Kernel Regression Residual Decomposition Method to Detect Rolling Element Bearing Faults

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1. Introduction

The device of rotating machinery is entirely dependent on the health condition of the rolling element bearings, which accounts for almost 40–50% of these equipment failures [1, 2]. Such failures could be disastrous or could lead to the shutdown of the entire production lines, potentially causing huge losses [3]. Therefore, fault detection of bearing has been a great challenge in recent decades [4]. In order to obtain the fault feature, the vibration signal is one of the most important sources of information for monitoring conditions of rolling element bearing. However, the vibration signals are often contaminated by various noises, and sometimes, it is difficult to obtain high-fidelity signals. Therefore, signal denoising is one of the significant tasks to detect faults in rolling element bearings [5].

In the past few decades, a lot of novelty approaches have been proposed [6–8]. Time-frequency (TF) method techniques have been developed to allow access to the time-frequency energy behavior of nonstationary signals. TF representations can give insight into the complex structure of signals consisting of several components [9]. It maps a 1D signal to a 2D function of time and frequency, so a TF representation can be obtained to characterize the signal in the time and frequency domains simultaneously [10]. Many signal time-frequency methods were applied to machine
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diagnostics in the last few decades. The wavelet decomposition method has been most commonly used [11–14], and an arbitrary signal can be decomposed into a set of wavelet coefficients. At each scale, the soft thresholding technique is performed to suppress noises, and finally, the wavelet reconstructed technique is employed to obtain the purified signals. Lei et al. [15] presented a method based on wavelet packets and Hilbert–Huang transform to improve productivity and part quality in the machining process. A remarkable advantage of the wavelet transform method is that the wavelet transform is sensitive to defects for a longer duration reflected by nonstationary signals [6]. However, wavelet transform suffers from disadvantages, one limitation worth noting that the usage of wavelet transforms to suppress noise will often lead to oscillation effects when dealing with the low signal-to-noise ratio (SNR) scenarios [16].

According to the multiscale signal decomposition principle, the well-known soft threshold technique proposed by Donoho [12, 36] can be applied to denoise signals at each scale.

The rest of the paper is organized as follows. Brief introductions of the basic idea of the KRRD algorithm, SD criterion, soft threshold technique, and KRRR algorithm are given in Section 2. In Section 3, a numerical simulation and an open-access faulty bearing data are carried out to validate the effectiveness of the present method, and the experimental investigations for three cases are further given in Section 4. Conclusions and further work are drawn in Section 5.

2. Kernel Regression Residual Decomposition Methodology

Regression model-based methods will be applied to decompose signals, according to the basic idea of EMD. In the present, KR is employed as a bridge to form an explicit KRRD method. Figure 1 shows the three steps of the proposed KRRD. Firstly, the residual signals of a raw signal are obtained using KR repeatedly, which can be called KRRD. Secondly, the soft thresholding technique and SD criterion are applied to obtain noise-suppressed residual signals. KRRR is then used to reconstruct the purified signal with high SNR. Finally, to obtain demodulation frequencies, the traditional Hilbert envelope spectrum analysis is applied in the paper and the fault types will be given.

### 2.1. KRRD and KRRR

The procedure of KRRD is simply listed as follows. The kernel regression method has been most commonly used; a signal \( f(t) \) can be represented by the summation of \( n \) residual signals as

\[
f(t) = \sum_{i=1}^{n} r_i(t) = f \sum_{i=1}^{n} 1(t),
\]

where \( r_i(t) (i = 1, 2, \ldots, n) \) is a residual signal characterizing the signal information at scale \( i \) and \( f \sum_{i=1}^{n} 1(t) \) is the summation of all residual signals at scale \([1,n]\), i.e., the original/raw signal \( f(t) \). In order to decompose the signal \( f(t) \) into a number of residual signals, a deep framework can be rewritten by

\[
f \sum_{i=1}^{n} 1(t) = f \sum_{i=1}^{n} 2(t) + r_1(t) = s_1,
\]

where \( f \sum_{i=1}^{n} 2(t) \) denote the summation of all residual signals at scale \([2,n]\). In this step, estimating \( f \sum_{i=1}^{n} 2(t) \) by given
$f \sum, 1(t)$ is the key to solve the problem. In the present, a KR strategy is used in conditional expectation. Therefore, the estimation of $f \sum, 2(t)$ can be represented by $f \sum, 1(t)$ as

$$f \sum, 2(t) = K_1(\sum f, 1(t) - \sum f, 1(t_j)) = s_2,$$  \hspace{1cm} (3)

where $K_1$ is the bandwidth of the $f(t)$ and $f \sum, 1(t)$ is the Gaussian kernel function center. It is worth pointing out that the RBF has the characteristic of unique best approximation to the unknown time series [37]. Since the residual signal at scale $n$ can be obtained by equation (4), a deep cascading framework can be expressed as follows:

$$r_j(t) = f \sum, j(t) - f \sum, j + 1(t) = s_j - s_{j + 1},$$  \hspace{1cm} (5)

in which the $(j+1)\text{th}$ smooth term $s_{j + 1} = K_1(\sum f, j + 1(t) - f \sum, j + 1(t_j))$ can also be estimated by nonparametric Nadaraya–Watson KR. The last residual signal can be expressed as follows:

$$r_n(t) = s_n.$$  \hspace{1cm} (6)

Finally, the signal decomposed realization of KRRD via kernel regression at each scale is completed, and the KRRD is simply the summation of all residual signals $r_1(t), r_2(t), \ldots, r_n(t)$ which is shown in Figure 2. Besides, the residual signals at the lower scales contain great characteristics of the signal, while the signal contained coarser signal characteristics.
2.2. Noise Suppression. In this paper, the work is to illustrate the feasibility of utilizing the KRRD for processing fault signals. However, at each scale, there is some low but not useful information. The soft thresholding technique [12, 36] is the commonly used tool to obtain greater content at each scale.

Since residual signals at different scales are obtained. Considering some unusual data in the signal, we use the median absolute deviation function to estimate the noise threshold \( \theta_j \) at scale \( j \), using adaptive noise estimate. The threshold strategy can be expressed as follows:

\[
\theta_j = \frac{\text{MAD}(r_j)}{0.6745},
\]

where MAD is the median absolute deviation and 0.6745 is the normal inverse cumulative distribution function of 3/4 [38]. Based on soft thresholding, \( C_j(t) \) can be obtained by

\[
C_j(t) = \begin{cases} 
0, & \text{if } |r_j(t)| < \theta_j, \\
|r_j(t)| - \theta_j, & \text{otherwise}.
\end{cases}
\]

The sifting usually has to be implemented more times, but to get the most useful information and improve the computational efficiency of this method. In practice, we used the standard deviation (SD) criterion [39] based on the EMD method to determine whether or not \( C_j(t) \) well satisfies the IMF properties; the SD can be expressed as follows:

\[
SD(k) = \sum_{t=0}^{T} \frac{[C_{j-1}(t) - C_j(t)]^2}{C_{j-1}(t)},
\]

where \( T \) is the length of data. This procedure should be repeatedly used for \( n \) times until the last residue \( C_n(t) \) satisfies the formula. In KRRD, the process of calculation is usually set to 0.2, and if SD is less than the threshold, the process stops and the decomposition procedure is finished.

Since the circulated shift is invertible, the reverse shift is simply the summation of all residual signals, which can be represented by

\[
f(t) = R^{-1}(C_1(t), C_2(t), \ldots, C_n(t)) = \sum_{j=1}^{n} C_j(t).
\]

The framework of KRRR is shown in Figure 3. Finally, the inverse KRRD called KRRR is used to reconstruct the new purified signal.

2.3. Fault Detection. The traditional Hilbert envelope spectrum analysis [18, 40] is then applied to determine demodulation frequencies.

Compared with the theoretical feature frequencies of faults in bearings with demodulation frequencies, the type of bearing faults could finally be determined.

3. Simulation Analysis

In this section, to validate the effectiveness of the proposed method, a simulated signal which contains random noise is constructed [41, 42]. The model of the signal is expressed as follows:

\[
f(t) = x(t) + R(t),
\]

where \( x(t) \) is the periodic impulse signal without noise, \( R(t) \) is the random noise, and \( f(t) \) is the signal simulation. Suppose the sampling frequency is 20kHz and the sampling points are 8192, the impulse signal and the noise-contaminated signal are shown in Figures 4(a) and 4(b), respectively. It is clear to see that the signal is immersed in heavy noises.

The decomposition signals using KRRD are shown in Figures 4(c)–4(e). As shown in Figures 4(c)–4(e), the KRRD method can effectively distinguish the narrow band impulses (see Figures 4(c)–4(e)) from the purified signal (see Figure 4(f)). It can also be observed that the noise process is well characterized at the low scales and the structural characteristics of the simulation signal are well characterized as the scale \( j \) increases. It indicates that the proposed method can be regarded as a promising method to decompose signals. To further clarify the denoising process of the KRRD method, the faulty bearing data of Case Western Reserve University [43] are used in this paper. The proposed method is applied to detect the bearing with inner race fault, the sampling frequency \( f_s \) is set to 25.6kHz, the collected data length is 12288 points, the shaft rotating frequency is \( f_r = 29.5 \) Hz, and bearing pass frequency of inner race (BPFI) is 159.93 Hz. The bearing fault data are added with Gaussian noises for SNR = 4 dB to illustrate the effectiveness of this method and the results are shown in Figure 5.

From Figure 5(a), the raw signal \( S_1 \) is divided into four residual signals using kernel regression function. By comparing with Figure 5(b), it can be observed that in Figure 5(b), it is clear that each residual signal is decomposed using noise suppression.

The raw signal and the corresponding denoising signal are shown in Figure 6. And the frequency spectrum of raw signal and denoising signal can be seen in Figure 7. It is
worth noting that the fault frequency of 154.2 Hz is shown in Figure 7(b), which is more accurate than Figure 7(a) and the fault frequency of 154.2 Hz is matching with the theoretical calculation value of 159.93 Hz. Based on the above investigation, we conclude that the present approach can be employed to determine the faults of bearings.

4. Experimental Evaluation

In this section, the proposed approach is evaluated by rolling element bearings with inner race, outer race, and compound fault [12, 44]. Generally, the vibration response of a bearing with faults consists of a series of impulses, whereas the frequencies in these impulses are the bearing characteristic frequencies (BCFs). Figure 8 shows the defects at the outer race, inner race, and ball. It is noted that a deep groove bearing (ER-12K) is used in the experiment [45]. The fault simulator (MFS-MG, manufactured by Spectraquest Inc.) and the test system are shown in Figure 9. The test system includes speed indicator, manual speed governor, acceleration sensors, speed sensor, motor, spindle, bearings, etc. The bearing parameters are the number of rolling elements \( n = 8 \), ball diameter \( d = 0.3125 \) inches, pitch diameter \( D = 1.318 \) inches, and the contact angle \( \phi = 0° \). During the experiment, the data are acquired by an accelerometer mounted on the top of the bearing holder on the left side, and the sampling frequency \( f_s \) is set to 25.6 kHz.

4.1. Outer Race Fault Detection. In this section, the proposed method is applied to detect the bearing with outer race fault. For the bearing with outer race fault, the collected data length is 12288 points and the shaft rotating frequency \( f_r = 29.87 \) Hz. According to equation (12), BPFO can be calculated as

\[
BPFO = \frac{n f_r}{2} \left[ 1 - \frac{d}{D} \cos \phi \right],
\]

in which \( f_r \) is the shaft speed (Hz), \( n \) is the number of rolling elements, \( \phi \) is the angle of the load from the radial plane, \( d \) is the ball diameter, and \( D \) is the pitch diameter.

The fault simulator (MFS-MG, manufactured by Spectraquest Inc.) and the test system are shown in Figure 9. The test system includes speed indicator, manual speed governor, acceleration sensors, speed sensor, motor, spindle, bearings, etc. The bearing parameters are the number of rolling elements \( n = 8 \), ball diameter \( d = 0.3125 \) inches, pitch diameter \( D = 1.318 \) inches, and the contact angle \( \phi = 0° \). During the experiment, the data are acquired by an accelerometer mounted on the top of the bearing holder on the left side, and the sampling frequency \( f_s \) is set to 25.6 kHz.

\[BPFO = \frac{n f_r}{2} \left[ 1 - \frac{d}{D} \cos \phi \right] = 91.15 \text{Hz}.\]

From Figure 10(a), it can be seen that, due to the defect present in the rolling bearing, the vibration signal presents the impacts feature, but there exist very serious noises. Applying the KRRD method to the raw signal, the denoised signal is shown in Figure 10(b). By comparing with Figure 10(a), the ambient noises are effectively suppressed. The fault characteristic frequency (87.5 Hz) is clearly revealed. The comparisons show that the above method
Figure 5: The KRRD method using Case Western Reserve University data. (a) Raw signal $S_1$ is decomposed into four residual signals $S_2$, $S_3$, $S_4$, and $S_5$. (b) Each residual signal is decomposed into $C_1$, $C_2$, $C_3$, $C_4$, and $C_5$ using noise suppression.

Figure 6: The KRRD method using Case Western Reserve University data. (a) Raw signal. (b) Denoising signal.
proposed in this paper is more effective. Therefore, the bearing with outer race fault will be clearly detected.

### 4.2. Inner Race Fault Detection

For the bearing with inner race fault, the total collected data are 12288 points; the shaft rotating frequency is \( f_r = 29.87 \text{ Hz} \). The ball pass frequency of the outer race (BPFI) is given by

\[
BPFI = \frac{n f_r}{2} \left[ 1 + \frac{d}{D} \cos \phi \right] = 147.8 \text{Hz}. \tag{15}
\]

In this section, the proposed method is applied to detect the bearing with inner race fault. By comparing Figures 11(a) and 11(b), we can see that the impact characteristic is not clearly shown in the former graphic but clearly shown in the latter graphic. And the fault frequency of the inner race in
In Figure 11(a), the shaft rotating frequency is 29.87 Hz, the second harmonic frequency is 59.26 Hz, and its feature frequency of 147.9 Hz is matching with the theoretical calculation value of 147.8 Hz, and we conclude that there is a fault in the inner race of the bearing.

4.3. Compound Fault Detection. For the bearing with compound fault (the combination of BPFO and BPFI), the total collected data are 12288 points, the shaft rotating frequency is $f_r = 39.82$ Hz, and the corresponding BPFO is given by

$$\text{BPFO} = \frac{n f_r}{2} \left[ 1 - \frac{d}{D} \cos \phi \right] = 121.5 \text{Hz}. \quad (16)$$

The bearing pass frequency of inner race (BPFI) is given by

$$\text{BPFI} = \frac{n f_r}{2} \left[ 1 + \frac{d}{D} \cos \phi \right] = 197.1 \text{Hz}. \quad (17)$$

The ball spin frequency (BSF) is given by

$$\text{BSF} = \frac{D f_r}{2d} \left[ 1 - \left( \frac{d}{D} \cos \phi \right)^2 \right] = 79.3 \text{Hz}. \quad (18)$$

In this section, the proposed method is applied to detect the bearing with compound fault. The original signal is illustrated in Figure 12(a) and the denoised signal using the KRRD method is shown in Figure 12(b). In Figure 12(a), only the input shafting frequency of 40 Hz is shown, while the other
harmonics are submerged by other unknown frequency components; it is not clear what kind of fault is. From Figure 12(b), the input shafting rotating frequency (40 Hz) and its inner race fault (202.5 Hz), outer race fault (120 Hz), and the second harmonic frequency of ball fault (79.25 Hz) are matching with the theoretical calculation values. The results demonstrate that the proposed approach can effectively extract the fault features of defective bearings.

5. Conclusion

In this paper, aiming at the shortcoming of the conventional denoising method for bearing fault signal under variation conditions, a new KRRD method is proposed. Like the EMD method, the KRRD method is used to decompose a signal into a number of residual signals at different scales. It allows the noise suppression method to get rid of the noise information while preserving the important signal characteristics. The method is verified by the feature extraction of the faulty bearing of the outer race, inner race, and compound fault. From the detection results of the numerical simulation and experiment investigations from both open access data and mechanical fault simulator, it can be observed that the present method is suitable to detect faults in the mechanical systems. Because KR is a nonparametric mathematical tool with the support of the strictly mathematical theory, KRRD might be superior to EMD. The decomposition procedures are similar to EMD, and the application area can be enlarged if more research works are further performed. Moreover, it is expected that the procedures of the present method are surely suitable to other regression models.

Data Availability

The data used to support the findings of this study are available on the Case Western Reserve University Bearing Data Center website (http://csegroups.case.edu/bearingdatacenter/home).

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

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