2)\right) &\geq \sum_{j=1}^n \sum_{k=1}^{q(j)} (a_{1jk}a_{2jk} + b_{1jk}b_{2jk} + \alpha_{1jk}\alpha_{2jk}) \\
\sum_{j=1}^n \sum_{k=1}^{q(j)} (a_{1jk} \vee a_{2jk} + b_{1jk} \vee b_{2jk} + \alpha_{1jk} \vee \alpha_{2jk}) &\geq \sum_{j=1}^n \sum_{k=1}^{q(j)} (a_{1jk} \wedge a_{2jk} + b_{1jk} \wedge b_{2jk} + \alpha_{1jk} \wedge \alpha_{2jk}).
\end{aligned} \quad (6)$$

Thus, $C_2(M_1, M_2) \in [0, 1]$ and $C_3(M_1, M_2) \in [0, 1]$ can exist based on the above inequalities.

Therefore, the verification of Proposition 1 is completed.

If we consider the importance of MFCVs m_{1j} and m_{2j} ($j = 1, 2, \dots, n$) in M_1 and M_2 , one can give the weight $\beta_j \in [0, 1]$ of m_{1j}

and m_{2j} with $\sum_{j=1}^n \beta_j = 1$. Thus, the weighted correlation coefficients of MFCVs M_1 and M_2 can be expressed by the following formulae:

$$C_{w1}(M_1, M_2) = \frac{N_w(M_1, M_2)}{\sqrt{N_w(M_1, M_1)}\sqrt{N_w(M_2, M_2)}} = \frac{\sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} (a_{1jk}a_{2jk} + b_{1jk}b_{2jk} + \alpha_{1jk}\alpha_{2jk})}{\sqrt{\sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} (a_{1jk}^2 + b_{1jk}^2 + \alpha_{1jk}^2)}\sqrt{\sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} (a_{2jk}^2 + b_{2jk}^2 + \alpha_{2jk}^2)}}, \quad (7)$$

$$C_{w2}(M_1, M_2) = \frac{N_w(M_1, M_2)}{\max(N_w(M_1, M_1), N_w(M_2, M_2))} = \frac{\sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} (a_{1jk}a_{2jk} + b_{1jk}b_{2jk} + \alpha_{1jk}\alpha_{2jk})}{\max(\sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} (a_{1jk}^2 + b_{1jk}^2 + \alpha_{1jk}^2), \sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} (a_{2jk}^2 + b_{2jk}^2 + \alpha_{2jk}^2))}, \quad (8)$$

$$C_{w3}(M_1, M_2) = \frac{\sum_{j=1}^n \beta_j (m_{1j} \wedge m_{2j})}{\sum_{j=1}^n \beta_j (m_{1j} \vee m_{2j})} = \frac{\sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} (a_{1jk} \wedge a_{2jk} + b_{1jk} \wedge b_{2jk} + \alpha_{1jk} \wedge \alpha_{2jk})}{\sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} (a_{1jk} \vee a_{2jk} + b_{1jk} \vee b_{2jk} + \alpha_{1jk} \vee \alpha_{2jk})}. \quad (9)$$

Similarly, the weighted correlation coefficients $C_{wp}(M_1, M_2)$ ($p = 1, 2, 3$) also indicate the following proposition. $\square$

Proposition 2. *The weighted correlation coefficients $C_{wp}(M_1, M_2)$ ($p = 1, 2, 3$) contain the following properties:*

- (P1) $C_{wp}(M_1, M_2) = 1$ for $M_1 = M_2$
- (P2) $C_{wp}(M_1, M_2) = C_{wp}(M_2, M_1)$
- (P3) $C_{wp}(M_1, M_2) \in [0, 1]$

Similar to the verification of Proposition 1, the verification of Proposition 2 is straightforward. Hence, it is not repeated here.

Remark 1

- (i) When there exist all $[a_{1jk}, b_{1jk}] = [a_{2jk}, b_{2jk}] = [0, 0]$ ($k = 1, 2, \dots, q(j); j = 1, 2, \dots, n$) in M_1 and M_2 , the weighted correlation coefficients of equations (7)–(9) are reduced to the following weighted correlation coefficients of MFSSs:

$$C_{w4}(M_1, M_2) = \frac{N_w(M_1, M_2)}{\sqrt{N_w(M_1, M_1)}\sqrt{N_w(M_2, M_2)}} = \frac{\sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} \alpha_{1jk} \alpha_{2jk}}{\sqrt{\sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} \alpha_{1jk}^2} \sqrt{\sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} \alpha_{2jk}^2}}, \quad (10)$$

$$C_{w5}(M_1, M_2) = \frac{N_w(M_1, M_2)}{\max(N_w(M_1, M_1), N_w(M_2, M_2))} = \frac{\sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} \alpha_{1jk} \alpha_{2jk}}{\max(\sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} \alpha_{1jk}^2, \sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} \alpha_{2jk}^2)}, \quad (11)$$

$$C_{w6}(M_1, M_2) = \frac{\sum_{j=1}^n \beta_j (m_{1j} \wedge m_{2j})}{\sum_{j=1}^n \beta_j (m_{1j} \vee m_{2j})} = \frac{\sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} (\alpha_{1jk} \wedge \alpha_{2jk})}{\sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} (\alpha_{1jk} \vee \alpha_{2jk})}. \quad (12)$$

- (ii) When there exist all $\alpha_{1jk} = \alpha_{2jk} = 0$ ($k = 1, 2, \dots, q(j); j = 1, 2, \dots, n$), the weighted correlation coefficients

of equations (7)–(9) are reduced to the following weighted correlation coefficients of MIVFSs:

$$C_{w7}(M_1, M_2) = \frac{N_w(M_1, M_2)}{\sqrt{N_w(M_1, M_1)}\sqrt{N_w(M_2, M_2)}} = \frac{\sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} (a_{1jk} a_{2jk} + b_{1jk} b_{2jk})}{\sqrt{\sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} (a_{1jk}^2 + b_{1jk}^2)} \sqrt{\sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} (a_{2jk}^2 + b_{2jk}^2)}}, \quad (13)$$

$$C_{w8}(M_1, M_2) = \frac{N_w(M_1, M_2)}{\max(N_w(M_1, M_1), N_w(M_2, M_2))} = \frac{\sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} (a_{1jk} a_{2jk} + b_{1jk} b_{2jk})}{\max(\sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} (a_{1jk}^2 + b_{1jk}^2), \sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} (a_{2jk}^2 + b_{2jk}^2))}, \quad (14)$$

$$C_{w9}(M_1, M_2) = \frac{\sum_{j=1}^n \beta_j (m_{1j} \wedge m_{2j})}{\sum_{j=1}^n \beta_j (m_{1j} \vee m_{2j})} = \frac{\sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} (a_{1jk} \wedge a_{2jk} + b_{1jk} \wedge b_{2jk})}{\sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} (a_{1jk} \vee a_{2jk} + b_{1jk} \vee b_{2jk})}. \quad (15)$$

- (iii) Since the above-weighted correlation coefficients of MFSSs and MFCSSs are special cases of the weighted correlation coefficients of MFCSSs, the three weighted correlation coefficients of MFCSSs are more general and more useful in actual applications.

5. GDM Methods Using the Weighted Correlation Coefficients of MFCSSs

This section develops multicriteria GDM methods using the weighted correlation coefficients in the environment of MFCSSs.

In the GDM process, there usually exists a set of alternatives $M = \{M_1, M_2, \dots, M_s\}$ to be assessed by a set of criteria $E = \{e_1,$

$e_2, \dots, e_n\}$ regarding a multicriteria GDM problem. Then, a weight vector $\beta = (\beta_1, \beta_2, \dots, \beta_n)$ is specified to consider the importance weights of various criteria e_k ($j = 1, 2, \dots, n$) in E . Due to certain and uncertain situations of decision-makers' cognition regarding the different criteria, the satisfactory assessment values of the alternatives M_j ($j = 1, 2, \dots, s$) over the criteria e_j ($j = 1, 2, \dots, n$) are given by MFCVs $m_{ij} = \langle [a_{ij1}, a_{ij1}], \alpha_{ij1} \rangle, \langle [a_{ij2}, a_{ij2}], \alpha_{ij2} \rangle, \dots, \langle [a_{ijq(j)}, a_{ijq(j)}], \alpha_{ijq(j)} \rangle$ ($i = 1, 2, \dots, s; j = 1, 2, \dots, n$) for $[a_{ijk}, a_{ijk}] \subseteq [0, 1]$ and $\alpha_{ijk} \in [0, 1]$ ($k = 1, 2, \dots, q(j)$). Thus, all the assessed MFCVs can be constructed as the decision matrix of MFCSSs $M = (m_{ij})_{s \times n}$.

In the multicriteria GDM problem with the information of MFCSSs, the decision algorithm of the multicriteria GDM methods is presented by the following steps:

Step 1: regarding the decision matrix of MFCSSs $M = (m_{ij})_{s \times n}$, one first yields an ideal alternative $M^* = \{m_1^*, m_2^*, \dots, m_n^*\}$ by the following formula:

$$m_j^* = \left\langle ([a_{j1}^*, b_{j1}^*], \alpha_{j1}^*), ([a_{j2}^*, b_{j2}^*], \alpha_{j2}^*), \dots, ([a_{jq(j)}^*, b_{jq(j)}^*], \alpha_{jq(j)}^*) \right\rangle, \\ \left(\left[\max_i \left(\max(a_{ij1}), \max(a_{ij2}), \dots, \max(a_{ijq(j)}) \right), \max_i \left(\max(b_{ij1}), \max(b_{ij2}), \dots, \max(b_{ijq(j)}) \right) \right], \max_i \left(\max(\alpha_{ij1}), \max(\alpha_{ij2}), \dots, \max(\alpha_{ijq(j)}) \right) \right) \right\rangle, \\ = \left\langle \left(\left[\max_i \left(\max(a_{ij1}), \max(a_{ij2}), \dots, \max(a_{ijq(j)}) \right), \max_i \left(\max(b_{ij1}), \max(b_{ij2}), \dots, \max(b_{ijq(j)}) \right) \right], \max_i \left(\max(\alpha_{ij1}), \max(\alpha_{ij2}), \dots, \max(\alpha_{ijq(j)}) \right) \right) \right\rangle, \\ \dots, \\ \left(\left[\max_i \left(\max(a_{ij1}), \max(a_{ij2}), \dots, \max(a_{ijq(j)}) \right), \max_i \left(\max(b_{ij1}), \max(b_{ij2}), \dots, \max(b_{ijq(j)}) \right) \right], \max_i \left(\max(\alpha_{ij1}), \max(\alpha_{ij2}), \dots, \max(\alpha_{ijq(j)}) \right) \right) \right\rangle \quad (16)$$

Step 2: the values of the weighted correlation coefficient are obtained by using one of the three weighted

correlation coefficients $C_{wp}(M_i, M^*)$ ($p = 1, 2, 3; i = 1, 2, \dots, s$):

$$C_{w1}(M_i, M^*) = \frac{\sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} (a_{ijk} a_{jk}^* + b_{ijk} b_{jk}^* + \alpha_{ijk} \alpha_{jk}^*)}{\sqrt{\sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} (a_{1jk}^2 + b_{1jk}^2 + \alpha_{1jk}^2)} \sqrt{\sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} [(a_{jk}^*)^2 + (b_{jk}^*)^2 + (\alpha_{jk}^*)^2]}} \quad (17)$$

$$C_{w2}(M_i, M^*) = \frac{\sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} (a_{ijk} a_{jk}^* + b_{ijk} b_{jk}^* + \alpha_{ijk} \alpha_{jk}^*)}{\sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} [(a_{jk}^*)^2 + (b_{jk}^*)^2 + (\alpha_{jk}^*)^2]} \quad (18)$$

$$C_{w3}(M_i, M^*) = \frac{\sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} (a_{ijk} + b_{ijk} + \alpha_{ijk})}{\sum_{j=1}^n \beta_j \sum_{k=1}^{q(j)} (a_{jk}^* + b_{jk}^* + \alpha_{jk}^*)} \quad (19)$$

Step 3: ranking of the alternatives is given in descending order of the values of the weighted correlation coefficient and the best one is selected.

Step 4: end.

6. Illustrative Example and Comparative Analysis

6.1. GDM Example on the Selection Problem of Equipment Suppliers. A manufacturing company wants to enhance the competitiveness of products and to enlarge the production capacity, the company plans to replace an existing equipment supplier to improve the product quality. Thus, the decision department has to select four potential suppliers preliminarily as a set of four alternatives $M = \{M_1, M_2, M_3, M_4\}$. Then, the company must make a decision by the suitable assessment of

the four alternatives over the requirements of three criteria: cost (e_1), quality (e_2), and supplier's reputation (e_3). The importance of the three criteria is taken into account by the weight vector $\beta = (0.35, 0.4, 0.25)$. Thus, the company invites three experts/decision-makers to give the assessment values of the four alternatives over the three criteria based on the partial certain and partial uncertain judgments of their cognition, which are presented in the form of MFCVs $m_{ij} = \langle ([a_{ij1}, b_{ij1}], \alpha_{ij1}), ([a_{ij2}, b_{ij2}], \alpha_{ij2}), ([a_{ij3}, b_{ij3}], \alpha_{ij3}) \rangle$ for $[a_{ijk}, b_{ijk}] \subseteq [0, 1]$ and $\alpha_{ijk} \in [0, 1]$ ($j, k = 1, 2, 3; i = 1, 2, 3, 4$). Thus, all the assessed MFCVs can be constructed as the decision matrix of MFCSSs $M = (m_{ij})_{4 \times 3}$ in Table 1.

By applying equation (16) for the decision matrix of Table 1, we first give the ideal alternative:

$M^* = \{m_1^*, m_2^*, m_3^*\} = \{ \langle ([0.8, 0.9], 0.9), ([0.8, 0.9], 0.9), ([0.8, 0.9], 0.9) \rangle, \langle ([0.8, 0.9], 0.9), ([0.8, 0.9], 0.9), ([0.8, 0.9], 0.9) \rangle, \langle ([0.8, 0.95], 0.9), ([0.8, 0.95], 0.9), ([0.8, 0.95], 0.9) \rangle \}$.

TABLE 1: The decision matrix of MFCSSs.

	e_1	e_2	e_3
M_1	$\langle ([0.7, 0.8], 0.7), ([0.6, 0.8], 0.7), ([0.6, 0.7], 0.6) \rangle$	$\langle ([0.7, 0.9], 0.8), ([0.7, 0.8], 0.7), ([0.7, 0.8]), 0.7) \rangle$	$\langle ([0.8, 0.9], 0.9), ([0.8, 0.9]), 0.8), ([0.7, 0.8], 0.7) \rangle$
M_2	$\langle ([0.8, 0.9], 0.8), ([0.7, 0.9]), 0.8), ([0.7, 0.8], 0.7) \rangle$	$\langle ([0.8, 0.9], 0.9), ([0.8, 0.9], 0.8), ([0.7, 0.9]), 0.7) \rangle$	$\langle ([0.7, 0.9], 0.7), ([0.7, 0.8]), 0.7), ([0.7, 0.7], 0.7) \rangle$
M_3	$\langle ([0.7, 0.9], 0.8), ([0.7, 0.8]), 0.7), ([0.7, 0.8], 0.7) \rangle$	$\langle ([0.7, 0.85], 0.75), ([0.7, 0.8], 0.7), ([0.6, 0.8]), 0.7) \rangle$	$\langle ([0.7, 0.9], 0.8), ([0.7, 0.8]), 0.7), ([0.7, 0.8], 0.7) \rangle$
M_4	$\langle ([0.8, 0.9], 0.9), ([0.8, 0.9]), 0.8), ([0.7, 0.8], 0.7) \rangle$	$\langle ([0.7, 0.8], 0.8), ([0.6, 0.8], 0.7), ([0.5, 0.7]), 0.6) \rangle$	$\langle ([0.8, 0.95], 0.9), ([0.8, 0.9]), 0.8), ([0.7, 0.9], 0.8) \rangle$

By using one of equations (17)–(19), we obtain the values of $C_{wp}(M_i, M^*)$ ($p = 1, 2, 3; i = 1, 2, 3, 4$). Then, their decision results are indicated in Table 2.

In the decision results of Table 2, the ranking orders based on the two weighted correlation coefficients $C_{w2}(M_i, M^*)$ and $C_{w3}(M_i, M^*)$ are identical in the GDM example, but different from the ranking order based on the weighted correlation coefficient $C_{w1}(M_i, M^*)$. Then, the best alternative is M_2 or M_3 . However, the different weighted correlation coefficients can affect the ranking orders of the four alternatives; then the final decision result depends on the weighted correlation coefficient used by decision-makers.

6.2. Comparative Analysis of GDM Methods. This part presents the comparative analysis of the developed GDM methods and the related methods to demonstrate the practicability and suitability of the developed GDM methods.

First, we only consider the multicriteria GDM problems in the environments of MFSs and MIVFSs as two special cases of the above illustrative example in the environment of MFCSSs. Regarding the GDM problems of the two special cases, if there exist either all $[a_{ijk}, b_{ijk}] = [0, 0]$ or all $\alpha_{ijk} = 0$ ($j, k = 1, 2, 3$ and $i = 1, 2, 3, 4$) in the decision matrix of MFCSSs $M = (m_{ij})_{4 \times 3}$, the decision matrix $M = (m_{ij})_{4 \times 3}$ is reduced to the decision matrices of MFSs or MIVFSs:

$$M_{\text{MFS}} = \begin{bmatrix} M_{\text{MFS1}} \\ M_{\text{MFS2}} \\ M_{\text{MFS3}} \\ M_{\text{MFS4}} \end{bmatrix} = \begin{bmatrix} (0.7, 0.7, 0.6) & (0.8, 0.7, 0.7) & (0.9, 0.8, 0.7) \\ (0.8, 0.8, 0.7) & (0.9, 0.8, 0.7) & (0.7, 0.7, 0.7) \\ (0.7, 0.7, 0.6) & (0.75, 0.7, 0.7) & (0.8, 0.7, 0.7) \\ (0.9, 0.8, 0.7) & (0.8, 0.7, 0.6) & (0.9, 0.8, 0.8) \end{bmatrix}, \quad (20)$$

$$M_{\text{MIVFS}} = \begin{bmatrix} M_{\text{MIVFS1}} \\ M_{\text{MIVFS2}} \\ M_{\text{MIVFS3}} \\ M_{\text{MIVFS4}} \end{bmatrix} = \begin{bmatrix} ([0.7, 0.8], [0.6, 0.8], [0.6, 0.7]) & ([0.7, 0.9], [0.7, 0.8], [0.7, 0.8]) & ([0.8, 0.9], [0.8, 0.9], [0.7, 0.8]) \\ ([0.8, 0.9], [0.7, 0.9], [0.7, 0.8]) & ([0.8, 0.9], [0.8, 0.9], [0.7, 0.9]) & ([0.7, 0.9], [0.7, 0.8], [0.7, 0.7]) \\ ([0.7, 0.8], [0.6, 0.8], [0.6, 0.7]) & ([0.7, 0.85], [0.7, 0.8], [0.6, 0.8]) & ([0.7, 0.9], [0.7, 0.8], [0.7, 0.8]) \\ ([0.8, 0.9], [0.8, 0.9], [0.7, 0.8]) & ([0.7, 0.8], [0.6, 0.8], [0.5, 0.7]) & ([0.8, 0.95], [0.8, 0.9], [0.7, 0.9]) \end{bmatrix}. \quad (21)$$

Based on the decision matrices of MFSs and MIVFSs, the two ideal alternatives are yielded in the following:

$$M_1^* = \{m_1^*, m_2^*, m_3^*\} = \{(0.9, 0.9, 0.9), (0.9, 0.9, 0.9), (0.9, 0.9, 0.9)\},$$

$$M_2^* = \{m_1^*, m_2^*, m_3^*\} = \{([0.8, 0.9], [0.8, 0.9], [0.8, 0.9]), ([0.8, 0.9], [0.8, 0.9], [0.8, 0.9]), ([0.8, 0.95], [0.8, 0.95], [0.8, 0.95])\}.$$

Thus, by applying equations (10)–(15), we obtain all decision results of the weighted correlation coefficients of MFSs and MIVFSs, which are shown in Table 3.

In the decision results of Table 3, the ranking orders of the four alternatives based on different weighted correlation

coefficients under different environments of MFSs and MIVFSs indicate different ranking results, and then the best alternative is M_2 or M_3 or M_4 .

Regarding the decision results of Tables 2 and 3, it is clear that the developed multifuzzy cubic GDM methods contain the multifuzzy and multi-interval-valued fuzzy GDM methods under the specific environments of MFSs and MIVFSs. However, their ranking orders and the best ones show the partial same and partial difference based on different weighted correlation coefficients under different environments of MFCSSs, MFSs, and MIVFSs, which show that either different weighted correlation coefficients or different information environments can impact on the ranking orders

TABLE 2: Decision results of the weighted correlation coefficients between M_i and M^* .

Weighted correlation coefficient	Value of weighted correlation coefficient	Ranking	The best one
$C_{w1}(M_i, M^*)$	0.9951, 0.9954, 0.9966, 0.9929	$M_3 > M_2 > M_1 > M_4$	M_3
$C_{w2}(M_i, M^*)$	0.8567, 0.9043, 0.8309, 0.8840	$M_2 > M_4 > M_1 > M_3$	M_2
$C_{w3}(M_i, M^*)$	0.8568, 0.9053, 0.8306, 0.8833	$M_2 > M_4 > M_1 > M_3$	M_2

TABLE 3: Decision results of the weighted correlation coefficients between M_{MFSi} and M_1^* and between M_{MIVFSi} and M_2^* .

Weighted correlation coefficient	Value of weighted correlation coefficient	Ranking	The best one
$C_{w4}(M_{MFSi}, M_1^*)$	0.9944, 0.9957, 0.9977, 0.9926	$M_3 > M_2 > M_1 > M_4$	M_3
$C_{w5}(M_{MFSi}, M_1^*)$	0.8074, 0.8481, 0.7815, 0.8537	$M_4 > M_2 > M_1 > M_3$	M_4
$C_{w6}(M_{MFSi}, M_1^*)$	0.8074, 0.8481, 0.7815, 0.8537	$M_4 > M_2 > M_1 > M_3$	M_4
$C_{w7}(M_{MIVFSi}, M_2^*)$	0.9968, 0.9967, 0.9976, 0.9935	$M_3 > M_1 > M_2 > M_4$	M_3
$C_{w8}(M_{MIVFSi}, M_2^*)$	0.8838, 0.9352, 0.8581, 0.9006	$M_2 > M_4 > M_1 > M_3$	M_2
$C_{w9}(M_{MIVFSi}, M_2^*)$	0.8827, 0.9353, 0.8564, 0.8988	$M_2 > M_4 > M_1 > M_3$	M_2

of alternatives in the GDM example. Since MFCSSs consist of both MFSs and MIVFSs, the multifuzzy and multi-interval-valued fuzzy GDM methods introduced in this study are only special cases of the multifuzzy cubic DM methods. Therefore, the developed multifuzzy cubic DM methods are more general and more reasonable in the information expressions and decision usability, which show their highlighting advantages in the setting of MFCSSs.

Since hesitant cubic FSs [26, 27], CHFSSs [28–30], CHNNs [31], FCCNs [32], and cubic IFSs [33–35] cannot express the MFCSS information, their decision-making methods [26–35] cannot also carry out the GDM problems in the setting of MFCSSs. In the situations, it is obvious that the developed multifuzzy cubic GDM methods also indicate their superiority and usability in the setting of MFCSSs.

7. Conclusion

Regarding the hybrid expression problem of MFS and MIVFS in the multifuzzy setting of uncertainty and certainty, this paper presented a MFCSS concept as the extension of FCS in order to adequately depict the opinion of several experts/decision-makers by using a union/sequence of the same and/or different FCVs. Then, we proposed three correlation coefficients of MFCSSs and after that introduced the correlation coefficients of MFSs and MIVFSs as the special cases of the three correlation coefficients of MFCSSs. Further, multicriteria GDM methods using the three weighted correlation coefficients of MFCSSs were developed under the environment of MFCSSs, which contain the multifuzzy and multi-interval-valued fuzzy GDM methods. Lastly, these multicriteria GDM methods were applied in an illustrative example on the selection problem of equipment suppliers; then their decision results and comparative analysis indicated that the developed GDM methods are more practicable and effective and reflected that either different correlation coefficients or different information expressions can impact on the ranking orders of alternatives.

As future work, we shall extend this original study to clustering analysis, pattern recognition, and image processing under the environment of MFCSSs and further

develop new similarity measures of MFCSSs for GDM, medical diagnosis, pattern recognition, and so on.

Data Availability

No data were used to support the results in this study.

Conflicts of Interest

The authors declare no conflicts of interest.

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