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Research Article

Some Similarity and Distance Measures between Complex Interval-Valued *q*-Rung Orthopair Fuzzy Sets Based on Cosine Function and their Applications

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The purpose of this paper is to present a new method to solve the decision-making algorithm based on the cosine similarity and distance measures by utilizing the uncertain and vague information. A complex interval-valued q-rung orthopair fuzzy set (CIVQROFS) is a reliable and competent technique for handling the uncertain information with the help of the complex-valued membership grades. To address the degree of discrimination between the pairs of the sets, cosine similarity measures (CSMs) and distance measures (DMs) are an accomplished technique. Driven by these, in this manuscript, we defined some CSMs and DMs for the pairs of CIVQROFSs and investigated their several properties. Choosing that the CSMs do not justify the axiom of the similarity measure (SM), then we investigate a technique to developing other CIVQROFSs-based SMs using the explored CSMs and Euclidean DMs, and it fulfills the axiom of the SMs. In addition, we find the cosine DMs (CDMs) by considering the interrelationship between the SM and DMs; then, we have modified the procedure for the rank of partiality by similarity to the ideal solution method for the CDMs under investigation, which can deal with the associated decision-making problems not only individually from the argument of the opinion of geometry but also the fact of the opinion of algebra. Finally, we provide a numerical example to demonstrate the practicality and effectiveness of the proposed procedure, which is also in line with existing procedures. Graphical representations of the measures developed are also used in this manuscript.

1. Introduction

Decision-making is one of the most difficult processes in our day-to-day life in which we make decisions at all times. Uncertainty, however, is an unavoidable phenomenon, as a result of which the rating for assessing a given object is not clearly appraisable. There is therefore an excessive need for theory to extract the information more precisely. For example, when the institute decides whether to enroll the tutoring team, the ten-member committee of the authorities has assessed the persons selected, seven of whom have agreed to employ the person, two of whom have expressed disapproval of the job, and the additional one has not given a clear judgment. It is therefore difficult, in such circumstances, to take a decision into account. To characterize the excess of knowledge, Atanassov's intuitionistic fuzzy set (IFS) [1] was investigated by including the term falsity in the fuzzy set (FS) environment [2] in order to deal with unreliable and problematic information in decision-making issues. The term "Truth" and the term "Falsity" in IFS contains the rule that the sum of the two terms is restarted to [0, 1]. The IFS deals with the crisp numbers and fails to deal with the number of the intervals. To address this, Atanassov [3] has developed a theory of interval-assessed IFS containing the degree of truth and falsehood in the form of a subinterval of unit interval. The idea of IFS has received a great deal of attention from separate scholars and has been widely used by many scholars in a diverse environment (for

more details, we refer to [4–7] and their references). Due to some complications, the IFS is unable to manage some of the daily life issues, for example, if a person gives 0.6 for true grade and 0.5 for falsehood, then the sum of the two values is extended from [0, 1], and the IFS is unable to explain these types of information precisely. The theory of Pythagorean FS (PFS) has therefore been investigated by Yager [8], which is a competent and capable technique to handle unreliable and problematic information in decision-making problems. The term "Truth" and the term "Falsity" in PFS contains the rule that the sum of the squares of both terms is restarted to [0, 1]. He theory of interval-valued PFS was developed by Garg [9], which contains the degree of truth and falsehood in the form of a subinterval of the unit interval. Since its existence, many scholars have addressed the problems of the decision-making proves using PFS features [10–14]. However, sometimes, the PFS is unable to manage ratings of the expert during the evaluation process. For example, if a person gives 0.9 for true grade and 0.8 for falsehood, then the sum of the squares of both values is extended from [0, 1], and the PFS is unable to explain these types of information precisely. The q-rung orthopair FS (QROFS) theory has therefore been investigated by Yager [15] as a competent and capable technique for dealing with decision-making issues. In QROFS, the term "Truth" and the term "Falsity" contain the rule that the sum of the q-powers of both terms is restarted to [0, 1]. The theory of interval-valued QROFS was developed by Joshi et al. [16], which includes the degree of truth and falsehood in the form of a subinterval of the unit interval. The idea of QROFS has received a great deal of attention from separate scholars and has been widely used by many scholars in the environment of different fields [17–19].

All of the abovementioned algorithms have been widely used by researchers in the decision-making process, but these approaches are limited in access to manage their variations during the given period of time. For example, data from the "Medical Investigation, Biometric, and Facial Recognition Database" continuously transform at the same time as the period passes. Thus, in order to deal with these types of problems, the range of truth and falsehood degrees is changed from the actual subset to the unit disk of the complex plane by Alkouri and Salleh [20], and thus, the notion of the complex IFS (CIFS) has been established by extending the complex FS (CFS) [21] to decision-making issues. The term "Truth" and the term "Falsity" in the CIFS contain the rule that the sum of the real parts (also for imaginary parts) of both terms lies at [0, 1]. The theory of complex interval-valued IFS was developed by Garg and Rani [22] which contains the degree of truth and falsity in the form of a subinterval of the unit interval. The idea of the CIFS has received a great deal of attention from the scholars [23–26] and has been widely used in the different areas. The CIFS is limited in access due to their restriction on the sum of the degrees to be 1, for instance, if a person gives $0.6e^{i2\pi(7)}$ for truth grade and $0.5e^{i2\pi(6)}$ for falsity, then clearly $0.6 + 0.5 > 1$ and hence does not satisfy the properties of CIFS. Thus, such types of the ratings are not accessed under the CIFS environment. The theory of complex PFS (CPFS) has therefore been investigated by Ullah et al. [27] which is a competent and capable technique for dealing with unreliable and problematic information on decision-making issues. The term of truth and the term of falsity contains in CPFS with a rule, that is, the sum of the squares of the real parts (also for imaginary parts) of both terms is restarted to [0, 1]. The idea of CPFS has received extensive attention from separated scholars, and numerous scholars have widely utilized it in the environment of different areas [28]. Due to some complications, the CPFS is not able to manage some daily life issues, for instance, if a person gives $0.9e^{i2\pi(8)}$ for truth grade and $0.8e^{i2\pi(7)}$ for falsity, then the sum of the squares of the real parts (also for imaginary parts) of both values is extended from [0, 1], and the CPFS is not able to explain such types of information accurately. Therefore, the theory of complex QROFS (CQROFS) was investigated by Liu et al. [29, 30] which is a proficient and capable technique to handle unreliable and problematic information in decision-making issues. The term of truth and the term of falsity contain in CQROFS with a rule, that is, the sum of the q-powers of the squares of the real parts (also for imaginary parts) of both terms is restarted to [0, 1]. The theory of complex interval-valued QROFS was developed by Garg et al. [31] which contains the grade of truth and falsity in the form of the subinterval of unit interval. The idea of CQROFS has received extensive attention from separated scholars, and numerous scholars have widely utilized it in the environment of different areas [32–37].

In the event of a conflict, the SM is a competent tool for examining the interrelationships between any number of CQROFSs and a number of scholars who have used it in separate areas [35]. Garg and Rani [38] also explored information measures based on the IFCS. Garg and Rani [39] have developed the theory of a robust correlation coefficient based on the CYPS. However, to date, SMs among CQROFSs have not been investigated. Keeping in mind the advantages of SMs, the CQROFSs theory is a reliable technique for managing awkward and unreliable information on daily issues. The main investigation of this manuscript is summarized as follows:

(1) The CSMs and Euclidean DMs (EDMs) by using CQROFSs and their properties are investigated

(2) Choosing that the CSMs do not justify the axiom of similarity measure (SM), we investigate a technique to develop other SMs based on CQROFSs using the explored CSMs and EDMs, and it fulfills the axiom of the SMs

(3) To find a cosine DMs (CDMs) based on CQROFSs, by considering the inter-relationship among the SM and DMs, we modified the procedure for the rank of partiality by similarity to the ideal solution method to the investigated CDMs, which can be arranged with the associated decision-making troubles not only individually from the argument of the opinion of geometry but also the fact of the opinion of algebra

(4) A sensible example to demonstrate the practicality and efficiency of the suggested procedure is

provided, which is also matched with additional existing procedures

(5) The graphical representations of the developed measures are also utilized in this manuscript

The purpose of this manuscript is summarized as follows. In Section 2, we briefly recall the concept of CIFSs, CPFSs, and CQROFSs and their fundamental laws. In Section 3, we developed the idea of CSMs and DMs by using CQROFNs. In Section 4, we utilized the TOPSIS method in the environment of the MADM procedure to find the reliability and effectiveness of the investigated measures. In Section 5, we discussed the comparative analysis of the proposed work with some existing approaches. The conclusion of this manuscript is discussed in Section 6.

2. Preliminaries

In this investigation work, we recall the main ideas of CIVIFSs, CIVPFSs, and CIVQROFSs and their fundamental laws. In this study, we use the symbol $\widehat{\mathcal{O}}$ for universal sets and the truth and falsity and degrees are shown by $\mathfrak{M}_{\mathfrak{C}_{CQ}}$ and $\mathfrak{N}_{\mathfrak{C}_{CQ}}$, where $\mathfrak{M}_{\mathfrak{C}_{CQ}}(\widehat{\mathcal{O}}) = [\mathfrak{M}^{-}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}}), \mathfrak{M}^{+}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}})]\ e^{i2\pi([\mathfrak{M}^{-}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}}), \mathfrak{M}^{+}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}})])}$ and $\mathfrak{N}_{\mathfrak{C}_{CQ}}(\widehat{\mathcal{O}}) = [\mathfrak{N}^{-}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}}), \mathfrak{N}^{+}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}})]e^{i2\pi([\mathfrak{N}^{-}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}}), \mathfrak{N}^{+}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}})])}$.

Definition 1 (see [22]). A CIVIFS $\mathfrak{C}_{CQ}$ is stated by

$$\mathfrak{C}_{CQ} = \left\{\left(\mathfrak{M}_{\mathfrak{C}_{CQ}}(\widehat{\mathcal{O}}), \mathfrak{N}_{\mathfrak{C}_{CQ}}(\widehat{\mathcal{O}})\right): \widehat{\mathcal{O}} \in \frown\mathcal{O}\right\}, \tag{1}$$

where

$$\begin{aligned}\mathfrak{M}_{\mathfrak{C}_{CQ}}(\widehat{\mathcal{O}}) &= \left[\mathfrak{M}^{-}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}}), \mathfrak{M}^{+}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}})\right]e^{i2\pi\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}}), \mathfrak{M}^{+}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}})\right]\right)},\\ \mathfrak{N}_{\mathfrak{C}_{CQ}}(\widehat{\mathcal{O}}) &= \left[\mathfrak{N}^{-}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}}), \mathfrak{N}^{+}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}})\right]e^{i2\pi\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}}), \mathfrak{N}^{+}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}})\right]\right)},\end{aligned} \tag{2}$$

represent the degrees of agreement and disagreement with the conditions that $0 \le \mathfrak{M}^{+}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}}) + \mathfrak{N}^{+}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}}) \le 1$ and $0 \le \mathfrak{M}^{+}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}}) + \mathfrak{N}^{+}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}}) \le 1$. Moreover, the terms

$$\begin{aligned}\mathcal{I}_{\mathfrak{C}_{CQ}}(\widehat{\mathcal{O}}) &= \left[\mathcal{I}^{-}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}}), \mathcal{I}^{+}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}})\right]e^{i2\pi\left(\left[\mathcal{I}^{-}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}}), \mathcal{I}^{+}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}})\right]\right)}\\ &= \left[\left(1 - \mathfrak{M}^{+}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}}) - \mathfrak{N}^{+}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}})\right), \left(1 - \mathfrak{M}^{-}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}}) - \mathfrak{N}^{-}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}})\right)\right]e^{i2\pi\left(\left[1-\mathfrak{M}^{+}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}})-\mathfrak{N}^{+}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}}), 1-\mathfrak{M}^{-}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}})-\mathfrak{N}^{-}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}})\right]\right)},\end{aligned} \tag{3}$$

is the degree of indeterminacy.

Definition 2. A CIVPFS $\mathfrak{C}_{CQ}$ is demonstrated by

$$\mathfrak{C}_{CQ} = \left\{\left(\mathfrak{M}_{\mathfrak{C}_{CQ}}(\widehat{\mathcal{O}}), \mathfrak{N}_{\mathfrak{C}_{CQ}}(\widehat{\mathcal{O}})\right): \widehat{\mathcal{O}} \in \widehat{\mathcal{O}}\right\}, \tag{4}$$

where

$$\begin{aligned}\mathfrak{M}_{\mathfrak{C}_{CQ}}(\widehat{\mathcal{O}}) &= \left[\mathfrak{M}^{-}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}}), \mathfrak{M}^{+}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}})\right]e^{i2\pi\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}}), \mathfrak{M}^{+}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}})\right]\right)},\\ \mathfrak{N}_{\mathfrak{C}_{CQ}}(\widehat{\mathcal{O}}) &= \left[\mathfrak{N}^{-}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}}), \mathfrak{N}^{+}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}})\right]e^{i2\pi\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}}), \mathfrak{N}^{+}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}})\right]\right)},\end{aligned} \tag{5}$$

represent the degrees of agreement and disagreement with the conditions that $0 \le \mathfrak{M}^{+2}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}}) + \mathfrak{N}^{+2}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}}) \le 1$ and $0 \le \mathfrak{M}^{+2}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}}) + \mathfrak{N}^{+2}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}}) \le 1$. Moreover, the term

$$\begin{aligned}\mathcal{I}_{\mathfrak{C}_{CQ}}(\widehat{\mathcal{O}}) &= \left[\mathcal{I}^{-}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}}), \mathcal{I}^{+}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}})\right]e^{i2\pi\left(\left[\mathcal{I}^{-}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}}), \mathcal{I}^{+}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}})\right]\right)}\\ &= \left[\left(\begin{matrix}1 - \mathfrak{M}^{+2}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}}) - \\ \mathfrak{N}^{+2}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}})\end{matrix}\right)^{(1/2)}, \left(\begin{matrix}1 - \mathfrak{M}^{-2}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}}) - \\ \mathfrak{N}^{-2}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}})\end{matrix}\right)^{(1/2)}\right]e^{i2\pi\left[\begin{matrix}\left(1 - \mathfrak{M}^{+2}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}}) - \mathfrak{N}^{+2}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}})\right)^{(1/2)}\\ \left(1 - \mathfrak{M}^{-2}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}}) - \mathfrak{N}^{-2}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}})\right)^{(1/2)}\end{matrix}\right]},\end{aligned} \tag{6}$$

is expressed as the degree of indeterminacy.

Definition 3 (see [31]). A CIVQROFS $\mathfrak{C}_{CQ}$ is described by

$$\mathfrak{C}_{CQ} = \left\{\left(\mathfrak{M}_{\mathfrak{C}_{CQ}}(\widehat{\mathcal{O}}), \mathfrak{N}_{\mathfrak{C}_{CQ}}(\widehat{\mathcal{O}})\right): \widehat{\mathcal{O}} \in \widehat{\mathcal{O}}\right\}, \tag{7}$$

where

$$\mathfrak{M}_{\mathfrak{C}_{CQ}}(\widehat{\mathcal{O}}) = \left[\mathfrak{M}^{-}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}}), \mathfrak{M}^{+}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}})\right] e^{i2\pi\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}}), \mathfrak{M}^{+}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}})\right]\right)},$$
$$\mathfrak{N}_{\mathfrak{C}_{CQ}}(\widehat{\mathcal{O}}) = \left[\mathfrak{N}^{-}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}}), \mathfrak{N}^{+}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}})\right] e^{i2\pi\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}}), \mathfrak{N}^{+}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}})\right]\right)}, \tag{8}$$

represent the degrees of agreement and disagreement with the conditions that $0 \le \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}}) + \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}}) \le 1$ and $0 \le \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}}) + \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}}) \le 1, q_{CQ} \ge 1$. Moreover, the term

$$\mathcal{I}_{\mathfrak{C}_{CQ}}(\widehat{\mathcal{O}}) = \left[\mathcal{I}^{-}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}}), \mathcal{I}^{+}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}})\right] e^{i2\pi\left(\left[\mathcal{I}^{-}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}}), \mathcal{I}^{+}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}})\right]\right)}$$
$$= \left[\begin{pmatrix} 1 - \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}}) - \\ \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}}) \end{pmatrix}^{(1/q_{CQ})}, \begin{pmatrix} 1 - \mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}}) - \\ \mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP}}(\widehat{\mathcal{O}}) \end{pmatrix}^{(1/q_{CQ})}\right] e^{i2\pi\begin{bmatrix} \left(1 - \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}}) - \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}})\right)^{(1/q_{CQ})} \\ \left(1 - \mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}}) - \mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP}}(\widehat{\mathcal{O}})\right)^{(1/q_{CQ})} \end{bmatrix}}, \tag{9}$$

is expressed as the degree of indeterminacy.

Throughout, this manuscript, the complex interval-valued q-rung orthopair fuzzy numbers (CIVQROFNs) are represented by $\mathfrak{C}_{CQ} = \begin{pmatrix} [\mathfrak{M}^{-}_{\mathfrak{C}_{RP}}, \mathfrak{M}^{+}_{\mathfrak{C}_{RP}}] e^{i2\pi([\mathfrak{M}^{-}_{\mathfrak{C}_{IP}}, \mathfrak{M}^{+}_{\mathfrak{C}_{IP}}])}, \\ [\mathfrak{N}^{-}_{\mathfrak{C}_{RP}}, \mathfrak{N}^{+}_{\mathfrak{C}_{RP}}] e^{i2\pi([\mathfrak{N}^{-}_{\mathfrak{C}_{IP}}, \mathfrak{N}^{+}_{\mathfrak{C}_{IP}}])} \end{pmatrix}$.

Additionally, by using this CIVQROFN, we define the score and accuracy values such that

$$\mathfrak{S}_{CQ}(\mathfrak{C}_{CQ}) = \frac{1}{4}\begin{pmatrix} \mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP}} + \mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP}} - \mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP}} - \mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP}} \\ + \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP}} + \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP}} - \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP}} - \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP}} \end{pmatrix}, \quad \mathfrak{S}_{CQ}(\mathfrak{C}_{CQ}) \in [-1, 1], \tag{10}$$

$$\mathfrak{H}_{CQ}(\mathfrak{C}_{CQ}) = \frac{1}{4}\begin{pmatrix} \mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP}} + \mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP}} + \mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP}} + \mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP}} \\ + \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP}} + \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP}} + \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP}} + \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP}} \end{pmatrix}, \quad \mathfrak{H}_{CQ}(\mathfrak{C}_{CQ}) \in [0, 1]. \tag{11}$$

To find the relationships between any number two CIVQROFNs,

$$\mathfrak{C}_{CQ-1} = \begin{pmatrix} [\mathfrak{M}^{-}_{\mathfrak{C}_{RP-1}}, \mathfrak{M}^{+}_{\mathfrak{C}_{RP-1}}] e^{i2\pi\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP-1}}, \mathfrak{M}^{+}_{\mathfrak{C}_{IP-1}}\right]\right)}, \\ [\mathfrak{N}^{-}_{\mathfrak{C}_{RP-1}}, \mathfrak{N}^{+}_{\mathfrak{C}_{RP-1}}] e^{i2\pi\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP-1}}, \mathfrak{N}^{+}_{\mathfrak{C}_{IP-1}}\right]\right)} \end{pmatrix},$$
$$\mathfrak{C}_{CQ-2} = \begin{pmatrix} [\mathfrak{M}^{-}_{\mathfrak{C}_{RP-2}}, \mathfrak{M}^{+}_{\mathfrak{C}_{RP-2}}] e^{i2\pi\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP-2}}, \mathfrak{M}^{+}_{\mathfrak{C}_{IP-2}}\right]\right)}, \\ [\mathfrak{N}^{-}_{\mathfrak{C}_{RP-2}}, \mathfrak{N}^{+}_{\mathfrak{C}_{RP-2}}] e^{i2\pi\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP-2}}, \mathfrak{N}^{+}_{\mathfrak{C}_{IP-2}}\right]\right)} \end{pmatrix}, \tag{12}$$

we use the following rules:

(1) If $\mathfrak{S}_{CQ}(\mathfrak{C}_{CQ-1}) > \mathfrak{S}_{CQ}(\mathfrak{C}_{CQ-2}) \Longrightarrow \mathfrak{C}_{CQ-1} > \mathfrak{C}_{CQ-2}$.

(2) If $\mathfrak{S}_{CQ}(\mathfrak{C}_{CQ-1}) < \mathfrak{S}_{CQ}(\mathfrak{C}_{CQ-2}) \Longrightarrow \mathfrak{C}_{CQ-1} < \mathfrak{C}_{CQ-2}$.

(3) If $\mathfrak{S}_{CQ}(\mathfrak{C}_{CQ-1}) = \mathfrak{S}_{CQ}(\mathfrak{C}_{CQ-2})$.

(a) If $\mathfrak{H}_{CQ}(\mathfrak{C}_{CQ-1}) > \mathfrak{H}_{CQ}(\mathfrak{C}_{CQ-2}) \Longrightarrow \mathfrak{C}_{CQ-1} > \mathfrak{C}_{CQ-2}$.

(b) If $\mathfrak{H}_{CQ}(\mathfrak{C}_{CQ-1}) < \mathfrak{H}_{CQ}(\mathfrak{C}_{CQ-2}) \Longrightarrow \mathfrak{C}_{CQ-1} < \mathfrak{C}_{CQ-2}$.

3. Cosine Similarity Measures and Distance Measures between CIVQROFSs

In this section, we investigate some CSMs ("cosine similarity measures") and DMs ("distance measures") between the pairs of the CIVQROFNs. Some cases of the presented works are also discussed.

Definition 4. For any two CIVQROFNs,

$$\mathfrak{C}_{CQ-1} = \left(\begin{array}{c} \left[\mathfrak{M}^{-}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i), \mathfrak{M}^{+}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\right] e^{i2\pi\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i), \mathfrak{M}^{+}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\right]\right)}, \\ \left[\mathfrak{N}^{-}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\right] e^{i2\pi\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\right]\right)} \end{array} \right),$$
$$\mathfrak{C}_{CQ-2} = \left(\begin{array}{c} \left[\mathfrak{M}^{-}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i), \mathfrak{M}^{+}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right] e^{i2\pi\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i), \mathfrak{M}^{+}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right]\right)}, \\ \left[\mathfrak{N}^{-}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right] e^{i2\pi\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right]\right)} \end{array} \right), \quad i = 1, 2, \ldots, \tilde{n}, \tag{13}$$

are based on a universal set $\widehat{\mathcal{O}} = \{\widehat{\mathcal{O}}_1, \widehat{\mathcal{O}}_2, \ldots, \widehat{\mathcal{O}}_{\tilde{n}}\}$; then, the CSM, denoted by $\mathrm{CSM}_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2})$, is defined by

$$\mathrm{CSM}_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = \frac{1}{\tilde{n}} \sum_{i=1}^{\tilde{n}} \left(\frac{\left(\begin{array}{c} \mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \\ \mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \\ \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \\ \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) \end{array} \right)}{\left(\begin{array}{c} \sqrt{\left(\begin{array}{c} \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i) + \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \\ \mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i) + \mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) \end{array} \right)} \times \\ \sqrt{\left(\begin{array}{c} \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \\ \mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) \end{array} \right)} \end{array} \right)} \right). \tag{14}$$

Theorem 1. *For any two CIVQROFNs,*

$$\mathfrak{C}_{CQ-1} = \left(\begin{array}{c} \left[\mathfrak{M}^{-}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i), \mathfrak{M}^{+}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\right] e^{i2\pi\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i), \mathfrak{M}^{+}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\right]\right)}, \\ \left[\mathfrak{N}^{-}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\right] e^{i2\pi\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\right]\right)} \end{array} \right),$$
$$\mathfrak{C}_{CQ-2} = \left(\begin{array}{c} \left[\mathfrak{M}^{-}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i), \mathfrak{M}^{+}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right] e^{i2\pi\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i), \mathfrak{M}^{+}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right]\right)}, \\ \left[\mathfrak{N}^{-}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right] e^{i2\pi\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right]\right)} \end{array} \right), \quad i = 1, 2, \ldots, \tilde{n}, \tag{15}$$

are based on a universal set $\widehat{\mathcal{O}} = \{\widehat{\mathcal{O}}_1, \widehat{\mathcal{O}}_2, \ldots, \widehat{\mathcal{O}}_{\tilde{n}}\}$; then, the CSM $CSM_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2})$ holds the following conditions:

(1) $0 \le CSM_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}) \le 1$

(2) $CSM_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}) = CSM_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2})$

(3) $CSM_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}) = 1$ *if* $\mathfrak{C}_{CQ-1} = \mathfrak{C}_{CQ-2}$, *that is,* $\mathfrak{M}^{-}_{\mathfrak{C}_{RP-1}} = \mathfrak{M}^{-}_{\mathfrak{C}_{RP-2}}$, $\mathfrak{M}^{-}_{\mathfrak{C}_{IP-1}} = \mathfrak{M}^{-}_{\mathfrak{C}_{IP-2}}$, $\mathfrak{N}^{-}_{\mathfrak{C}_{RP-1}} = \mathfrak{N}^{-}_{\mathfrak{C}_{RP-2}}$, $\mathfrak{N}^{-}_{\mathfrak{C}_{IP-1}} = \mathfrak{N}^{-}_{\mathfrak{C}_{IP-2}}$ *and* $\mathfrak{M}^{+}_{\mathfrak{C}_{RP-1}} = \mathfrak{M}^{+}_{\mathfrak{C}_{RP-2}}$, $\mathfrak{M}^{+}_{\mathfrak{C}_{IP-1}} = \mathfrak{M}^{+}_{\mathfrak{C}_{IP-2}}$, $\mathfrak{N}^{+}_{\mathfrak{C}_{RP-1}} = \mathfrak{N}^{+}_{\mathfrak{C}_{RP-2}}$, $\mathfrak{N}^{+}_{\mathfrak{C}_{IP-1}} = \mathfrak{N}^{+}_{\mathfrak{C}_{IP-2}}$

Proof. Based on Definition (10), conditions (1) and (4) are straightforward. Moreover, if we choose the $\mathfrak{C}_{CQ-1} = \mathfrak{C}_{CQ-2}$, that is,

$$\begin{aligned} &\mathfrak{M}^{-}_{\mathfrak{C}_{RP-1}} = \mathfrak{M}^{-}_{\mathfrak{C}_{RP-2}}, \mathfrak{M}^{-}_{\mathfrak{C}_{IP-1}} = \mathfrak{M}^{-}_{\mathfrak{C}_{IP-2}}, \mathfrak{N}^{-}_{\mathfrak{C}_{RP-1}} = \mathfrak{N}^{-}_{\mathfrak{C}_{RP-2}}, \mathfrak{N}^{-}_{\mathfrak{C}_{IP-1}} = \mathfrak{N}^{-}_{\mathfrak{C}_{IP-2}}, \\ &\mathfrak{M}^{+}_{\mathfrak{C}_{RP-1}} = \mathfrak{M}^{+}_{\mathfrak{C}_{RP-2}}, \mathfrak{M}^{+}_{\mathfrak{C}_{IP-1}} = \mathfrak{M}^{+}_{\mathfrak{C}_{IP-2}}, \mathfrak{N}^{+}_{\mathfrak{C}_{RP-1}} = \mathfrak{N}^{+}_{\mathfrak{C}_{RP-2}}, \mathfrak{N}^{+}_{\mathfrak{C}_{IP-1}} = \mathfrak{N}^{+}_{\mathfrak{C}_{IP-2}}, \end{aligned} \tag{16}$$

then

$$\begin{aligned} \mathrm{CSM}_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) &= \frac{1}{\tilde{n}} \sum_{i=1}^{\tilde{n}} \left(a \frac{\begin{pmatrix} \mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\mathcal{O}}_i)\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-2}}(\tilde{\mathcal{O}}_i) + \mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\mathcal{O}}_i)\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP-2}}(\tilde{\mathcal{O}}_i) + \\ \mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\mathcal{O}}_i)\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-2}}(\tilde{\mathcal{O}}_i) + \mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\mathcal{O}}_i)\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-2}}(\tilde{\mathcal{O}}_i) + \\ \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\mathcal{O}}_i)\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\tilde{\mathcal{O}}_i) + \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\mathcal{O}}_i)\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\tilde{\mathcal{O}}_i) + \\ \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\mathcal{O}}_i)\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\tilde{\mathcal{O}}_i) + \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\mathcal{O}}_i)\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\tilde{\mathcal{O}}_i) \end{pmatrix}}{\left(\begin{array}{c} \sqrt{\begin{pmatrix} \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\mathcal{O}}_i) + \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\tilde{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\tilde{\mathcal{O}}_i) + \\ \mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\mathcal{O}}_i) + \mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\tilde{\mathcal{O}}_i) + \mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\mathcal{O}}_i) + \mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\tilde{\mathcal{O}}_i) \end{pmatrix}} \times \\ \sqrt{\begin{pmatrix} \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\tilde{\mathcal{O}}_i) + \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\tilde{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\tilde{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\tilde{\mathcal{O}}_i) + \\ \mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\tilde{\mathcal{O}}_i) + \mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\tilde{\mathcal{O}}_i) + \mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\tilde{\mathcal{O}}_i) + \mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\tilde{\mathcal{O}}_i) \end{pmatrix}} \end{array} \right)} \right) \\ &= \frac{1}{\tilde{n}} \sum_{i=1}^{\tilde{n}} \left(\frac{\begin{pmatrix} \mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\mathcal{O}}_i)\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\mathcal{O}}_i) + \mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\mathcal{O}}_i)\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\mathcal{O}}_i) + \\ \mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\mathcal{O}}_i)\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\mathcal{O}}_i) + \mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\mathcal{O}}_i)\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\mathcal{O}}_i) + \\ \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\mathcal{O}}_i)\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\mathcal{O}}_i) + \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\mathcal{O}}_i)\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\mathcal{O}}_i) + \\ \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\mathcal{O}}_i)\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\mathcal{O}}_i) + \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\mathcal{O}}_i)\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\mathcal{O}}_i) \end{pmatrix}}{\left(\begin{array}{c} \sqrt{\begin{pmatrix} \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\mathcal{O}}_i) + \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\mathcal{O}}_i) + \\ \mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\mathcal{O}}_i) + \mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\mathcal{O}}_i) + \mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\mathcal{O}}_i) + \mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\mathcal{O}}_i) \end{pmatrix}} \times \\ \sqrt{\begin{pmatrix} \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\mathcal{O}}_i) + \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\mathcal{O}}_i) + \\ \mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\mathcal{O}}_i) + \mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\mathcal{O}}_i) + \mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\mathcal{O}}_i) + \mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\mathcal{O}}_i) \end{pmatrix}} \end{array} \right)} \right) \\ &= \frac{1}{\tilde{n}} \sum_{i=1}^{\tilde{n}} \left(\frac{\begin{pmatrix} \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\mathcal{O}}_i)\mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\mathcal{O}}_i) + \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\mathcal{O}}_i)\mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\mathcal{O}}_i) + \\ \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\mathcal{O}}_i)\mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\mathcal{O}}_i)\mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\mathcal{O}}_i) \end{pmatrix}}{\left(\begin{pmatrix} \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\mathcal{O}}_i)\mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\mathcal{O}}_i)\mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\mathcal{O}}_i) + \\ \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\mathcal{O}}_i)\mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\tilde{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\mathcal{O}}_i)\mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\tilde{\mathcal{O}}_i) \end{pmatrix}^{(1/2)+(1/2)} \right)} \right) = 1. \end{aligned} \tag{17}$$

Hence, we obtain $\mathrm{CSM}_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}) = 2$.

By using the weight vector $\Omega_{WV} = \{\Omega_{WV-1}, \Omega_{WV-2}, \ldots, \Omega_{WV-\tilde{n}}\}$ with a rule, that is, $\sum_{i=1}^{n} \Omega_{WV-i} = 1, \Omega_{WV-i} \in [0,1]$, then the WCSM ("weighted cosine similarity measure") is defined as follows.

Definition 5. For any two CIVQROFNs,

$$\mathfrak{C}_{CQ-1} = \begin{pmatrix} \left[\mathfrak{M}^{-}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i), \mathfrak{M}^{+}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\right] + e^{i2\pi}\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{+}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\right]\right), \\ \left[\mathfrak{N}^{-}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\right] + e^{i2\pi}\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\right]\right) \end{pmatrix},$$
$$\mathfrak{C}_{CQ-2} = \begin{pmatrix} \left[\mathfrak{M}^{-}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i), \mathfrak{M}^{+}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right] + e^{i2\pi}\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{+}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right]\right)10,0 \\ \left[\mathfrak{N}^{-}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right] + e^{i2\pi}\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right]\right) \end{pmatrix}, \quad i = 1, 2, \ldots, \tilde{n}, \tag{18}$$

are based on a universal set $\widehat{\mathcal{O}} = \{\widehat{\mathcal{O}}_1, \widehat{\mathcal{O}}_2, \ldots, \widehat{\mathcal{O}}_{\tilde{n}}\}$; then, the $\mathrm{WCSM}_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2})$ is demonstrated by

$$\mathrm{WCSM}_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = \sum_{i=1}^{\tilde{n}} \Omega_{WV-i} \left(a \frac{\begin{pmatrix} \mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \\ \mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \\ \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \\ \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) \end{pmatrix}}{\begin{pmatrix} \sqrt{\begin{pmatrix} \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i) + \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i) + \\ \mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i) + \mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i) \end{pmatrix}} \times \\ \sqrt{\begin{pmatrix} \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \\ \mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) \end{pmatrix}} \end{pmatrix}} \right). \tag{19}$$

For any two CIVQROFNs,

$$\mathfrak{C}_{CQ-1} = \begin{pmatrix} \left[\mathfrak{M}^{-}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i), \mathfrak{M}^{+}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\right] + e^{i2\pi}\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{+}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\right]\right), \\ \left[\mathfrak{N}^{-}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\right] + e^{i2\pi}\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\right]\right) \end{pmatrix},$$
$$\mathfrak{C}_{CQ-2} = \begin{pmatrix} \left[\mathfrak{M}^{-}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i), \mathfrak{M}^{+}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right] + e^{i2\pi}\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{+}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right]\right)10,0 \\ \left[\mathfrak{N}^{-}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right] + e^{i2\pi}\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right]\right) \end{pmatrix}, \quad i = 1, 2, \ldots, \tilde{n}, \tag{20}$$

are based on a universal set $\widehat{\mathcal{O}}$; if we choose the value of weight vector $\Omega_{WV} = \{\Omega_{WV-1}, \Omega_{WV-2}, \ldots, \Omega_{WV-\tilde{n}}\} = ((1/n), (1/n), \ldots, (1/n))$, then the $\mathrm{WCSM}_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2})$ is reduced to $\mathrm{CSM}_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2})$.

Theorem 2. *For any two CIVQROFNs,*

$$\mathfrak{C}_{CQ-1}=\left(\begin{array}{c}\left[\mathfrak{M}^{-}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i),\mathfrak{M}^{+}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\right]+e^{i2\pi}\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{+}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\right]\right),\\ \left[\mathfrak{N}^{-}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i),\mathfrak{N}^{+}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\right]+e^{i2\pi}\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i),\mathfrak{N}^{+}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\right]\right)\end{array}\right),\frac{\delta y}{\delta x}$$

$$\mathfrak{C}_{CQ-2}=\left(\begin{array}{c}\left[\mathfrak{M}^{-}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i),\mathfrak{M}^{+}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right]+e^{i2\pi}\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{+}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right]\right)10,0\\ \left[\mathfrak{N}^{-}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i),\mathfrak{N}^{+}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right]+e^{i2\pi}\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i),\mathfrak{N}^{+}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right]\right)\end{array}\right),\quad i=1,2,\ldots,\tilde{n}, \tag{21}$$

are based on a universal set $\widehat{\mathcal{O}}=\{\widehat{\mathcal{O}}_1,\widehat{\mathcal{O}}_2,\ldots,\widehat{\mathcal{O}}_{\tilde{n}}\}$*; then, the* $WCSM_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})$ *holds the following conditions:*

(1) $0\le WCSM_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})\le 1$

(2) $WCSM_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})=WCSM_{CQ}(\mathfrak{C}_{CQ-2},\mathfrak{C}_{CQ-1})$

(3) $WCSM_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})=1$ *if* $\mathfrak{C}_{CQ-1}=\mathfrak{C}_{CQ-2}$*, that is,* $\mathfrak{M}^{-}_{\mathfrak{C}_{RP-1}}=\mathfrak{M}^{-}_{\mathfrak{C}_{RP-2}},\mathfrak{M}^{-}_{\mathfrak{C}_{IP-1}}=\mathfrak{M}^{-}_{\mathfrak{C}_{RP-2}},\mathfrak{N}^{-}_{\mathfrak{C}_{RP-1}}=\mathfrak{N}^{-}_{\mathfrak{C}_{RP-2}},\mathfrak{N}^{-}_{\mathfrak{C}_{IP-1}}=\mathfrak{N}^{-}_{\mathfrak{C}_{IP-2}}$ *and* $\mathfrak{M}^{+}_{\mathfrak{C}_{RP-1}}=\mathfrak{M}^{+}_{\mathfrak{C}_{RP-2}},\mathfrak{M}^{+}_{\mathfrak{C}_{IP-1}}=\mathfrak{M}^{+}_{\mathfrak{C}_{RP-2}},\mathfrak{N}^{+}_{\mathfrak{C}_{RP-1}}=\mathfrak{N}^{+}_{\mathfrak{C}_{RP-2}},\mathfrak{N}^{+}_{\mathfrak{C}_{IP-1}}=\mathfrak{N}^{+}_{\mathfrak{C}_{IP-2}}$

Proof. Straightforward.

Lemma 1. *For any two fuzzy sets* $\mathfrak{C}_{CQ-1}$ *and* $\mathfrak{C}_{CQ-2}$*, if a measure* $SM_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})$ *holds the following axioms,*

(1) $0\le SM_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})\le 1$

(2) $SM_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})=SM_{CQ}(\mathfrak{C}_{CQ-2},\mathfrak{C}_{CQ-1})$

(3) $SM_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})=1$*, if* $\mathfrak{C}_{CQ-1}=\mathfrak{C}_{CQ-2}$

then, we say that $SM_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})$ *is called the similarity measure. On the contrary, a distance measure is a complement to the similarity measure stated as* $DM_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})=1-SM_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})$*. Moreover, we develop the idea of EDM ("Euclidean distance measure"), which is demonstrated below.*

Definition 6. For any two CIVQROFNs,

$$\mathfrak{C}_{CQ-1}=\left(\begin{array}{c}\left[\mathfrak{M}^{-}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i),\mathfrak{M}^{+}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\right]+e^{i2\pi}\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{+}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\right]\right),\\ \left[\mathfrak{N}^{-}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i),\mathfrak{N}^{+}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\right]+e^{i2\pi}\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i),\mathfrak{N}^{+}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\right]\right)\end{array}\right),$$

$$\mathfrak{C}_{CQ-2}=\left(\begin{array}{c}\left[\mathfrak{M}^{-}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i),\mathfrak{M}^{+}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right]+e^{i2\pi}\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{+}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right]\right)10,0\\ \left[\mathfrak{N}^{-}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i),\mathfrak{N}^{+}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right]+e^{i2\pi}\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i),\mathfrak{N}^{+}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right]\right)\end{array}\right),\quad i=1,2,\ldots,\tilde{n}, \tag{22}$$

are based on a universal set $\widehat{\mathcal{O}}=\{\widehat{\mathcal{O}}_1,\widehat{\mathcal{O}}_2,\ldots,\widehat{\mathcal{O}}_{\tilde{n}}\}$; then, the $\mathrm{EDM}_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})$ is demonstrated by

$$\mathrm{EDM}_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)=\left(\frac{1}{8\tilde{n}}\le\sum_{\widehat{\mathcal{O}}_i\in\widehat{\mathcal{O}}}\left(\begin{array}{c}\left|\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\text{-}\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right|^2+\left|\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\text{-}\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right|^2+\\ \left|\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\text{-}\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right|^2+\left|\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\text{-}\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right|^2+\\ \left|\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\text{-}\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right|^2+\left|\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\text{-}\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right|^2+\\ \left|\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\text{-}\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right|^2+\left|\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\text{-}\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right|^2\end{array}\right)\right)^{(1/2)}. \tag{23}$$

By using the weight vector $\Omega_{WV} = \{\Omega_{WV-1}, \Omega_{WV-2}, \ldots, \Omega_{WV-\tilde{n}}\}$ with a rule, that is, $\sum_{i=1}^{\tilde{n}} \Omega_{WV-i} = 1$, $\Omega_{WV-i} \in [0,1]$, then the WEDM ("weighted Euclidean distance measure") $\mathrm{WEDM}_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2})$ is demonstrated below:

$$\mathrm{WEDM}_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = \left(\frac{1}{8} \leq \sum_{\widetilde{\mathcal{O}}_i \in \widetilde{\mathcal{O}}} \Omega_{WV-i} \left(\begin{array}{c} \left|\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i) - \mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)\right|^2 + \left|\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i) - \mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)\right|^2 + \\ \left|\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i) - \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)\right|^2 + \left|\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i) - \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)\right|^2 + \\ \left|\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i) - \mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)\right|^2 + \left|\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i) - \mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)\right|^2 + \\ \left|\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i) - \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)\right|^2 + \left|\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i) - \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)\right|^2 \end{array} \right)\right)^{(1/2)}. \tag{24}$$

Theorem 3. *For any two CIVQROFNs,*

$$\begin{aligned} \mathfrak{C}_{CQ-1} &= \left(\begin{array}{c} \left[\mathfrak{M}^{-}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i), \mathfrak{M}^{+}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i)\right] + e^{i2\pi}\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i)\mathfrak{M}^{+}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i)\right]\right), \\ \left[\mathfrak{N}^{-}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i)\right] + e^{i2\pi}\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i)\right]\right) \end{array} \right), \\ \mathfrak{C}_{CQ-2} &= \left(\begin{array}{c} \left[\mathfrak{M}^{-}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i), \mathfrak{M}^{+}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)\right] + e^{i2\pi}\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)\mathfrak{M}^{+}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)\right]\right)10, 0 \\ \left[\mathfrak{N}^{-}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)\right] + e^{i2\pi}\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)\right]\right) \end{array} \right), \\ & \qquad i = 1, 2, \ldots, \tilde{n}, \end{aligned} \tag{25}$$

are based on a universal set $\widetilde{\mathcal{O}} = \{\widetilde{\mathcal{O}}_1, \widetilde{\mathcal{O}}_2, \ldots, \widetilde{\mathcal{O}}_{\tilde{n}}\}$*; then, the* $WEDM_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2})$ *holds the following conditions:*

(1) $0 \leq WEDM_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}) \leq 1$

(2) $WEDM_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}) = WEDM_{CQ}(\mathfrak{C}_{CQ-2}, \mathfrak{C}_{CQ-1})$

(3) $WEDM_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}) = 0$ *if* $\mathfrak{C}_{CQ-1} = \mathfrak{C}_{CQ-2}$*, that is,* $\mathfrak{M}^{-}_{\mathfrak{C}_{RP-1}} = \mathfrak{M}^{-}_{\mathfrak{C}_{RP-2}}, \mathfrak{M}^{-}_{\mathfrak{C}_{IP-1}} = \mathfrak{M}^{-}_{\mathfrak{C}_{RP-2}}, \mathfrak{N}^{-}_{\mathfrak{C}_{RP-1}} = \mathfrak{N}^{-}_{\mathfrak{C}_{RP-2}}, \mathfrak{N}^{-}_{\mathfrak{C}_{IP-1}} = \mathfrak{N}^{-}_{\mathfrak{C}_{IP-2}}$ *and* $\mathfrak{M}^{+}_{\mathfrak{C}_{RP-1}} = \mathfrak{M}^{+}_{\mathfrak{C}_{RP-2}}, \mathfrak{M}^{+}_{\mathfrak{C}_{IP-1}} = \mathfrak{M}^{+}_{\mathfrak{C}_{RP-2}}, \mathfrak{N}^{+}_{\mathfrak{C}_{RP-1}} = \mathfrak{N}^{+}_{\mathfrak{C}_{RP-2}}, \mathfrak{N}^{+}_{\mathfrak{C}_{IP-1}} = \mathfrak{N}^{+}_{\mathfrak{C}_{IP-2}}$

Proof. Based on Definition (14), we obtain the following.

(1) We know that $0 \leq \left(\mathfrak{M}^{-}_{\mathfrak{C}_{RP-1}} = \mathfrak{M}^{-}_{\mathfrak{C}_{RP-2}}, \mathfrak{M}_{\mathfrak{C}_{IP-}} 1^{-} = \mathfrak{M}^{-}_{\mathfrak{C}_{IP-2}}, \mathfrak{N}^{-}_{\mathfrak{C}_{RP-1}} = \mathfrak{N}^{-}_{\mathfrak{C}_{RP-2}}, \mathfrak{N}^{-}_{\mathfrak{C}_{IP-1}} = \mathfrak{N}^{-}_{\mathfrak{C}_{IP-2}}, \mathfrak{M}^{+}_{\mathfrak{C}_{RP-1}} = \mathfrak{M}^{+}_{\mathfrak{C}_{RP-2}}, \mathfrak{M}^{+}_{\mathfrak{C}_{IP-1}} = \mathfrak{M}^{+}_{\mathfrak{C}_{IP-2}}, \mathfrak{N}^{+}_{\mathfrak{C}_{RP-1}} = \mathfrak{N}^{+}_{\mathfrak{C}_{RP-2}}, \mathfrak{N}^{+}_{\mathfrak{C}_{IP-1}} = \mathfrak{N}^{+}_{\mathfrak{C}_{IP-2}}\right) \leq 1$ for the value of the parameter $q_{CQ} \geq 1$; then, $0 \leq |\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i) - \mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)|^2$, $|\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i) - \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)|^2 \leq 1$:

$$\begin{aligned} &0 \leq \left|\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i) - \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)\right|^2, \left|\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i) - \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)\right|^2 \leq 1, \\ &0 \leq \left|\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i) - \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)\right|^2, \left|\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i) - \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)\right|^2 \leq 1, \\ &0 \leq \left|\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i) - \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)\right|^2, \left|\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i) - \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)\right|^2 \leq 1. \end{aligned} \tag{26}$$

Therefore, $0 \leq \mathrm{WEDM}_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}) \leq (1/8)^{(1/2)} (8\sum_{\widetilde{\mathcal{O}}_i \in \widetilde{\mathcal{O}}} \Omega_{WV-i})^{(1/2)} = 1$.

(2) By using Definition (14), we can easily obtain

$$\mathrm{WEDM}_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = \mathrm{WEDM}_{CQ}\left(\mathfrak{C}_{CQ-2}, \mathfrak{C}_{CQ-1}\right). \tag{27}$$

(3) $\mathrm{WEDM}_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}) = 0$:

$$\begin{aligned}
\Leftrightarrow \left|\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)-\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right|^2, \left|\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)-\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right|^2 &= 0,\\
\left|\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)-\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right|^2, \left|\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)-\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right|^2 &= 0,\\
\left|\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)-\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right|^2, \left|\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)-\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right|^2 &= 0,\\
\left|\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)-\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right|^2, \left|\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)-\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right|^2 &= 0,
\end{aligned} \tag{28}$$

that is, $\mathfrak{M}^-_{\mathfrak{C}_{RP-1}} = \mathfrak{M}^-_{\mathfrak{C}_{RP-2}}, \mathfrak{M}^-_{\mathfrak{C}_{IP-1}} = \mathfrak{M}^-_{\mathfrak{C}_{IP-2}}, \mathfrak{N}^-_{\mathfrak{C}_{RP-1}} = \mathfrak{N}^-_{\mathfrak{C}_{RP-2}}$, $\mathfrak{N}^-_{\mathfrak{C}_{IP-1}} = \mathfrak{N}^-_{\mathfrak{C}_{IP-2}}$ and $\mathfrak{M}^+_{\mathfrak{C}_{RP-1}} = \mathfrak{M}^+_{\mathfrak{C}_{RP-2}}, \mathfrak{M}^+_{\mathfrak{C}_{IP-1}} = \mathfrak{M}^+_{\mathfrak{C}_{IP-2}}$, $\mathfrak{N}^+_{\mathfrak{C}_{RP-1}} = \mathfrak{N}^+_{\mathfrak{C}_{RP-2}}, \mathfrak{N}^+_{\mathfrak{C}_{IP-1}} = \mathfrak{N}^+_{\mathfrak{C}_{IP-2}}$.

Definition 7. For any two CIVQROFNs,

$$\begin{aligned}
\mathfrak{C}_{CQ-1} &= \begin{pmatrix} \left[\mathfrak{M}^-_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i), \mathfrak{M}^+_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\right] + e^{i2\pi}\left(\left[\mathfrak{M}^-_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^+_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\right]\right), \\ \left[\mathfrak{N}^-_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i), \mathfrak{N}^+_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\right] + e^{i2\pi}\left(\left[\mathfrak{N}^-_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i), \mathfrak{N}^+_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\right]\right) \end{pmatrix}, \\
\mathfrak{C}_{CQ-2} &= \begin{pmatrix} \left[\mathfrak{M}^-_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i), \mathfrak{M}^+_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right] + e^{i2\pi}\left(\left[\mathfrak{M}^-_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^+_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right]\right)10, 0 \\ \left[\mathfrak{N}^-_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i), \mathfrak{N}^+_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right] + e^{i2\pi}\left(\left[\mathfrak{N}^-_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i), \mathfrak{N}^+_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right]\right) \end{pmatrix}, \\
& \qquad i = 1, 2, \ldots, \widetilde{n},
\end{aligned} \tag{29}$$

are based on a universal set $\widehat{\mathcal{O}} = \{\widehat{\mathcal{O}}_1, \widehat{\mathcal{O}}_2, \ldots, \widehat{\mathcal{O}}_{\widetilde{n}}\}$; then, the NSM ("a new similarity measure") $\mathrm{NSM}_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2})$ is defined as

$$\mathrm{NSM}_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = \frac{CSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) + 1 - EDM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right)}{2}, \tag{30}$$

where

$$\mathrm{CSM}_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = \frac{1}{\widetilde{n}}\sum_{i=1}^{\widetilde{n}}\left(a\frac{\begin{pmatrix} \mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \\ \mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \\ \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \\ \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) \end{pmatrix}}{\left(\sqrt{\begin{pmatrix} \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i) + \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i) + \\ \mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i) + \mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i) \end{pmatrix}} \times \sqrt{\begin{pmatrix} \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \\ \mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) \end{pmatrix}}\right)}\right), \tag{31}$$

$$\mathrm{EDM}_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)=\left(\frac{1}{8}\sum_{\widetilde{\mathcal{O}}_i\in\widetilde{\mathcal{O}}}\left(\begin{array}{c}\left|\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i)-\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)\right|^2+\left|\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i)-\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)\right|^2+\\ \left|\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i)-\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)\right|^2+\left|\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i)-\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)\right|^2+\\ \left|\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i)-\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)\right|^2+\left|\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i)-\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)\right|^2+\\ \left|\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i)-\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)\right|^2+\left|\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i)-\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)\right|^2\end{array}\right)\right)^{(1/2)}. \tag{31}$$

By using the weight vector $\Omega_{WV}=\{\Omega_{WV-1},\ \Omega_{WV-2},\ldots,\Omega_{WV-\widetilde{n}}\}$ with a rule, that is, $\sum_{i=1}^{n}\Omega_{WV-i}=1$, $\Omega_{WV-i}\in[0,1]$, then the WNSM $\mathrm{WNSM}_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})$ is demonstrated below.

Definition 8. For any two CIVQROFNs,

$$\begin{aligned}\mathfrak{C}_{CQ-1}&=\left(\begin{array}{c}\left[\mathfrak{M}^{-}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i),\mathfrak{M}^{+}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i)\right]+e^{i2\pi}\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i)\mathfrak{M}^{+}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i)\right]\right),\\ \left[\mathfrak{N}^{-}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i),\mathfrak{N}^{+}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i)\right]+e^{i2\pi}\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i),\mathfrak{N}^{+}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i)\right]\right)\end{array}\right),\\ \mathfrak{C}_{CQ-2}&=\left(\begin{array}{c}\left[\mathfrak{M}^{-}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i),\mathfrak{M}^{+}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)\right]+e^{i2\pi}\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)\mathfrak{M}^{+}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)\right]\right)10,0\\ \left[\mathfrak{N}^{-}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i),\mathfrak{N}^{+}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)\right]+e^{i2\pi}\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i),\mathfrak{N}^{+}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)\right]\right)\end{array}\right),\\ &\qquad i=1,2,\ldots,\widetilde{n},\end{aligned} \tag{32}$$

are based on a universal set $\widetilde{\mathcal{O}}=\{\widetilde{\mathcal{O}}_1,\widetilde{\mathcal{O}}_2,\ldots,\widetilde{\mathcal{O}}_{\widetilde{n}}\}$; then, the WNSM ("weighted new similarity measure") $\mathrm{WNSM}_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})$ is demonstrated by

$$\mathrm{NSM}_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)=\frac{WCSM_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)+1-\mathrm{WEDM}_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)}{2}, \tag{33}$$

where

$$\mathrm{WCSM}_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)=\sum_{i=1}^{\widetilde{n}}\Omega_{WV-i}\left(\frac{\left(\begin{array}{c}\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i)\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)+\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i)\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)+\\ \mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i)\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)+\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i)\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)+\\ \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i)\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)+\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i)\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)+\\ \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i)\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)+\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i)\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)\end{array}\right)}{\left(\begin{array}{c}\sqrt{\left(\begin{array}{c}\mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i)+\mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i)+\mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i)+\mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i)+\\ \mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i)+\mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i)+\mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i)+\mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i)\end{array}\right)}\times\\ \sqrt{\left(\begin{array}{c}\mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)+\mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)+\mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)+\mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)+\\ \mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)+\mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)+\mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)+\mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)\end{array}\right)}\end{array}\right)\right),$$

$$\mathrm{WEDM}_{CQ}\left(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2}\right)=\left(\frac{1}{8}\sum_{\widetilde{\mathcal{O}}_i\in\widetilde{\mathcal{O}}}\Omega_{WV-i}\left(\begin{array}{c}\left|\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i)-\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)\right|^2+\left|\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i)-\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)\right|^2+\\ \left|\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i)-\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)\right|^2+\left|\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i)-\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)\right|^2+\\ \left|\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i)-\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)\right|^2+\left|\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i)-\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)\right|^2+\\ \left|\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i)-\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)\right|^2+\left|\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i)-\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)\right|^2\end{array}\right)\right)^{(1/2)}. \tag{34}$$

If we choose the value of the weight vector $\Omega_{WV}=\{\Omega_{WV-1},\Omega_{WV-2},\ldots,\Omega_{WV-\tilde{n}}\}=((1/n),(1/n),\ldots,(1/n))$, then the $\mathrm{WNSM}_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})$ is reduced to $\mathrm{NSM}_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})$.

Theorem 4. *For any two CIVQROFNs,*

$$\begin{aligned}\mathfrak{C}_{CQ-1}&=\left(\begin{array}{c}\left[\mathfrak{M}^{-}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i),\mathfrak{M}^{+}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i)\right]+e^{i2\pi}\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i)\mathfrak{M}^{+}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i)\right]\right),\\ \left[\mathfrak{N}^{-}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i),\mathfrak{N}^{+}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i)\right]+e^{i2\pi}\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i),\mathfrak{N}^{+}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i)\right]\right)\end{array}\right),\\ \mathfrak{C}_{CQ-2}&=\left(\begin{array}{c}\left[\mathfrak{M}^{-}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i),\mathfrak{M}^{+}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)\right]+e^{i2\pi}\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)\mathfrak{M}^{+}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)\right]\right)10,0\\ \left[\mathfrak{N}^{-}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i),\mathfrak{N}^{+}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)\right]+e^{i2\pi}\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i),\mathfrak{N}^{+}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)\right]\right)\end{array}\right),\\ &\qquad i=1,2,\ldots,\tilde{n},\end{aligned} \tag{35}$$

are based on a universal set $\widetilde{\mathcal{O}}=\{\widetilde{\mathcal{O}}_1,\widetilde{\mathcal{O}}_2,\ldots,\widetilde{\mathcal{O}}_{\tilde{n}}\}$*; then,* $WNSM_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})$ *holds the following conditions:*

(1) $0\le WNSM_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})\le 1$

(2) $WNSM_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})=WNSM_{CQ}(\mathfrak{C}_{CQ-2},\mathfrak{C}_{CQ-1})$

(3) $WNSM_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})=1$ *if* $\mathfrak{C}_{CQ-1}=\mathfrak{C}_{CQ-2}$*, that is,* $\mathfrak{M}^{-}_{\mathfrak{C}_{RP-1}}=\mathfrak{M}^{-}_{\mathfrak{C}_{RP-2}},\mathfrak{M}^{-}_{\mathfrak{C}_{IP-1}}=\mathfrak{M}^{-}_{\mathfrak{C}_{RP-2}},\mathfrak{N}^{-}_{\mathfrak{C}_{RP-1}}=\mathfrak{N}^{-}_{\mathfrak{C}_{RP-2}},\mathfrak{N}^{-}_{\mathfrak{C}_{IP-1}}=\mathfrak{N}^{-}_{\mathfrak{C}_{IP-2}}$ *and* $\mathfrak{M}^{+}_{\mathfrak{C}_{RP-1}}=\mathfrak{M}^{+}_{\mathfrak{C}_{RP-2}},\mathfrak{M}^{+}_{\mathfrak{C}_{IP-1}}=\mathfrak{M}^{+}_{\mathfrak{C}_{RP-2}},\mathfrak{N}^{+}_{\mathfrak{C}_{RP-1}}=\mathfrak{N}^{+}_{\mathfrak{C}_{RP-2}},\mathfrak{N}^{+}_{\mathfrak{C}_{IP-1}}=\mathfrak{N}^{+}_{\mathfrak{C}_{IP-2}}$

Proof. Based on Definition 8, we obtain the following:

(1) By using Theorem 2, we know that $0\le\mathrm{WNSM}_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})\le 1$ for the value of the parameter and then $0\le\mathrm{WNSM}_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})\le 1$; then, by using Lemma 1, we obtain $0\le(\mathrm{WCSM}_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})+1-\mathrm{WEDM}_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})/2)\le 1$ implies that $0\le\mathrm{WNSM}_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})\le 1$.

(2) By using Definition 6 and with the help of Theorem 2 and Theorem 3, we easily obtain $\mathrm{WNSM}_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})=\mathrm{WNSM}_{CQ}(\mathfrak{C}_{CQ-2},\mathfrak{C}_{CQ-1})$.

(3) When $\mathfrak{C}_{CQ-1}=\mathfrak{C}_{CQ-2}$, we know that $\mathrm{WCSM}_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})=1$ and $\mathrm{WEDM}_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})=0$ and then $\mathrm{WNSM}_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})=1$. In contrast, we have $\mathrm{WCSM}_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})=1$ and then $\mathrm{WCSM}_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})+1-\mathrm{WEDM}_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})=1+1-0=2$, such that $\mathrm{CSM}_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})=1-\mathrm{WEDM}_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})$. Since all CQROFNs $0\le\mathrm{WCSM}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})\le 1$ and $0\le\mathrm{WEDM}_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})\le 1$ exists continuously, then $\mathrm{WCSM}_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})=1$ and $\mathrm{WEDM}_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})=0$, by using in Theorem 3; if $\mathrm{WEDM}_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})=0$, then it is obviously $\mathfrak{C}_{CQ-1}=\mathfrak{C}_{CQ-2}$. Hence, $\mathrm{WNSM}_{CQ}(\mathfrak{C}_{CQ-1},\mathfrak{C}_{CQ-2})$ iff $\mathfrak{C}_{CQ-1}=\mathfrak{C}_{CQ-2}$, that is, $\mathfrak{M}^{-}_{\mathfrak{C}_{RP-1}}=\mathfrak{M}^{-}_{\mathfrak{C}_{RP-2}},\mathfrak{M}^{-}_{\mathfrak{C}_{IP-1}}=\mathfrak{M}^{-}_{\mathfrak{C}_{RP-2}},\mathfrak{N}^{-}_{\mathfrak{C}_{RP-1}}=\mathfrak{N}^{-}_{\mathfrak{C}_{RP-2}},\mathfrak{N}^{-}_{\mathfrak{C}_{IP-1}}=\mathfrak{N}^{-}_{\mathfrak{C}_{IP-2}}$ and $\mathfrak{M}^{+}_{\mathfrak{C}_{RP-1}}=\mathfrak{M}^{+}_{\mathfrak{C}_{RP-2}},\mathfrak{M}^{+}_{\mathfrak{C}_{IP-1}}=\mathfrak{M}^{+}_{\mathfrak{C}_{RP-2}},\mathfrak{N}^{+}_{\mathfrak{C}_{RP-1}}=\mathfrak{N}^{+}_{\mathfrak{C}_{RP-2}},\mathfrak{N}^{+}_{\mathfrak{C}_{IP-1}}=\mathfrak{N}^{+}_{\mathfrak{C}_{IP-2}}$

Definition 9. For any two CIVQROFNs,

$$\mathfrak{C}_{CQ-1} = \begin{pmatrix} \left[\mathfrak{M}^{-}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i), \mathfrak{M}^{+}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\right] + e^{i2\pi}\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{+}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\right]\right), \\ \left[\mathfrak{N}^{-}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\right] + e^{i2\pi}\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\right]\right) \end{pmatrix},$$
$$\mathfrak{C}_{CQ-2} = \begin{pmatrix} \left[\mathfrak{M}^{-}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i), \mathfrak{M}^{+}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right] + e^{i2\pi}\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{+}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right]\right)10, 0 \\ \left[\mathfrak{N}^{-}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right] + e^{i2\pi}\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right]\right) \end{pmatrix}, \quad i = 1, 2, \ldots, \tilde{n}, \tag{36}$$

are based on a universal set $\widehat{\mathcal{O}} = \{\widehat{\mathcal{O}}_1, \widehat{\mathcal{O}}_2, \ldots, \widehat{\mathcal{O}}_{\tilde{n}}\}$; then, the WDM ("weighted distance measure") $\mathrm{WNSM}_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2})$ is defined as

$$\mathrm{WDM}_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = 1 - WNSM_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right)\frac{1 - \mathrm{WCSM}_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) + \mathrm{WEDM}_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right)}{2}, \tag{37}$$

where

$$\mathrm{WCSM}_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = \sum_{i=1}^{\tilde{n}} \Omega_{WV-i}\left(\frac{\begin{pmatrix} \mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \\ \mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \\ \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \\ \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) \end{pmatrix}}{\left(\begin{array}{c} \sqrt{\begin{pmatrix} \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i) + \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i) + \\ \mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i) + \mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i) \end{pmatrix}} \times \\ \sqrt{\begin{pmatrix} \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \\ \mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) \end{pmatrix}} \end{array}\right)}\right),$$

$$\mathrm{WEDM}_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = \left(\frac{1}{8}\sum_{\widehat{\mathcal{O}}_i \in \widehat{\mathcal{O}}} \Omega_{WV-i}\begin{pmatrix} \left|\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\text{-}\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right|^2 + \left|\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\text{-}\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right|^2 + \\ \left|\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\text{-}\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right|^2 + \left|\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\text{-}\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right|^2 + \\ \left|\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\text{-}\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right|^2 + \left|\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\text{-}\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right|^2 + \\ \left|\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\text{-}\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right|^2 + \left|\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\text{-}\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right|^2 \end{pmatrix}\right)^{(1/2)}. \tag{38}$$

If we choose the value of the weight vector $\Omega_{WV} = \{\Omega_{WV-1}, \Omega_{WV-2}, \ldots, \Omega_{WV-\tilde{n}}\} = ((1/n), (1/n), \ldots, (1/n))$, then $\mathrm{WEDM}_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2})$ is reduced to $\mathrm{DM}_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2})$.

Definition 10. For any two CIVQROFNs,

$$\mathfrak{C}_{CQ-1} = \begin{pmatrix} \left[\mathfrak{M}^{-}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i), \mathfrak{M}^{+}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\right] + e^{i2\pi}\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{+}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\right]\right), \\ \left[\mathfrak{N}^{-}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\right] + e^{i2\pi}\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\right]\right) \end{pmatrix},$$
$$\mathfrak{C}_{CQ-2} = \begin{pmatrix} \left[\mathfrak{M}^{-}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i), \mathfrak{M}^{+}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right] + e^{i2\pi}\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{+}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right]\right)10, 0 \\ \left[\mathfrak{N}^{-}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right] + e^{i2\pi}\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right]\right) \end{pmatrix}, \quad i = 1, 2, \ldots, \tilde{n}, \tag{39}$$

are based on a universal set $\widehat{\mathcal{O}} = \{\widehat{\mathcal{O}}_1, \widehat{\mathcal{O}}_2, \ldots, \widehat{\mathcal{O}}_{\tilde{n}}\}$; then, the weighted DM $\mathrm{DM}_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2})$ is demonstrated by

$$\mathrm{DM}_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = 1 - \mathrm{NSM}_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = \frac{1 - \mathrm{CSM}_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) + \mathrm{EDM}_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right)}{2}, \tag{40}$$

where

$$\mathrm{CSM}_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = \sum_{i=1}^{\tilde{n}} \left(\frac{\begin{pmatrix} \mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \\ \mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \\ \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \\ \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i)\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) \end{pmatrix}}{\left(\begin{array}{l} \sqrt{\begin{pmatrix} \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i) + \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i) + \\ \mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i) + \mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i) \end{pmatrix}} \times \\ \sqrt{\begin{pmatrix} \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{M}^{-2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{-2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \\ \mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{M}^{+2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i) + \mathfrak{N}^{+2q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i) \end{pmatrix}} \end{array} \right)} \right), \tag{41}$$

$$\mathrm{EDM}_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}\right) = \left(\frac{1}{8\tilde{n}} \sum_{\widehat{\mathcal{O}}_i \in \widehat{\mathcal{O}}} \begin{pmatrix} \left|\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i) - \mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right|^2 + \left|\mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i) - \mathfrak{M}^{-q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right|^2 + \\ \left|\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i) - \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right|^2 + \left|\mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i) - \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right|^2 + \\ \left|\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i) - \mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right|^2 + \left|\mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i) - \mathfrak{N}^{-q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right|^2 + \\ \left|\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-1}}(\widehat{\mathcal{O}}_i) - \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-2}}(\widehat{\mathcal{O}}_i)\right|^2 + \left|\mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-1}}(\widehat{\mathcal{O}}_i) - \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-2}}(\widehat{\mathcal{O}}_i)\right|^2 \end{pmatrix} \right)^{(1/2)}.$$

Theorem 5. *For any two CIVQROFNs,*

$$\mathfrak{C}_{CQ-1} = \begin{pmatrix} \left[\mathfrak{M}^{-}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i), \mathfrak{M}^{+}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i)\right] + e^{i2\pi}\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i)\mathfrak{M}^{+}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i)\right]\right), \\ \left[\mathfrak{N}^{-}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{RP-1}}(\widetilde{\mathcal{O}}_i)\right] + e^{i2\pi}\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{IP-1}}(\widetilde{\mathcal{O}}_i)\right]\right) \end{pmatrix},$$
$$\mathfrak{C}_{CQ-2} = \begin{pmatrix} \left[\mathfrak{M}^{-}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i), \mathfrak{M}^{+}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)\right] + e^{i2\pi}\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)\mathfrak{M}^{+}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)\right]\right)10, 0 \\ \left[\mathfrak{N}^{-}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{RP-2}}(\widetilde{\mathcal{O}}_i)\right] + e^{i2\pi}\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i), \mathfrak{N}^{+}_{\mathfrak{C}_{IP-2}}(\widetilde{\mathcal{O}}_i)\right]\right) \end{pmatrix}, \quad i = 1, 2, \ldots, \tilde{n}, \tag{42}$$

are based on a universal set $\widetilde{\mathcal{O}} = \{\widetilde{\mathcal{O}}_1, \widetilde{\mathcal{O}}_2, \ldots, \widetilde{\mathcal{O}}_{\tilde{n}}\}$*; then,* $WDM_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2})$ *holds the following conditions:*

(1) $0 \le WNSM_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}) \le 1$

(2) $WNSM_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}) = WNSM_{CQ}(\mathfrak{C}_{CQ-2}, \mathfrak{C}_{CQ-1})$

(3) $WNSM_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}) = 1$ *if* $\mathfrak{C}_{CQ-1} = \mathfrak{C}_{CQ-2}$*, that is,* $\mathfrak{M}^{-}_{\mathfrak{C}_{RP-1}} = \mathfrak{M}^{-}_{\mathfrak{C}_{RP-2}}, \mathfrak{M}^{-}_{\mathfrak{C}_{IP-1}} = \mathfrak{M}^{-}_{\mathfrak{C}_{RP-2}}, \mathfrak{N}^{-}_{\mathfrak{C}_{RP-1}} = \mathfrak{N}^{-}_{\mathfrak{C}_{RP-2}}, \mathfrak{N}^{-}_{\mathfrak{C}_{IP-1}} = \mathfrak{N}^{-}_{\mathfrak{C}_{IP-2}}$ *and* $\mathfrak{M}^{+}_{\mathfrak{C}_{RP-1}} = \mathfrak{M}^{+}_{\mathfrak{C}_{RP-2}}, \mathfrak{M}^{+}_{\mathfrak{C}_{IP-1}} = \mathfrak{M}^{+}_{\mathfrak{C}_{RP-2}}, \mathfrak{N}^{+}_{\mathfrak{C}_{RP-1}} = \mathfrak{N}^{+}_{\mathfrak{C}_{RP-2}}, \mathfrak{N}^{+}_{\mathfrak{C}_{IP-1}} = \mathfrak{N}^{+}_{\mathfrak{C}_{IP-2}}$

Proof. Based on Theorem 4, we obtain $\mathrm{WDM}_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}) = 1 - \mathrm{WNSM}_{CQ}(\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2})$; by using Theorem 4, we easily obtain the proof of Theorem 5.

4. TOPSIS Procedure to MADM with CIVQROFS Information

In this section, we explore the idea of the TOPSIS technique to MADM with CIVQROFSs. To resolve such types of troubles, we assume the family of alternatives $\mathfrak{E}_{Al} = \{\mathfrak{E}_{Al-1}, \mathfrak{E}_{Al-2}, \ldots, \mathfrak{E}_{Al-\tilde{m}}\}$, which is evaluated by the decision maker concerning the family of attributes $\mathfrak{P}_{At} = \{\mathfrak{P}_{At-1}, \mathfrak{P}_{At-2}, \ldots, \mathfrak{P}_{At-\tilde{n}}\}$. For this, we consider the family of CIVQROFNs which is shown by $\mathfrak{C}_{CQ-ij} = ([\mathfrak{M}^{-}_{\mathfrak{C}_{RP-ij}}, \mathfrak{M}^{+}_{\mathfrak{C}_{RP-ij}}]e^{i2\pi([\mathfrak{M}_{\mathfrak{C}_{I}} \quad P-ij^{-}, \mathfrak{M}^{+}_{\mathfrak{C}_{IP-ij}}])}, [\mathfrak{N}^{-}_{\mathfrak{C}_{RP-ij}}, \mathfrak{N}^{+}_{\mathfrak{C}_{RP-ij}}]e^{i2\pi([\mathfrak{N}^{-}_{\mathfrak{C}_{IP-ij}}, \mathfrak{N}^{+}_{\mathfrak{C}_{IP-ij}}])})$ with a rule, that is, $0 \le \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{RP-ij}} + \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{RP-ij}} \le 1$ and $0 \le \mathfrak{M}^{+q_{CQ}}_{\mathfrak{C}_{IP-ij}} + \mathfrak{N}^{+q_{CQ}}_{\mathfrak{C}_{IP-ij}} \le 1, q_{CQ} \ge 1$ concerning weight vector $\Omega_{WV} = \{\Omega_{WV-1}, \Omega_{WV-2}, \ldots, \Omega_{WV-\tilde{n}}\}$ by using the condition, that is, $\sum_{i=1}^{\tilde{n}} \Omega_{WV-i} = 1, \Omega_{WV-i} \in [0, 1]$. Then, the complex interval-valued q-rung orthopair fuzzy decision matrix (CIVQROFDM) $\mathcal{Q}_{DM} = (\mathfrak{E}_{Al-ij})_{\tilde{m}\times\tilde{n}} = ([\mathfrak{M}^{-}_{\mathfrak{C}_{RP-ij}}, \mathfrak{M}^{+}_{\mathfrak{C}_{RP-ij}}] \quad e^{i2\pi([\mathfrak{M}_{\mathfrak{C}_{IP-}} \quad ij^{-}, \mathfrak{M}^{+}_{\mathfrak{C}_{IP-ij}}])}, [\mathfrak{N}^{-}_{\mathfrak{C}_{RP-ij}}, \mathfrak{N}^{+}_{\mathfrak{C}_{RP-ij}}] \; e^{i2\pi([\mathfrak{N}^{-}_{\mathfrak{C}_{IP-ij}}, \mathfrak{N}^{+}_{\mathfrak{C}_{IP-ij}}])})_{\tilde{m}\times\tilde{n}}$ is expressed as follows:

$$\mathcal{Q}_{DM} = \begin{bmatrix} \mathfrak{E}_{Al-11} & \mathfrak{E}_{Al-12} & \mathfrak{E}_{Al-12} & \cdots & \mathfrak{E}_{Al-1\tilde{n}} \\ \mathfrak{E}_{Al-21} & \mathfrak{E}_{Al-22} & \mathfrak{E}_{Al-23} & \cdots & \mathfrak{E}_{Al-2\tilde{n}} \\ \mathfrak{E}_{Al-31} & \mathfrak{E}_{Al-32} & \mathfrak{E}_{Al-33} & \cdots & \mathfrak{E}_{Al-3\tilde{n}} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \mathfrak{E}_{Al-\tilde{m}1} & \mathfrak{E}_{Al-\tilde{m}2} & \mathfrak{E}_{Al-\tilde{m}3} & \cdots & \mathfrak{E}_{Al-\tilde{m}\tilde{n}} \end{bmatrix}, \tag{43}$$

where $\mathfrak{E}_{Al-ij}, i = 1, 2, \ldots, \tilde{m}$ and $j = 1, 2, \ldots, \tilde{n}$, expressed the family of CIVQROFNs. By using the investigated CSMs, the steps of the developed decision-making procedure are as follows:

Step 1: the CIVQROFDM $\mathcal{Q}_{DM} = (\mathfrak{E}_{Al-ij})_{\tilde{m}\times\tilde{n}} = \begin{pmatrix} [\mathfrak{M}^{-}_{\mathfrak{C}_{RP-ij}}, \mathfrak{M}^{+}_{\mathfrak{C}_{RP-ij}}]e^{i2\pi([\mathfrak{M}^{-}_{\mathfrak{C}_{IP-ij}}, \mathfrak{M}^{+}_{\mathfrak{C}_{IP-ij}}])}, \\ [\mathfrak{N}^{-}_{\mathfrak{C}_{RP-ij}}, \mathfrak{N}^{+}_{\mathfrak{C}_{RP-ij}}]e^{i2\pi([\mathfrak{N}^{-}_{\mathfrak{C}_{IP-ij}}, \mathfrak{N}^{+}_{\mathfrak{C}_{IP-ij}}])} \end{pmatrix}_{\tilde{m}\times\tilde{n}}$ is normalized. If all criteria are benefit kinds, then we cannot do anything, but if in all criteria at least one criterion is cost kind, then we will convert the cost kind criteria into benefits, by using the following formula, such that

$$\widetilde{\mathfrak{E}_{Al-ij}} = \begin{pmatrix} \left[\widetilde{\mathfrak{M}}^{-}_{\mathfrak{C}_{RP-ij}}, \widetilde{\mathfrak{M}}^{+}_{\mathfrak{C}_{RP-ij}}\right]e^{i2\pi\left(\left[\widetilde{\mathfrak{M}}^{-}_{\mathfrak{C}_{IP-ij}}, \widetilde{\mathfrak{M}}^{+}_{\mathfrak{C}_{IP-ij}}\right]\right)}, \\ \left[\widetilde{\mathfrak{N}}^{-}_{\mathfrak{C}_{RP-ij}}, \widetilde{\mathfrak{N}}^{+}_{\mathfrak{C}_{RP-ij}}\right]e^{i2\pi\left(\left[\widetilde{\mathfrak{N}}^{-}_{\mathfrak{C}_{IP-ij}}, \widetilde{\mathfrak{N}}^{+}_{\mathfrak{C}_{IP-ij}}\right]\right)} \end{pmatrix},$$

$$= \begin{cases} \left(\begin{array}{c} \left[\mathfrak{M}^{-}_{\mathfrak{C}_{RP-ij}}, \mathfrak{M}^{+}_{\mathfrak{C}_{RP-ij}}\right] e^{i2\pi\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP-ij}}, \mathfrak{M}^{+}_{\mathfrak{C}_{IP-ij}}\right]\right)}, \\ \left[\mathfrak{N}^{-}_{\mathfrak{C}_{RP-ij}}, \mathfrak{N}^{+}_{\mathfrak{C}_{RP-ij}}\right] e^{i2\pi\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP-ij}}, \mathfrak{N}^{+}_{\mathfrak{C}_{IP-ij}}\right]\right)} \end{array} \right) & \text{for benefit types,} \\ \left(\begin{array}{c} \left[\mathfrak{N}^{-}_{\mathfrak{C}_{RP-ij}}, \mathfrak{N}^{+}_{\mathfrak{C}_{RP-ij}}\right] e^{i2\pi\left(\left[\mathfrak{N}^{-}_{\mathfrak{C}_{IP-ij}}, \mathfrak{N}^{+}_{\mathfrak{C}_{IP-ij}}\right]\right)}, \\ \left[\mathfrak{M}^{-}_{\mathfrak{C}_{RP-ij}}, \mathfrak{M}^{+}_{\mathfrak{C}_{RP-ij}}\right] e^{i2\pi\left(\left[\mathfrak{M}^{-}_{\mathfrak{C}_{IP-ij}}, \mathfrak{M}^{+}_{\mathfrak{C}_{IP-ij}}\right]\right)} \end{array} \right) & \text{for cost types.} \end{cases} \tag{44}$$

Step 2: moreover, by using all alternatives with the same criteria, the positive ideal solution (PIS) $\mathfrak{E}^{+}_{Al} = \left\{\mathfrak{E}^{+}_{Al-1}, \mathfrak{E}^{+}_{Al-2}, \ldots, \mathfrak{E}^{+}_{Al-\tilde{n}}\right\}$ and negative ideal solution (NIS) $\mathfrak{E}^{-}_{Al} = \left\{\mathfrak{E}^{-}_{Al-1}, \mathfrak{E}^{-}_{Al-2}, \ldots, \mathfrak{E}^{-}_{Al-\tilde{n}}\right\}$ are examined with the help of score values, which is demonstrated in Definition 3, such that

$$\begin{aligned} \mathfrak{E}^{+}_{Al-j} &= \max\left\{\mathfrak{S}_{CQ}\left(\mathfrak{E}_{Al-1j}\right), \mathfrak{S}_{CQ}\left(\mathfrak{E}_{Al-2j}\right), \ldots, \mathfrak{S}_{CQ}\left(\mathfrak{E}_{Al-\tilde{m}j}\right)\right\}, \quad j = 1, 2, \ldots, \tilde{n}, \\ \mathfrak{E}^{-}_{Al-j} &= \min\left\{\mathfrak{S}_{CQ}\left(\mathfrak{E}_{Al-1j}\right), \mathfrak{S}_{CQ}\left(\mathfrak{E}_{Al-2j}\right), \ldots, \mathfrak{S}_{CQ}\left(\mathfrak{E}_{Al-\tilde{m}j}\right)\right\}, \quad j = 1, 2, \ldots, \tilde{n}. \end{aligned} \tag{45}$$

If the value of all CIVQROFNs is the same, then we use the accuracy values, which are demonstrated in Definition 3.

Step 3: by using the PIS and NIS combined with the family of alternatives and resolved it with the help of investigated measures such as $\mathrm{WDM}_{CQ}(\mathfrak{C}_{CQ-i}, \mathfrak{E}^{+}_{Al})$, $\mathrm{WNSM}_{CQ}(\mathfrak{C}_{CQ-i}, \mathfrak{E}^{+}_{Al})$ and $\mathrm{WDM}_{CQ}(\mathfrak{C}_{CQ-i}, \mathfrak{E}^{-}_{Al})$, $\mathrm{WNSM}_{CQ}(\mathfrak{C}_{CQ-i}, \mathfrak{E}^{-}_{Al})$ and by using these investigated measures, we examine the closeness index Ψ_{CI-i}, which is demonstrated below

:

$$\begin{aligned} \Psi_{CI-i} &= \frac{\mathrm{WDM}_{CQ}\left(\mathfrak{C}_{CQ-i}, \mathfrak{E}^{+}_{Al}\right)}{\mathrm{WDM}_{CQ}\left(\mathfrak{C}_{CQ-i}, \mathfrak{E}^{+}_{Al}\right) + \mathrm{WDM}_{CQ}\left(\mathfrak{C}_{CQ-i}, \mathfrak{E}^{-}_{Al}\right)}, \quad j = 1, 2, \ldots, \tilde{m}, \\ \hat{\Psi}_{CI-i} &= \frac{\mathrm{WNSM}_{CQ}\left(\mathfrak{C}_{CQ-i}, \mathfrak{E}^{+}_{Al}\right)}{\mathrm{WNSM}_{CQ}\left(\mathfrak{C}_{CQ-i}, \mathfrak{E}^{+}_{Al}\right) + \mathrm{WNSM}_{CQ}\left(\mathfrak{C}_{CQ-i}, \mathfrak{E}^{-}_{Al}\right)}, \quad j = 1, 2, \ldots, \tilde{m}. \end{aligned} \tag{46}$$

Step 4: by using the value of the closeness index, we examine the ranking results and find the best one in all alternatives $\mathfrak{C}_{CQ-i}$.

Example 1. To develop the application of the investigated procedure in the environment of the MADM technique, we choose the real MADM example from [40]. To increase our monthly income, an enterprise wants to invest money in the market. For this, we choose four potential companies which are denoted by $\left\{\mathfrak{C}_{CQ-1}, \mathfrak{C}_{CQ-2}, \mathfrak{C}_{CQ-3}, \mathfrak{C}_{CQ-4}\right\}$ expressed by the family of alternatives. Moreover, for these alternatives, we choose the family of attributes, whose expressions and shapes are discussed below:

$\mathfrak{P}_{At-1}$: risk analysis

$\mathfrak{P}_{At-2}$: growth analysis

$\mathfrak{P}_{At-3}$: social impact

$\mathfrak{P}_{At-4}$: environment Impact

To resolve the above issues, we considered the weight vector for the attributes is demonstrated by $(0.4, 0.3, 0.2, 0.1)^{T}$; then, the CIVQROFDM is expressed in the form of Table 1.

$\mathfrak{E}_{Al-ij}, i = 1, 2, \ldots, \tilde{m}$ and $j = 1, 2, \ldots, \tilde{n}$, expressed the family of CIVQROFNs. By using the investigated CSMs, the steps of the developed decision-making procedure, then the procedure of the TOPSIS method is discussed in the following ways:

Step 1: the CIVQROFDM $\mathcal{Q}_{DM} = (\mathfrak{E}_{Al-ij})_{\tilde{m}\times\tilde{n}} = \left(\begin{array}{c} [\mathfrak{M}^{-}_{\mathfrak{C}_{RP-ij}}, \mathfrak{M}^{+}_{\mathfrak{C}_{RP-ij}}] e^{i2\pi([\mathfrak{M}^{-}_{\mathfrak{C}_{IP-ij}}, \mathfrak{M}^{+}_{\mathfrak{C}_{IP-ij}}])}, \\ [\mathfrak{N}^{-}_{\mathfrak{C}_{RP-ij}}, \mathfrak{N}^{+}_{\mathfrak{C}_{RP-ij}}] e^{i2\pi([\mathfrak{N}^{-}_{\mathfrak{C}_{IP-ij}}, \mathfrak{N}^{+}_{\mathfrak{C}_{IP-ij}}])} \end{array} \right)_{\tilde{m}\times\tilde{n}}$ is normalized. If all criteria are benefit kinds, then we cannot do anything, but if in all criteria at least one criterion is

Table 1: Decision matrix in terms of CIVQROFNs.

	$P_{\mathbf{At}-1}$	$P_{\mathbf{At}-2}$	$P_{\mathbf{At}-3}$	$P_{\mathbf{At}-4}$
$C_{\mathbf{CQ}-1}$	$\left(\begin{matrix}[0.7,0.71]e^{i2\pi([0.6,0.61])},\\ [0.9,0.91]e^{i2\pi([0.8,0.81])}\end{matrix}\right)$	$\left(\begin{matrix}[0.91,0.92]e^{i2\pi([0.81,0.82])},\\ [0.71,0.72]e^{i2\pi([0.61,0.62])}\end{matrix}\right)$	$\left(\begin{matrix}[0.92,0.93]e^{i2\pi([0.82,0.83])},\\ [0.72,0.73]e^{i2\pi([0.62,0.63])}\end{matrix}\right)$	$\left(\begin{matrix}[0.93,0.94]e^{i2\pi([0.83,0.84])},\\ [0.73,0.74]e^{i2\pi([0.63,0.64])}\end{matrix}\right)$
$C_{\mathbf{CQ}-2}$	$\left(\begin{matrix}[0.8,0.81]e^{i2\pi([0.7,0.71])},\\ [0.85,0.86]e^{i2\pi([0.89,0.9])}\end{matrix}\right)$	$\left(\begin{matrix}[0.86,0.87]e^{i2\pi([0.9,0.91])},\\ [0.81,0.82]e^{i2\pi([0.71,0.72])}\end{matrix}\right)$	$\left(\begin{matrix}[0.87,0.88]e^{i2\pi([0.91,0.92])},\\ [0.82,0.83]e^{i2\pi([0.72,0.73])}\end{matrix}\right)$	$\left(\begin{matrix}[0.88,0.89]e^{i2\pi([0.92,0.93])},\\ [0.83,0.84]e^{i2\pi([0.73,0.74])}\end{matrix}\right)$
$C_{\mathbf{CQ}-3}$	$\left(\begin{matrix}[0.6,0.61]e^{i2\pi([0.9,0.91])},\\ [0.7,0.71]e^{i2\pi([0.8,0.81])}\end{matrix}\right)$	$\left(\begin{matrix}[0.71,0.72]e^{i2\pi([0.81,0.82])},\\ [0.61,0.62]e^{i2\pi([0.91,0.92])}\end{matrix}\right)$	$\left(\begin{matrix}[0.72,0.73]e^{i2\pi([0.82,0.83])},\\ [0.62,0.63]e^{i2\pi([0.92,0.93])}\end{matrix}\right)$	$\left(\begin{matrix}[0.73,0.74]e^{i2\pi([0.83,0.84])},\\ [0.63,0.64]e^{i2\pi([0.93,0.94])}\end{matrix}\right)$
$C_{\mathbf{CQ}-4}$	$\left(\begin{matrix}[0.81,0.82]e^{i2\pi([0.61,0.62])},\\ [0.85,0.86]e^{i2\pi([0.7,0.71])}\end{matrix}\right)$	$\left(\begin{matrix}[0.86,0.87]e^{i2\pi([0.71,0.72])},\\ [0.82,0.83]e^{i2\pi([0.62,0.63])}\end{matrix}\right)$	$\left(\begin{matrix}[0.87,0.88]e^{i2\pi([0.72,0.73])},\\ [0.83,0.84]e^{i2\pi([0.63,0.64])}\end{matrix}\right)$	$\left(\begin{matrix}[0.88,0.89]e^{i2\pi([0.73,0.74])},\\ [0.84,0.85]e^{i2\pi([0.64,0.65])}\end{matrix}\right)$

cost kind, then we will convert the cost kind criteria into benefits, and the normalized decision matrix is discussed in the form of Table 2.

Step 2: moreover, by using all alternatives with the same criteria, PIS $\mathfrak{C}^{+}_{Al} = \left\{\mathfrak{C}^{+}_{Al-1}, \mathfrak{C}^{+}_{Al-2}, \ldots, \mathfrak{C}^{+}_{Al-\tilde{n}}\right\}$ and NIS $\mathfrak{C}^{-}_{Al} = \left\{\mathfrak{C}^{-}_{Al-1}, \mathfrak{C}^{-}_{Al-2}, \ldots, \mathfrak{C}^{-}_{Al-\tilde{n}}\right\}$ are examined with the help of score values, which is demonstrated in Definition 3, such that

$$\begin{aligned}
\mathfrak{C}^{+}_{Al-j} &= \left\{\begin{matrix}\left(\begin{matrix}[0.93,0.94]e^{i2\pi([0.83,0.84])},\\ [0.73,0.74]e^{i2\pi([0.63,0.64])}\end{matrix}\right), \left(\begin{matrix}[0.88,0.89]e^{i2\pi([0.92,0.93])},\\ [0.83,0.84]e^{i2\pi([0.73,0.74])}\end{matrix}\right),\\ \left(\begin{matrix}[0.70,0.71]e^{i2\pi([0.80,0.81])},\\ [0.60,0.61]e^{i2\pi([0.90,0.91])}\end{matrix}\right), \left(\begin{matrix}[0.88,0.89]e^{i2\pi([0.73,0.74])},\\ [0.84,0.85]e^{i2\pi([0.64,0.65])}\end{matrix}\right)\end{matrix}\right\},\\
\mathfrak{C}^{-}_{Al-j} &= \left\{\begin{matrix}\left(\begin{matrix}[0.90,0.91]e^{i2\pi([0.8,0.81])},\\ [0.70,0.71]e^{i2\pi([0.6,0.61])}\end{matrix}\right), \left(\begin{matrix}[0.85,0.86]e^{i2\pi([0.89,0.9])},\\ [0.80,0.81]e^{i2\pi([0.7,0.71])}\end{matrix}\right),\\ \left(\begin{matrix}[0.73,0.74]e^{i2\pi([0.83,0.84])},\\ [0.63,0.64]e^{i2\pi([0.93,0.94])}\end{matrix}\right), \left(\begin{matrix}[0.85,0.86]e^{i2\pi([0.7,0.71])},\\ [0.81,0.82]e^{i2\pi([0.61,0.62])}\end{matrix}\right)\end{matrix}\right\}.
\end{aligned} \tag{47}$$

Step 3: by using the PIS and NIS combined with the family of alternatives and resolved it with the help of investigated measures such as $\mathrm{WDM}_{CQ}(\mathfrak{C}_{CQ-i}, \mathfrak{C}^{+}_{Al})$, $\mathrm{WNSM}_{CQ}(\mathfrak{C}_{CQ-i}, \mathfrak{C}^{+}_{Al})$ and $\mathrm{WDM}_{CQ}(\mathfrak{C}_{CQ-i}, \mathfrak{C}^{-}_{Al})$, $\mathrm{WNSM}_{CQ}(\mathfrak{C}_{CQ-i}, \mathfrak{C}^{-}_{Al})$ and by using the values of the parameter $q_{CQ} = 6$,

$$\begin{aligned}
\mathrm{WDM}_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}^{+}_{Al}\right) &= 0.5875\ \mathrm{WDM}_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}^{-}_{Al}\right) = 0.5875,\\
\mathrm{WDM}_{CQ}\left(\mathfrak{C}_{CQ-2}, \mathfrak{C}^{+}_{Al}\right) &= 0.5839\ \mathrm{WDM}_{CQ}\left(\mathfrak{C}_{CQ-2}, \mathfrak{C}^{-}_{Al}\right) = 0.5872,\\
\mathrm{WDM}_{CQ}\left(\mathfrak{C}_{CQ-3}, \mathfrak{C}^{+}_{Al}\right) &= 0.6495\ \mathrm{WDM}_{CQ}\left(\mathfrak{C}_{CQ-3}, \mathfrak{C}^{-}_{Al}\right) = 0.6331,\\
\mathrm{WDM}_{CQ}\left(\mathfrak{C}_{CQ-4}, \mathfrak{C}^{+}_{Al}\right) &= 0.6042\ \mathrm{WDM}_{CQ}\left(\mathfrak{C}_{CQ-4}, \mathfrak{C}^{-}_{Al}\right) = 0.5968,\\
\mathrm{WNSM}_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}^{+}_{Al}\right) &= 0.4134\ \mathrm{WNSM}_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}^{-}_{Al}\right) = 0.4134,\\
\mathrm{WNSM}_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}^{+}_{Al}\right) &= 0.4134\ \mathrm{WNSM}_{CQ}\left(\mathfrak{C}_{CQ-1}, \mathfrak{C}^{-}_{Al}\right) = 0.4134,\\
\mathrm{WNSM}_{CQ}\left(\mathfrak{C}_{CQ-2}, \mathfrak{C}^{+}_{Al}\right) &= 0.4169\ \mathrm{WNSM}_{CQ}\left(\mathfrak{C}_{CQ-2}, \mathfrak{C}^{-}_{Al}\right) = 0.4138,\\
\mathrm{WNSM}_{CQ}\left(\mathfrak{C}_{CQ-3}, \mathfrak{C}^{+}_{Al}\right) &= 0.3515\ \mathrm{WNSM}_{CQ}\left(\mathfrak{C}_{CQ-3}, \mathfrak{C}^{-}_{Al}\right) = 0.3679,\\
\mathrm{WNSM}_{CQ}\left(\mathfrak{C}_{CQ-4}, \mathfrak{C}^{+}_{Al}\right) &= 0.3968\ \mathrm{WNSM}_{CQ}\left(\mathfrak{C}_{CQ-4}, \mathfrak{C}^{-}_{Al}\right) = 0.4041.
\end{aligned} \tag{48}$$

Table 2: Normalized decision matrix.

	$P_{\mathbf{At}-1}$	$P_{\mathbf{At}-2}$	$P_{\mathbf{At}-3}$	$P_{\mathbf{At}-4}$
$C_{\mathbf{CQ}-1}$	$\left(\begin{matrix}[0.9,0.91]e^{i2\pi([0.8,0.81])}\\ [0.7,0.71]e^{i2\pi([0.6,0.61])}\end{matrix}\right)$	$\left(\begin{matrix}[0.91,0.92]e^{i2\pi([0.81,0.82])}\\ [0.71,0.72]e^{i2\pi([0.61,0.62])}\end{matrix}\right)$	$\left(\begin{matrix}[0.92,0.93]e^{i2\pi([0.82,0.83])}\\ [0.72,0.73]e^{i2\pi([0.62,0.63])}\end{matrix}\right)$	$\left(\begin{matrix}[0.93,0.94]e^{i2\pi([0.83,0.84])}\\ [0.73,0.74]e^{i2\pi([0.63,0.64])}\end{matrix}\right)$
$C_{\mathbf{CQ}-2}$	$\left(\begin{matrix}[0.85,0.86]e^{i2\pi([0.89,0.9])}\\ [0.8,0.81]e^{i2\pi([0.7,0.71])}\end{matrix}\right)$	$\left(\begin{matrix}[0.86,0.87]e^{i2\pi([0.9,0.91])}\\ [0.81,0.82]e^{i2\pi([0.71,0.72])}\end{matrix}\right)$	$\left(\begin{matrix}[0.87,0.88]e^{i2\pi([0.91,0.92])}\\ [0.82,0.83]e^{i2\pi([0.72,0.73])}\end{matrix}\right)$	$\left(\begin{matrix}[0.88,0.89]e^{i2\pi([0.92,0.93])}\\ [0.83,0.84]e^{i2\pi([0.73,0.74])}\end{matrix}\right)$
$C_{\mathbf{CQ}-3}$	$\left(\begin{matrix}[0.7,0.71]e^{i2\pi([0.8,0.81])}\\ [0.6,0.61]e^{i2\pi([0.9,0.91])}\end{matrix}\right)$	$\left(\begin{matrix}[0.71,0.72]e^{i2\pi([0.81,0.82])}\\ [0.61,0.62]e^{i2\pi([0.91,0.92])}\end{matrix}\right)$	$\left(\begin{matrix}[0.72,0.73]e^{i2\pi([0.82,0.83])}\\ [0.62,0.63]e^{i2\pi([0.92,0.93])}\end{matrix}\right)$	$\left(\begin{matrix}[0.73,0.74]e^{i2\pi([0.83,0.84])}\\ [0.63,0.64]e^{i2\pi([0.93,0.94])}\end{matrix}\right)$
$C_{\mathbf{CQ}-4}$	$\left(\begin{matrix}[0.85,0.86]e^{i2\pi([0.7,0.71])}\\ [0.81,0.82]e^{i2\pi([0.61,0.62])}\end{matrix}\right)$	$\left(\begin{matrix}[0.86,0.87]e^{i2\pi([0.71,0.72])}\\ [0.82,0.83]e^{i2\pi([0.62,0.63])}\end{matrix}\right)$	$\left(\begin{matrix}[0.87,0.88]e^{i2\pi([0.72,0.73])}\\ [0.83,0.84]e^{i2\pi([0.63,0.64])}\end{matrix}\right)$	$\left(\begin{matrix}[0.88,0.89]e^{i2\pi([0.73,0.74])}\\ [0.84,0.85]e^{i2\pi([0.64,0.65])}\end{matrix}\right)$

Then, by using these investigated measures, we examine the closeness index Ψ_{CI-i}, which is demonstrated below:

$$\begin{aligned}&\Psi_{CI-1}=0.5,\Psi_{CI-2}=0.4987,\Psi_{CI-3}=0.5065,\Psi_{CI-4}=0.5032,\\&\widehat{\Psi}_{CI-1}=0.5,\widehat{\Psi}_{CI-2}=0.5019,\widehat{\Psi}_{CI-3}=0.4887,\widehat{\Psi}_{CI-4}=0.4955.\end{aligned}\tag{49}$$

The graphical shown of the developed values are demonstrated with the help of Figure 1.

Step 4: by using the value of the closeness index, we examine the ranking results and find the best one in all alternatives $\mathfrak{C}_{CQ-i}$, such that

$$\begin{aligned}&\Psi_{CI-3}\geq\Psi_{CI-4}\geq\Psi_{CI-1}\geq\Psi_{CI-2},\\&\widehat{\Psi}_{CI-2}\geq\widehat{\Psi}_{CI-1}\geq\widehat{\Psi}_{CI-4}\geq\widehat{\Psi}_{CI-3}.\end{aligned}\tag{50}$$

By using the idea of WDM and WNSM are given different results such that the best alternatives are Ψ_{CI-3} and Ψ_{CI-2}. In Example 1, we considered the CIVQROFNs and resolved them by using investigated measures. Moreover, we choose the complex interval-valued Pythagorean fuzzy information (CIVPFIs) and complex interval-valued intuitionistic fuzzy information (CIVIFIs) and resolve it by using the investigated measures. To discuss the above issues, we illustrate the following examples.

Example 2. To develop the application of the investigated procedure in the environment of the MADM technique, we choose the real MADM example from [40]. Moreover, the needed information is discussed in Example 1. To resolve the above issues, we considered the weight vector for the attributes is demonstrated by $(0.4,0.3,0.2,0.1)^T$; then, the CIVQROFDM is expressed in the form of Table 1. By using the investigated CSMs, the steps of the developed decision-making procedure, then the procedure of the TOPSIS method is discussed in the following ways. The CIVQROFDM $\mathcal{Q}_{DM}=(\mathfrak{C}_{Al-ij})_{\widetilde{m}\times\widetilde{n}}=\left(\begin{matrix}[\mathfrak{M}^-_{\mathfrak{C}_{RP-ij}},\mathfrak{M}^+_{\mathfrak{C}_{RP-ij}}]e^{i2\pi([\mathfrak{M}^-_{\mathfrak{C}_{IP-ij}},\mathfrak{M}^+_{\mathfrak{C}_{IP-ij}}])}\\ [\mathfrak{N}^-_{\mathfrak{C}_{RP-ij}},\mathfrak{N}^+_{\mathfrak{C}_{RP-ij}}]e^{i2\pi([\mathfrak{N}^-_{\mathfrak{C}_{IP-ij}},\mathfrak{N}^+_{\mathfrak{C}_{IP-ij}}])}\end{matrix}\right)_{\widetilde{m}\times\widetilde{n}}$ is normalized. If all criteria are benefit kinds, then we cannot do anything, but if in all criteria at least one criterion is cost kind, then we will convert the cost kind criteria into benefits, and the normalized decision matrix is discussed in the form of Table 3.

Then, by using these investigated measures, we examine the closeness index Ψ_{CI-i}, which is demonstrated below:

$$\begin{aligned}&\Psi_{CI-1}=0.5010,\Psi_{CI-2}=0.4996,\Psi_{CI-3}=0.5053,\Psi_{CI-4}=0.5047,\\&\widehat{\Psi}_{CI-1}=0.4995,\widehat{\Psi}_{CI-2}=0.5004,\widehat{\Psi}_{CI-3}=0.4962,\widehat{\Psi}_{CI-4}=0.4969.\end{aligned}\tag{51}$$

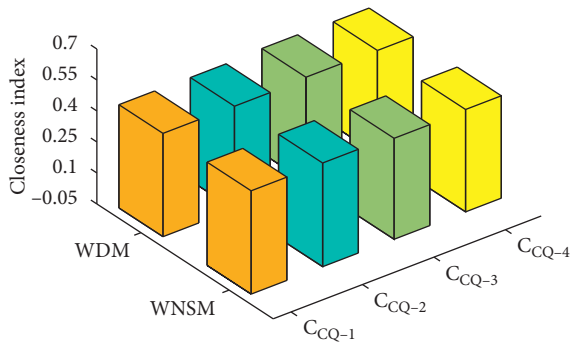

Figure 1: Graphical representation of the closeness index for Example 1.

Table 3: Normalized decision matrix of Example 2.

	$P_{\mathbf{At}-1}$	$P_{\mathbf{At}-2}$	$P_{\mathbf{At}-3}$	$P_{\mathbf{At}-4}$
$C_{\mathbf{CQ}-1}$	$\left(\begin{matrix}[0.9,0.91]e^{i2\pi([0.8,0.81])},\\ [0.1,0.11]e^{i2\pi([0.2,0.21])}\end{matrix}\right)$	$\left(\begin{matrix}[0.91,0.92]e^{i2\pi([0.81,0.82])},\\ [0.11,0.12]e^{i2\pi([0.21,0.22])}\end{matrix}\right)$	$\left(\begin{matrix}[0.92,0.93]e^{i2\pi([0.82,0.83])},\\ [0.12,0.13]e^{i2\pi([0.22,0.23])}\end{matrix}\right)$	$\left(\begin{matrix}[0.93,0.94]e^{i2\pi([0.83,0.84])},\\ [0.13,0.14]e^{i2\pi([0.23,0.24])}\end{matrix}\right)$
$C_{\mathbf{CQ}-2}$	$\left(\begin{matrix}[0.85,0.86]e^{i2\pi([0.89,0.9])},\\ [0.2,0.21]e^{i2\pi([0.1,0.11])}\end{matrix}\right)$	$\left(\begin{matrix}[0.86,0.87]e^{i2\pi([0.9,0.91])},\\ [0.21,0.22]e^{i2\pi([0.11,0.12])}\end{matrix}\right)$	$\left(\begin{matrix}[0.87,0.88]e^{i2\pi([0.91,0.92])},\\ [0.22,0.23]e^{i2\pi([0.12,0.13])}\end{matrix}\right)$	$\left(\begin{matrix}[0.88,0.89]e^{i2\pi([0.92,0.93])},\\ [0.23,0.24]e^{i2\pi([0.13,0.14])}\end{matrix}\right)$
$C_{\mathbf{CQ}-3}$	$\left(\begin{matrix}[0.7,0.71]e^{i2\pi([0.8,0.81])},\\ [0.3,0.31]e^{i2\pi([0.3,0.31])}\end{matrix}\right)$	$\left(\begin{matrix}[0.71,0.72]e^{i2\pi([0.81,0.82])},\\ [0.31,0.32]e^{i2\pi([0.31,0.32])}\end{matrix}\right)$	$\left(\begin{matrix}[0.72,0.73]e^{i2\pi([0.82,0.83])},\\ [0.32,0.33]e^{i2\pi([0.32,0.33])}\end{matrix}\right)$	$\left(\begin{matrix}[0.73,0.74]e^{i2\pi([0.83,0.84])},\\ [0.33,0.34]e^{i2\pi([0.33,0.34])}\end{matrix}\right)$
$C_{\mathbf{CQ}-4}$	$\left(\begin{matrix}[0.85,0.86]e^{i2\pi([0.7,0.71])},\\ [0.2,0.21]e^{i2\pi([0.3,0.31])}\end{matrix}\right)$	$\left(\begin{matrix}[0.86,0.87]e^{i2\pi([0.71,0.72])},\\ [0.21,0.22]e^{i2\pi([0.31,0.32])}\end{matrix}\right)$	$\left(\begin{matrix}[0.87,0.88]e^{i2\pi([0.72,0.73])},\\ [0.22,0.23]e^{i2\pi([0.32,0.33])}\end{matrix}\right)$	$\left(\begin{matrix}[0.88,0.89]e^{i2\pi([0.73,0.74])},\\ [0.23,0.24]e^{i2\pi([0.33,0.34])}\end{matrix}\right)$

The graphical representation of the obtained results is shown in Figure 2.

Step 5: by using the value of the closeness index, we examine the ranking results and find the best one in all alternatives $\mathfrak{C}_{CQ-i}$, such that

$$\begin{aligned}&\Psi_{CI-3} \geq \Psi_{CI-4} \geq \Psi_{CI-1} \geq \Psi_{CI-2},\\ &\widehat{\Psi}_{CI-2} \geq \widehat{\Psi}_{CI-1} \geq \widehat{\Psi}_{CI-4} \geq \widehat{\Psi}_{CI-3}.\end{aligned} \tag{52}$$

By using the idea of WDM and WNSM, given different results, such that the best alternatives are Ψ_{CI-3} and Ψ_{CI-2}, in Example 2, we considered the CIVPFIs and resolved them by using investigated measures. Moreover, we choose the complex interval-valued intuitionistic fuzzy information (CIVIFIs) and resolve it by using the investigated measures. To discuss the above issues, we illustrate the following examples.

Example 3. To develop the application of the investigated procedure in the environment of the MADM technique, we choose the real MADM example from [40]. Moreover, the needed information is discussed in Example 1. To resolve the above issues, we considered the weight vector for the attributes is demonstrated by $(0.4, 0.3, 0.2, 0.1)^T$; then, the CIVQROFDM is expressed in the form of Table 1. By using the investigated CSMs, the steps of the developed decision-making procedure, then the procedure of the TOPSIS method is discussed in the following ways. The CIVQROFDM $\mathcal{Q}_{DM} = (\mathfrak{E}_{Al-ij})_{\widetilde{m}\times\widetilde{n}} = \left(\begin{matrix}[\mathfrak{M}^-_{\mathfrak{C}_{RP-ij}}, \mathfrak{M}^+_{\mathfrak{C}_{RP-ij}}]e^{i2\pi([\mathfrak{M}^-_{\mathfrak{C}_{IP-ij}}, \mathfrak{M}^+_{\mathfrak{C}_{IP-ij}}])},\\ [\mathfrak{N}^-_{\mathfrak{C}_{RP-ij}}, \mathfrak{N}^+_{\mathfrak{C}_{RP-ij}}]e^{i2\pi([\mathfrak{N}^-_{\mathfrak{C}_{IP-ij}}, \mathfrak{N}^+_{\mathfrak{C}_{IP-ij}}])}\end{matrix}\right)_{\widetilde{m}\times\widetilde{n}}$ is normalized. If all criteria are benefit kinds, then we cannot do anything, but if in all criteria at least one criterion is cost kind, then we will convert the cost kind criteria into benefits, and the normalized decision matrix is discussed in the form of Table 4.

The graphical representation of the developed values are demonstrated in Figure 3.

Then, by using these investigated measures, we examine the closeness index Ψ_{CI-i}, which is demonstrated below:

$$\begin{aligned}&\Psi_{CI-1} = 0.5052, \Psi_{CI-2} = 0.4982, \Psi_{CI-3} = 0.4938, \Psi_{CI-4} = 0.4974,\\ &\widehat{\Psi}_{CI-1} = 0.4997, \widehat{\Psi}_{CI-2} = 0.5003, \widehat{\Psi}_{CI-3} = 0.5008, \widehat{\Psi}_{CI-4} = 0.5004.\end{aligned} \tag{53}$$

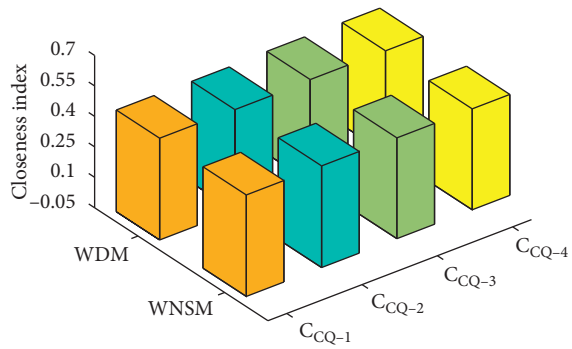

Figure 2: Graphical representation of the closeness index for Example 2.

Table 4: Normalized decision matrix of Example 3.

	$P_{\mathbf{At-1}}$	$P_{\mathbf{At-2}}$	$P_{\mathbf{At-3}}$	$P_{\mathbf{At-4}}$
$C_{\mathbf{CQ-1}}$	$\left(\begin{matrix}[0.7,0.71]e^{i2\pi([0.6,0.61])},\\ [0.1,0.11]e^{i2\pi([0.2,0.21])}\end{matrix}\right)$	$\left(\begin{matrix}[0.71,0.72]e^{i2\pi([0.61,0.62])},\\ [0.11,0.12]e^{i2\pi([0.21,0.22])}\end{matrix}\right)$	$\left(\begin{matrix}[0.72,0.73]e^{i2\pi([0.62,0.63])},\\ [0.12,0.13]e^{i2\pi([0.22,.0.23])}\end{matrix}\right)$	$\left(\begin{matrix}[0.73,0.74]e^{i2\pi([0.63,0.64])},\\ [0.13,0.14]e^{i2\pi([0.23,0.24])}\end{matrix}\right)$
$C_{\mathbf{CQ-2}}$	$\left(\begin{matrix}[0.6,0.61]e^{i2\pi([0.8,0.81])},\\ [0.2,0.21]e^{i2\pi([0.1,0.11])}\end{matrix}\right)$	$\left(\begin{matrix}[0.61,0.62]e^{i2\pi([0.81,0.82])},\\ [0.21,0.22]e^{i2\pi([0.11,0.12])}\end{matrix}\right)$	$\left(\begin{matrix}[0.62,0.63]e^{i2\pi([0.82,0.83])},\\ [0.22,0.23]e^{i2\pi([0.12,0.13])}\end{matrix}\right)$	$\left(\begin{matrix}[0.63,0.64]e^{i2\pi([0.83,0.84])},\\ [0.23,0.24]e^{i2\pi([0.13,0.14])}\end{matrix}\right)$
$C_{\mathbf{CQ-3}}$	$\left(\begin{matrix}[0.5,0.51]e^{i2\pi([0.5,0.51])},\\ [0.3,0.31]e^{i2\pi([0.3,0.31])}\end{matrix}\right)$	$\left(\begin{matrix}[0.51,0.52]e^{i2\pi([0.51,0.52])},\\ [0.31,0.32]e^{i2\pi([0.31,0.32])}\end{matrix}\right)$	$\left(\begin{matrix}[0.52,0.53]e^{i2\pi([0.52,0.53])},\\ [0.32,0.33]e^{i2\pi([0.32,0.33])}\end{matrix}\right)$	$\left(\begin{matrix}[0.53,0.54]e^{i2\pi([0.53,0.54])},\\ [0.33,0.34]e^{i2\pi([0.33,0.34])}\end{matrix}\right)$
$C_{\mathbf{CQ-4}}$	$\left(\begin{matrix}[0.7,0.71]e^{i2\pi([0.4,0.41])},\\ [0.2,0.21]e^{i2\pi([0.3,0.31])}\end{matrix}\right)$	$\left(\begin{matrix}[0.71,0.72]e^{i2\pi([0.41,0.42])},\\ [0.22,0.23]e^{i2\pi([0.32,0.33])}\end{matrix}\right)$	$\left(\begin{matrix}[0.72,0.73]e^{i2\pi([0.42,0.43])},\\ [0.23,0.24]e^{i2\pi([0.33,0.34])}\end{matrix}\right)$	$\left(\begin{matrix}[0.73,0.74]e^{i2\pi([0.43,0.44])},\\ [0.24,0.25]e^{i2\pi([0.34,0.35])}\end{matrix}\right)$

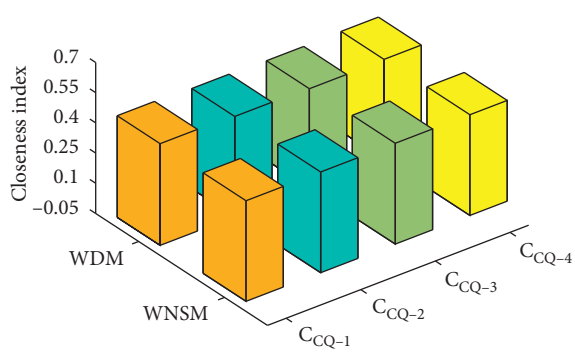

Figure 3: Graphical representation of the closeness index for Example 3.

Step 6: by using the value of the closeness index, we examine the ranking results and find the best one in all alternatives $\mathfrak{C}_{CQ-i}$, such that

$$\begin{aligned}&\Psi_{CI-1} \geq \Psi_{CI-2} \geq \Psi_{CI-4} \geq \Psi_{CI-3},\\ &\widehat{\Psi}_{CI-4} \geq \widehat{\Psi}_{CI-4} \geq \widehat{\Psi}_{CI-2} \geq \widehat{\Psi}_{CI-1}.\end{aligned} \tag{54}$$

By using the idea of WDM and WNSM, given different results, such that the best alternatives are Ψ_{CI-3} and Ψ_{CI-2}, therefore, the investigated measures based on CIVQROFSs are extensively useful to manage difficult data in daily issues.

5. Comparative Analysis

By using some existing measures, we compare the investigated measures with the help of numerical examples, which are discussed in the above section. To improve the quality of the research work and examine the validity and capability of the presented approaches, the information about existing ideas is discussed as follows: Wei and Zhang [41] developed CSMs based on IVIFSs and Kumar et al. [42] explored CSMs for IVPFS, compared with CSMs for IVQROFSs, compared with the investigated SMs for CIVIFSs, and compared with explored DMs for CIVPFSs. By using the information of Example 2, the comparative analysis of the presented works with some existing ideas is discussed in the form of Tables 5 and 6.

The graphical representation of the developed values in Tables 5 and 6 is demonstrated with the help of Figures 4 and 5 which contains graphical expressions of six different types of measures, and each measure contains four alternatives.

By using the information of Example 3, the comparative analysis of the presented works with some existing ideas is discussed in the form of Tables 7 and 8.

For the existing measures, we choose another set for finding measures such as $\mathfrak{C}_{CQ} = \{([1,1]e^{i2\pi([1,1])}, [0,0]e^{i2\pi([0.0,0.0])}), ([1,1]e^{i2\pi([1,1])}, [0,0]e^{i2\pi([0.0,0.0])}), ([1,1]e^{i2\pi([1,1])}, [0,0]e^{i2\pi([0.0,0.0])}), ([1,1]e^{i2\pi([1,1])}, [0,0]e^{i2\pi([0.0,0.0])})\}$.

Table 5: Comparative analysis of the proposed and existing ideas for distance measures.

Methods	Score values/measures' values	Ranking values
Wei and Zhang [41]	Cannot be able to resolve it	Cannot be able to resolve it
Kumar et al. [42]	Cannot be able to resolve it	Cannot be able to resolve it
For IVQROFSs	Cannot be able to resolve it	Cannot be able to resolve it
For CIVIFSs	Cannot be able to resolve it	Cannot be able to resolve it
For CIVPFSs	Cannot be able to resolve it	Cannot be able to resolve it
Proposed WDM	$\Psi_{CI-1} = 0.5, \Psi_{CI-2} = 0.4987, \Psi_{CI-3} = 0.5065$, and $\Psi_{CI-4} = 0.5032$	$\Psi_{CI-3} \geq \Psi_{CI-4} \geq \Psi_{CI-1} \geq \Psi_{CI-2}$

Table 6: Comparative analysis of the proposed and existing ideas for similarity measures.

Methods	Score values/measures' values	Ranking values
Wei and Zhang [41]	Cannot be able to resolve it	Cannot be able to resolve it
Kumar et al. [42]	Cannot be able to resolve it	Cannot be able to resolve it
For IVQROFSs	Cannot be able to resolve it	Cannot be able to resolve it
For CIVIFSs	Cannot be able to resolve it	Cannot be able to resolve it
For CIVPFSs	Cannot be able to resolve it	Cannot be able to resolve it
Proposed WDM	$\widehat{\Psi}_{CI-1} = 0.5, \widehat{\Psi}_{CI-2} = 0.5019, \widehat{\Psi}_{CI-3} = 0.4887$, and $\widehat{\Psi}_{CI-4} = 0.4955$	$\widehat{\Psi}_{CI-2} \geq \widehat{\Psi}_{CI-1} \geq \widehat{\Psi}_{CI-4} \geq \widehat{\Psi}_{CI-3}$

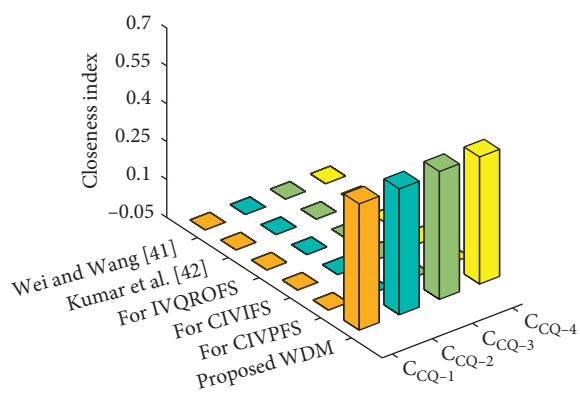

Figure 4: Graphical representation of the result shown in Table 5.

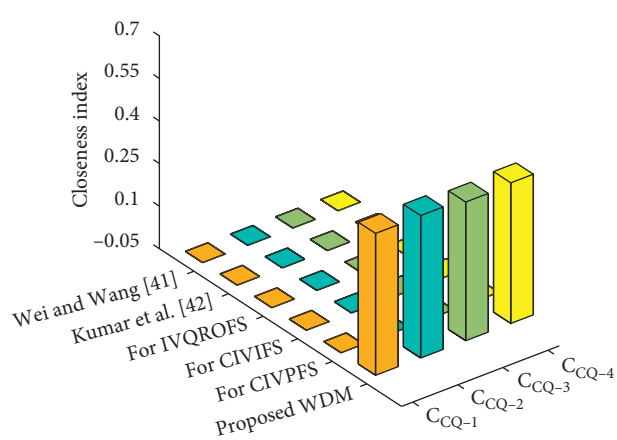

Figure 5: Graphical representation of the result shown in Table 6.

The graphical representation of the information given in Tables 7 and 8 is demonstrated with the help of Figures 6 and 7, respectively.

By using the information of Example 3, the comparative analysis of the presented works with some existing ideas is discussed in the form of Tables 9 and 10.

Table 7: Comparative analysis of the proposed and existing ideas for distance measures.

Methods	Score values/measures' values	Ranking values
Wei and Zhang [41]	Cannot be able to resolve it	Cannot be able to resolve it
Kumar et al. [42]	Cannot be able to resolve it	Cannot be able to resolve it
For IVQROFSs	Cannot be able to resolve it	Cannot be able to resolve it
For CIVIFSs	Cannot be able to resolve it	Cannot be able to resolve it
For CIVPFSs	$\Psi_{CI-1} = 0.6172, \Psi_{CI-2} = 0.6004, \Psi_{CI-3} = 0.6279,$ and $\Psi_{CI-4} = 0.6190$	$\Psi_{CI-3} \geq \Psi_{CI-4} \geq \Psi_{CI-1} \geq \Psi_{CI-2}$
Proposed WDM	$\Psi_{CI-1} = 0.5010, \Psi_{CI-2} = 0.4996, \Psi_{CI-3} = 0.5053,$ and $\Psi_{CI-4} = 0.5047$	$\Psi_{CI-3} \geq \Psi_{CI-4} \geq \Psi_{CI-1} \geq \Psi_{CI-2}$

Table 8: Comparative analysis of the proposed and existing ideas for similarity measures.

Methods	Score values/measures' values	Ranking values
Wei and Zhang [41]	Cannot be able to resolve it	Cannot be able to resolve it
Kumar et al. [42]	Cannot be able to resolve it	Cannot be able to resolve it
For IVQROFSs	Cannot be able to resolve it	Cannot be able to resolve it
For CIVIFSs	Cannot be able to resolve it	Cannot be able to resolve it
For CIVPFSs	$\widehat{\Psi}_{CI-1} = 0.5635, \widehat{\Psi}_{CI-2} = 0.5850, \widehat{\Psi}_{CI-3} = 0.5157,$ and $\widehat{\Psi}_{CI-4} = 0.5399$	$\widehat{\Psi}_{CI-2} \geq \widehat{\Psi}_{CI-1} \geq \widehat{\Psi}_{CI-4} \geq \widehat{\Psi}_{CI-3}$
Proposed WDM	$\widehat{\Psi}_{CI-1} = 0.4995, \widehat{\Psi}_{CI-2} = 0.5004, \widehat{\Psi}_{CI-3} = 0.4962,$ and $\widehat{\Psi}_{CI-4} = 0.4969$	$\widehat{\Psi}_{CI-2} \geq \widehat{\Psi}_{CI-1} \geq \widehat{\Psi}_{CI-4} \geq \widehat{\Psi}_{CI-3}$

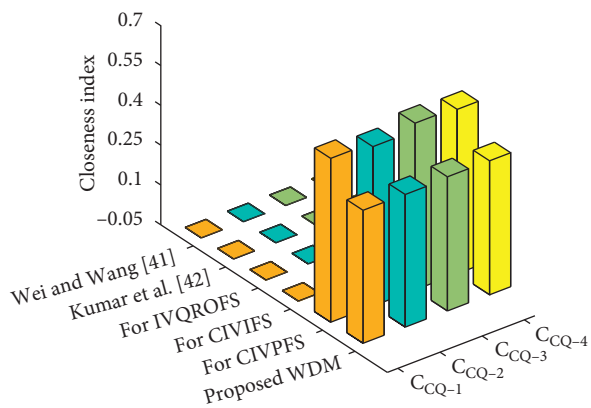

Figure 6: Graphical representation of the information given in Table 7.

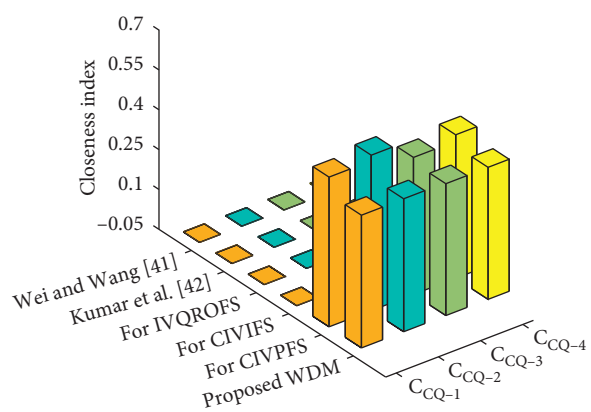

Figure 7: Graphical representation of the information given in Table 8.

The graphical representation of the developed values in Tables 9 and 10 is demonstrated with the help of Figures 8 and 9 which contain graphical expressions of six different types of measures, and each measure contains four alternatives. From the above discussion, we obtain that if we choose the CIVQRIFIs, then the existing measures based on CIVIFSs, CIVPFSs, and their special cases are not able to cope with it. However, if we choose the existing types of data,

Table 9: Comparative analysis of the proposed and existing ideas for distance measures.

Methods	Score values/measures' values	Ranking values
Wei and Zhang [41]	Cannot be able to resolve it	Cannot be able to resolve it
Kumar et al. [42]	Cannot be able to resolve it	Cannot be able to resolve it
For IVQROFSs	Cannot be able to resolve it	Cannot be able to resolve it
For CIVIFSs	$\widehat{\Psi}_{CI-1} = 0.5039, \widehat{\Psi}_{CI-2} = 0.4979, \widehat{\Psi}_{CI-3} = 0.3927$, and $\widehat{\Psi}_{CI-4} = 0.4956$	$\Psi_{CI-1} \geq \Psi_{CI-2} \geq \Psi_{CI-4} \geq \Psi_{CI-3}$
For CIVPFSs	$\widehat{\Psi}_{CI-1} = 0.5116, \widehat{\Psi}_{CI-2} = 0.4992, \widehat{\Psi}_{CI-3} = 0.5044$, and $\widehat{\Psi}_{CI-4} = 0.5026$	$\Psi_{CI-1} \geq \Psi_{CI-3} \geq \Psi_{CI-4} \geq \Psi_{CI-2}$
Proposed WDM	$\Psi_{CI-1} = 0.5052, \Psi_{CI-2} = 0.4982, \Psi_{CI-3} = 0.4938$, and $\Psi_{CI-4} = 0.4974$	$\Psi_{CI-1} \geq \Psi_{CI-2} \geq \Psi_{CI-4} \geq \Psi_{CI-3}$

Table 10: Comparative analysis of the proposed and existing ideas for similarity measures.

Methods	Score values/measures' values	Ranking values
Wei and Zhang [41]	Cannot be able to resolve it	Cannot be able to resolve it
Kumar et al. [42]	Cannot be able to resolve it	Cannot be able to resolve it
For IVQROFSs	Cannot be able to resolve it	Cannot be able to resolve it
For CIVIFSs	$\widehat{\Psi}_{CI-1} = 0.4986, \widehat{\Psi}_{CI-2} = 0.4992, \widehat{\Psi}_{CI-3} = 0.3999$, and $\widehat{\Psi}_{CI-4} = 0.4994$	$\widehat{\Psi}_{CI-2} \geq \widehat{\Psi}_{CI-1} \geq \widehat{\Psi}_{CI-4} \geq \widehat{\Psi}_{CI-3}$
For CIVPFSs	$\widehat{\Psi}_{CI-1} = 0.4992, \widehat{\Psi}_{CI-2} = 0.4998, \widehat{\Psi}_{CI-3} = 0.5003$, and $\widehat{\Psi}_{CI-4} = 0.4999$	$\widehat{\Psi}_{CI-2} \geq \widehat{\Psi}_{CI-1} \geq \widehat{\Psi}_{CI-4} \geq \widehat{\Psi}_{CI-3}$
Proposed WDM	$\widehat{\Psi}_{CI-1} = 0.4997, \widehat{\Psi}_{CI-2} = 0.5003, \widehat{\Psi}_{CI-3} = 0.5008$, and $\widehat{\Psi}_{CI-4} = 0.5004$	$\widehat{\Psi}_{CI-3} \geq \widehat{\Psi}_{CI-4} \geq \widehat{\Psi}_{CI-2} \geq \widehat{\Psi}_{CI-1}$

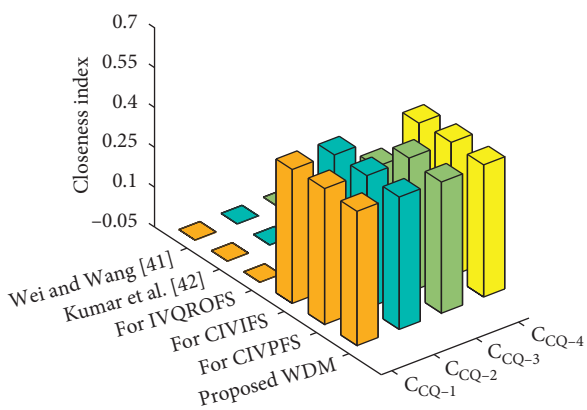

Figure 8: Graphical representation of the information given in Table 9.

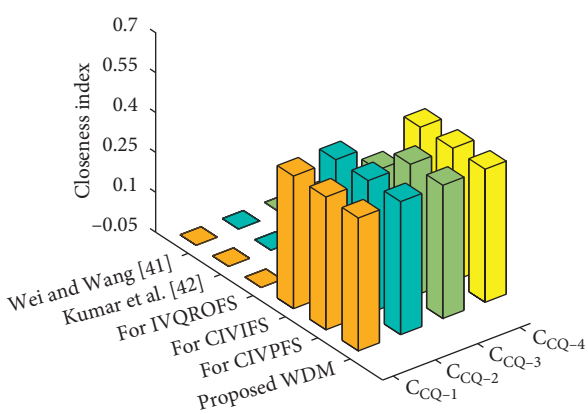

Figure 9: Graphical representation of the information given in Table 10.

then the investigated measures based on CIVQROFSs can resolve it, see Tables 5 to 10. Therefore, the investigated measures based on CIVQROFSs are more useful to resolve daily life issues.

6. Conclusion

As a modification of the IVQROFSs, CIVQROFSs is a reliable and competent technique for the realization of incorrect knowledge by the use of complex-valued truth grades with an additional term, referred to as the phase term. CSMs and DMs are accomplished techniques for verifying the degree of discrimination between the two sets. In this manuscript, we choose some CSMs and DMs from CIVQROFS. CSMs and EDMs are investigated using CIVQROFS and their properties. Choosing that the CSMs do not justify the axiom of SM, then we investigate the technique of developing other CIVQROFS-based SMs using the explored CSMs and EDMs, and it fulfills the axiom of the SMs. In addition, we find CDMs based on CIVQROFSs by considering the inter-relationship between the SM and the DMs, and then, we modified the procedure for the rank of partiality by similarity to the ideal solution method of the CDMs under investigation, which could deal with the associated decision-making problems not only individually from the argument of the opinion of the geometry but also from the fact of the opinion of the algebra. Finally, we provide a sensible example to demonstrate the practicality and effectiveness of the proposed procedure, which is also in line with existing additional procedures. Graphic representations of the measures developed are also used in this manuscript. Future work can focus on extending the proposed approach in a different fuzzy environment to solve the problems related to decision-making, medical diagnosis, pattern recognition, etc. [43–46].

Data Availability

No data were used to support this study.

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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