# Abundant Explicit Solutions to Fractional Order Nonlinear Evolution Equations 

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We utilize the modified Riemann-Liouville derivative sense to develop careful arrangements of time-fractional simplified modified Camassa-Holm (MCH) equations and generalized ( $3+1$ )-dimensional time-fractional Camas-sa-Holm-Kadomtsev-Petviashvili ( $\mathrm{gCH}-\mathrm{KP}$ ) through the potential double ( $G^{\prime} / G, 1 / G$ )-expansion method ( DEM ). The mentioned equations describe the role of dispersion in the formation of patterns in liquid drops ensued in plasma physics, optical fibers, fluid flow, fission and fusion phenomena, acoustics, control theory, viscoelasticity, and so on. A generalized fractional complex transformation is appropriately used to change this equation to an ordinary differential equation; thus, many precise logical arrangements are acquired with all the freer parameters. At the point when these free parameters are taken as specific values, the traveling wave solutions are transformed into solitary wave solutions expressed by the hyperbolic, the trigonometric, and the rational functions. The physical significance of the obtained solutions for the definite values of the associated parameters is analyzed graphically with 2D, 3D, and contour format. Scores of solitary wave solutions are obtained such as kink type, periodic wave, singular kink, dark solitons, bright-dark solitons, and some other solitary wave solutions. It is clear to scrutinize that the suggested scheme is a reliable, competent, and straightforward mathematical tool to discover closed form traveling wave solutions.

## 1. Introduction

In recent years, fractional calculus (FC) assumed a basic part of a capable, catalyst, and rudimentary hypothetical structure for more sufficient displaying of multifaceted powerful cycles. FC and nonlinear fractional differential equations (NLFDEs) have recently been used to solve problems in plasma physics, protein chemistry, cell biology, mechanical engineering, signal processing and systems recognition, electrical transmission, control theory, economics, and fractional dynamics. FDE has a wide range of applications in fields such as magnetism, sound waves propagation in rigid porous materials, cardiac tissue electrode interface, principle of viscoelasticity, fluid dynamics, lateral and longitudinal regulation of autonomous vehicles, ultrasonic wave
propagation in human cancerous bone, wave propagation in viscoelastic horn, heat transfer, RLC electric circuit, modeling of earthquake, and some other areas [1-5]. The highly prepared polylayer portion of the human body is a particularly capable model system for using fractional calculus. As a result, researchers are increasingly interested in seeking exact solutions to NLFDEs, which play a significant role in nonlinear science. Wave shape has an effect on sediment transport and beach morphodynamics, while wave skewness has an impact on radar altimetry signals and asymmetry has an impact on ship responses to wave impacts. Traveling wave solutions is a special class of analytical solutions for nonlinear evolution equations (NLEEs). Solitary waves are transmitted traveling waves with constant speeds and shapes that achieve asymptotically zero at distant locations. In order
to know the inner mechanism of the mentioned complex tangible phenomena, investigation of exact solutions of NLFDEs are very much important. In this way, numerous authors have been interested in studying the FC and finding precise and productive techniques for comprehending nonlinear fractional partial differential equations (NFPDEs). In the previous few decades, numerous strategies have been produced for illuminating NFPDEs, for example, nearby variational iteration method [6], the F-expansion method [7], homotopy perturbation method [8], Kudryashov method [9], improved $\left(G^{\prime} / G\right)$-expansion method [10], and the DEM [11-13].

As of late, a clear and succinct method called the DEM, which is presented in [14], and is exhibited as a mighty method for looking at analytical solutions of NLDEs. The DEM is a reliable technique, which provides different types of solitary wave solutions (SWS), namely, the hyperbolic, the trigonometric, and the rational functions. The proposed MCH equation has been researched for its precise diagnostic arrangements through the $\left(G^{\prime} / G\right)$-expansion method [15], exp-function method [16], modified simple equation method [17], and so on. Also, the proposed (gCH-KP) equation has been investigated for its exact analytic solutions through Agrawal's method [18] and the bifurcation method [19]. To the best of our knowledge, the recommended
condition has not been concentrated through the DEM [12]. So, the point of this investigation is to build up some fresh and further broad precise solutions for the previously mentioned condition utilizing the DEM.

The rest of of the article is planned as follows. In Section 2, we have presented the definition and primers. In Section 3, the DEM has been depicted. In Section 4, we have built up the specific answer for the proposed equation by the previously mentioned method. In Section 5, we have uncovered the graphical portrayal and conversation, and in Section 6, comparison of results has been drawn. In Section 7, the conclusion is given.

## 2. Definition and Primers

Jumarie offered a mRL. With such a fractional derivative and some accommodating ways, we can change over fractional differential equations (FDEs) into integer-order differential equations applying variable transformation [20]. In this section, we first provide a couple of features and definitions of the mRL subsidiary which is used further in this study. Acknowledge that $f: R \longrightarrow R, x \longrightarrow f(x)$ implies a continuous, however, not really differentiable function. Jumarie's mRL having order $a$ is defined by the articulation

$$
D_{x}^{\alpha} f(x)= \begin{cases}\frac{1}{\Gamma(-\alpha)} \int_{0}^{x}(x-\xi)^{-\alpha-1}[f(\xi)-f(0)] \mathrm{d} \xi, & \alpha<0,  \tag{1}\\ \frac{1}{\Gamma(1-\alpha)} \frac{\mathrm{d}}{\mathrm{~d} x} \int_{0}^{x}(x-\xi)^{-\alpha}[f(\xi)-f(0)] \mathrm{d} \xi, & 0<\alpha<1, \\ \left(f^{(n)}(x)\right)^{(a-n)}, & n \leq \alpha \leq n+1, n>1 .\end{cases}
$$

Two or three features of the mRL were concise and four acclaimed conditions of them are as follows:

$$
\begin{equation*}
D_{t}^{\alpha} t^{\gamma}=\frac{\Gamma(1+\gamma)}{\Gamma(1+\gamma-\alpha)} t^{(\gamma-\alpha)}, \quad \gamma>0 \tag{2}
\end{equation*}
$$

$D_{t}^{\alpha}(a f(t)+b g(t))=a D_{t}^{\alpha} f(t)+b D_{t}^{\alpha} g(t)$,
wherever a and $b$ stand for constants and

$$
\begin{align*}
& D_{x}^{\alpha} f[u(x)]=f_{u}^{\alpha}(u) D_{x}^{\alpha} u(x)  \tag{4}\\
& D_{x}^{\alpha} f[u(x)]=D_{u}^{\alpha} f(u)\left(u^{\prime}(x)\right)^{\alpha} \tag{5}
\end{align*}
$$

which are the immediate results of

$$
\begin{equation*}
d^{\alpha} x(t)=\Gamma(1+\alpha) \mathrm{d} x(t) \tag{6}
\end{equation*}
$$

This holds for nondifferentiable function. Among equations (3)-(5), $u(x)$ is nondifferentiable in equations (3) and (4) but differentiable in equation (5). The function $u(x)$ is nondifferentiable, and $f(u)$ is differentiable in equation
(4) and no differentiable in equation (5). So, the explanation equations (3)-(5) should be used mindfully.

## 3. The Double-Expansion Method

In this part, the center aspect of the DEM to assess the specific traveling wave solution of the NFPDEs has been represented. Let us guess the standard differential equation of order two:

$$
\begin{equation*}
G^{\prime \prime}(\xi)+\lambda G(\xi)=\mu \tag{7}
\end{equation*}
$$

Also, the accompanying relations

$$
\begin{align*}
& \phi=\frac{G^{\prime}}{G},  \tag{8}\\
& \psi=\frac{1}{G}
\end{align*}
$$

Subsequently, it gives

$$
\begin{align*}
& \phi^{\prime}=-\phi^{2}+\mu \psi-\lambda,  \tag{9}\\
& \psi^{\prime}=-\phi \psi,
\end{align*}
$$

The solution for equation (7) relies upon $\lambda$ as $\lambda<0, \lambda>0$, and $\lambda=0$.

For $\lambda<0$, the complete solution of equation (7) will be

$$
\begin{equation*}
G(\xi)=C_{1} \sinh (\sqrt{-\lambda} \xi)+C_{2} \cosh (\sqrt{-\lambda} \xi)+\frac{\mu}{\lambda} \tag{10}
\end{equation*}
$$

Take into account that we obtain

$$
\begin{equation*}
\psi^{2}=\frac{-\lambda}{\lambda^{2} \sigma+\mu^{2}}\left(\phi^{2}-2 \mu \psi+\lambda\right) \tag{11}
\end{equation*}
$$

where $\sigma=C_{1}^{2}-C_{2}^{2}$.
On the off chance that $\lambda>0$, the solution for equation (7) is as follows:

$$
\begin{equation*}
G(\xi)=C_{1} \sin (\sqrt{\lambda} \xi)+C_{2} \cos (\sqrt{\lambda} \xi)+\frac{\mu}{\lambda} \tag{12}
\end{equation*}
$$

Considering that we acquire

$$
\begin{equation*}
\psi^{2}=\frac{\lambda}{\lambda^{2} \sigma-\mu^{2}}\left(\phi^{2}-2 \mu \psi+\lambda\right), \tag{13}
\end{equation*}
$$

where $\sigma=C_{1}^{2}+C_{2}^{2}$, when $\lambda=0$, the overall solution for condition (7) is as follows:

$$
\begin{equation*}
G(\xi)=\frac{\mu}{2} \xi^{2}+C_{1} \xi+C_{2} . \tag{14}
\end{equation*}
$$

Taking into account that we acquire

$$
\begin{equation*}
\psi^{2}=\frac{1}{C_{1}^{2}-2 \mu C_{2}}\left(\phi^{2}-2 \mu \psi\right) \tag{15}
\end{equation*}
$$

where $C_{1}$ and $C_{2}$ stand for constants and those are arbitrary, in this section, we talk about the principle part of proposed methods to take exact traveling wave solutions to the NLFDE is as the form

$$
\begin{equation*}
p\left(u, D_{t}^{\alpha} u, D_{x}^{\beta} u, D_{t}^{\alpha} D_{t}^{\alpha} u, D_{t}^{\alpha} D_{x}^{\beta} u, D_{x}^{\beta} D_{x}^{\beta}, \ldots,\right)=0, \quad 0<\alpha \leq 1,0<\beta \leq 1, \tag{16}
\end{equation*}
$$

where $u$ speaks to an unidentified function of spatial subordinate $x$ and transient subsidiary $t$ and speaks to a polynomial of $u(x, t)$ and its derivatives wherein the most maximal order of derivatives and nonlinear terms of the maximal order are related.

Step 1: take into account the traveling wave transformation:

$$
\begin{equation*}
\xi=L x+V \frac{t^{\alpha}}{\Gamma(1+\alpha)} \tag{17}
\end{equation*}
$$

where $c$ and $k$ are nonzero abstract constant.
Applying this wave transformation in (16), it is reworked as

$$
\begin{equation*}
Q\left(u, u^{\prime}, u^{\prime \prime}, u^{\prime \prime \prime}, \ldots,\right)=0 \tag{18}
\end{equation*}
$$

where the prime speaks to the ordinary derivative of $u$ regarding $\xi$.
Step 2 : take the arrangement of equation (9) which have been uncovered as polynomial in $\phi$ and $\psi$ of the endorse type:

$$
\begin{equation*}
u(\xi)=\sum_{i=0}^{N} a_{i} \phi^{i}+\sum_{i=1}^{N} b_{i} \phi^{i-1} \psi \tag{19}
\end{equation*}
$$

where $a_{i}$ and $b_{i}$ stand for constants which will be calculated later.
Step 3 : in equation (18), " N " will be calculated using homogeneous balance principal which determines equation (19).

Step 4 : put (19) in (18) along with (9) and (11), and it decreases to a polynomial in $\phi$, where the degree is one. Contrasting the polynomial of similar terms with zero, a game plan of logarithmic conditions that are examined by using computational programming produces the estimations of $a_{i}, b_{i}, \mu, C_{1}, C_{2}$, and $\lambda$ where $\lambda<0$, which give hyperbolic function arrangements.
Step 5 : in a similar fashion, we explore the estimations of $a_{i}, b_{i}, \mu, C_{1}, C_{2}$, and $\lambda$, where $\lambda>0$ and $\lambda=0$ which are giving trigonometric and rational function results correspondingly.

## 4. Formulation of Exact Solution

4.1. The Exact Solutions to the Space-Time Fractional MCH Equation. This equation was presented by Camassa and Holm [21] in 1993 which describes shallow water waves with peakon solutions. The peakon solution is a special solitary wave solution which is peaked in the limiting case, and the first derivatives are discontinuous in the peaks [22] and pseudospherical surfaces, and therefore, its integrability properties can be studied by geometrical means [23].

First, take the space-time fractional MCH equation [15] in the form

$$
\begin{equation*}
D_{t}^{\alpha}+2 \delta u_{x}-u_{x x t}+\gamma u^{2} u_{x}=0 \tag{20}
\end{equation*}
$$

where $u(x, t)$ is the velocity of the fluid, $\delta$ is the coefficient related to the critical shallow water wave speed, and $\gamma$ is a nonzero constant. Employing transformation (17), equation (20) reduced an ODE as follows:

$$
\begin{equation*}
V u^{\prime}+2 L \delta u^{\prime}-L^{2} V u^{\prime \prime \prime}+\gamma L u^{2} u^{\prime}=0, \tag{21}
\end{equation*}
$$

where $V, L$, and $\gamma$ are nonzero constants and $\delta$ is the coefficient related to the critical shallow water wave speed.

Integrating (21) once and taking the constant of the integration as zero, it becomes

$$
\begin{equation*}
u(v+2 L \delta)-L^{2} V u^{\prime \prime}+\gamma L \frac{u^{3}}{3}=0 \tag{22}
\end{equation*}
$$

Balancing the maximal order derivative term $u^{\prime \prime}$ with the most order nonlinear term $u^{3}$, the adjusting number is resolved to be $N=1$. At that point, expect the specific arrangement of equation (22) as

$$
\begin{equation*}
u(\xi)=a_{0}+a_{1} \phi+b_{1} \psi \tag{23}
\end{equation*}
$$

wherever $a_{0}, a_{1}$, and $b_{1}$ are constants to be resolved.
Case 1 : for $\lambda<0$, setting equation (23) in (22) and by using (9) and (11), we get the following solution:

$$
\begin{align*}
& a_{0}=0, \\
& a_{1}=b_{1} \sqrt{-\frac{\lambda}{\lambda^{2} \sigma+\mu^{2}}}, \\
& b_{1}=b_{1},  \tag{24}\\
& L=b_{1} \sqrt{\frac{2 \gamma \lambda}{6 \mu^{2} \delta+\gamma b_{1}^{2} \lambda^{2}+6 \delta \lambda^{2} \sigma}}, \\
& V=-\frac{2 \gamma b_{1} \lambda}{3\left(\lambda^{2} \sigma+\mu^{2}\right)} \sqrt{\frac{6 \mu^{2} \delta+\gamma b_{1}^{2} \lambda^{2}+6 \delta \lambda^{2} \sigma}{2 \gamma \lambda}} .
\end{align*}
$$

Substituting these values in (23), we find to the solution for the MCH equation (20) as the structure:

$$
\begin{align*}
u_{1_{1}}(x, t)= & b_{1} \sqrt{-\frac{\lambda}{\lambda^{2} \sigma+\mu^{2}}} \\
& \times \frac{C_{1} \sqrt{-\lambda} \cosh (\sqrt{-\lambda} \xi)+C_{2} \sqrt{-\lambda} \sinh (\sqrt{-\lambda} \xi)}{C_{1} \sinh (\sqrt{-\lambda} \xi)+C_{2} \cosh (\sqrt{-\lambda} \xi)+(\mu / \lambda)} \\
& +\frac{b_{1}}{C_{1} \sinh (\sqrt{-\lambda} \xi)+C_{2} \cosh (\sqrt{-\lambda} \xi)+(\mu / \lambda)} \tag{25}
\end{align*}
$$

wherever

$$
\begin{align*}
\xi= & b_{1} \sqrt{\frac{2 \gamma \lambda}{6 \mu^{2} \delta+\gamma b_{1}^{2} \lambda^{2}+6 \delta \lambda^{2} \sigma}} x \\
& -\frac{2 \gamma b_{1} \lambda}{3\left(\lambda^{2} \sigma+\mu^{2}\right)} \sqrt{\frac{6 \mu^{2} \delta+\gamma b_{1}^{2} \lambda^{2}+6 \delta \lambda^{2} \sigma}{2 \gamma \lambda}} \frac{t^{\alpha}}{\Gamma(1+\alpha)} \\
\sigma= & C_{1}^{2}-C_{2}^{2} \tag{26}
\end{align*}
$$

Since $C_{1}$ and $C_{2}$ are arbitrary constants, it may be self-assertively picked. In the event that we pick $C_{1}=\mu=0$ and $C_{2} \neq 0$ in equation (25), we get the solitary wave solution:
$u_{1_{2}}(x, t)=b_{1} \sqrt{-\frac{\lambda}{\lambda^{2} \sigma}} \times \sqrt{-\lambda} \tanh (\sqrt{-\lambda} \xi)+b_{1} \operatorname{sech}(\sqrt{-\lambda} \xi)$.

Again, if we choose $C_{1} \neq 0$ and $C_{2}=\mu=0$ in equation (25), we will find the solitary wave solution:
$u_{1_{3}}(x, t)=b_{1} \sqrt{-\frac{\lambda}{\lambda^{2} \sigma}} \times \sqrt{-\lambda} \operatorname{coth}(\sqrt{-\lambda} \xi)+b_{1} \operatorname{cosech}(\sqrt{-\lambda} \xi)$,
wherever

$$
\begin{align*}
\xi= & b_{1} \sqrt{\frac{2 \gamma \lambda}{6 \mu^{2} \delta+\gamma b_{1}^{2} \lambda^{2}+6 \delta \lambda^{2} \sigma}} x \\
& -\frac{2 \gamma b_{1} \lambda}{3\left(\lambda^{2} \sigma+\mu^{2}\right)} \sqrt{\frac{6 \mu^{2} \delta+\gamma b_{1}^{2} \lambda^{2}+6 \delta \lambda^{2} \sigma}{2 \gamma \lambda}} \frac{t^{\alpha}}{\Gamma(1+\alpha)} \tag{29}
\end{align*}
$$

Case 2 : for $\lambda>0$, setting equation (23) in (22) by using (9) and (13), we get the resulting result:

$$
\begin{align*}
& a_{0}=0, \\
& a_{1}=b_{1} \sqrt{-\frac{\lambda}{\left(-\lambda^{2} \sigma+\mu^{2}\right)}}, \\
& b_{1}=b_{1},  \tag{30}\\
& L=b_{1} \sqrt{\frac{2 \gamma \lambda}{6 \mu^{2} \delta+\gamma b_{1}^{2} \lambda^{2}-6 \delta \lambda^{2} \sigma}} \\
& V=-\frac{2 \gamma b_{1} \lambda}{3\left(-\lambda^{2} \sigma+\mu^{2}\right)} \sqrt{\frac{6 \mu^{2} \delta+\gamma b_{1}^{2} \lambda^{2}-6 \delta \lambda^{2} \sigma}{2 \gamma \lambda}} .
\end{align*}
$$

Substituting these values in (23), we find the solution for the MCH equation (20) as the structure:

$$
\begin{align*}
u_{1_{4}}(x, t)= & b_{1} \sqrt{-\frac{\lambda}{\left(-\lambda^{2} \sigma+\mu^{2}\right)}} \\
& \times \frac{C_{1} \sqrt{\lambda} \cos (\sqrt{-\lambda} \xi)-C_{2} \sqrt{\lambda} \sin (\sqrt{-\lambda} \xi)}{C_{1} \sin (\sqrt{\lambda} \xi)+C_{2} \cos (\sqrt{\lambda} \xi)+(\mu / \lambda)} \\
& +\frac{b_{1}}{C_{1} \sinh (\sqrt{\lambda} \xi)+C_{2} \cosh (\sqrt{\lambda} \xi)+(\mu / \lambda)} \tag{31}
\end{align*}
$$

wherever

$$
\begin{align*}
\xi= & b_{1} \sqrt{\frac{2 \gamma \lambda}{6 \mu^{2} \delta+\gamma b_{1}^{2} \lambda^{2}-6 \delta \lambda^{2} \sigma}} x \\
& -\frac{2 \gamma b_{1} \lambda}{3\left(-\lambda^{2} \sigma+\mu^{2}\right)} \sqrt{\frac{6 \mu^{2} \delta+\gamma b_{1}^{2} \lambda^{2}-6 \delta \lambda^{2} \sigma}{2 \gamma \lambda}} \frac{t^{\alpha}}{\Gamma(1+\alpha)}, \\
\sigma= & C_{1}^{2}+C_{2}^{2} \tag{32}
\end{align*}
$$

It can be chosen arbitrarily, since $C_{1}$ and $C_{2}$ are arbitrary constants. We get the solitary wave solution by choosing $C_{1}=\mu=0$ and $C_{2} \neq 0$ in equation (31):

$$
\begin{equation*}
u_{1_{5}}(x, t)=b_{1} \sqrt{-\frac{\lambda}{\left(-\lambda^{2} \sigma\right)}} \times \sqrt{\lambda} \tan (\sqrt{\lambda} \xi)+b_{1} \sec (\sqrt{\lambda} \xi) \tag{33}
\end{equation*}
$$

$a_{0}=\frac{\lambda C_{2}+\sqrt{\lambda^{2} C_{2}^{2}+C_{1}^{2} \lambda}}{C_{1}^{2}} b_{1}$,
$a_{1}=0$,
$b_{1}=b_{1}$,
$L=\frac{2 \gamma b_{1}}{48 C_{2}\left(\lambda C_{2}+\sqrt{\lambda^{2} C_{2}^{2}+C_{1}^{2} \lambda}\right)-24 C_{1}^{2} \lambda-96 C_{1}^{2} \lambda \delta}$,
$V=\frac{2 \gamma b_{1}}{576 C_{1}^{2} C_{2}\left(\lambda C_{2}+\sqrt{\lambda^{2} C_{2}^{2}+C_{1}^{2} \lambda}\right)-24 C_{1}^{2} \lambda-96 C_{1}^{2} \lambda \delta}\left(2 \gamma b_{1}^{2} \lambda C_{2} \frac{\lambda C_{2}+\sqrt{\lambda^{2} C_{2}^{2}+C_{1}^{2} \lambda}}{C_{1}^{2}} 24 \delta C_{1}^{2}+b_{1}^{2} \gamma \lambda\right)$.

Substituting these values in (23), we achieve to the rational function solution for the MCH equation (20) as the structure:

$$
\begin{equation*}
u_{1_{7}}(x, t)=\frac{\lambda C_{2}+\sqrt{\lambda^{2} C_{2}^{2}+C_{1}^{2} \lambda}}{C_{1}^{2}} b_{1}+\frac{b_{1}}{(\mu / 2) \xi^{2}+C_{1} \xi+C_{2}} \tag{37}
\end{equation*}
$$

wherever

$$
\begin{align*}
\xi= & \frac{2 \gamma b_{1}}{48 C_{2}\left(\lambda C_{2}+\sqrt{\lambda^{2} C_{1}^{2}+C_{1}^{2} \lambda}\right)-24 C_{1}^{2} \lambda-96 C_{1}^{2} \lambda \delta} x \\
& -\frac{2 \gamma b_{1}}{576 C_{1}^{2} C_{2}\left(\lambda C_{2}+\sqrt{\lambda^{2} C_{2}^{2}+C_{1}^{2} \lambda}\right)-24 C_{1}^{2} \lambda-96 C_{1}^{2} \lambda \delta}  \tag{38}\\
& \left(2 \gamma b_{1}^{2} \lambda C_{2} \frac{\lambda C_{2}+\sqrt{\lambda^{2} C_{2}^{2}+C_{1}^{2} \lambda}}{C_{1}^{2}}+24 \delta C_{1}^{2}+b_{1}^{2} \gamma \lambda\right) \frac{t^{\alpha}}{\Gamma(1+\alpha)}
\end{align*}
$$

It is observable to see that the traveling wave arrangements $u_{1_{1}}-u_{1_{7}}$ of our proposed MCH equation are broadly new and general. These picked up arrangements have not been checked in the previous investigation. These arrangements are advantageous to assign the above expressed wonders.

### 4.2. Generalized $(3+1)$-Dimensional $g C H-K P$ Equation.

$$
\begin{equation*}
\left(D_{t}^{\alpha} u+a u_{x}+b u u_{x}+c D_{t}^{\alpha} u_{x x}\right)_{x}+c_{1} u_{y y}+c_{2} u_{z z}=0 \tag{39}
\end{equation*}
$$

describes the role of dispersion in the formation of patterns in liquid drops, where $a, b, c, c_{1}$, and $c_{2}$ are nonzero constants and $D_{t}^{\alpha}$ is the Riemann-Liouville fractional derivative of $u(t, x, y, z), 0<\alpha<1$ [20].

Introduce the following fractional transformation:

$$
\begin{equation*}
\xi=k x+l y+m z-\frac{n t^{\alpha}}{\Gamma(1+\alpha)} \tag{40}
\end{equation*}
$$

Applying equation (40) in (39), we have

$$
\begin{equation*}
k\left(-n u^{\prime}+a k u^{\prime}+b k u u^{\prime}-c n k^{2} u^{\prime \prime \prime}\right)^{\prime}+c_{1} l^{2} u^{\prime \prime}+c_{2} m^{2} u^{\prime \prime}=0 . \tag{41}
\end{equation*}
$$

Integrating equation (41) two times and taking integrating constant as zero, we obtain

$$
\begin{equation*}
c n k^{3}+n k u-a k^{2} u-\frac{b k^{2}}{2} u^{2}-c_{1} l^{2} u-c_{2} m^{2} u=0 \tag{42}
\end{equation*}
$$

Balancing linear and nonlinear higher-order term, we get $N=2$, which implies using (19) that

$$
\begin{equation*}
u(\xi)=a_{0}+a_{1} \phi+a_{2} \phi^{2}+b_{1} \psi+b_{2} \phi \psi \tag{43}
\end{equation*}
$$

where $a_{0}, a_{1}, a_{2}, b_{1}$, and $b_{2}$ are constants to be resolved.
Case 1 : for $\lambda<0$, setting equation (43) in (42), close to (9) and (11), generates an arrangement of mathematical equations by utilizing computerbased math such as maple, and we get the subsequent result.

## Set 1:

$a_{0}=\frac{4 k \lambda n c}{b}$,
$a_{1}=0$,
$a_{2}=\frac{6 k n c}{b}$,
$b_{1}=-\frac{6 k n c \mu}{b}$,
$b_{2}=\frac{k n c}{b} \sqrt{\frac{-36\left(\mu^{2}+\lambda^{2} \sigma\right)}{\lambda}}, \quad k=k$,

$$
\begin{equation*}
l=\sqrt{\frac{-a k^{2}+n k-c_{2} m^{2}+k^{3} \lambda n c}{c_{1}}}, \quad m=m \text { and } n=n \tag{44}
\end{equation*}
$$

Set 2:

$$
\begin{align*}
a_{0} & =\frac{6 k \lambda n c}{b} \\
a_{1} & =0 \\
a_{2} & =\frac{6 k n c}{b} \\
b_{1} & =-\frac{6 k n c \mu}{b}, \\
b_{2} & =\frac{k n c}{b} \sqrt{\frac{-36\left(\mu^{2}+\lambda^{2} \sigma\right)}{\lambda}}, \\
k & =k \\
l & =\sqrt{\frac{-a k^{2}+n k-c_{2} m^{2}+k^{3} \lambda n c}{c_{1}}}, \quad m=m \text { and } n=n . \tag{45}
\end{align*}
$$

For Set 1 , substituting these values in (43), we get the solution for the $\mathrm{gCH}-\mathrm{KP}$ equation (39) as the structure

$$
\begin{aligned}
u_{2_{1}}(x, t)= & \frac{4 k \lambda n c}{b}+\frac{6 k n c}{b} \times\left(\frac{C_{1} \sqrt{-\lambda} \cosh (\sqrt{-\lambda} \xi)+C_{2} \sqrt{-\lambda} \sinh (\sqrt{-\lambda} \xi)}{C_{1} \sinh (\sqrt{-\lambda} \xi)+C_{2} \cosh (\sqrt{-\lambda} \xi)+(\mu / \lambda)}\right)^{2} \\
& -\frac{6 k n c \mu}{b} \times \frac{1}{C_{1} \sinh (\sqrt{-\lambda} \xi)+C_{2} \cosh (\sqrt{-\lambda} \xi)+(\mu / \lambda)}+\frac{k n c}{b} \times\left(\sqrt{\frac{-36\left(\mu^{2}+\lambda^{2} \sigma\right)}{\lambda}}\right) \\
& \times \frac{C_{1} \sqrt{-\lambda} \cosh (\sqrt{-\lambda} \xi)+C_{2} \sqrt{-\lambda} \sinh (\sqrt{-\lambda} \xi)}{\left(C_{1} \sinh (\sqrt{-\lambda} \xi)+C_{2} \cosh (\sqrt{-\lambda} \xi)+(\mu / \lambda)\right)^{2}}
\end{aligned}
$$

where
$\xi=k x+\sqrt{\left(\left(-a k^{2}+n k-c_{2} m^{2}+k^{3} \lambda n c\right) / c_{1}\right)} y+$ $m z-\left(n t^{\alpha} /(\Gamma(1+\alpha))\right)$ and $\sigma=C_{1}^{2}-C_{2}^{2}$.

Since $C_{1}$ and $C_{2}$ are arbitrary constants, it may be self-assertively picked. We get the following solitary wave solution by choosing $C_{1}=\mu=0$ and $C_{2} \neq 0$ in equation (46):

$$
\begin{equation*}
u_{2_{2}}(x, t)=\frac{4 k \lambda n c}{b}+\frac{6 k n c}{b}(\sqrt{-\lambda})^{2} \tanh ^{2}(\sqrt{-\lambda} \xi)+\frac{k n c}{b}\left(\sqrt{\frac{-36\left(\lambda^{2} \sigma\right)}{\lambda}}\right) \sqrt{-\lambda} \tanh (\sqrt{-\lambda} \xi) \operatorname{sech}(\sqrt{-\lambda} \xi) . \tag{47}
\end{equation*}
$$

Again, we get the following solitary wave solution by choosing $C_{1} \neq 0$ and $C_{2}=\mu=0$ in equation (46):

$$
\begin{align*}
u_{2_{3}}(x, t)= & \frac{4 k \lambda n c}{b}+\frac{6 k n c}{b}(\sqrt{-\lambda})^{2} \operatorname{coth}^{2}(\sqrt{-\lambda} \xi)+\frac{k n c}{b} \\
& \times\left(\sqrt{\frac{-36\left(\lambda^{2} \sigma\right)}{\lambda}}\right) \sqrt{-\lambda} \operatorname{coth}(\sqrt{-\lambda} \xi) \operatorname{cosech}(\sqrt{-\lambda} \xi), \tag{48}
\end{align*}
$$

where
$\xi=k x+\sqrt{\left(\left(-a k^{2}+n k-c_{2} m^{2}+k^{3} \lambda n c\right) / c_{1}\right)} y+$ $m z-\left(n t^{\alpha} /(\Gamma(1+\alpha))\right)$ and $\sigma=C_{1}^{2}-C_{2}^{2}$.

Similarly, for Set 2, substituting these values in (43), we get the solution for the $\mathrm{gCH}-\mathrm{KP}$ equation (39) as the structure:

$$
\begin{align*}
u_{2_{4}}(x, t)= & \frac{6 k \lambda n c}{b}+\frac{6 k n c}{b} \times\left(\frac{C_{1} \sqrt{-\lambda} \cosh (\sqrt{-\lambda} \xi)+C_{2} \sqrt{-\lambda} \sinh (\sqrt{-\lambda} \xi)}{C_{1} \sinh (\sqrt{-\lambda} \xi)+C_{2} \cosh (\sqrt{-\lambda} \xi)+(\mu / \lambda)}\right)^{2} \\
& -\frac{6 k n c \mu}{b} \times \frac{1}{C_{1} \sinh (\sqrt{-\lambda} \xi)+C_{2} \cosh (\sqrt{-\lambda} \xi)+(\mu / \lambda)}+\frac{k n c}{b}  \tag{49}\\
& \times\left(\sqrt{\frac{-36\left(\mu^{2}+\lambda^{2} \sigma\right)}{\lambda}}\right) \times \frac{\left(C_{1} \sqrt{-\lambda} \cosh (\sqrt{-\lambda} \xi)+C_{2} \sqrt{-\lambda} \sinh (\sqrt{-\lambda} \xi)\right)}{\left(C_{1} \sinh (\sqrt{-\lambda} \xi)+C_{2} \cosh (\sqrt{-\lambda} \xi)+(\mu / \lambda)\right)^{2}}
\end{align*}
$$

where $\xi=k x+\sqrt{\left(\left(-a k^{2}+n k-c_{2} m^{2}+k^{3} \lambda n c\right)\right.}$ $\left./ c_{1}\right) y+m z-\left(n t^{\alpha} /(\Gamma(1+\alpha))\right)$ and $\sigma=C_{1}^{2}-C_{2}^{2}$.

Since $C_{1}$ and $C_{2}$ are arbitrary constants, it may be self-assertively picked. We get the following
solitary wave solution by choosing $C_{1}=\mu=0$,
and $C_{2} \neq 0$ in equation (49):

$$
\begin{equation*}
u_{2_{5}}(x, t)=\frac{6 k \lambda n c}{b}+\frac{6 k n c}{b}(\sqrt{-\lambda})^{2} \tanh ^{2}(\sqrt{-\lambda} \xi)+\frac{k n c}{b}\left(\sqrt{\frac{-36\left(\lambda^{2} \sigma\right)}{\lambda}}\right) \sqrt{-\lambda} \tanh (\sqrt{-\lambda} \xi) \operatorname{sech}(\sqrt{-\lambda} \xi) \tag{50}
\end{equation*}
$$

Again, we get the following solitary wave solution by choosing $C_{1} \neq 0$ and $C_{2}=\mu=0$ in equation (49):

$$
\begin{equation*}
u_{2_{6}}(x, t)=\frac{6 k \lambda n c}{b}+\frac{6 k n c}{b}(\sqrt{-\lambda})^{2} \operatorname{coth}^{2}(\sqrt{-\lambda} \xi)+\frac{k n c}{b} \times\left(\sqrt{\frac{-36\left(\lambda^{2} \sigma\right)}{\lambda}}\right) \sqrt{-\lambda} \operatorname{coth}(\sqrt{-\lambda} \xi) \cos e c h(\sqrt{-\lambda} \xi) \tag{51}
\end{equation*}
$$

where
$\xi=k x+\sqrt{\left(\left(-a k^{2}+n k-c_{2} m^{2}+k^{3} \lambda n c\right) / c_{1}\right)} y+$ $m z-\left(n t^{\alpha} /(\Gamma(1+\alpha))\right)$ and $\sigma=C_{1}^{2}-C_{2}^{2}$.
Case 2: with the same system, when $\lambda>0$, putting equation (43) in (42) close by (9) and (13)
generates an arrangement of mathematical equations by utilizing computer-based math such as maple, we get the result as follows.

$$
\begin{align*}
a_{0} & =\frac{4 k \lambda n c}{b} \\
a_{1} & =0 \\
a_{2} & =\frac{6 k n c}{b}, \\
b_{1} & =-\frac{6 k n c \mu}{b},  \tag{52}\\
b_{2} & =\frac{k n c}{b} \sqrt{\frac{36\left(\lambda^{2} \sigma-\mu^{2}\right)}{\lambda}}, \quad k=k, \\
l & =\sqrt{\frac{-a k^{2}+n k-c_{2} m^{2}+k^{3} \lambda n c}{c_{1}}}, \quad m=m \text { and } n=n .
\end{align*}
$$

Set 2:

$$
\begin{align*}
a_{0} & =\frac{6 k \lambda n c}{b} \\
a_{1} & =0 \\
a_{2} & =\frac{6 k n c}{b} \\
b_{1} & =-\frac{6 k n c \mu}{b},  \tag{53}\\
b_{2} & =\frac{k n c}{b} \sqrt{\frac{36\left(\lambda^{2} \sigma-\mu^{2}\right)}{\lambda}}, \quad k=k \\
l & =\sqrt{\frac{-a k^{2}+n k-c_{2} m^{2}+k^{3} \lambda n c}{c_{1}}}, \quad m=m \text { and } n=n
\end{align*}
$$

For Set 1, substituting these values in (43), we get the solution for the $\mathrm{gCH}-\mathrm{KP}$ equation (39) as the structure:

$$
\begin{align*}
u_{2_{7}}= & \frac{4 k \lambda n c}{b}+\frac{6 k n c}{b} \times\left(\frac{C_{1} \sqrt{\lambda} \cos (\sqrt{\lambda} \xi)-C_{2} \sqrt{\lambda} \sin (\sqrt{\lambda} \xi)}{C_{1} \sinh (\sqrt{-\lambda} \xi)+C_{2} \cosh (\sqrt{-\lambda} \xi)+(\mu / \lambda)}\right)^{2}-\frac{6 k n c \mu}{b} \\
& \times \frac{1}{C_{1} \sin (\sqrt{\lambda} \xi)+C_{2} \cos (\sqrt{\lambda} \xi)+(\mu / \lambda)}+\frac{k n c}{b} \times\left(\sqrt{\frac{36\left(\lambda^{2} \sigma-\mu^{2}\right)}{\lambda}}\right)  \tag{54}\\
& \times \frac{C_{1} \sqrt{\lambda} \cos (\sqrt{\lambda} \xi)-C_{2} \sqrt{\lambda} \sin (\sqrt{\lambda} \xi)}{\left(C_{1} \sin (\sqrt{\lambda} \xi)+C_{2} \cos (\sqrt{\lambda} \xi)+(\mu / \lambda)\right)^{2}},
\end{align*}
$$

where
$\xi=k x+\sqrt{\left(\left(-a k^{2}+n k-c_{2} m^{2}+k^{3} \lambda n c\right) / c_{1}\right)} y+$ $m z-\left(n t^{\alpha} /(\Gamma(1+\alpha))\right)$ and $\sigma=C_{1}^{2}+C_{2}^{2}$.

Since $C_{1}$ and $C_{2}$ are arbitrary constants, it might be self-assertively picked. The following solitary wave solution can be found by choosing $C_{1}=$ $\mu=0$ and $C_{2} \neq 0$ in equation (54):

$$
\begin{equation*}
u_{2_{8}}(x, t)=\frac{4 k \lambda n c}{b}+\frac{6 k \lambda n c}{b} \tan ^{2}(\sqrt{\lambda} \xi)-\frac{k n c}{b}\left(\sqrt{\frac{36\left(\lambda^{2} \sigma\right)}{\lambda}}\right) \sqrt{\lambda} \tan (\sqrt{\lambda} \xi) \sec (\sqrt{\lambda} \xi) \tag{55}
\end{equation*}
$$

Again, by choosing $C_{1} \neq 0$ and $C_{2}=\mu=0$ in equation (54), the following solitary wave solution can be obtained:

$$
\begin{equation*}
u_{2_{9}}(x, t)=\frac{4 k \lambda n c}{b}+\frac{6 k \lambda n c}{b} \cot ^{2}(\sqrt{\lambda} \xi)+\frac{k n c}{b}\left(\sqrt{\frac{36\left(\lambda^{2} \sigma\right)}{\lambda}}\right) \sqrt{\lambda} \cot (\sqrt{\lambda} \xi) \operatorname{cosec}(\sqrt{\lambda} \xi) \tag{56}
\end{equation*}
$$

where $\xi=k x+\sqrt{\left(\left(-a k^{2}+n k-c_{2} m^{2}+k^{3} \lambda n c\right)\right.}$ $\left./ c_{1}\right) y+m z-\left(n t^{\alpha} /(\Gamma(1+\alpha))\right)$ and $\sigma=C_{1}^{2}+C_{2}^{2}$.
Case 3 : at last, when $\lambda=0$, putting equation (43) in (42) along with equations (9) and (15), we will
reach a set of mathematical equations having the solutions.

Set 1 :

$$
\begin{align*}
& a_{0}=-\frac{1}{b k^{2}}\left(a k^{2}-n k+c_{2} m^{2}-11 c n \lambda k^{3}+\sqrt{-c_{1}\left(\sqrt{73} c n k^{3} \lambda+c_{2} m^{2}-n k+a k^{2}\right)}\right), \\
& a_{1}=0, \\
& a_{2}=\frac{12 k n c}{b}, \\
& b_{1}=-\frac{12 k n c}{b}\left(\lambda C_{2}+\sqrt{\lambda^{2} C_{2}^{2}-\lambda C_{1}^{2}}\right),  \tag{57}\\
& b_{2}=0, \\
& k=k, \\
& l=\frac{\sqrt{c_{1}\left(\sqrt{73} c n k^{3} \lambda-c_{2} m^{2} \mp n k-a k^{2}\right)}}{c_{1}}, \quad m=m \text { and } n=n .
\end{align*}
$$

Set 2:
$a_{0}=a_{0}$,
$a_{1}=a_{1}$,
$a_{2}=\frac{b_{1} C_{2}\left(b b_{1} C_{2}+c n k C_{1}^{2}\right)}{k C_{1}^{4} c n}$,
$b_{1}=b_{1}$,
$b_{2}=0$.

For set 1, substituting these values into (43), we get the solution for the $\mathrm{gCH}-\mathrm{KP}$ equation (39) as the structure:

$$
\begin{align*}
u_{20}= & -\frac{1}{b k^{2 \times}}\left(a k^{2}-n k+c_{2} m^{2}-11 c n \lambda k^{3}+\sqrt{-c_{1}\left(\sqrt{73} c n k^{3} \lambda+c_{2} m^{2}-n k+a k^{2}\right)}\right) \\
& +\frac{12 k n c}{b} \times\left(\frac{\mu \xi+C_{1}}{(\mu / 2) \xi^{2}+C_{1} \xi+C_{2}}\right)^{2}-\frac{12 k n c}{b} \times\left(\lambda C_{2}+\sqrt{\lambda^{2} C_{2}^{2}-\lambda C_{1}^{2}}\right) \times\left(\frac{1}{(\mu / 2) \xi^{2}+C_{1} \xi+C_{2}}\right) \tag{59}
\end{align*}
$$

where
$\xi=k x+\left(\left(\sqrt{c_{1}\left(\sqrt{73} c n k^{3} \lambda-c_{2} m^{2} \mp n k-a k^{2}\right)}\right)\right.$ $\left./ c_{1}\right) y+m z-\left(n t^{\alpha} /(\Gamma(1+\alpha))\right)$.
It is essential to see that, for the aftereffect of the constants given in set 2 for both in (case 2, and case 3), we achieve new and simpler solitary wave solutions whose are additionally valuable to examine the above-stated matter. For plainness, the solutions have been excluded from this section.

## 5. Brief Discussion and Graphical Representation Discussion

The specific arrangements accomplished from the current method are novel and not quite the same as the existing procedure which is built by different authors. We utilized proposed DEM to get general arrangements. In this study, a group of traveling wave arrangements as obscure boundaries are acquired. Achieved traveling wave solutions show various types of solitary waves when particular values are given


Figure 1: 3D (left section), 2D (middle section), and contour (right section) for $u_{1}(x, t)$ when $C_{1}=0, C_{2}=1, \mu=0, \lambda=-1, b_{1}=1, \sigma=1$, $L=1, V=1, \alpha=(1 / 2), 0 \leq t \leq 10$, and $0 \leq x \leq 10$. (a) Three-dimensional plotline. (b) Two-dimensional plotline. (c) Plot of contour.


Figure 2: 3D (left section), 2D (middle section), and contour (right section) plots represent to the kink wave solution of $u_{1_{3}}(x, t)$ when $C_{1}=1, C_{2}=0, \mu=0, \lambda=-1, b_{1}=1, \sigma=1, L=1, V=1, \alpha=(1 / 2), 0 \leq t \leq 10$, and $0 \leq x \leq 10$. (a) Three-dimensional plotline. (b) Two-dimensional plotline. (c) Plot of contour.


Figure 3: 3D (left section), 2D (middle section), and contour (right section) plots represent to the periodic wave solution of $u_{1_{5}}(x, t)$ when $C_{1}=0, C_{2}=1, \mu=0, \lambda=1, b_{1}=1, \sigma=1, L=1, V=1, \alpha=(1 / 2), 0 \leq t \leq 10$, and $0 \leq x \leq 10$. (a) Three-dimensional plotline. (b) Two-dimensional plotline. (c) Plot of contour.


Figure 4: 3D (left section), 2D (middle section), and contour (right section) plots represent solitary wave solution of $u_{1_{6}}(x, t)$ when $C_{1}=1, C_{2}=0, \mu=0, \lambda=1, b_{1}=1, \sigma=1, L=1, V=1, \alpha=(1 / 2), 0 \leq t \leq 10$, and $0 \leq x \leq 10$. (a) Three-dimensional plotline. (b) Two-dimensional plotline. (c) Plot of contour.


Figure 5: 3D (left section), 2D (middle section), and contour (right section) plots represent bright-dark wave solution of $u_{1_{7}}(x, t)$ when $C_{1}=1, C_{2}=0, \mu=0, \lambda=0, a_{1}=0, a_{2}=0, b_{2}=0, b_{1}=1, L=1, V=1, \alpha=(1 / 2), 0 \leq t \leq 10$, and $0 \leq x \leq 10$. (a) Three-dimensional plotline. (b) Two-dimensional plotline. (c) Plot of contour.

(a)

(b)

(c)

Figure 6: 3D (left section), 2D (middle section), and contour (right section) plots represent dark soliton solution of $u_{2_{2}}(x, t)$ when $C_{1}=0, C_{2}=1, \mu=0, c=1, k=1, n=1, \lambda=-1, b=1, \sigma=1, l=1, m=1, \alpha=(1 / 2), y=z=0,0 \leq t \leq 10$, and $0 \leq x \leq 10$. (a) Three-dimensional plotline. (b) Two-dimensional plotline. (c) Plot of contour.


Figure 7: 3D (left section), 2D (middle section), and contour (right section) plots represent bright soliton solution of $u_{2_{3}}(x, t)$ when $C_{1}=1, C_{2}=0, \mu=0, c=1, k=1, \quad n=1, \lambda=-1, b=1, \sigma=1, l=1, m=1, \alpha=(1 / 2), y=z=0,0 \leq t \leq 10$, and $0 \leq x \leq 10$. (a) Three-dimensional plotline. (b) Two-dimensional plotline. (c) Plot of contour.


Figure 8: 3 D (left section), 2D (middle section), and contour (right section) plots represent periodic wave solution of $u_{2_{5}}(x, t)$ when $C_{1}=0, C_{2}=1, \mu=0, c=1, k=1, n=1, \lambda=1, b=1, \sigma=1, l=1, m=1, \alpha=(1 / 2), y=z=0,0 \leq t \leq 10$, and $0 \leq x \leq 10$. (a) Three-dimensional plotline. (b) Two-dimensional plotline. (c) Plot of contour.


Figure 9: 3D (left section), 2D (middle section), and contour (right section) plots represent periodic solitary wave solution of $u_{2_{6}}(x, t)$ when $C_{1}=1, C_{2}=0, \mu=0, c=1, k=1, n=1, \lambda=1, b=1, \sigma=1, l=1, m=1, \alpha=(1 / 2), y=z=0,0 \leq t \leq 10$ and $0 \leq x \leq 10$. (a) Three-dimensional plotline. (b) Two-dimensional plotline. (c) Plot of contour.


Figure 10: 3D (left section), 2D (middle section), and contour (right section) plots represent singular kink type wave solution of $u_{2_{7}}(x, t)$ when $C_{1}=-1, C_{2}=0, \mu=0, c=1, k=1, \quad n=1, \lambda=0, b=1, \sigma=1, l=1, m=1, a=1, \quad \alpha=(1 / 2), y=z=0,0 \leq t \leq 10$, and $0 \leq x \leq 10$. (a) Three-dimensional plotline. (b) Two-dimensional plotline. (c) Plot of contour.

Table 1: Comparison between Liu et al. [23] solutions and our solutions to the $\mathrm{gCH}-\mathrm{KP}$ equation.

| Liu et al. [23] | Obtained solutions |
| :---: | :---: |
| If $A=3$ and $B=2$, then equation (16) becomes $u(t, x, y, z)=1-\tanh ^{2}\left((1 / \sqrt{2})\left\|k x+l y+m z-\left(n t^{\alpha} /(\Gamma(1+\alpha))\right)\right\|\right)$. | If $C_{1}=\mu=\sigma=0, \lambda=-1, c=1, b=1$, and $C_{2}=1$, then the obtained solution $u_{2,}(x, t)$ becomes <br> $u_{2}(x, t)=-4 k+6 k n \tanh ^{2}\left(k x+l y+m z-\left(n t^{\alpha} /(\Gamma(1+\alpha))\right)\right)$ |
| If $A=1$ and $B=-2$, then equation (23) becomes $u(t, x, y, z)=1-3 \tanh ^{2}\left((1 / \sqrt{2})\left\|k x+l y+m z-\left(n t^{\alpha} /(\Gamma(1+\alpha))\right)\right\|\right)$ | If $C_{1}=\mu=\sigma=0, \lambda=-1, c=1, b=1$, and $C_{2}=1$, then the obtained solution $u_{25}(x, t)$ becomes <br> $u_{2_{2}}(x, t)=-6 k+6 k n \tanh ^{2}\left(k x+l y+m z-\left(n t^{\alpha} /(\Gamma(1+\alpha))\right)\right)$ |
| If $A=6$ and $B=0$, then equation (25) becomes $u(t, x, y, z)=\left(1 /\left(k x+l y+m z-\left(n t^{\alpha} /(\Gamma(1+\alpha))\right)\right)^{2}\right)$ | If $b=1, a=0, C_{1}=-1, c=1, \lambda=0, C_{2}=0$, and $\mu=0$, then the obtained solution $u_{27}(x, t)$ becomes $u_{2_{10}}(x, t)=\left(k n /\left(k x+l y+m z-\left(n t^{\alpha} /(\Gamma(1+\alpha))\right)\right)^{2}\right)$ |

to its unknown parameters such as Kink wave and singular kink wave, single soliton, periodic wave, bright soliton, dark soliton, and combined dark-bright solitary wave solutions in Figures 1-10. From attained solutions, some solutions cannot be created by other methods such as the exp $(-\psi(\xi))$-expansion method [24] and modified simple equation method [25]. Therefore, some solutions are novel from earlier constructed solutions in the literature. We also demonstrate all the figures in this study which have been represented in three arrangements such as 3D plot, 2D plot, and contour plot within the specified domain $0 \leq t \leq 10$ and $-10 \leq x \leq 10$ (see Figures 1-10). Mathematica, a computation package application, was used to construct all of the figures. In order to observe the physical appearance of these models, the structure of figures is depicted via giving suitable values of parameters.

## 6. Results' Comparison

It is amazing to observe that some of the achieved solutions demonstrate good similarity with earlier established solutions. A comparison of the solutions of Liu et al. [23] and obtained solutions is presented in Table 1.

The hyperbolic and rational function solutions alluded to in the above table are comparative, and for setting the definite values of the arbitrary constants, they are
indistinguishable. In a nutshell, it is substantial to realize that the TWS $u_{2_{1}}(x, t), u_{2_{3}}(x, t), u_{2_{4}}(x, t), u_{2_{6}}(x, t), \quad u_{2_{7}}(x, t)$, $u_{2_{8}}(x, t)$, and SWS $u_{2_{9}}(x, t)$ of the fractional gCH-KP equation all are recent and very much significant, which were not originally in the previous works. The time-fractional gCH-KP equation is also solved by the bilinear and RBF method [19]. It can be seen from here that the RBF method gives a high-precision numerical solution of the fractional differential equation. Applying our proposed DEM on the mentioned equation, we acquire hyperbolic, trigonometric, and rational function solution containing parameters which are fresh and further general. The obtained solutions are capable to examine the role of dispersion in the formation of patterns in liquid drops and shallow water waves with peakon solutions ensued in plasma physics, optical fibers, fluid flow, fission and fusion phenomena, control theory, and some other areas.

## 7. Conclusion

In this study, we have successfully established the more and further general stable solitary wave solitary wave solutions with assorted physical structures which appeal wide attention to physicist, engineers, and mathematicians to the new solutions of space-time fractional MCH and space-time fractional gCH-KP equation in the light of

Riemann-Liouville fractional derivative by implementing the novel approach DEM. The depiction of the solutions are in the form of hyperbolic, trigonometric, and rational functions including kink wave, antikink wave, dark, bright, singular, combined, optical solitons, periodic wave, and traveling wave, and some new types of solitary wave solutions are discovered which expose the phenomena relating to plasma physics, optical fibers, fluid flow, fission and fusion phenomena, acoustics, control theory, viscoelasticity, geophysics, nonlinear mechanics, protein chemistry, and chemical kinematics. The physical significance of the obtained solutions for the definite values of the associated parameters is analyzed graphically with 2D, 3D, and contour shape. The solutions achieved in this study have been observed with maple by placing them back into NLFDEs and found precise. It is possible to conclude that the adopted method is direct, reliable, effective, and conformable and provides many new physical model solutions to NLPFEEs that arise in mathematical physics, applied mathematics, and engineering.

## Data Availability

The data used to support the findings of the study are included within the article.

## Conflicts of Interest

The authors author declare that they have no conflicts of interest.

## Authors' Contributions

M. Ayesha Khatun helped with software, curated the data, and wrote the manuscript. Mohammad Asif Arefin helped with software, curated the data, and carried out formal analysis. M. Hafiz Uddin conceptualized and supervised the study, reviewed and edited the manuscript, and validated the study. Mustafa Inc reviewed and edited and investigated the study.

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