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A rough based multi-criteria evaluation method for healthcare waste disposal location decisions

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ABSTRACT

Healthcare waste disposal management is one of the biggest day-to-day challenges faced by healthcare providers and urban municipalities. Poor management of healthcare waste can cause serious problems for healthcare workers, patients, and the general public. Healthcare providers and urban planners usually struggle with the action of locating an appropriate waste disposal center in a municipal area. Healthcare waste disposal location planning is a difficult task due to complexities inherent in the evaluation of alternative locations according to multiple and often competing criteria. We propose a new best-worst method with interval rough numbers (IRN) for healthcare waste disposal location decisions. A new IRN Dombi-Bonferroni (IRNDBM) means the operator is also introduced to process the rough data because of the unavailability of precise information. A case study at a private hospital in Madrid is presented to demonstrate the applicability and exhibit the efficacy of the proposed multi-criteria evaluation method.

1. Introduction

Medical waste management is inherently a complex problem with multiple and often conflicting criteria. Medical waste is potentially infectious unused/unwanted materials generated at health care facilities such as hospitals, physician's offices, blood banks, and medical laboratories and research facilities (Windfeld & Brooks, 2015). Awodele, Adewoye, and Oparah (2016) show that the following items from healthcare systems (hospitals, laboratories, and general or dental clinics) are the fundamental sources of waste: things that are soaked in blood, human or animal tissues, infectious diseases, the waste in patient's rooms with communicable diseases, and the discarded vaccines (Alam, Alam, Ayub, & Siddiqui, 2019). Different types of medical wastes are demonstrated as Biomedical, Clinical or dental, Bio-hazardous, Infectious Medicals, and Healthcare waste. Potentially, medical wastes are considerable sources of risk of infection or injury to healthcare staff, workers, patients, and public health if they are not collected, disposed, and controlled systematically. The U.S. hospital association reported an estimated 5.9 million tons of biohazardous and other medical waste every year. Almost 85% of all medical waste is classified as non-hazardous while 15% is hazardous and may be infectious, radioactive, or toxic (Windfeld & Brooks, 2015).

A healthcare management system is the body responsible for treating on-site or off-site operations employing various practices. Hazardous waste can cause microorganisms, radiation burns, poisoning, and serious pollution (Rabbani, Heidari, Farrokhi-Asl, & Rahimi, 2018; Yu & Solvang, 2016). Unfortunately, an improperly and ill-developed waste treatment system can damage our environment, contaminate drinking water, and finally causing a disaster. To prevent such kinds of events and reduce the risk of imposing medical wastes, as well as establish a fruitful condition for the entire society, one strategy is to find an optimal location for collecting, controlling, and imposing various wastes. This is a very critical issue that has yet to be effectively addressed in the literature. A limited number of studies proposed methods related to the sustainable assessment of the waste disposal center establishment (Kazimieras Zavadskas, Baušys, & Lazauskas,

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2015; Margallo, Taddei, Hernández-Pellón, Aldaco, & Irabien, 2015).

Location selection traditionally is defined as a problem of comparing alternative locations (airports, logistics and distribution centers, and commercial centers) by referring to a set of objectives (economic, environmental factors, accessibility, regional regulations, and other factors) through a predetermined plan. Multi-criteria decision making (MCDM) consists of a set of techniques for determining the priority of alternatives using strategic tools that deliver optimal consequences and satisfy decision-makers. MCDM methods can evaluate different alternatives not only by taking into account the data for various criteria, but also the nature of the data (i.e., crisp, fuzzy, interval, and rough, among others). Goulart Coelho, Lange, and Coelho (2017) report that the majority of the MCDM models proposed for waste management used the analytic hierarchy process (AHP) to solve the problem. Our motivations in this study are, therefore, to (i) develop a quantitative methodology that can simultaneously assess different indicators and dimensions in a healthcare waste management system, (ii) use of a weight assessment model based on the combination of data-centric approaches and stakeholder opinions, and (iii) confirm the stability and robustness of the results through a comprehensive and elaborate sensitivity analysis.

The best-worst method (BWM) is preferred to other MCDM methods such as AHP and decision-making trial and evaluation laboratory (DEMATEL) for weight elicitation because it is easier, requires less pairwise comparisons, and produces more consistent results. However, since it is unlikely that decision-makers will have complete knowledge of all the aspects of the problem of location selection for waste disposal, uncertainties must be taken into consideration in the modeling and decision-making process. Advanced algorithms may further help decision-makers analyze the uncertainties inherent in real-world problems and finding the most suitable solution.

Unlike fuzzy theory, grey theory, and other interval-valued approaches; the rough set theory (Pawlak, 1982), is a very powerful tool for the treatment of imprecision and uncertainty without the impact of subjectivism. In the rough approach, the borders are determined based on border approximation areas and the uncertainty that governs them. While in traditional fuzzy theory, the degree of uncertainty is defined based on assumptions, in the rough approach, uncertainty is determined based on approximation, which is the basic concept of rough numbers (RNs) (Song, Ming, Wu, & Zhu, 2014). In the application of RNs, instead of different additional/external parameters, only the structure of the given data is used. This leads to the objective indicators contained in the data. The basic logic of RNs is that the actual data should speak for themselves. RN eliminates the shortcomings of the traditional fuzzy approach relating to the interval borders since, for every rating of the expert, unique interval borders are formed. This means that the interval borders do not depend on subjective assessment, but rather are defined based on uncertainty and imprecision in the data. In this paper, we propose a hybrid MCDM model with these features:

- The interval RNs (IRNs) MCDM approach proposed here relies exclusively on internal knowledge (i.e., operational data) and requires no superficial or unrealistic assumptions.
- The interval borders are formed based on uncertainty and imprecision in the data, not subjective assessments.
- The proposed model is structured, flexible, and takes into consideration the interaction and the interdependencies among the decision criteria.
- The proposed model considers the interconnection among the attributes and eradicates the impact of information uncertainties.

The three-phase model is different from the conventional methods of waste disposal location selection. In Phase 1, we use a modified Best-Worst Method with IRNs for criteria weight estimation. In Phase 2, we use the IRN Dombi-Bonferroni (D'Bonferroni) approach to formulating the problem, and in Phase 3, we use a multi-dimensional sensitivity analysis to evaluate the robustness of our model. The remainder of the paper is organized as follows. In Section 2, we present the relevant literature on waste management and location planning. In Section 3, we present the background theories and mathematical relations. Section 4 presents the proposed model. In Section 5, we present a case study to demonstrate the applicability of the proposed model. In Section 6, we exhibit the efficacy of our model through sensitivity analysis, performance comparison, and discussion of the outcomes. Finally, Section 7 presents our conclusions, implications, and future research directions.

2. Literature review

In this section, we review the relevant literature on the rough-based decision-making structure and location selection studies and waste disposal system. We conclude this section by highlighting our research novelty and contribution.

2.1. Rough-based decision-making structure

Uncertainty is associated with vagueness, fuzziness, and roughness in MCDM. Grey theory, fuzzy sets, and rough sets have been widely used in MCDM for uncertainty analysis. In fuzzy set theory, an element has a degree of membership to a set, which is expressed as a number between zero (no membership) and one (full membership). Fuzzy numbers can be described by verbal phrases. In grey theory, uncertainty is represented with black, white, or gray ranges (numbers). The shade of a number represents the extent of uncertainty where black numbers indicate no knowledge, white numbers indicate complete knowledge, and gray numbers are between these two extremes. Similar to fuzzy sets, gray numbers can be described by verbal phrases. Rough set theory (RST) is a new method for uncertainty representation proposed by Pawlak (1982). The borders in RST are determined based on border approximation areas and their associated uncertainties. In contrast, fuzzy sets are characterized by subjectivity when defining the borders of the sets (Roy, Chatterjee, Bandhopadhyay, & Kar, 2016). While the degree of uncertainty is defined based on the traditional fuzzy and probability theory assumptions, RST determines uncertainty on the basis of approximation, which is the central concept in RNs. The rough approach uses exclusively internal knowledge (i.e., operational data), and there is no need to rely on assumptions. In other words, in the application of RNs, the structure of the given data is used instead of different additional/external parameters. RNs eliminate the drawbacks of the traditional fuzzy approaches with regards to the interval borders because specific unique interval borders are formed for specific expert ratings. This means the interval borders do not depend on any subjective assessments. Instead, they are defined according to the uncertainties and imprecision in the data. The integration of RNs into traditional MCDM models exploits the subjectivity and unclear assessment of the experts and avoids assumptions, unlike the fuzzy theory. In this study, we use rough sets because they do not need any additional or prior information about data (Pawlak, 1982).

Relevant to RS decision making, several studies have been developed by past researchers. Song et al. (2014) used the Rough Technique for Order of Preference by Similarity to Ideal Solution (RTOPSIS) approach for failure mode and effects analysis in uncertain environments. Roy et al. (2016), Roy, Chatterjee, Bandyopadhyay, and Kar (2017)) adopted an integration of Rough AHP (RAHP) and Multi-Attributive Border Approximation Area Comparison (MABAC) methods for the assessment of medical Tourism sites, while Stević, Pamučar, Vasiljević, Stojić, & Korica (2017), Stević, Pamučar, Kazimieras Zavadskas, Ćirović, & Prentkovskis (2017) applied the rough BWM model for the determination of criteria weights for selecting the most suitable wagon in a logistics company. Pamučar, Gigović, Bajić, and Janošević (2017), Pamučar, Mihajlović, Obradović, & Atanasković (2017) propounded a hybrid multi-criteria model based on IRNs and demonstrated the application for a bidder selection process in the state administration public procurement system. Gigović, Pamučar, Bajić, and Drobnjak (2017) proposed a combined interval of rough AHP and GIS for flood hazard mapping. Song et al. (2014) show the integration of RNs with MCDM methods is promising and produces reliable and consistent results. Thus far, the rough BWM has been used to solve several real-world problems (Cao & Song, 2016; Pamučar, Gigović, et al. (2017); Stević, Pamučar, Vasiljević et al., 2017, Stević, Pamučar, Kazimieras et al., 2017; Vasiljević, Fazlollahtabar, Stević, & Vesković, 2018). Wang, Kuang, Wang, and Zhang (2016) developed an extended ELECTRE III approach for rough stochastic MCDM problems. The rough BWM model in Pamučar, Gigović, et al. (2017) is used for determining weight coefficients of the criteria for the selection of wind farm locations. Stević, Pamučar, Kazimieras Zavadskas, et al. (2017) used this integration to determine the importance of the criteria in selecting a wagon for a logistics company. Pamučar, Petrović, & Ćirović (2018), Pamučar, Stević, & Kazimieras Zavadskas (2018) applied interval rough fuzzy BWM to select the most suitable firefighting helicopters. Pamučar, Petrović, et al. (2018), Pamučar, Stević, et al. (2018) integrated interval RAHP and interval rough MABAC methods for evaluating university websites. Stojić, Stević, Antuchevičienė, Pamučar, and Vasiljević (2018) proposed a supplier selection model for a polyvinyl chloride carpentry firm using RAHP and the rough weighted aggregated sum product assessment method.

Vasiljević et al. (2018) extended the AHP model in a rough environment to determine criteria weights in an MCDM model for supplier selection in the automotive industry. Xuerui, Suihuai, and Jianjie (2018) proposed a novel hybrid MCDM integrating the rough analytic network process (ANP) and RTOPSIS for the optimal selection of cloud manufacturing services. Pamučar, Chatterjee, and Kazimieras Zavadskas (2019) investigated the ability to apply a roughly based multi-attribute decision-making approach for choosing third-party logistics providers. The main decision-making tools used were BWM, MABAC, and the weighted aggregated sum product assessment methods. The authors also highlighted some major advantages of using a rough-set in logistics operations. Roy, Adhikary, Kar, and Pamucar (2018) proposed a rough strength DEMATEL method for analyzing the individual priorities of key success factors in hospitals. Chen, Ming, Zhang, Yin, and Sun (2019) developed a hybrid model by encompassing fuzzy set theory, RST, DEMATEL, and ANP methods for evaluating the sustainable value requirement in an excavator product-service system.

Furthermore, Song and Sakao (2018) proposed a rough DEMATEL model to manipulate the interactions of vague user preferences in multi-criteria weight determination. Kazimieras Zavadskas, Stević, Tanackov, and Prentkovskis (2018) proposed the rough step-wise weight assessment ratio analysis method (R-SWARA) and its application in logistics. Stanković, Gladović, and Popović (2019) compared the significance of particular criteria for traffic accessibility using the fuzzy AHP and the rough AHP methods. In addition to understanding and appreciating uncertainty and imprecision, MCDM models need to fulfill these additional requirements when solving real-world problems: (1) simplicity and reliability of models for determining criteria weights; (2) possibility of validation of the criteria weights; (3) appreciation of the relationships between criteria, and (4) eliminating the awkward given by the initial decision matrix. Table 1 shows a comparative overview of the rough MCMD models that have been developed in the last five years and the fulfillment of the above conditions.

As shown in Table 1, the traditional MCDM models extended by RNs do not consider the relationship between attributes. In addition, to the best of our knowledge, no MCDM model has been developed to date, which is capable of eliminating awkward data. On the other hand, the rough BWM-Dombi-Bonferroni model proposed in this study meets all four conditions in Table 1. Therefore, the development of the proposed model is a logical step towards fulfilling the rough MCDM gap in the literature.

Table 1
Overview of rough MCDM models and their characteristics.

Reference	Rough MCDM model	Consideration the relationship between attributes	Elimination of awkward data	A small number of pairwise comparisons	Checking consistency of data
Song et al. (2014), Xuerui et al. (2018)	Rough TOPSIS	No	No	No	No
Roy et al. (2016, 2017)	Rough AHP-MABAC	No	No	No	Yes
Wang et al. (2016)	Rough ELECTRE III	No	No	Yes	No
Stević, Pamučar, Vasiljević et al. (2017), Stević, Pamučar, Kazimieras et al. (2017), Stojić et al. (2018)	Rough BWM-SAW	No	No	Yes	Yes
Pamučar, Gigović, et al. (2017)	Rough BWM	No	No	No	Yes
Pamučar, Gigović, et al. (2017), Pamučar, Mihajlović, et al. (2017)	Rough DEMATEL-ANPMAIRCA	Yes	No	Yes	Yes
Gigović et al. (2017)	Rough AHP and GIS	No	No	No	Yes
Pamučar, Gigović et al. (2017), Pamučar, Mihajlović, et al. (2017), Roy et al. (2018)	Rough DEMATEL	Yes	No	No	No
Cao and Song (2016)	Rough ANP	Yes	No	No	Yes
Stević, Pamučar, Vasiljević, et al. (2017), Vasiljević et al. (2018), Stanković et al. (2019)	Rough AHP	Yes	No	No	Yes
Pamučar, Petrović, et al. (2018), Pamučar, Stević, et al. (2018), Pamučar et al. (2019)	Rough BWM-MABAC	Yes	No	No	Yes
Chen et al. (2019)	Rough DEMATEL-ANP	Yes	No	No	Yes
Kazimieras Zavadskas et al. (2018)	Rough SWARA	No	No	Yes	No
Proposed model	Rough BWM-Dombi-Bonferroni	Yes	Yes	Yes	Yes

2.2. Location selection studies & waste disposal system

A wide range of studies focused on site selection, location evaluation, and selection, facility location selection using various tools and methods focus on mathematical tools, optimization models, and decision-making approaches. Those methods were proposed in particular for selecting a manufacturing site, an electric vehicle charging station, a logistics center, an airport transportation facility, a shelter site for the prevention of disasters, and a waste and landfill location, among others. For instance, Song, Zhou, and Song (2019) have demonstrated the application of rough-based computation with a qualitative flexible multiple criteria method tool in natural disaster treatment and the selection of the best place for shelter. The case study took place in the Province of Wenchuan County in China. Using a formulation of sustainable dimensions, Ju, Ju, Gonzalez, Giannakis, and Wang (2019) defined an analytical platform to resolve the complex problem of selecting charging stations for electric vehicles. They utilized the interaction of picture fuzzy numbers and the picture fuzzy weighted interaction geometric operator plus fuzzy AHP to obtain the weights of sustainable factors. Then, a grey relational projection method was applied to rank the list of stations in Beijing, China. Sennaroglu and Celebi (2018) incorporated five multi-attribute decision-making techniques to solve the problem of finding a safe and suitable place for a military airport in Turkey and compared the results. The role of AHP is to weight the site selection factors, while Preference Ranking Organization METHod for Enrichment of Evaluations (PROMETHEE), Vlse Kriterijumska Optimizacija Kompromisno Resenje (VIKOR), MABAC, and Complex Proportional Assessment (COPRAS) derived the list of optimal locations. In Isfahan, a province in Iran, Zoghi, Ehsani, Sadat, Amiri, and Karimi (2017) built a structure to evaluate and position the solar energy generation sites by using AHP weighted linear combination under fuzzy sets. In order to find the best location for logistics operations, a model based on fuzzy Delphi and TOPSIS was proposed to select the most suitable solution (Pham, Ma, & Yeo, 2017).

The idea of finding the most suitable place to establish a center for waste collection and disposal can be modeled using a logical MCDM system. The essential issue is how to deal with uncertainty, a topic that scientific communities and business schools often struggle with using fuzzy or probabilistic sets. On the other hand, the study of facility location has involved the study of solid municipal wastes, medical wastes, and factory wastes. Ekmekçioğlu, Kaya, and Kahraman (2010) formulated a fuzzy-based system for evaluating municipal waste disposal methods in Istanbul using a fuzzy AHP to rate decision factors and fuzzy TOPSIS to determine the score of each method. They used criteria like cost, reliability, feasibility, pollution and emission levels and a set of alternatives such as landfilling, composting conventional incineration, and refuse-derived fuel combustion. In Turkey, an investigation was oriented toward landfill site assessment using the Geographical Information System (GIS) and AHP, and criteria such as surface water, groundwater depth, lithology, lineaments, and distance to roads were incorporated in the body of the study (Şener, Sener, & Karagüzel, 2011). To aid in selecting an optimal location for disposing of the municipal wet waste landfill with less risk of uncertainty, Mokhtarian, Sadi-Nezhad, and Makui (2014) elaborated a fuzzy decision-making system by using an interval set with the VIKOR method. Kahraman et al. (2017) introduced an intuitionistic fuzzy Evaluation based on distance from the average solution method for finding an appropriate site for solid waste disposal. In a real project, to determine the optimal location for infectious waste disposal in Thailand, the fuzzy analytical hierarchy and a goal programming approaches were used. The authors used the three factors infrastructure, geological conditions, social and environmental concerns during their decision process (Wichapa & Khokhajaikiat, 2017). An investigation team used a multi-attribute hierarchy process to rank 40 hospitals in Ahvaz, Iran to determine how much waste each hospital must receive from a total of 13 tons/day of hospital waste (Karamouz, Zahraie, Kerachian, Jaafarzadeh, & Mahjouri, 2007). As a result of this project, it was ascertained that it was more advantageous to improve management techniques than to build new facilities. With the integration of the three dimensions environmental, social and economic and considering eleven measures (land-use pattern, distance from the settlement, depth of groundwater table, wind orientation, etc.), Khan and Samadder (2015) illustrated a model containing a GIS and AHP for the assessment of the potential landfill sites for municipal wastes in India.

Using life cycle analysis for the objective of hospital waste disposal optimal location in Pakistan, Ali, Wang, and Chaudhry (2016) found out that waste segregation can be an issue of substitution rather than incineration in other places. Aydi, Abichou, Nasr, Louati, and Zairi (2016) proposed an intelligent GIS to evaluate the suitability of several sites for olive mill wastewater collection and disposal in a region in Tunisia. The authors have relied on the AHP and Weighted linear combination under fuzzy with regards to the factors such as Groundwater depth, Soil permeability, etc. Several pieces of research took into account the benefit of GIS in the application of finding an accurate position for landfill site selection. For example, Bahrani, Ebadi, Ehsani, Yousefi, and Maknoon (2016) modeled a decision-making structure based on multiple attributes using fuzzy theory, AHP, weighted linear combination, and GIS to select among the candidate landfill sites in Shabestar, Iran. This action helps to isolate solid wastes and keep them away from any danger for the environment and the community. Beskese, Demir, Ozcan, and Okten (2015) used FAHP and FTOPSIS to choose among landfill centers for the city of Istanbul in a similar study. The criteria used in the study were soil conditions, land availability, economic issues, and climate.

Chauhan and Singh (2016) used a hybrid multi-criteria decision-making model comprised of interpretive structural modeling, FAHP, and FTOPSIS to select a suitable location for a healthcare waste disposal facility. The hybrid model was used to identify the interrelationship among sustainable factors, AHP to weigh each factor, and TOPSIS to rank locations based on the overall score. Based on the factors of land use, soil type, distance from roads, rainfall, research reports the selection among landfill positions in the Gaza strip, Palestine using an analytical hierarchy process and GIS (El Baba, Kayastha, & De Smedt, 2015). Eskandari, Homaee, and Falamaki (2016) proposed a similar approach using AHP and GIS to measure the optimality of the landfill location for solid waste in a mountain basis of a southern province in Iran by implementing sustainable elements. Gergin, Tunçbilek, and Esnaf (2019) suggested the use of the Artificial Bee Colony (ABC) clustering algorithm to locate the best waste disposal site for a Hospital in the great district of Istanbul. In another study in Kenya, a decision support system involving GIS and several MCDA tools was proposed to find the best area for healthcare system waste disposal. GIS filters the eight items as they satisfy the initial requirements. After that, AHP, VIKOR, and PROMETHEE techniques were used to analyze the alternatives based on a set of triple-bottom-line (TBL) variables (interconnecting financial, environmental, and social performance scores), and compare the results of the three different methods techniques (Hariz, Dönmez, & Sennaroglu, 2017). An approach was reported to rely on the fuzzy TOPSIS-PROMETHEE method to select among the ten solid waste disposal locations considering eighteen evaluation criteria. This was a case study managed by the Istanbul Environmental Management Industry and Trade, Turkey (Arıkan, Şimşit-Kalender, & Vayvay, 2017).

Similar to several previous studies, Torabi-Kaveh, Babazadeh, Mohammadi, and Zaresefat (2016) studied the performance of GIS and AHP with fuzzy values to choose the best landfill site. Rahmat et al. (2017) developed a study to find the location for exposure of solid waste in the city of Behbahan in Iran and applied GIS to analyze maps and geographical information. The role of the simple additive weighting tool is to gather information and obtain the weights of decision criteria such as distance to groundwater. Then all the locations were rated and analyzed using AHP to find the best one of the group.

Considering a sustainable evaluation of waste incineration plants, Kazimieras Zavadskas et al. (2015) used an interval-valued Neutrosophic extension of the WASPAS method. They articulated four essential dimensions in their process called engineering, social, economic, and environmental aspects. A recent project related to the healthcare waste disposal system concluded that factors as transportation and risk, government regulations, expert accessibility, and environmental awareness and economic issues could directly influence the attitude of leaders and policymakers for optimal decision making. This is part of the investigation was presented in India, and the authors conducted grey measures by AHP (Thakur & Ramesh, 2017). Municipal waste treatment and providing strategies for its reduction is an emergent topic in rural and urban design. Therefore, rather than a multi-criteria approach, a multi-objective mixed-integer program has been adopted for waste collection centers, including operational costs, greenhouse gas emissions, and environmental impacts (Yu & Solvang, 2017).

2.3. Research novelty and contribution

The majority of researchers validated and concluded their evaluation process using contemporary and classical MCDM tools such as AHP, PROMETHEE, and TOPSIS with or without fuzzy approaches. Since we know that waste elimination and disposal is a very vital task in every community, there is a big gap to be filled by many relevant and updated models. None of the studies we encountered (2014–2018) could define an aggregator such as the Dombi and rough set demonstration in waste management of healthcare systems. We used the BWM with the rough set approach for the first time to check its validation in finding criteria weights.

Generally, aggregation operators are important tools for integrating information in MCDM problems. The most widely used operators in uncertainty theory are the min and the max operator. With this, we would like to emphasize their main advantages: (i) They are easy to calculate, and (ii) They can be extended into a lattice structure. However, in the case of min-max operators, the main disadvantages are: (i) The result is determined only by one variable, and the other has no influence, and (ii) Their second derivative is not continuous (Dombi, 2009). These disadvantages of traditional min-max operators in a fuzzy environment are successfully eliminated by a generalized Dombi operator class. In addition, Dombi T-norms (TN) and T-conorms (TCN) have general parameters of general TN and TCN, and this can make the decision-making process more flexible. Because of this capability of Dombi TN and TCN, a logical step is to use Dombi TN and TCN for the development of the hybrid MCDM model.

However, one of the Dombi TN and TCN limitations is the manipulation with the numbers in the interval [0,1]. Thus far, Dombi TN and TCN have only been used to transform uncertain numbers meeting this requirement. In real decision-making systems, the attributes are often represented by values that are not within the interval [0,1], such as, for example, when it is necessary to measure the distance (in kilometers) between two cities. The transformation of such attributes into MCDM models by using traditional Dombi TN and TCN is not possible. To eliminate this limitation, in this paper, Dombi TN and TCN are modified with the purpose of the aggregation of IRNs regardless of their values. Up to now, there is no research on how to use the D'Bonferroni operator to aggregate the IRNs. Therefore, a logical goal and motivation for this study are to show how the hybrid D'Bonferroni aggregator can be used for the transformation by IRN. In addition, because IRNs can describe imprecise information easier, there is an increasing demand to combine the BM operator and the Dombi operations to deal with IRNs when dealing with MCDM problems. Taking into account the above information and the previously mentioned advantages, the authors of the present paper propose a rough-based framework of BWM and D'Bonferroni aggregators, which provides deeper insights into decision-makers' perceptions in the waste management perspective. Thus, the contribution and novelty of this study are fourfold:

(a) A new rough based BWM and D'Bonferroni aggregators' model that provides a structured and systematic approach to expert judgment evaluation in a subjective environment,
(b) An improved MCDM methodology with a powerful model for selecting the most suitable location for waste disposal facilities,
(c) A platform with a robust MCDM model for solving problems with subjective and non-quantitative data, and
(d) A flexible and yet structured decision-making framework for solving waste disposal problems in healthcare and other public or private organizations.

Making decisions in real systems requires a rational understanding of the relationship between attributes and the elimination of the impact of awkward data. For this purpose, Bonferroni (1950) introduced the Bonferroni Mean (BM) operator to allow for the presentation of interconnections between elements and their fusion into a unique score function. Bonferroni and Dombi aggregators can successfully achieve this goal. However, BM cannot be processed by Dombi operations. To consider the advantages of Dombi and BM operators together, we proposed some hybrid D'Bonferroni operators by combining the Dombi operator and the BM operator. The D'Bonferroni operator can take advantage of the Dombi and BM operators.

Hence, this paper has four major research implications: (1) The first objective is to undergo a comprehensive literature review on waste management methodologies in healthcare; (2) The second objective is to select and evaluate the optimal location for a waste disposal facility in an uncertain environment using a hybrid methodology; (3) Third is to propose a new rough based framework of BWM and D'Bonferroni aggregators model for processing imprecise (rough) information in MCDM problems and (4) The fourth goal of this paper is to show the real application in locating a waste disposal center.

3. Research methods, preliminaries and arithmetic operations

In this section, the fundamental elements of the IRN, Dombi TNs/TCNs and BM operators are briefly represented in the field.

3.1. IRNs and operations on IRNs

In group decision-making problems, the priorities are defined on the aggregated decisions of multi-experts and the process of the subjective evaluation of the decisions of experts. RNs consisting of upper, lower, and boundary intervals, respectively, determine intervals of their evaluations without requiring additional information by relying only on original data (Zhai, Khoo, & Zhong, 2008). Hence, obtained expert decision-makers' perceptions objectively present and improve their decision making process. According to Zhai, Khoo, and Zhong (2009), the definition of a rough number is shown below.

Suppose U is the universe that contains all the objects, Y is an arbitrary object of U, R is a set of t classes $(G_1; G_2; \dots; G_t)$, which include all the objects in U, R $(G_1; G_2; \dots; G_t)$. If these classes are ordered as $G_1 < G_2 < \dots < G_t$, then $\forall\, Y \in U,\ G_q \in R,\ 1 \le q \le t$, $1 \le q \le t\ \forall\, Y \in U,\ G_q \in R$, the lower approximation $\left(\underline{Apr}(G_q)\right)$, upper approximation $(\overline{Apr}(G_q))$ and boundary region $(Bnd(G_q))$ of class G_q are, according to Zhai et al. (2009), defined as:

$$\underline{Apr}(G_q) = \cup_{1 \leqslant q \leqslant t}\{Y \in U/R(Y) \leqslant G_q\}$$
$$\bar{Apr}(G_q) = \cup_{1 \leqslant q \leqslant t}\{Y \in U/R(Y) \geqslant G_q\} \tag{1}$$

$$\begin{aligned} Bnd(G_q) &= \bigcup_{1 \leqslant q \leqslant t}\{Y \in U/R(Y) \neq G_q\} = \{Y \in U/R(Y) \geqslant G_q\} \\ &= \bigcup_{1 \leqslant q \leqslant t}\{Y \in U/R(Y) \leqslant G_q\} \end{aligned} \tag{2}$$

Then G_q can be shown as a rough number $(RN(G_q))$, which is determined by its corresponding lower limit $\left(\underline{Lim}(G_q)\right)\left(\underline{\mathrm{Lim}}(\mathrm{G_q})\right)$ and upper limit $(\overline{Lim}(G_q))(\overline{\mathrm{Lim}}(\mathrm{G_q}))$, where:

$$\underline{Lim}\,(G_q) = \frac{1}{M_L}\sum_{q=1}^{t} R(Y)|Y \in \underline{Apr}(G_q)$$

$$\bar{Lim}\,(G_q) = \frac{1}{M_U}\sum_{q=1}^{t} R(Y)|Y \in \bar{Apr}(G_q) \tag{3}$$

$$RN(G_q) = [\underline{Lim}\,(G_q),\ \bar{Lim}\,(G_q)] \tag{4}$$

where M_L, and M_U are the numbers of objects that contained in $\underline{Apr}(G_q)$ and $\overline{Apr}(G_q)$, respectively. The difference between them is expressed as a rough boundary interval ($IRBnd(G_q)$):

$$IRBnd(G_q) = \bar{Lim}\,(G_q) - \underline{Lim}\,(G_q) \tag{5}$$

where M_L and M_U represent the sum of objects contained in the lower and upper object approximation of J_q, respectively.

Since RNs belong to the group of interval numbers, arithmetic operations applied to interval numbers are also appropriate for RNs. If A and B presents two RNs $RN(A) = [\underline{a}, \bar{a}]$ and $RN(B) = [\underline{b}, \bar{b}]$, k denotes constant, $k > 0$, then the arithmetic operations with $RN(A)$, $RN(B)$ and k are as follows:

- Addition of RNs"+"

$$RN(A) + RN(B) = [\underline{a}, \bar{a}] + [\underline{b}, \bar{b}] = [\underline{a} + \underline{b}, \bar{a} + \bar{b}] \tag{6}$$

- Subtraction of RNs"-"

$$RN(A) - RN(B) = [\underline{a}, \bar{a}] - [\underline{b}, \bar{b}] = [\underline{a} - \bar{b}, \bar{a} - \underline{b}] \tag{7}$$

- Multiplication of RNs"×"

$$RN(A) \times RN(B) = [\underline{a}, \bar{a}] \times [\underline{b}, \bar{b}] = [\underline{a} \times \underline{b}, \bar{a} \times \bar{b}] \tag{8}$$

- Dividing of RNs"/"

$$RN(A)/RN(B) = [\underline{a}, \bar{a}]/[\underline{b}, \bar{b}] = [\underline{a}/\bar{b}, \bar{a}/\underline{b}] \tag{9}$$

- Scalar multiplication of RNs, where$k > 0$

$$k \times RN(A) = k \times [\underline{a}, \bar{a}] = [k \times \underline{a}, k \times \bar{a}] \tag{10}$$

3.2. Dombi operations of GN

Definition 1.. Let p and q be any two real numbers. Then, the Dombi *TN* and *TCN* between p and q are defined as follows (Dombi, 1982):

$$O_D(p, q) = \frac{1}{1 + \left\{\left(\frac{1-p}{p}\right)^{\rho} + \left(\frac{1-q}{q}\right)^{\rho}\right\}^{1/\rho}} \tag{11}$$

$$O_D^c(p, q) = 1 - \frac{1}{1 + \left\{\left(\frac{p}{1-p}\right)^{\rho} + \left(\frac{q}{1-q}\right)^{\rho}\right\}^{1/\rho}} \tag{12}$$

where $\rho > 0$ and$(p, q) \in [0, 1]$. According to the Dombi *TN* and *TCN*, we define the Dombi operations of IRNs.

Definition 2.. Suppose $RN(\xi_1) = [\underline{\xi}_1, \bar{\xi}_1]$ and $RN(\xi_2) = [\underline{\xi}_2, \bar{\xi}_2]$ are two IRN, $\rho, \gamma > 0$and let $f(RN(\xi_i)) = [f(\underline{\xi}_i), f(\bar{\xi}_i)] = \left[\frac{\underline{\xi}_i}{\sum_{i=1}^{n}\underline{\xi}}, \frac{\bar{\xi}_i}{\sum_{i=1}^{n}\bar{\xi}_i}\right]$ be a grey function, then some operational lows of RNs-based on the Dombi TN and TCN can be defined as follows:

(1) Addition "+"

$$RN(\xi_1) + RN(\otimes\xi_2)$$

$$= \left[\sum_{j=1}^{2}\underline{\xi}_j - \frac{\sum_{j=1}^{2}\underline{\xi}_j}{1 + \left\{\left(\frac{f(\underline{\xi}_1)}{1-f(\underline{\xi}_1)}\right)^{\rho} + \left(\frac{f(\underline{\xi}_2)}{1-f(\underline{\xi}_2)}\right)^{\rho}\right\}^{1/\rho}},\ \sum_{j=1}^{2}\bar{\xi}_j - \frac{\sum_{j=1}^{2}\bar{\xi}_j}{1 + \left\{\left(\frac{f(\bar{\xi}_1)}{1-f(\bar{\xi}_1)}\right)^{\rho} + \left(\frac{f(\bar{\xi}_2)}{1-f(\bar{\xi}_2)}\right)^{\rho}\right\}^{1/\rho}}\right] \tag{13}$$

(2) Multiplication "×"

$$RN(\xi_1) \times RN(\xi_2)$$

$$= \left[\frac{\sum_{j=1}^{2}\underline{\xi}_j}{1 + \left\{\left(\frac{1-f(\underline{\xi}_1)}{f(\underline{\xi}_1)}\right)^{\rho} + \left(\frac{1-f(\underline{\xi}_2)}{f(\underline{\xi}_2)}\right)^{\rho}\right\}^{1/\rho}}, \frac{\sum_{j=1}^{2}\bar{\xi}_j}{1 + \left\{\left(\frac{1-f(\bar{\xi}_1)}{f(\bar{\xi}_1)}\right)^{\rho} + \left(\frac{1-f(\bar{\xi}_2)}{f(\bar{\xi}_2)}\right)^{\rho}\right\}^{1/\rho}}\right] \tag{14}$$

(3) Scalar multiplication, where$\gamma > 0$

$$\gamma RN(\xi_1) = \left[\underline{\xi}_1 - \frac{\underline{\xi}_1}{1 + \left\{\gamma\left(\frac{\underline{\xi}_1}{1-\underline{\xi}_1}\right)^{\rho}\right\}^{1/\rho}},\ \bar{\xi}_1 - \frac{\bar{\xi}_1}{1 + \left\{\gamma\left(\frac{\bar{\xi}_1}{1-\bar{\xi}_1}\right)^{\rho}\right\}^{1/\rho}}\right] \tag{15}$$

(4) Power, where$\gamma > 0$

$$\{RN(\xi_1)\}^{\gamma} = \left[\frac{\underline{\xi}_1}{1 + \left\{\gamma\left(\frac{1-\underline{\xi}_1}{\underline{\xi}_1}\right)^{\rho}\right\}^{1/\rho}}, \frac{\bar{\xi}_1}{1 + \left\{\gamma\left(\frac{1-\bar{\xi}_1}{\bar{\xi}_1}\right)^{\rho}\right\}^{1/\rho}}\right] \tag{16}$$

Based on the IRN operators (1)–(5), we propose the Dombi Geometric Bonferroni Mean (DGBM) and Dombi Normalized Geometric Bonferroni Mean (DNGBM) operators, respectively.

3.3. IRN Dombi geometric Bonferroni mean operator

Definition 3 ((*Zhu, Xu, & Xia, 2012*)**:).** Let p,$q \geq 0$ and (a_1,a_2,…,a_n) be a set of non-negative numbers. If

$$GB^{p,q}(a_1, a_2, \ldots, a_n) = \left(\frac{1}{n}\sum_{i,j=1}^{n} a_i^{\,p}\left(\prod_{j=1}^{n} a_j^{q}\right)^{\frac{1}{n-1}}\right)^{\frac{1}{p+q}} \tag{17}$$

Then $GB^{p,q}$is called a geometric Bonferroni mean operator.

Definition 4 ((*Sun & Liu, 2013*)**:).** Let (a_1,a_2,…,a_n) be a set of non-negative numbers and p, $q \geq 0$. If

$$NGB^{p,q}(a_1, a_2, \ldots, a_n) = \frac{1}{p+q}\prod_{i,j=1}^{n}(pa_i + qa_j)^{\frac{w_i w_j}{1-w_i}} \tag{18}$$

Then $NGB^{p,q}$ is called a normalized weighted geometric Bonferroni mean operator.

Theorem 1.. *Assuming that* $RN(\xi_j) = \left[\underline{\xi}_j, \bar{\xi}_j\right]$; *is a collection of IRNs in R, then the Dombi Geometric Bonferroni Mean operator is defined as follows*:

$$DGBM^{p,q,\rho}\{RN(\xi_1), RN(\xi_2), ..., RN(\xi_n)\} = \left(\frac{1}{n}\sum_{i,j=1}^{n} RN(\xi_i^p)\left(\prod_{j=1}^{n} RN\left(\xi_j^q\right)\right)^{\frac{1}{n-1}}\right)^{\frac{1}{p+q}}$$

$$= \left[\frac{\sum_{i=1}^{n}\underline{\xi}_i}{1+\left\{\frac{1}{p+q}\frac{n}{\frac{1}{\sum_{i=1, i\neq j}^{n}\left(p\left(\frac{1-f(\underline{\xi}_i)}{f(\underline{\xi}_i)}\right)^{\rho}+\frac{q}{n-1}\sum_{j=1, j\neq i}^{n}\left(\frac{1-f(\underline{\xi}_j)}{f(\underline{\xi}_j)}\right)^{\rho}\right)}}\right\}^{1/\rho}}, \frac{\sum_{i=1}^{n}\bar{\xi}_i}{1+\left\{\frac{1}{p+q}\frac{n}{\frac{1}{\sum_{i=1, i\neq j}^{n}\left(p\left(\frac{1-f(\bar{\xi}_i)}{f(\bar{\xi}_i)}\right)^{\rho}+\frac{q}{n-1}\sum_{j=1, j\neq i}^{n}\left(\frac{1-f(\bar{\xi}_j)}{f(\bar{\xi}_j)}\right)^{\rho}\right)}}\right\}^{1/\rho}}\right] \quad (19)$$

where $f(RN(\xi_i)) = \begin{cases} f(\underline{\xi}_i) = \frac{\underline{\xi}_i}{\sum_{i=1}^{n}\underline{\xi}_i}; \\ f(\bar{\xi}_i) = \frac{\bar{\xi}_i}{\sum_{i=1}^{n}\bar{\xi}_i}. \end{cases}$ *represents a grey function.*

See Appendix A for the proof of Theorem 1.

Example 1.. Let $RN(\xi_1) = [2, 3]$, $RN(\xi_2) = [3, 4]$, $RN(\xi_3) = [3, 4]$ and $RN(\xi_4) = [2, 3]$ be four IRNs and $p = q = \rho = 1$, then we can show the following calculations:

(1) $f(\underline{\xi}_1) = 2/10 = 0.2$; $f(\underline{\xi}_2) = 3/10 = 0.3$; $f(\underline{\xi}_3) = 3/10 = 0.3$; $f(\underline{\xi}_4) = 2/10 = 0.2$; $f(\bar{\xi}_1) = 3/14 = 0.214$; $f(\bar{\xi}_2) = 4/14 = 0.286$; $f(\bar{\xi}_3) = 4/14 = 0.286$ and $f(\bar{\xi}_4) = 3/14 = 0.214$.

(2) $\frac{1-f(\underline{\xi}_1)}{f(\underline{\xi}_1)} = 4.0$; $\frac{1-f(\underline{\xi}_2)}{f(\underline{\xi}_2)} = 2.33$; $\frac{1-f(\underline{\xi}_3)}{f(\underline{\xi}_3)} = 2.33$; $\frac{1-f(\underline{\xi}_4)}{f(\underline{\xi}_4)} = 4.0$; $\frac{1-f(\bar{\xi}_1)}{f(\bar{\xi}_1)} = 3.67$; $\frac{1-f(\bar{\xi}_2)}{f(\bar{\xi}_2)} = 2.5$; $\frac{1-f(\bar{\xi}_3)}{f(\bar{\xi}_3)} = 2.5$ and $\frac{1-f(\bar{\xi}_4)}{f(\bar{\xi}_4)} = 3.67$.

(3)

$DGBM^{1,1,1}\{[2, 3]; [3, 4]; [3, 4]; [2, 3]\}=$

$$= \left[\frac{2+3+3+4}{1+\left\{\frac{1}{1+1}\frac{4}{\left(\frac{1}{4}+\frac{1}{4-1}\frac{1}{2.33+2.33+4}\right)^{1}+\left(\frac{1}{2.33}+\frac{1}{4-1}\frac{1}{4+2.33+4}\right)^{1}+\left(\frac{1}{2.33}+\frac{1}{4-1}\frac{1}{4+2.33+4}\right)^{1}+\left(\frac{1}{4}+\frac{1}{4-1}\frac{1}{4+2.33+2.33}\right)^{1}}\right\}^{1/1}}, \frac{3+4+4+3}{1+\left\{\frac{1}{1+1}\frac{4(4-1)}{\left(\frac{1}{3.67}+\frac{1}{4-1}\frac{1}{2.5+2.5+3.67}\right)^{1}+\left(\frac{1}{2.5}+\frac{1}{4-1}\frac{1}{3.67+2.5+3.67}\right)^{1}+\left(\frac{1}{2.5}+\frac{1}{4-1}\frac{1}{3.67+2.5+3.67}\right)^{1}+\left(\frac{1}{3.67}+\frac{1}{4-1}\frac{1}{3.67+2.5+2.5}\right)^{1}}\right\}^{1/1}}\right]$$

$= [2.40, 3.43]$

In the following, we shall explore some desirable properties of the DGBM operator.

Theorem 2 ((*Idempotency):*). *Assuming that* $RN(\xi_i) = [\underline{\xi}_i, \bar{\xi}_i]$; $(j = 1, 2, ..., n)$ *is a collection of IRNs in R, if* $RN(\xi_i) = RN(\xi)$, *then:*

$$DGBM^{p,q,\rho}\{RN(\xi_1), RN(\xi_2), ..., RN(\xi_n)\} = DGBM^{p,q,\rho}\{RN(\xi), RN(\xi), ..., RN(\xi)\}.$$

See Appendix B for the proof of Theorem 2.

Theorem 3 ((*Boundedness):*). *Assuming that* $RN(\xi_i) = [\underline{\xi}_i, \bar{\xi}_i]$; $(j = 1, 2, ..., n)$ *is a collection of IRNs in R, and letting* $RN(\xi^-) = [\min \underline{\xi}_i, \min \bar{\xi}_i]$ *and* $RN(\xi^+) = [\max \underline{\xi}_i, \max \bar{\xi}_i]$, *then:*

$RN(\xi^-) \leqslant DGBM^{p,q,\rho}(RN(\xi_1), RN(\xi_2), ..., RN(\xi_n)) \leqslant RN(\xi^+)$.

See Appendix B for the proof of Theorem 3.According to the inequalities shown above, it can be concluded that $RN(\xi^-) \leqslant DGBM^{p,q,\rho}(RN(\xi_1), RN(\xi_2), ..., RN(\xi_n)) \leqslant RN(\xi^+)$ *holds.*

Theorem 4 ((*Commutativity):*). *Let the grey set* $(RN(\xi_1'), RN(\xi_2'), ..., RN(\xi_n'))$ *be any permutation of*$(RN(\xi_1), RN(\xi_2), ..., RN(\xi_n))$ *then:*

$$DGBM^{p,q,\rho}(RN(\xi_1), RN(\xi_2), ..., RN(\xi_n)) = DGBM^{p,q,\rho}(RN(\xi_1'), RN(\xi_2'), ..., RN(\xi_n')).$$

Proof:. This property is obvious.□

3.4. IRN Dombi normalized weighted geometric Bonferroni-mean operator

Based on the IRN operators (1)–(5), we propose the IRN Normalized Weighted Dombi-Bonferroni Mean (DNGBM) operator.

Theorem 5.. *Assuming that* $RN(\xi_j) = \left[\underline{\xi}_j, \bar{\xi}_j\right]$; $(j = 1, 2, ..., n)$ *is a collection of GNs in R, then the DNGBM operator is defined as follows:*

$$DNGBM^{p,q,\rho}(RN(\xi_1), RN(\xi_2), ..., RN(\xi_n)) = \frac{1}{p+q}\prod_{i,j=1}^{n}(pRN(\xi_i) + qRN(\xi_j))^{\frac{w_i w_j}{1-w_i}}$$

$$= \left[\sum_{i=1}^{n} \underline{\xi}_i - \frac{\sum_{i=1}^{n} \underline{\xi}_i}{\left\{1 + \left\{\frac{1}{(p+q)w_i w_j}\frac{1-w_i}{\sum_{\substack{i,j=1\\ i\neq j}}^{n}\left(\frac{1}{p\left(\frac{f(\underline{\xi}_i)}{1-f(\underline{\xi}_i)}\right)^{\rho} + q\left(\frac{f(\underline{\xi}_j)}{1-f(\underline{\xi}_j)}\right)^{\rho}}\right)}\right\}\right\}^{1/\rho}}, \sum_{i=1}^{n} \bar{\xi}_i - \frac{\sum_{i=1}^{n} \bar{\xi}_i}{\left\{1 + \left\{\frac{1}{(p+q)w_i w_j}\frac{1-w_i}{\sum_{\substack{i,j=1\\ i\neq j}}^{n}\left(\frac{1}{p\left(\frac{f(\bar{\xi}_i)}{1-f(\bar{\xi}_i)}\right)^{\rho} + q\left(\frac{f(\bar{\xi}_j)}{1-f(\bar{\xi}_j)}\right)^{\rho}}\right)}\right\}\right\}^{1/\rho}} \right] \tag{20}$$

where $w_j \in [0, 1]$ represents the weight coefficient of $RN(\xi_i)$, $i = 1, 2, ..., n$, $\sum_{j=1}^{n} w_j = 1$ and $f(RN(\xi_i)) = \begin{cases} f(\underline{\xi}_i) = \frac{\underline{\xi}_i}{\sum_{i=1}^{n}\underline{\xi}_i}; \\ f(\bar{\xi}_i) = \frac{\bar{\xi}_i}{\sum_{i=1}^{n}\bar{\xi}_i}. \end{cases}$ represents a grey function.

See Appendix C for the proof of Theorem 5.

Example 2.. Let $RN(\xi_1) = [2, 3]$, $RN(\xi_2) = [3, 4]$, $RN(\xi_3) = [3, 4]$ and $RN(\xi_4) = [2, 3]$ be four IRNs, $p = q = \rho = 1$ and $w_j = (0.18, 0.32, 0.33, 0.17)$, then we show the following calculations:

(1) $f(\underline{\xi}_1) = 2/10 = 0.2$; $f(\underline{\xi}_2) = 3/10 = 0.3$; $f(\underline{\xi}_3) = 3/10 = 0.3$; $f(\underline{\xi}_4) = 2/10 = 0.2$; $f(\bar{\xi}_1) = 3/14 = 0.214$; $f(\bar{\xi}_2) = 4/14 = 0.286$; $f(\bar{\xi}_3) = 4/14 = 0.286$ and $f(\bar{\xi}_4) = 3/14 = 0.214$.

(2) $\frac{f(\underline{\xi}_1)}{1-f(\underline{\xi}_1)} = 0.25$; $\frac{f(\underline{\xi}_2)}{1-f(\underline{\xi}_2)} = 0.43$; $\frac{f(\underline{\xi}_3)}{1-f(\underline{\xi}_3)} = 0.43$; $\frac{f(\underline{\xi}_4)}{1-f(\underline{\xi}_4)} = 0.25$; $\frac{f(\bar{\xi}_1)}{1-f(\bar{\xi}_1)} = 0.27$; $\frac{f(\bar{\xi}_2)}{1-f(\bar{\xi}_2)} = 0.4$; $\frac{f(\bar{\xi}_3)}{1-f(\bar{\xi}_3)} = 0.4$ and $\frac{f(\bar{\xi}_4)}{1-f(\bar{\xi}_4)} = 0.27$.

$DNGBM_w^{p,q,\rho}\{[2, 3]; [3, 4]; [3, 4]; [2, 3]\}=$

(3) $$= \left[\begin{array}{l} 10 - \frac{2+3+3+4}{\left\{1 + \left\{\frac{1}{1+1}\frac{1}{\frac{0.18\cdot 0.32}{1-0.18}\frac{1}{0.25^1+0.43^1} + \frac{0.18\cdot 0.33}{1-0.18}\frac{1}{0.25^1+0.43^1} + \frac{0.18\cdot 0.17}{1-0.18}\frac{1}{0.25^1+0.25^1} + \frac{0.32\cdot 0.18}{1-0.32}\frac{1}{0.43^1+0.25^1} + \frac{0.32\cdot 0.33}{1-0.32}\frac{1}{0.43^1+0.43^1} + \ldots + \frac{0.17\cdot 0.33}{1-0.17}\frac{1}{0.25^1+0.43^1}}\right\}\right\}^{1/1}}, \\ 14 - \frac{3+4+4+3}{\left\{1 + \left\{\frac{1}{1+1}\frac{1}{\frac{0.18\cdot 0.32}{1-0.18}\frac{1}{0.27^1+0.40^1} + \frac{0.18\cdot 0.33}{1-0.18}\frac{1}{0.27^1+0.40^1} + \frac{0.18\cdot 0.17}{1-0.18}\frac{1}{0.27^1+0.27^1} + \frac{0.32\cdot 0.18}{1-0.32}\frac{1}{0.40^1+0.27^1} + \frac{0.32\cdot 0.33}{1-0.32}\frac{1}{0.40^1+0.40^1} + \ldots + \frac{0.17\cdot 0.33}{1-0.17}\frac{1}{0.27^1+0.40^1}}\right\}\right\}^{1/1}} \end{array} \right]$$

$= [2.61, 3.61]$

In the following, we shall explore some desirable properties of the DNGBM operator.

Theorem 6.. *Let it be $w_j = (1/n, 1/n, ..., 1/n)$, $(j = 1, 2, ..., n)$. Then:*

$$DNGBM_w^{p,q,\rho}(RN(\xi_1), RN(\xi_2), ..., RN(\xi_n)) = DBM^{p,q,\rho}(RN(\xi_1), RN(\xi_2), ..., RN(\xi_n)),$$

where $DBM^{p,q,\rho}$ represents the IRN Dombi-Bonferroni mean operator.

See Appendix D for the proof of Theorem 6.

In the following, we shall explore some desirable properties of IRNGDBM operator. The IRNDNWGBM operator also contains the following properties:

(1) ***Idempotency***: Assuming that $RN(\xi_i) = [\underline{\xi}_i, \bar{\xi}_i]$; $(j = 1, 2, ..., n)$ is a collection of IRNs in R, if $RN(\xi_i) = RN(\xi)$, then:

$$DNGBM^{p,q,\rho}(RN(\xi_1), RN(\xi_2), .., RN(\xi_n)) = DNGBM^{p,q,\rho}(RN(\xi), RN(\xi), .., RN(\xi))$$

(2) ***Boundedness***: Assuming that $RN(\xi_i) = [\underline{\xi}_i, \bar{\xi}_i]$; $(j = 1, 2, ..., n)$ is a collection of IRNs in R, and letting $RN(\xi^-) = [\min \underline{\xi}_i, \min \bar{\xi}_i]$ and $RN(\xi^+) = [\max \underline{\xi}_i, \max \bar{\xi}_i]$, then:

$$RN(\xi^-) \leqslant DNGBM^{p,q,\rho}(RN(\xi_1), RN(\xi_2), ..., RN(\xi_n)) \leqslant RN(\xi^+)$$

(3) ***Commutativity***: Let the grey set $(RN(\xi_1'), RN(\xi_2'), ..., RN(\xi_n'))$ be any permutation of $(RN(\xi_1), RN(\xi_2), .., RN(\xi_n))$.

The proof of these properties is the same as that for the DNGBM operator, and because of that, it is omitted here. In the following, some special cases of the GDBM operator will be discussed.

(1) **If $q = 0$, then:**

(a) **Eq. (19)** reduces to the IRN Dombi generalized average operator:

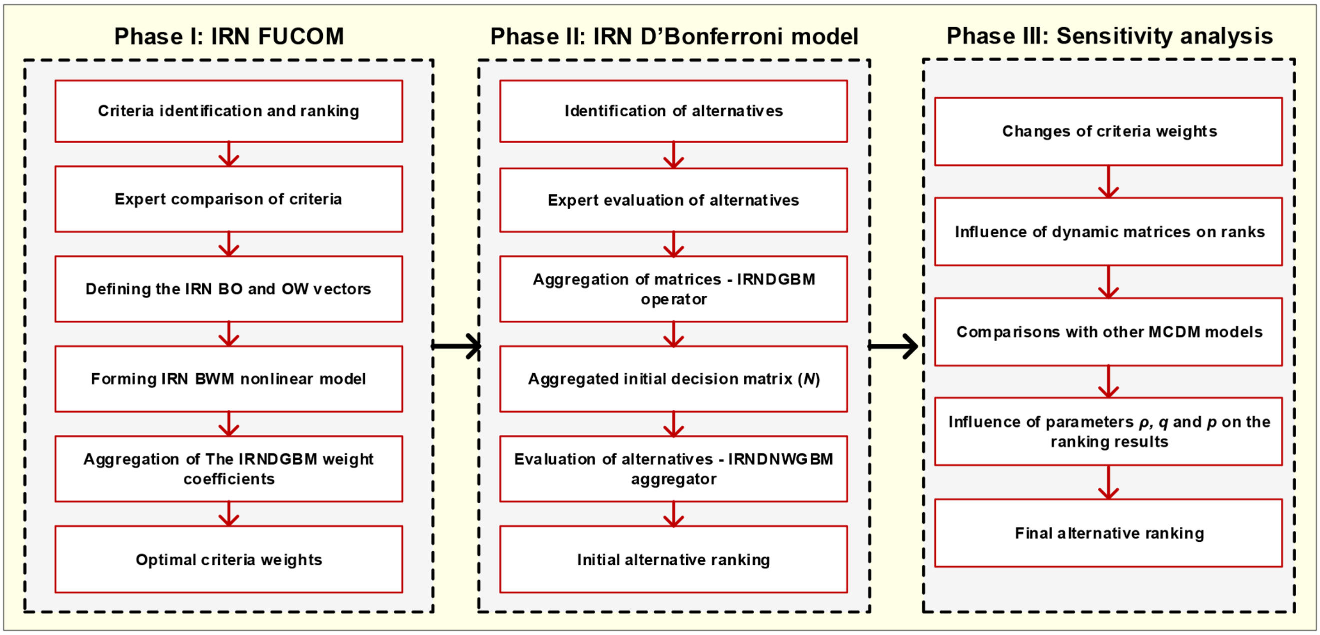

Fig. 1. IRN BWM D'Bonferroni multi-criteria model.

$$D^{p,0,\rho}(RN(\xi_1), RN(\xi_2), ..., RN(\xi_n)) = \left(\frac{1}{n}\sum_{i=1}^{n} RN(\xi_i^{\,p})\right)^{\frac{1}{p}} =$$

$$= \left[\frac{\sum_{i=1}^{n}\underline{\xi}_i}{1+\left\{\frac{1}{p}\frac{1}{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{p\left(\frac{1-f(\underline{\xi}_i)}{f(\underline{\xi}_i)}\right)^{\rho}}}\right\}^{1/\rho}}, \frac{\sum_{i=1}^{n}\bar{\xi}_i}{1+\left\{\frac{1}{p}\frac{1}{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{p\left(\frac{1-f(\bar{\xi}_i)}{f(\bar{\xi}_i)}\right)^{\rho}}}\right\}^{1/\rho}}\right]$$

See Appendix E for the proof of IRN Dombi generalized average operator.

(b) **Eq.** (20) reduces to the IRN Dombi generalized weighted geometric operator:

$$DG^{1,0,\rho}(RN(\xi_1), RN(\xi_2), ..., RN(\xi_n)) = \frac{1}{p}\prod_{i=1}^{n}(RN(\xi_i))^{w_i}$$

$$= \left[\sum_{i=1}^{n}\underline{\xi}_i - \frac{\sum_{i=1}^{n}\underline{\xi}_i}{1+\left\{\frac{1}{p}\frac{1}{w_i\sum_{i=1}^{n}\left(\frac{1-f(\underline{\xi}_i)}{f(\underline{\xi}_i)}\right)^{\rho}}\right\}^{1/\rho}}, \sum_{i=1}^{n}\bar{\xi}_i - \frac{\sum_{i=1}^{n}\bar{\xi}_i}{1+\left\{\frac{1}{p}\frac{1}{w_i\sum_{i=1}^{n}\left(\frac{1-f(\bar{\xi}_i)}{f(\bar{\xi}_i)}\right)^{\rho}}\right\}^{1/\rho}}\right]$$

See Appendix E for the proof of IRN Dombi generalized weighted geometric operator.

(2) **If $p = 1$ and $q = 0$, then:**
(a) **Eq.** (19) reduces to the IRN Dombi arithmetic average operator:

$$DA^{1,0,\rho}(RN(\xi_1), RN(\xi_2), ..., RN(\xi_n)) = \frac{1}{n}\sum_{i=1}^{n} RN(\xi_i)$$

$$= \left[\sum_{i=1}^{n}\underline{\xi}_i - \frac{\sum_{i=1}^{n}\underline{\xi}_i}{1+\left\{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{f(\underline{\xi}_i)}{1-f(\underline{\xi}_i)}\right)^{\rho}\right\}^{1/\rho}}, \sum_{i=1}^{n}\bar{\xi}_i - \frac{\sum_{i=1}^{n}\bar{\xi}_i}{1+\left\{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{f(\bar{\xi}_i)}{1-f(\bar{\xi}_i)}\right)^{\rho}\right\}^{1/\rho}}\right]$$

(b) **Eq.** (20) reduces to the IRN Dombi weighted geometric operator:

$$GW^{1,0,\rho}(RN(\xi_1), RN(\xi_2), ..., RN(\xi_n)) = \prod_{i=1}^{n}(RN(\xi_i))^{w_i}$$

$$= \left[\frac{\sum_{i=1}^{n}\underline{\xi}_i}{1+\left\{w_i\sum_{i=1}^{n}\left(\frac{1-f(\underline{\xi}_i)}{f(\underline{\xi}_i)}\right)^{\rho}\right\}^{1/\rho}}, \frac{\sum_{i=1}^{n}\bar{\xi}_i}{1+\left\{w_i\sum_{i=1}^{n}\left(\frac{1-f(\bar{\xi}_i)}{f(\bar{\xi}_i)}\right)^{\rho}\right\}^{1/\rho}}\right]$$

(3) **If $p \to 0$ and $q = 0$, then**
(a) **Eq.** (19) reduces to the IRN Dombi weighted geometric average operator:

$$\lim_{p\to 0} DGW_w^{p,0,\rho}(RN(\xi_1), RN(\xi_2), ..., RN(\xi_n)) = \left(\prod_{i=1}^{n} RN(\xi_i)\right)^{1/n}$$

$$= \left[\frac{\sum_{i=1}^{n}\underline{\xi}_i}{1+\left\{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{1-f(\underline{\xi}_i)}{f(\underline{\xi}_i)}\right)^{\rho}\right\}^{1/\rho}}, \frac{\sum_{i=1}^{n}\bar{\xi}_i}{1+\left\{\frac{1}{n}\sum_{i=1}^{n}\left(\frac{1-f(\bar{\xi}_i)}{f(\bar{\xi}_i)}\right)^{\rho}\right\}^{1/\rho}}\right]$$

(b) **Eq.** (20) reduces to the IRN Dombi weighted average operator:

$$\lim_{p\to 0} DWA_w^{p,0,\rho}(RN(\xi_1), RN(\xi_2), ..., RN(\xi_n)) = \sum_{i=1}^{n} w_i RN(\xi_i)$$

$$= \left[\sum_{i=1}^{n}\underline{\xi}_i - \frac{\sum_{i=1}^{n}\underline{\xi}_i}{1+\left\{w_i\sum_{i=1}^{n}\left(\frac{f(\underline{\xi}_i)}{1-f(\underline{\xi}_i)}\right)^{\rho}\right\}^{1/\rho}}, \sum_{i=1}^{n}\bar{\xi}_i - \frac{\sum_{i=1}^{n}\bar{\xi}_i}{1+\left\{w_i\sum_{i=1}^{n}\left(\frac{f(\bar{\xi}_i)}{1-f(\bar{\xi}_i)}\right)^{\rho}\right\}^{1/\rho}}\right]$$

4. MCDM model

The rough BWM-D'Bonferroni multi-criteria model is implemented in two phases, as shown in Fig. 1. In the first phase, weight coefficients are calculated using IRN BWM. The weight coefficients obtained in the first phase of the model are further used in the D'Bonferroni model for the evaluation of alternatives. A detailed presentation of the steps of the IRNBWM-D'Bonferroni model is shown in the next section.

4.1. Phase I: IRN best-worst method

A modification of the BWM was carried out using IRN to more comprehensively take into account the imprecision that appears in the group decision-making process. By using RNs, the need for additional information to determine the uncertainty of the intervals of the numbers is eliminated. The approach presented in this chapter introduces a rough number, which secures a more objective evaluation of the criteria in cases where there is imprecision in the expert decisions. The proposed modification of BWM using a rough number (RBWM) makes it possible to consider doubts that arise during the expert evaluation of the criteria. The next section presents the algorithm for the RBWM that includes the following steps:

Step 1. Determining the set of evaluation criteria. This starts with the assumption that the process of decision making involves t experts. In this step, experts consider the set of evaluation criteria and select the final set of criteria, $C = \{c_1, c_2, ...c_n\}$, where n represents the total number of criteria.

Step 2. Determining the most significant (most influential) and worst (least significant) criteria. The experts decide on the best and the worst criteria from the set of criteria, $C = \{c_1, c_2, ...c_n\}$, (Rezaei, 2015).

Step 3. Determining the preferences of the most significant criteria (B) from the set Cover the remaining criteria from the defined set. Under the assumption that there are m experts and n criteria under consideration, each expert should determine the degree of influence of the best criterion B on the criteria j $(j = 1, 2, ...,n)$. This is how we obtain a comparison between the best criterion and the other criteria. The preference of criterion B compared to the j-th criterion defined by the e^{th} expert is denoted bya_{Bj}^e $(j = 1, 2, ...,n;\ 1 \leqslant e \leqslant t)$. The value of each pair a_{Bj}^etakes value from the predefined scale in the interval $a_{Bj}^e \in [1, 9]$. As a result, a Best-to-Others (BO) vector is obtained:

$$A_B^e = (a_{B1}^e,\ a_{B2}^e\ , ..., a_{Bn}^e);\ \ 1 \leqslant e \leqslant t \tag{21}$$

where a_{Bj}^e represents the influence (preference) of the best criterion B over criterion j, whereby $a_{BB}^e = 1$.

Step 4. Determining the preferences of the criteria from the set Cover the worst criterion (W) from the defined set. Each expert should determine the degree of influence of criterion j $(j = 1, 2, ...,n)$ concerning criterion W. The preference of criterion j in relation to criterion W defined by the e-th expert is denoted bya_{jW}^e $(j = 1, 2, ...,n; 1 \leqslant e \leqslant t)$. The value of each pair a_{jW}^e takes value from the predefined scale in the interval $a_{jW}^e \in [1, 9]$. As a result, an others-to-worst (OW) vector is obtained:

$$A_W^e = (a_{1W}^e,\ a_{2W}^e\ , ..., a_{nW}^e);\ \ 1 \leqslant e \leqslant t \tag{22}$$

where a_{jW}^e represents the influence (preference) of criterion j in relation to criterion W, whereby $a_{WW}^e = 1$.

Step 5. Determining the rough BO and OW matrices for the average answers of the experts. Based on the BO and OW matrices of the experts' answers$A_B^e = [a_{Bj}^e]_{1\times n}$and $A_W^e = [a_{jW}^e]_{1\times n}$, we form matrices of the aggregated sequences of expertsA_B^{*e} and A_W^{*e}

$$\begin{aligned} A_B^{*e} &= [a_{B1}^m, a_{B1}^2, ..., a_{B1}^k;\quad a_{B2}^1; a_{B2}^2; ...; a_{B2}^m,\ ...,\ a_{Bn}^1; a_{Bn}^2, ..., a_{Bn}^m]_{1\times n} \\ A_W^{*e} &= [a_{1W}^1, a_{1W}^2, ..., a_{1W}^m;\quad a_{2W}^1; a_{2W}^2; ...; a_{2W}^m,\ ...,\ a_{nW}^1; a_{nW}^2, ..., a_{nW}^m]_{1\times n} \end{aligned} \tag{23}$$

where $a_{Bj}^e = \{a_{Bj}^1, a_{Bj}^2, ..., a_{Bn}^m\}$ and $a_{jW}^e = \{a_{jW}^1, a_{jW}^2, ..., a_{nW}^m\}$represents sequences through which the relative significance of criteria B and W are described in relation to criterion j. Using Eqs. (1)–(5) each sequence a_{Bj}^e and $a_{jW}^e$is transformed into the rough sequences $RN(a_{Bj}^e) = [\underline{a}_{Bj}^e,\ \bar{a}_{Bj}^e]$ and $RN(a_{jW}^e) = [\underline{a}_{jW}^e,\ \bar{a}_{jW}^e]$, where $\underline{a}_{Bj}^e$ and $\underline{a}_{jW}^e$represents the lower limit and $\bar{a}_{Bj}^e$ and$\bar{a}_{jW}^e$represents the upper limit of the rough sequences $RN(a_{Bj}^e)$ and $RN(a_{jW}^e)$. By applying IRNDGBM, Eq. (19), we obtain the average rough sequence of the BO and OW matrices. We thus obtain the averaged rough BO and OW matrices of average responses $\bar{A}_B$ and $\bar{A}_W$

$$\begin{aligned} \bar{A}_B &= [\bar{a}_{B1}, \bar{a}_{B2}, ..., \bar{a}_{Bn}]_{1\times n} \\ \bar{A}_W &= [\bar{a}_{1W}, \bar{a}_{2W}, ..., \bar{a}_{nW}]_{1\times n} \end{aligned} \tag{24}$$

Step 6. Calculation of the optimal rough values of the weight coefficients of the criteria $[RN(w_1), RN(w_2), ..., RN(w_n)]$from the set C. For all values of the interval rough weight coefficients of the criteria, $RN(w_j) = [\underline{w}_j, \bar{w}_j]$the condition is met that $0 \leqslant \underline{w}_j \leqslant \bar{w}_j \leqslant 1$for each evaluation criterion$c_j \in C$. By solving model (25) we obtain the optimal values of the weight coefficients for the evaluation criteria $[RN(w_1), RN(w_2), ..., RN(w_n)]$and ξ^*:

$$\begin{aligned} &\min\max_j \left\{ \left| \frac{RN(w_B)}{RN(w_j)} - RN(a_{Bj}) \right|, \left| \frac{RN(w_j)}{RN(w_W)} - RN(w_{jW}) \right| \right\} \\ &s.\ t. \\ &\begin{cases} \sum_{j=1}^n \underline{w}_j \leqslant 1 \\ \sum_{j=1}^n \bar{w}_j \geqslant 1; \\ \underline{w}_j \leqslant \bar{w}_j, \quad \forall\ j = 1, 2, ..., n \\ \underline{w}_j, \bar{w}_j \geqslant 0, \quad \forall\ j = 1, 2, ..., n \end{cases} \end{aligned} \tag{25}$$

where $RN(w_j) = [\underline{w}_j, \bar{w}_j]$ is the rough weight coefficient of a criterion.

Since we obtain the values of $RN(\bar{a}_{BW})$, i.e., $\bar{a}_{BW}^U$ based on the aggregated decisions of the decision-maker, and these change the rough number interval, it is not possible to predefine the values of ξ. The values of ξ depend on uncertainties in the decisions, since uncertainties change the rough number interval. If the decision-makers agree on their preference for the best criterion over the worst, then a_{BW} represent the crisp value of $a_{BW}$from the defined scale and then the maximum values of ξ apply for different values of $a_{BW} \in \{1, 2, ..., 9\}$, as shown in Table 2.

We obtain the consistency ratio (CR) based on CI in Table 2:

$$CR = \frac{\xi^*}{CI} \tag{26}$$

4.2. Phase II: IRN Dombi-Bonferroni model

Step 1. Forming an initial decision-making matrix. Five experts who participated in this research evaluated the alternatives. Five experts participated in this research, who evaluated the alternatives. For every expert E_e $(1 \leqslant e \leqslant t)$is obtained a correspondent matrix $X^e = [\tilde{x}_{ij}^e]_{m\times n}$for $(1 \leqslant e \leqslant t)$. By the application of DGBM operator (19), it is performed the aggregation of experts' matrices, and it is formed the initial decision-making matrix $X = [\tilde{x}_{ij}]_{m\times n}$.

Step 2. Normalization of the initial decision-making matrix. Generally, in MCDM models, there are two types of criteria, benefit criteria and cost criteria. Therefore, it is necessary to normalize the initial decision making matrices, in that way, forming the normalized matrix$N = [\tilde{n}_{ij}]_{m\times n}$. The elements of the normalized matrix are obtained by applying the expression (27).

Table 2
Values of the consistency index (Rezaei, 2015).

a_{BW}	1	2	3	4	5	6	7	8	9
CI (max ξ)	0.00	0.44	1.00	1.63	2.30	3.00	3.73	4.47	5.23

$$RN(n_{ij}) = \begin{cases} RN(x_{ij})/\sum_{i=1}^{n} RN(x_{ij}) & if \quad j \in B \\ 1 - RN(x_{ij})/\sum_{i=1}^{n} RN(x_{ij}) & if \quad j \in C \end{cases} \tag{27}$$

Step 3. Determining the score function $S(\tilde{n}_i)$. By applying the DNGBM aggregator (20) the values of the score function $S(n_i) = DNGBM_w^{p,q,\rho}\{RN(n_1); RN(n_2); ..., RN(n_m)\}$ are obtained, presenting final values of preferences by alternatives. The DNGBM aggregator implies the application of the values of criteria meets the condition $\sum_{j=}^{n} w_j = 1$. In phase I of the application, the IRN values of the weights which meet the condition $\sum_{j=}^{n} \underline{w}_j \leqslant 1$ and $\sum_{j=}^{n} \bar{w}_j \geqslant 1$ are obtained. To meet the previously defined condition, we apply the Eqs. (28)–(30) to remove the roughness of the IRN values:

$$RN(\hat{w}_j) = [\underline{w}_j^n, \bar{w}_j^n] = \begin{cases} \underline{w}_j^n = \dfrac{\underline{w}_j - \min_j\{\underline{w}_j\}}{\Delta_{\min}^{\max}} \\ \bar{w}_j^n = \dfrac{\bar{w}_j - \min_j\{\underline{w}_j\}}{\Delta_{\min}^{\max}} \end{cases} \tag{28}$$

where $\underline{w}_j$ and $\bar{w}_j$ represent the lower limit and upper limit of the $RN(w_j)$, respectively; $\Delta_{\min}^{\max} = \max_j\{\bar{w}_j\} - \min_i\{\underline{w}_j\}$, $\underline{w}_j^n$ and $\bar{w}_j^n$ are the normalized forms of $\underline{w}_j$ and $\bar{w}_j$.

After normalization, we obtain a total normalized crisp value

$$\beta_i = \frac{\underline{w}_j^n \cdot \{1 - \underline{w}_j^n\} + \bar{w}_j^n \cdot \bar{w}_j^n}{1 - \underline{w}_j^n + \bar{w}_j^n} \tag{29}$$

Finally, the crisp form w_j for $RN(w_j)$ is obtained by applying Eq. (30).

$$w_j = \min_j\{\underline{w}_j\} + \beta_i \cdot [\max_j\{\bar{w}_j\} - \min_j\{\underline{w}_j\}] \tag{30}$$

After removing roughness, by applying additive normalization, values are normalized so that $\sum_{j=}^{n} w_j = 1$.

Step 4. Ranking alternatives. Ranking alternatives $\{A_1, A_1, ..., A_m\}$ and the selection of the best alternative from the considered set. Alternatives are ranked based on the values of the score function $S(n_i)$, wherein the highest possible value of the alternative $S(n_i)$ is more favorable.

4.3. Phase III: Sensitivity analysis

Sensitivity analysis is a fundamental concept for effective implementation of MCDM methods to evaluate the stability of the best alternative under changes in input parameters either due to lack of controllability or error in precise information estimation. Sensitivity analysis is a technique used to determine how different values of an independent variable will affect a particular dependent variable under a given set of assumptions. This technique is used within specific boundaries that will depend on one or more input variables. Sensitivity analysis is a way to predict the outcome of a decision if a situation turns varying or dynamic. More detail on sensitivity analysis tools has been discussed in Section 7.

5. Case study and decision-making problem

Healthcare Management (HM) in big cities is a trendy and major subject. Madrid is an international and very cosmopolitan capital in Europe with a diversity of cultures, traditions, and languages. The city is transforming to the main business hub and an attractive place for students, families, and immigrants. The medical system in Spain, like many countries, is divided into public and private systems. During recent years, the growth of private hospitals in Metropolitan cities is increasing. This is due to a very effective healthcare system, including fast and attentive personnel, innovative technologies, and because of that, the public system is not able to serve all of their clients. Therefore, the number of private healthcare and insurance companies is increasing. Hospitals like any other organizations (businesses, hotels, universities, etc.) generate pollution and a considerable volume of wastes. Part of those wastes in general are similar to other organizations; however, there are parts of contamination and infections that cannot be recycled or collected easily. They need to be separated and controlled for further actions. Therefore, identifying a place or location for hospital waste disposal is a very crucial task for the private sector.

The proposed model is applied to Madrid General Hospital (MGH)[1] to determine a suitable location for their medical waste disposal system under sustainable objectives. We have sought a real case to demonstrate the applicability of the method proposed in this study. MGH is a modern and private hospital in the center of Madrid and is interested in implementing this study to dispose of hazardous and infectious wastes. At the beginning of its activities, MGH had a contract with some service providers. These providers gather healthcare wastes from many hospitals and securely dispose of them. However, because of the expansion of the hospital and the continuing increase in the number of patients, the owners of the hospital decided to build disposal and mechanized center with modern facilities to replace the service providers and their associated costs. This decision was consistent with the long-term horizon of the hospital as well as the relevant legal consequences. The experts in the relevant industries in Madrid were consulted. The first expert is a senior professional in Environmental management systems with more than 12 years of experience and currently works in the healthcare sector. The second expert that agreed to join the team of DMs is 52 years old, has 20 years' experience in medical procurement systems, and is part of the management board of a private clinic, as well as a specialist and technician in HM. The aim is to propose a Rough based decision-making model to locate a waste disposal center and assure hospital managers of a secure disposal system. In a system of decision making, the decision-makers must handle several tasks. One of the duties that DMs are responsible for that is to figure out the relevant variables and criteria to aid us in location comparison and evaluation.

As we argued previously, several indicators and criteria are defined based on stakeholders and experts in this area. We have classified them into TBL variables to be understandable and build an effective, sustainable perspective.

The economic (technical) factors (D1) are presented as:

C1: Land Price (per square meter) in the specific zone,
C2: Cost (transportation & maintenance),
C3: Possibility of future expansion; environmental factors,

The environmental factors (D2) are presented as:

C4: Risk of the potential of intrusion and emission (degree of contamination),
C5: The proximity to the urban and city infrastructure (society),
C6: Distance to a complex of waste sorting,
C7: Geographic and geologic conditions,
C8: The existing environmental friendliness facilities (air, water, energy, and electricity supply).

The social factors (D3) are presented as:

C9: Availability of workforce,
C10: Local and territorial rules or regulations,
C11: Level of satisfaction among residents in relation to the site selection.

Among the mentioned criteria, price, cost, risk of emission, and distance to waste sorting complex are classified as non-beneficial factors. As a result, we assumed 3 decision factors and then 11 decision

[1] The name is changed to protect the anonymity and the hospital.

criteria (we call them indicators as well). We have reviewed studies (Arıkan et al., 2017; Chauhan & Singh, 2016; Ekmekçioğlu et al., 2010; Gergin et al., 2019; Kahraman et al., 2017; Kazimieras Zavadskas et al., 2015; Khadivi & Fatemi Ghomi, 2012; Marchettini, Ridolfi, & Rustici, 2007; Sala-ngam, Suzuki, Toyotani, Wakabayashi, & Watanabe, 2015) to gather effective information and details to adopt into the dimensional and geographical development of Madrid. The second task of the decision team is to locate the potential position for the objective of establishing a waste disposal center. The team consulted the regional government, municipal and relevant parties to leave a debate. The team consulted with the regional and municipal governments, as well as with other relevant parties. Finally, five districts in Madrid were considered: Fuenlabrada in the south, Alcobendas in the north, as well as Coslada, Carabanchel, and Villa de Vallecas. These five locations are chosen as decision alternatives (A_1, A_2,.., A_5). We developed a questionnaire and sent it to the experts through email to evaluate the five distinct locations under the sustainable variables. They are used to express their opinions about each factor and the relative importance of the criteria and, in the second level, assessment of each alternative over the available criteria.

However, the evaluation of decision factors to obtain the importance weights is performed in two stages. At first, the TBL variables for sustainable consideration are evaluated by DMs in pairs, and after the 11 indicators must be discussed.

Having defined the evaluation clusters/criteria within the framework of the clusters and each group of the criteria, the experts also determined the best (B) and the worst (W) clusters/criteria. On this basis, the experts determined the BO and the OW vectors, in which the preferences of the B and the W over the clusters/criteria were considered for the remaining clusters/criteria from the defined set. The evaluation of the clusters/criteria was carried out by using the scale [1–7], where 1 (a very low influence), 2 (a low influence),..., 6 (a high influence), and 7 (a very high influence). The BO and the OW vectors are presented in Table 3.

Using Eqs. (1)–(5) the crisp expert evaluation shown in the BO and

Table 3
The Best-to-Others (BO) and the Others-to-Worst (OW) vectors obtained by the experts' evaluations.

Dimensions (Model 1)			
Best-to-Others	Expert evaluation	Others-to-Worst	Expert evaluation
D1	1;6	D1	4;5
D2	3;1	D2	5;3
D3	6;4	D3	1;1
Criteria: C1, C2, and C3 (Model 2)			
Best-to-Others	Expert evaluation	Others-to-Worst	Expert evaluation
C1	1;1	C1	4;3
C2	4;2	C2	7;1
C3	5;6	C3	1;5
Criteria:C4, C5, C6, C7 and C8 (Model 3)			
Best-to-Others	Expert evaluation	Others-to-Worst	Expert evaluation
C4	5;2	C4	2;3
C5	3;2	C5	1;1
C6	6;4	C6	4;2
C7	6;1	C7	7;5
C8	1;5	C8	3;5
Criteria: C9, C10, and C11(Model 4)			
Best-to-Others	Expert evaluation	Others-to-Worst	Expert evaluation
C9	3;3	C9	1;3
C10	1;3	C10	2;1
C11	6;1	C11	5;6

Table 4
The average BO and OW vectors.

Dimensions			
Best-to-Others	Expert evaluation	Others-to-Worst	Expert evaluation
D1	[1.56,4.42]	D1	[4.24,4.74]
D2	[1.33,2.40]	D2	[3.43,4.44]
D3	[4.44,5.45]	D3	[1.00,1.00]
Criteria: C1, C2, and C3			
Best-to-Others	Expert evaluation	Others-to-Worst	Expert evaluation
C1	[1.00,1.00]	C1	[3.23,3.73]
C2	[2.40,3.43]	C2	[1.60,5.09]
C3	[5.24,5.74]	C3	[1.50,3.75]
Criteria: C4, C5, C6, C7, and C8			
Best-to-Others	Expert evaluation	Others-to-Worst	Expert evaluation
C4	[2.55,4.12]	C4	[2.22,2.73]
C5	[2.22,2.73]	C5	[1.00,1.00]
C6	[4.44,5.45]	C6	[2.40,3.43]
C7	[1.56,4.42]	C7	[5.45,6.46]
C8	[1.50,3.75]	C8	[3.43,4.44]
Criteria: C9, C10, and C11			
Best-to-Others	Expert evaluation	Others-to-Worst	Expert evaluation
C9	[3.00,3.00]	C9	[1.33,2.40]
C10	[1.33,2.40]	C10	[1.20,1.71]
C11	[1.56,4.42]	C11	[5.24,5.74]

OW vectors were transformed into RNs. After the transformation of the crisp values into a rough number, using IRNDGBM, Eq. (19), the rough BO and OW expert matrices were transformed into aggregated RBO and ROW vectors (see Table 4).

The optimal values of the weight coefficients of the dimensions/criteria were calculated in Table 4 based on the RBO and the ROW vectors for each group of the clusters/criteria. Table 4 displays the four RBO or ROW vectors based the four LP model formulations that were developed, using Eq. (25), for calculating the optimal values of the weight coefficients of the clusters/criteria. The developed LP models were then solved during LINDO, and the results (optimal values of the weight coefficients) are shown in Table 5. For the considered case study, CR values were found to be($CR_1 = 0.108$; $CR_2 = 0.134$; $CR_3 = 0.095$ and CR_4=0.078), respectively, for the four models:

Table 5
The optimal values (weights) of the criteria.

Dimensions/Criteria	Local Weights	Global Weights	Rank
D1	[0.2608,0.2613]	–	*2*
C1	[0.6197,0.6197]	[0.1616,0.1619]	3
C2	[0.2745,0.4760]	[0.0716,0.1244]	4
C3	[0.1058,0.1716]	[0.0276,0.0448]	9
D2	[0.6183,0.6242]	–	*1*
C4	[0.1010,0.1120]	[0.0624,0.0699]	6
C5	[0.1287,0.1674]	[0.0795,0.1045]	5
C6	[0.0673,0.0714]	[0.0416,0.0446]	8
C7	[0.3039,0.3635]	[0.1879,0.2269]	**1**
C8	[0.2795,0.2856]	[0.1728,0.1783]	2
D3	[0.0965,0.1145]	–	*3*
C9	[0.1749,0.2907]	[0.0169,0.0333]	11
C10	[0.2427,0.2427]	[0.0234,0.0278]	10
C11	[0.5824,0.5824]	[0.0562,0.0667]	7

Model 1 (D1-D3)

$$
\begin{aligned}
&\min \xi \\
&s.t. \\
&\left|\frac{\bar{w}_2}{\underline{w}_1} - 4.42\right| \leqslant \xi;\ \left|\frac{\bar{w}_2}{\underline{w}_2} - 2.40\right| \leqslant \xi;\ \left|\frac{\bar{w}_2}{\underline{w}_3} - 5.45\right| \leqslant \xi; \\
&\left|\frac{\underline{w}_2}{\bar{w}_1} - 1.56\right| \leqslant \xi;\ \left|\frac{\underline{w}_2}{\bar{w}_2} - 1.33\right| \leqslant \xi;\ \left|\frac{\underline{w}_2}{\bar{w}_3} - 4.44\right| \leqslant \xi; \\
&\left|\frac{\bar{w}_1}{\underline{w}_3} - 4.74\right| \leqslant \xi;\ \left|\frac{\bar{w}_2}{\underline{w}_3} - 4.44\right| \leqslant \xi;\ \left|\frac{\bar{w}_1}{\underline{w}_3} - 4.24\right| \leqslant \xi; \\
&\left|\frac{\bar{w}_2}{\underline{w}_3} - 3.43\right| \leqslant \xi; \\
&\textstyle\sum_{j=1}^{3} \bar{w}_j \geqslant 1;\ \ \sum_{j=1}^{3} \underline{w}_j \leqslant 1 \\
&\bar{w}_j,\ \underline{w}_j \geqslant 0,\ \ \forall\, j = 1,\ 2,\ 3
\end{aligned}
$$

Model 2 (C1-C3)

$$
\begin{aligned}
&\min \xi \\
&s.t. \\
&\left|\frac{\bar{w}_1}{\underline{w}_2} - 3.43\right| \leqslant \xi;\ \left|\frac{\bar{w}_1}{\underline{w}_3} - 5.74\right| \leqslant \xi;\ \left|\frac{\underline{w}_1}{\bar{w}_2} - 2.40\right| \leqslant \xi; \\
&\left|\frac{\underline{w}_1}{\bar{w}_3} - 5.24\right| \leqslant \xi;\ \left|\frac{\underline{w}_1}{\bar{w}_3} - 3.73\right| \leqslant \xi;\ \left|\frac{\underline{w}_2}{\bar{w}_3} - 5.09\right| \leqslant \xi; \\
&\left|\frac{\underline{w}_3}{\bar{w}_3} - 3.75\right| \leqslant \xi;\ \left|\frac{\bar{w}_1}{\underline{w}_3} - 3.23\right| \leqslant \xi;\ \left|\frac{\bar{w}_2}{\underline{w}_3} - 1.60\right| \leqslant \xi; \\
&\left|\frac{\bar{w}_3}{\underline{w}_3} - 1.50\right| \leqslant \xi; \\
&\textstyle\sum_{j=1}^{3} \bar{w}_j \geqslant 1; \sum_{j=1}^{3} \underline{w}_j \leqslant 1 \\
&\bar{w}_j,\ \underline{w}_j \geqslant 0,\ \ \forall\, j = 1,\ 2,\ 3
\end{aligned}
$$

Model 3 (C4-C8)

$$
\begin{aligned}
&\min \xi \\
&s.t. \\
&\left|\frac{\bar{w}_8}{\underline{w}_4} - 4.12\right| \leqslant \xi;\ \left|\frac{\bar{w}_8}{\underline{w}_5} - 2.73\right| \leqslant \xi;\ \left|\frac{\bar{w}_8}{\underline{w}_6} - 5.45\right| \leqslant \xi; \\
&\left|\frac{\bar{w}_8}{\underline{w}_7} - 4.42\right| \leqslant \xi;\ \left|\frac{\bar{w}_8}{\underline{w}_8} - 1.15\right| \leqslant \xi;\ \left|\frac{\underline{w}_8}{\bar{w}_4} - 2.55\right| \leqslant \xi; \\
&\left|\frac{\underline{w}_8}{\bar{w}_5} - 2.22\right| \leqslant \xi; \ldots; \left|\frac{\bar{w}_6}{\underline{w}_5} - 2.40\right| \leqslant \xi;\ \left|\frac{\bar{w}_7}{\underline{w}_5} - 5.45\right| \leqslant \xi; \\
&\left|\frac{\bar{w}_8}{\underline{w}_5} - 3.43\right| \leqslant \xi; \\
&\textstyle\sum_{j=1}^{8} \bar{w}_j \geqslant 1;\ \ \sum_{j=1}^{8} \underline{w}_j \leqslant 1 \\
&\bar{w}_j,\ \underline{w}_j \geqslant 0,\ \ \forall\, j = 1,\ 2,\ \ldots, 8
\end{aligned}
$$

Model 4 (C9-C11)

$$
\begin{aligned}
&\min \xi \\
&s.t. \\
&\left|\frac{\bar{w}_3}{\underline{w}_1} - 3.00\right| \leqslant \xi;\ \left|\frac{\bar{w}_3}{\underline{w}_2} - 2.40\right| \leqslant \xi;\ \left|\frac{\bar{w}_3}{\underline{w}_3} - 4.42\right| \leqslant \xi; \\
&\left|\frac{\underline{w}_3}{\bar{w}_1} - 2.55\right| \leqslant \xi;\ \left|\frac{\underline{w}_3}{\bar{w}_2} - 1.33\right| \leqslant \xi;\ \left|\frac{\underline{w}_3}{\bar{w}_3} - 1.56\right| \leqslant \xi; \\
&\left|\frac{\underline{w}_1}{\bar{w}_2} - 2.40\right| \leqslant \xi; \ldots; \left|\frac{\bar{w}_1}{\underline{w}_2} - 1.33\right| \leqslant \xi;\ \left|\frac{\bar{w}_2}{\underline{w}_2} - 1.20\right| \leqslant \xi; \\
&\left|\frac{\bar{w}_3}{\underline{w}_2} - 5.24\right| \leqslant \xi; \\
&\textstyle\sum_{j=1}^{3} \bar{w}_j \geqslant 1;\ \ \sum_{j=1}^{3} \underline{w}_j \leqslant 1 \\
&\bar{w}_j,\ \underline{w}_j \geqslant 0,\ \ \forall\, j = 1,\ 2,\ 3
\end{aligned}
$$

Fig. 2 presents the global and the local values of the rough weight coefficients of the criteria. The global weights of the criteria were obtained by multiplying the weight coefficients of the dimensions by the weight coefficients of the sub-criteria. The global weight criteria continue to be used for the evaluation of the alternatives in the multi-criteria model. Based on the results presented in Table 5, D2 (environmental factors) is the most significant factor, followed by D1 (economic factors) and D3 (social factors). It is inferred that among decision-making criteria, C7 (geographic and geologic conditions) is the most important criterion, while C9 (workforce availability) is the least important criterion.

The final values of the weight coefficients are used to evaluate and select the optimal alternative in the D'Bonferroni model. The evaluation of the alternatives was carried out using the scale [1–7] with 1 (very low influence), 2 (low influence),…, 6 (high influence), and 7 (very high influence). Table 6 indicates the judgments of two decision-makers when they try to compare alternatives regarding eleven indicators.

The crisp expert evaluations shown in Table 6 were transformed into the RNs given in Table 7. Each crisp element x_{ij}^{e} $(1 \leqslant e \leqslant t)$ from Table 6 is transformed into a rough number $RN(x_{ij}^{e}) = [\underline{Lim}(x_{ij}^{e}),\ \overline{Lim}(x_{ij}^{e})]$ using Eqs. (1)–(5), where $\underline{Lim}(x_{ij}^{e})$ and $\overline{Lim}(x_{ij}^{e})$are the lower and upper limit of$RN(x_{ij}^{e})$, respectively.

Using the DGBM and Eq. (19), the rough expert matrices were transformed into the aggregated initial rough matrix, as given in Table 8. This table presents the aggregated individual rough experts' evaluations of the alternatives that are shown in Table 7.

After obtaining the initial decision matrix (Table 8), Eq. (27) is used to obtain the elements $RN(n_{ij})$ from the normalized matrix (N) (Table 9). In the following steps, using the elements from the normalized matrix (Table 9), the score functions of the alternatives are calculated.

For the calculation of the score functions of the alternatives, the elements of the normalized matrix (Table 9) and crisp values of the weight coefficient of the criteria are used $(0.162, 0.072, 0.028, 0.062, 0.08, 0.042, 0.188, 0.173, 0.017, 0.023, 0.056)^{T}$. Using the DNGBM aggregator (20), final values of the score function are obtained$S(n_i)$. Based on the value$S(n_i)$, the alternatives are ranked, and the optimal alternative is selected from the set of considered alternatives. Score functions and ranking of the alternatives are shown in Table 10.

The alternatives are ranked based on the value of the score function $S(n_i)$, wherein it is more preferable for the alternative to have the highest possible value of $S(n_i)$. Thus, based on the obtained values of $S(n_i)$, the first rank is assigned to the alternative A2.

6. Sensitivity analysis, performance comparison, and discussion

Analysis of the stability of the obtained results is carried out in three parts. In the first part, the sensitivity analysis of the IRN BWM D'Bonferroni model is performed by changing the weight coefficients of the criteria. The analysis of the influence of the change in the weight coefficients of the criteria is made using 21 scenarios. In the second part, the analysis of the influence of dynamic matrices of decision making to the change of the rank of the alternatives is performed. The third part shows the analysis of the dependence of the obtained results on the change of ρ, *p,* and *q* parameters. A more detailed overview of the sections of the discussion on the results is shown in the next part of the paper.

6.1. Changing the weights of the criteria

After determining the weight coefficients of the criteria using IRN BWM, the "most influential criterion" is identified for the sensitivity analysis. The goal of the sensitivity analysis is to evaluate the influence of the most influential criterion on the ranking performance of the proposed model. Based on the recommendations of Kirkwood (1997) and Kahraman (2002), the proportionality of the weights of the criteria during the sensitivity analysis and the elasticity coefficient (Kahraman, 2002) are defined. The elasticity coefficient is used to express the relative compensation of the values of other weight coefficients concerning changes in the weight of the most important criterion.

In this research, the C7 criterion is identified as the most influential since it has the highest value of the weight coefficient $w_7 = 0.1879$. In the next step, the coefficient of weight elasticity (α_s)of the most significant criterion (Table 11) is determined, and the limits of change of the weight coefficient of the most significant criterion are defined.

The limit values for criterion C7 are obtained as $-0.1879 \leq \Delta x \leq 0.757$. The scenarios for the sensitivity analysis are defined based on the defined limit values of the change in the weight coefficient for the most important criterion. The interval

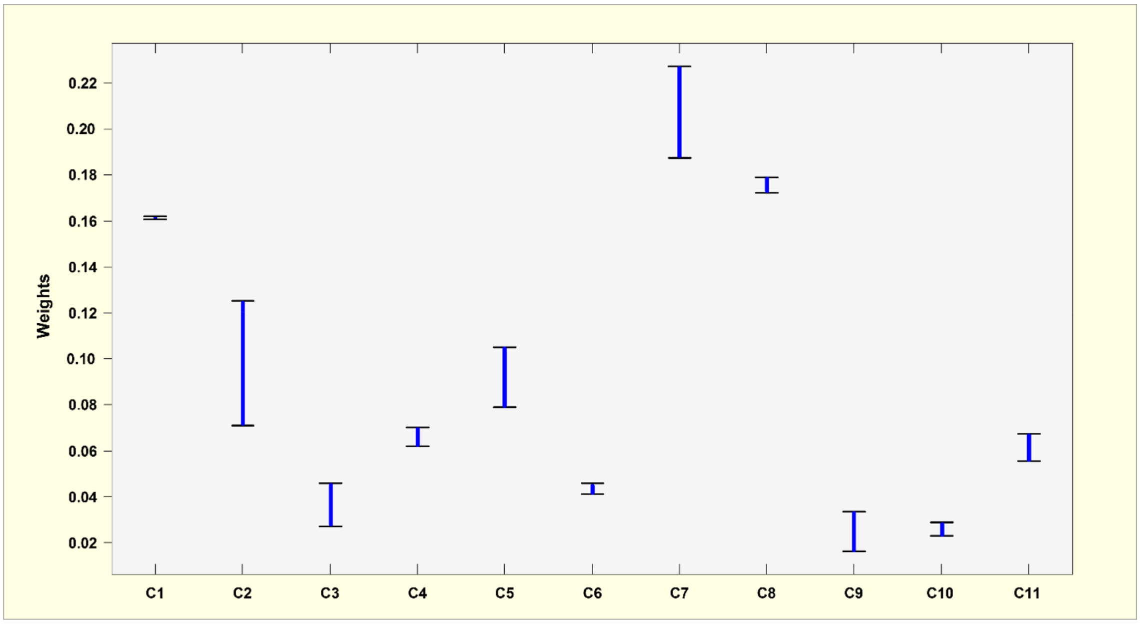

Fig. 2. Final values of the IRN weights.

Table 6
The alternative rating with respect to the sustainable factors.

DM1											
Alternative	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11
A1	6	3	5	5	5	4	5	1	4	1	6
A2	4	3	5	5	1	7	3	8	2	5	3
A3	2	4	1	3	5	5	2	5	7	5	2
A4	2	2	6	2	5	7	3	1	1	4	5
A5	5	3	3	1	3	3	4	2	5	2	1
DM2											
Alternative	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11
A1	5	4	3	5	3	1	5	3	4	2	7
A2	3	3	5	2	3	6	3	7	3	5	3
A3	2	1	7	4	4	5	1	4	3	3	1
A4	1	6	4	4	2	5	2	3	4	8	4
A5	3	3	2	4	2	5	2	2	5	2	3

Table 8
Initial decision making matrix.

Criteria	A1	A2	A3	A4	A5
C1	[5.24,5.74]	[3.23,3.73]	[2.00,2.00]	[1.2,1.71]	[3.43,4.44]
C2	[3.23,3.73]	[3.00,3.00]	[1.43,3.08]	[2.67,4.8]	[3.00,3.00]
C3	[3.43,4.44]	[5.00,5.00]	[1.60,5.09]	[4.44,5.45]	[2.22,2.73]
C4	[5.00,5.00]	[2.55,4.12]	[3.23,3.73]	[2.4,3.43]	[1.43,3.08]
C5	[3.43,4.44]	[1.33,2.40]	[4.24,4.74]	[2.55,4.12]	[2.22,2.73]
C6	[1.43,3.08]	[6.24,6.74]	[5.00,5.00]	[5.45,6.46]	[3.43,4.44]
C7	[5.00,5.00]	[3.00,3.00]	[1.20,1.71]	[2.22,2.73]	[2.40,3.43]
C8	[1.33,2.40]	[7.24,7.74]	[4.24,4.74]	[1.33,2.4]	[2.00,2.00]
C9	[4.00,4.00]	[2.22,2.73]	[3.75,5.83]	[1.43,3.08]	[5.00,5.00]
C10	[1.20,1.71]	[5.00,5.00]	[3.43,4.44]	[4.8,6.86]	[2.00,2.00]
C11	[6.24,6.74]	[3.00,3.00]	[1.20,1.71]	[4.24,4.74]	[1.33,2.40]

$-0.1879 \le \Delta x \le 0.757$ is divided into 21 sequences, which result in forming 21 scenarios. For every scenario, new weight coefficient values are formed. Consequently, 21 new groups of weight coefficients presented in Table 12 are obtained. The influence of the new values of the weight coefficients from Table 12 to the change of the ranks of

Table 7
Experts' rough matrices.

DM1											
Alternative	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11
A1	[5.5,6]	[3,3.5]	[4,5]	[5,5]	[4,5]	[2.5,4]	[5,5]	[1,2]	[4,4]	[1,1.5]	[6,6.5]
A2	[3.5,4]	[3,3]	[5,5]	[3.5,5]	[1,2]	[6.5,7]	[3,3]	[7.5,8]	[2,2.5]	[5,5]	[3,3]
A3	[2,2]	[2.5,4]	[1,4]	[3,3.5]	[4.5,5]	[5,5]	[1.5,2]	[4.5,5]	[5,7]	[4,5]	[1.5,2]
A4	[1.5,2]	[2,4]	[5,6]	[2,3]	[3.5,5]	[6,7]	[2.5,3]	[1,2]	[1,2.5]	[4,6]	[4.5,5]
A5	[4,5]	[3,3]	[2.5,3]	[1,2.5]	[2.5,3]	[3,4]	[3,4]	[2,2]	[5,5]	[2,2]	[1,2]
DM2											
Alternative	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11
A1	[5,5.5]	[3.5,4]	[3,4]	[5,5]	[3,4]	[1,2.5]	[5,5]	[2,3]	[4,4]	[1.5,2]	[6.5,7]
A2	[3,3.5]	[3,3]	[5,5]	[2,3.5]	[2,3]	[6,6.5]	[3,3]	[7,7.5]	[2.5,3]	[5,5]	[3,3]
A3	[2,2]	[1,2.5]	[4,7]	[3.5,4]	[4,4.5]	[5,5]	[1,1.5]	[4,4.5]	[3,5]	[3,4]	[1,1.5]
A4	[1,1.5]	[4,6]	[4,5]	[3,4]	[2,3.5]	[5,6]	[2,2.5]	[2,3]	[2.5,4]	[6,8]	[4,4.5]
A5	[3,4]	[3,3]	[2,2.5]	[2.5,4]	[2,2.5]	[4,5]	[2,3]	[2,2]	[5,5]	[2,2]	[2,3]

Table 9
Normalized matrix.

Criteria	A1	A2	A3	A4	A5
C1	[0.62,0.70]	[0.75,0.82]	[0.87,0.89]	[0.89,0.93]	[0.71,0.81]
C2	[0.72,0.82]	[0.77,0.83]	[0.77,0.92]	[0.64,0.85]	[0.77,0.83]
C3	[0.15,0.27]	[0.22,0.30]	[0.07,0.30]	[0.20,0.33]	[0.10,0.16]
C4	[0.66,0.74]	[0.72,0.87]	[0.74,0.83]	[0.77,0.88]	[0.79,0.93]
C5	[0.19,0.32]	[0.07,0.17]	[0.23,0.34]	[0.14,0.30]	[0.12,0.20]
C6	[0.86,0.94]	[0.69,0.76]	[0.77,0.81]	[0.70,0.79]	[0.79,0.87]
C7	[0.32,0.36]	[0.19,0.22]	[0.08,0.12]	[0.14,0.20]	[0.15,0.25]
C8	[0.07,0.15]	[0.38,0.48]	[0.22,0.29]	[0.07,0.15]	[0.10,0.12]
C9	[0.19,0.24]	[0.11,0.17]	[0.18,0.36]	[0.07,0.19]	[0.24,0.30]
C10	[0.90,0.94]	[0.70,0.75]	[0.73,0.83]	[0.58,0.76]	[0.88,0.90]
C11	[0.34,0.42]	[0.16,0.19]	[0.06,0.11]	[0.23,0.30]	[0.07,0.15]

Table 10
The rank of the alternatives.

Alternative	$S(n_i)$	Rank
A1	[0.345,0.449]	2
A2	[0.360,0.447]	1
A3	[0.282,0.398]	3
A4	[0.265,0.397]	4
A5	[0.259,0.359]	5

alternatives is presented in Fig. 3.

As shown in Fig. 3, assigning different weights to the criteria across the scenarios leads to the change in the ranks of individual alternatives, which confirms that the model is sensitive to changes in the weight coefficients. By comparing the first two alternatives in the rank (A2 and A1) across the scenarios, we can conclude that both first-ranked alternatives (A2 and A1) retain their ranks using 18 of the 21 scenarios. Just in the first three scenarios, the alternatives A2 and A1 change their rank. Alternatives A3 and A5 retain their ranks in 17 out of a total of 21 scenarios. Alternative A4 was in fourth place in all of the scenarios. Correlation of ranks is determined using Spearman's correlation coefficient. Spearman's coefficient (SCC) is used to determine the statistical significance of the difference between the ranks obtained across the scenarios (Noureddine & Ristic, 2019). By analyzing the obtained correlation values, we note that there is a high correlation of ranks since, in 17 out of 21 scenarios, the SCC exceeds 0.910. In the remaining two scenarios, the SCC values are 0.600. The mean value of the SCC across all the scenarios is 0.920, which shows a high correlation of ranks, respectively, and confirms the results shown in Table 9. From the above results, we can conclude that the rank of the alternative A2 is valid, and there is sufficient advantage of the mentioned alternative compared to the second-ranked (A1) and other alternatives. The results are also confirmed by the correlation of ranks across the scenarios.

6.2. Influence of dynamic matrices on changing the rank of alternatives

Internal changes in the decision-making matrix, such as the introduction of new ones or the elimination of the existing alternatives from the set of considered alternatives, can cause changes in final preferences. Accordingly, in this paper, the performance of the proposed model is analyzed under the conditions of the dynamic initial decision-making matrix. Three scenarios are formed. For every scenario, a change in the number of alternatives is made, and the ranks obtained are analyzed. The scenarios are formed by removing one inferior (the worst) alternative in every scenario from further consideration. At the same time, within the scenarios, the remaining alternatives are ranked according to the newly-obtained initial decision-making matrix.

The initial solution using the IRN BWM D'Bonferroni model is generated as $A2 > A1 > A3 > A4 > A5$. It is clear that the alternative A5 is the worst option, and in the first scenario, the alternative A5 is eliminated from the set, and a new decision-making matrix is obtained with a total of five alternatives. A new solution for the decision-making matrix is generated, and the rank $A2 > A1 > A3 > A4$ is obtained. The ranking in the first scenario shows that A2 is still the best alternative, while A4 is the worst alternative. Further implementation of the described procedure results in the following ranks using the remaining three scenarios: S2: $A2 > A1 > A3$ and S3: $A2 > A1$.

Using the modification of the initial matrix, which is done by the elimination of the worst alternative, we notice that there is no rank reversal among the alternatives in the IRN BWM D'Bonferroni model. The alternative A2 remained the best ranked across all of the scenarios, which confirmed the robustness and accuracy of the obtained ranks of the alternatives in a dynamic environment.

6.3. Influence of parameters ρ, p, and q on the ranking results

In the above steps, the values of the parameters p, q, and ρ were initially assumed to be 1. However, the effects of changing the value of p and q in the proposed IRN D'Bonferroni model can easily be observed.

Three scenarios are formed to consider the influence of the parameters p, q, and ρ on the obtained results. In the first scenario, the values of the parameter p are changed in the range from 0.5 to 50, while for the parameters p and q, the values are $\rho = q = 1$. In the second scenario (S2), the values of the parameter q are changed in the range from 0.5 to 50, while for the parameters ρ and p, the values are $\rho = q = 1$. In the third scenario (S3), the values of the parameter ρ are changed in the range from 0.5 to 50, while for the parameters q and p, the values are changed in the range from 0.5 to 5. Table 13 shows the influence of the parameters p, q, and ρ on the ranks of the alternatives.

Generally, the bigger the values of the parameters ρ, p and q, the more complex the calculation becomes, and more the interrelations between the attributes are emphasized. DMs usually choose the parameters ρ, p, and q according to their preferences (Fazlollahtabar, Smailbasic, & Stevic, 2019). In real decision making, we generally recommend that the values of the parameters be 1 from a practical point of view, which is not only intuitionistic and simple but also able to consider the inner connections between attributes. From Table 13, it can be revealed that when the parameters ρ, p and q have different values, the ranking orders of the considered alternatives remain almost the same. Small changes occur when the parameter q is changed while changing the parameters p, and ρ results in no change in the ranking. In the scenario S2, for the values $q = 30$, $q = 40$ and $q = 50$, the ranks $A2 > A1 > A3 > A4 > A5$ is obtained. The changes in ranks that occurred when the values of $q = 30$, $q = 40$, and $q = 50$ are minimal and did not affect the rank of alternatives A2 and A1 that were identified as the best alternatives. Table 13 shows when Parameter ρ has values within the 0 to 50 range, the ranking of the alternatives remains unaffected. Therefore, the changing values in Parameter ρ have no impact on the rankings. In all cases, A2 (Alcobendas) and A5 (Villa de Vallecas) remain the best and the worst alternative locations, respectively. Similarly, Table 13 shows Parameter p has no impact on the final

Table 11
Coefficient of elasticity for changing weights.

Criteria	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11
α_s	0.214	0.123	0.039	0.085	0.117	0.055	1.000	0.234	0.024	0.031	0.077

Table 12
New criteria weights.

Scenario	C1	C2	C3	C4	C5	C6	C7	C8	C9	C10	C11
S1	0.214	0.123	0.039	0.085	0.117	0.055	0.000	0.234	0.024	0.031	0.077
S2	0.203	0.117	0.037	0.081	0.111	0.053	0.050	0.222	0.023	0.030	0.073
S3	0.192	0.111	0.035	0.076	0.106	0.050	0.100	0.210	0.021	0.028	0.070
S4	0.182	0.105	0.033	0.072	0.100	0.047	0.150	0.199	0.020	0.027	0.066
S5	0.171	0.098	0.031	0.068	0.094	0.044	0.200	0.187	0.019	0.025	0.062
S6	0.160	0.092	0.029	0.064	0.088	0.042	0.250	0.175	0.018	0.023	0.058
S7	0.150	0.086	0.027	0.059	0.082	0.039	0.300	0.164	0.017	0.022	0.054
S8	0.139	0.080	0.026	0.055	0.076	0.036	0.350	0.152	0.016	0.020	0.050
S9	0.128	0.074	0.024	0.051	0.070	0.033	0.400	0.140	0.014	0.019	0.046
S10	0.118	0.068	0.022	0.047	0.065	0.031	0.450	0.129	0.013	0.017	0.043
S11	0.107	0.062	0.020	0.042	0.059	0.028	0.500	0.117	0.012	0.016	0.039
S12	0.096	0.055	0.018	0.038	0.053	0.025	0.550	0.105	0.011	0.014	0.035
S13	0.086	0.049	0.016	0.034	0.047	0.022	0.600	0.093	0.010	0.012	0.031
S14	0.075	0.043	0.014	0.030	0.041	0.019	0.650	0.082	0.008	0.011	0.027
S15	0.064	0.037	0.012	0.025	0.035	0.017	0.700	0.070	0.007	0.009	0.023
S16	0.053	0.031	0.010	0.021	0.029	0.014	0.750	0.058	0.006	0.008	0.019
S17	0.043	0.025	0.008	0.017	0.023	0.011	0.800	0.047	0.005	0.006	0.015
S18	0.032	0.018	0.006	0.013	0.018	0.008	0.850	0.035	0.004	0.005	0.012
S19	0.021	0.012	0.004	0.008	0.012	0.006	0.900	0.023	0.002	0.003	0.008
S20	0.011	0.006	0.002	0.004	0.006	0.003	0.950	0.012	0.001	0.002	0.004
S21	0.005	0.003	0.001	0.002	0.003	0.001	0.978	0.005	0.001	0.001	0.002

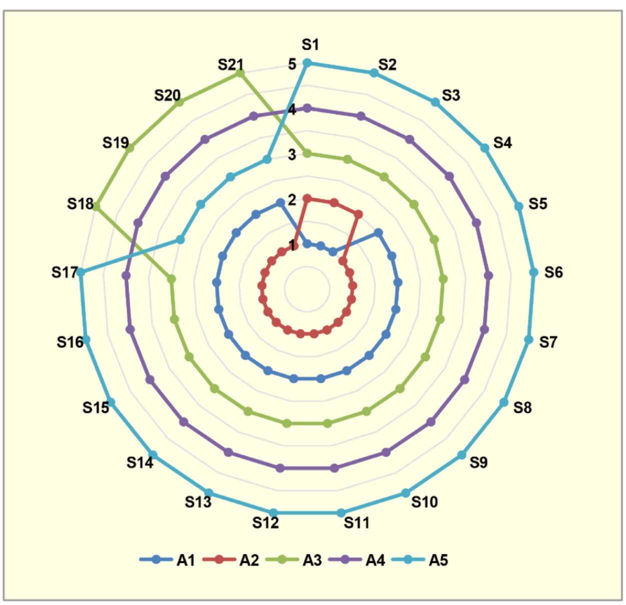

Fig. 3. Analysis of sensitivity of the ranks of the alternatives across 15 scenarios.

preferences. The parameter q has a small influence on the rank of certain alternatives, but it does not affect the final preferences when choosing the most influential alternatives, respectively, the alternatives A2 and A1.

7. Conclusions

Waste management is a convoluted issue, especially in the big cities and metropolitan areas, involving the fundamental interaction between various dimensions and endangers the maintenance of nature and natural life in the context of environmental pollution and protection acts. Thus, the analysis and control enforce perpetual challenges for policymakers. In the era that communities are worried about sustaining their surrounding environments, the private sector can be proactive in keeping a balance between the economic needs, social impact, and caring for environmental consciousness. In healthcare systems, any consequences from the output of medical services, hospitals, and clinics directly affect and damage human life, the environment and bring further disadvantages over time. The uncertainties associated with the healthcare waste disposal management, as addressed by this research would enable the implementation of systematic mitigation strategies and deployment of necessary resources for leveraging the efficiency of healthcare systems. Furthermore, the general ability of the proposed methodology and key insights developed will assist healthcare practitioners in policymaking at strategic and tactical levels for minimizing the harmful effects of improper waste disposal decisions. This paper investigates the eligibility for establishing a waste collection and disposal center for a private hospital. Defined by a case study, we have determined five places for the establishment of a center for waste disposal. Healthcare waste management has a discrete consequence in an integrated solid waste management system. In the planning of a healthcare waste management system, various parameters, including social impact, environmental concerns, and economic conditions, need utmost consideration. However, while devising disposal strategy for the healthcare waste management system, the policymakers often overlook some critical factors, causing environmental degeneration, lack of sanitation, and an overabundance of health issues. The structured and comprehensive MCDM model proposed in this study can effectively eliminate such problems.

In addition, to overcome the uncertainty of not having complete information and as well as the complexity in the evaluation process, we have configured a rough-based multi-attribute decision-making structure. Given this level of detail, a hybrid interval rough value BWM D'Bonferroni model is developed that has several benefits that conform to the study. We should note that MCDM models are heavily dependent on the criteria weights. The rankings in the model proposed in this study are also dependent on the Parameters p and q associated with the IRN D'Bonferroni model. Practicing managers and researchers must be careful using MCDM problems due to these dependencies. Obtaining reliable data for the proposed model requires some all-inclusive study; any addition or deletion in the decision matrix may affect the overall accuracy. For waste disposal site selection, it is expected that the information provided must be accurate and repeatedly revised, which ultimately leads to fewer opportunities for error. Another limitation of the method proposed in this study is the technical details of the model, which are complicated and potentially overwhelming for some decision-makers and managers. The analytical tools and techniques for complex models may require some technical assistance from the experts. In this study, we implemented the model with appropriate technical and decision support assistance.

Table 13
Ranking orders for varying values of the parameters ρ, *p*, and *q*.

Parameters	Score functions	Rank
S1: Changing parameter *p*		
$\rho = 1, p = 0.5, q = 1$	Q_i(A1) = [0.335,0.442]; Q_i(A2) = [0.351,0.439];Q_i(A3) = [0.274,0.389]; Q_i(A4) = [0.259,0.391]; Q_i(A5) = [0.254,0.352]	A2 > A1 > A3 > A4 > A5
$\rho = 1, p = 2, q = 1$	Q_i(A1) = [0.333,0.440]; Q_i(A2) = [0.352,0.439]; Q_i(A3) = [0.273,0.386]; Q_i(A4) = [0.257,0.388]; Q_i(A5) = [0.254,0.351]	A2 > A1 > A3 > A4 > A5
$\rho = 1, p = 3, q = 1$	Q_i(A1) = [0.319,0.428]; Q_i(A2) = [0.340,0.427]; Q_i(A3) = [0.261,0.371]; Q_i(A4) = [0.248,0.377]; Q_i(A5) = [0.246,0.341]	A2 > A1 > A3 > A4 > A5
$\rho = 1, p = 4, q = 1$	Q_i(A1) = [0.307,0.418]; Q_i(A2) = [0.330,0.418]; Q_i(A3) = [0.252,0.359]; Q_i(A4) = [0.240,0.367]; Q_i(A5) = [0.240,0.332]	A2 > A1 > A3 > A4 > A5
$\rho = 1, p = 5, q = 1$	Q_i(A1) = [0.297,0.411]; Q_i(A2) = [0.322,0.411]; Q_i(A3) = [0.244,0.350]; Q_i(A4) = [0.233,0.360]; Q_i(A5) = [0.234,0.325]	A2 > A1 > A3 > A4 > A5
$\rho = 1, p = 10, q = 1$	Q_i(A1) = [0.269,0.39]; Q_i(A2) = [0.298,0.390]; Q_i(A3) = [0.214,0.338]; Q_i(A4) = [0.220,0.322]; Q_i(A5) = [0.217,0.305]	A2 > A1 > A3 > A4 > A5
$\rho = 1, p = 20, q = 1$	Q_i(A1) = [0.247,0.374]; Q_i(A2) = [0.280,0.375]; Q_i(A3) = [0.198,0.321]; Q_i(A4) = [0.201,0.301]; Q_i(A5) = [0.204,0.289]	A2 > A1 > A3 > A4 > A5
$\rho = 1, p = 30, q = 1$	Q_i(A1) = [0.238,0.368]; Q_i(A2) = [0.272,0.369]; Q_i(A3) = [0.192,0.293]; Q_i(A4) = [0.191,0.315]; Q_i(A5) = [0.198,0.282]	A2 > A1 > A4 > A3 > A5
$\rho = 1, p = 40, q = 1$	Q_i(A1) = [0.233,0.364]; Q_i(A2) = [0.268,0.365]; Q_i(A3) = [0.188,0.288]; Q_i(A4) = [0.187,0.311]; Q_i(A5) = [0.195,0.278]	A2 > A1 > A4 > A3 > A5
$\rho = 1, p = 50, q = 1$	Q_i(A1) = [0.230,0.362]; Q_i(A2) = [0.265,0.363]; Q_i(A3) = [0.184,0.309]; Q_i(A4) = [0.192,0.276]; Q_i(A5) = [0.185,0.285];	A2 > A1 > A3 > A4 > A5
S2: Changing parameter *q*		
$\rho = 1, p = 1, q = 0.5$	Q_i(A1) = [0.333,0.440]; Q_i(A2) = [0.352,0.439]; Q_i(A3) = [0.273,0.386]; Q_i(A4) = [0.257,0.388]; Q_i(A5) = [0.254,0.351]	A2 > A1 > A3 > A4 > A5
$\rho = 1, p = 1, q = 2$	Q_i(A1) = [0.335,0.442]; Q_i(A2) = [0.351,0.439]; Q_i(A3) = [0.274,0.389]; Q_i(A4) = [0.259,0.391]; Q_i(A5) = [0.254,0.352]	A2 > A1 > A3 > A4 > A5
$\rho = 1, p = 1, q = 3$	Q_i(A1) = [0.321,0.431]; Q_i(A2) = [0.339,0.429]; Q_i(A3) = [0.264,0.376]; Q_i(A4) = [0.250,0.381]; Q_i(A5) = [0.246,0.342]	A2 > A1 > A3 > A4 > A5
$\rho = 1, p = 1, q = 4$	Q_i(A1) = [0.309,0.422]; Q_i(A2) = [0.329,0.420]; Q_i(A3) = [0.255,0.365]; Q_i(A4) = [0.243,0.373]; Q_i(A5) = [0.240,0.334]	A2 > A1 > A3 > A4 > A5
$\rho = 1, p = 1, q = 5$	Q_i(A1) = [0.300,0.415]; Q_i(A2) = [0.321,0.413]; Q_i(A3) = [0.247,0.357]; Q_i(A4) = [0.237,0.366]; Q_i(A5) = [0.235,0.327]	A2 > A1 > A3 > A4 > A5
$\rho = 1, p = 1, q = 10$	Q_i(A1) = [0.273,0.395]; Q_i(A2) = [0.297,0.392]; Q_i(A3) = [0.218,0.346]; Q_i(A4) = [0.224,0.330]; Q_i(A5) = [0.219,0.307]	A2 > A1 > A3 > A4 > A5
$\rho = 1, p = 1, q = 20$	Q_i(A1) = [0.251,0.380]; Q_i(A2) = [0.278,0.377]; Q_i(A3) = [0.202,0.330]; Q_i(A4) = [0.205,0.310]; Q_i(A5) = [0.205,0.292]	A2 > A1 > A3 > A4 > A5
$\rho = 1, p = 1, q = 30$	Q_i(A1) = [0.242,0.374]; Q_i(A2) = [0.271,0.371]; Q_i(A3) = [0.197,0.302]; Q_i(A4) = [0.195,0.324]; Q_i(A5) = [0.199,0.285]	A2 > A1 > A4 > A3 > A5
$\rho = 1, p = 1, q = 40$	Q_i(A1) = [0.237,0.371]; Q_i(A2) = [0.266,0.368]; Q_i(A3) = [0.192,0.297]; Q_i(A4) = [0.191,0.320]; Q_i(A5) = [0.196,0.282]	A2 > A1 > A4 > A3 > A5
$\rho = 1, p = 1, q = 50$	Q_i(A1) = [0.234,0.369]; Q_i(A2) = [0.264,0.366]; Q_i(A3) = [0.189,0.294]; Q_i(A4) = [0.189,0.318]; Q_i(A5) = [0.194,0.280]	A2 > A1 > A3 > A4 > A5
S3: Changing parameters *p*,*q*, and ρ		
$\rho = 0.5, p = 0.5, q = 0.5$	Q_i(A1) = [0.354,0.465]; Q_i(A2) = [0.379,0.468]; Q_i(A3) = [0.308,0.418]; Q_i(A4) = [0.290,0.422]; Q_i(A5) = [0.287,0.384]	A2 > A1 > A3 > A4 > A5
$\rho = 2, p = 2, q = 2$	Q_i(A1) = [0.321,0.409]; Q_i(A2) = [0.324,0.433]; Q_i(A3) = [0.220,0.349]; Q_i(A4) = [0.214,0.349]; Q_i(A5) = [0.206,0.310]	A2 > A1 > A3 > A4 > A5
$\rho = 3, p = 3, q = 3$	Q_i(A1) = [0.282,0.362]; Q_i(A2) = [0.297,0.414]; Q_i(A4) = [0.184,0.307]; Q_i(A3) = [0.174,0.297]; Q_i(A5) = [0.176,0.273]	A2 > A1 > A3 > A4 > A5
$\rho = 4, p = 4, q = 4$	Q_i(A1) = [0.253,0.325]; Q_i(A2) = [0.274,0.396]; Q_i(A3) = [0.160,0.250]; Q_i(A4) = [0.145,0.256]; Q_i(A5) = [0.166,0.281]	A2 > A1 > A3 > A4 > A5
$\rho = 5, p = 5, q = 5$	Q_i(A1) = [0.231,0.299]; Q_i(A2) = [0.255,0.382]; Q_i(A3) = [0.151,0.235]; Q_i(A4) = [0.128,0.226]; Q_i(A5) = [0.153,0.264]	A2 > A1 > A3 > A4 > A5
$\rho = 10, p = 0.5, q = 0.5$	Q_i(A1) = [0.173,0.245]; Q_i(A2) = [0.209,0.336]; Q_i(A3) = [0.130,0.197]; Q_i(A4) = [0.097,0.166]; Q_i(A5) = [0.112,0.228]	A2 > A1 > A3 > A4 > A5
$\rho = 20, p = 2, q = 2$	Q_i(A1) = [0.345,0.449]; Q_i(A2) = [0.360,0.447]; Q_i(A3) = [0.282,0.398]; Q_i(A4) = [0.265,0.397]; Q_i(A5) = [0.259,0.359]	A2 > A1 > A3 > A4 > A5
$\rho = 30, p = 3, q = 3$	Q_i(A1) = [0.345,0.449]; Q_i(A2) = [0.360,0.447]; Q_i(A3) = [0.282,0.398]; Q_i(A4) = [0.265,0.397]; Q_i(A5) = [0.259,0.359]	A2 > A1 > A3 > A4 > A5
$\rho = 40, p = 4, q = 4$	Q_i(A1) = [0.345,0.449]; Q_i(A2) = [0.360,0.447]; Q_i(A3) = [0.282,0.398]; Q_i(A4) = [0.265,0.397]; Q_i(A5) = [0.259,0.359]	A2 > A1 > A3 > A4 > A5
$\rho = 50, p = 5, q = 5$	Q_i(A1) = [0.345,0.449]; Q_i(A2) = [0.360,0.447]; Q_i(A3) = [0.282,0.398]; Q_i(A4) = [0.265,0.397]; Q_i(A5) = [0.259,0.359]	A2 > A1 > A3 > A4 > A5

Further research directions can be outlined as (1) implementing the recently developed MCDM methods (e.g., full consistency method (FUCOM) or combined compromise solution (CoCoSo)) under RST for criteria weighting and ranking; (2) designing decision support systems to facilitate the judgment processes; and (3) applying the proposed model in other healthcare management problems (e.g., sustainability and life cycle assessment, free-standing emergency clinic location planning). Researchers can use the proposed model to solve other MCDM problems such as airport, commercial center, warehouse or logistics hub, renewable energy plant, and port locations provided that the model is calibrated depending on the available decision variables, evaluation dimensions, and other relevant information.

CRediT authorship contribution statement

Morteza Yazdani: Conceptualization, Formal analysis, Methodology. **Madjid Tavana:** Methodology, Project administration, Supervision, Visualization, Writing - review & editing. **Dragan Pamučar:** Investigation, Methodology, Resources, Software, Validation. **Prasenjit Chatterjee:** Investigation, Methodology, Resources, Software, Validation.

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Appendix A

Theorem 1.. *Assuming that* $RN(\xi_j) = \left[\underline{\xi}_j, \bar{\xi}_j\right]$; $(j = 1, 2, \ldots, n)$ *is a collection of IRNs in R, then the DGBM operator is defined as follows:*

$$DGBM^{p,q,\rho}(RN(\xi_1), RN(\xi_2), \ldots, RN(\xi_n)) = \left(\frac{1}{n}\sum_{i,j=1}^{n} RN(\xi_i^{p})\left(\prod_{j=1}^{n} RN\left(\xi_j^{q}\right)\right)^{\frac{1}{n-1}}\right)^{\frac{1}{p+q}}$$

$$= \left[\frac{\sum_{i=1}^{n}\underline{\xi}_i}{1+\left\{\frac{1}{p+q}\frac{n}{\frac{1}{\sum_{\substack{i=1\\ i\neq j}}^{n}\left(p\left(\frac{1-f(\underline{\xi}_i)}{f(\underline{\xi}_i)}\right)^{\rho}+\frac{q}{n-1}\sum_{\substack{j=1\\ j\neq i}}^{n}\left(\frac{1-f(\underline{\xi}_j)}{f(\underline{\xi}_j)}\right)^{\rho}\right)}}\right\}^{1/\rho}}, \frac{\sum_{i=1}^{n}\bar{\xi}_i}{1+\left\{\frac{1}{p+q}\frac{n}{\frac{1}{\sum_{\substack{i=1\\ i\neq j}}^{n}\left(p\left(\frac{1-f(\bar{\xi}_i)}{f(\bar{\xi}_i)}\right)^{\rho}+\frac{q}{n-1}\sum_{\substack{j=1\\ j\neq i}}^{n}\left(\frac{1-f(\bar{\xi}_j)}{f(\bar{\xi}_j)}\right)^{\rho}\right)}}\right\}^{1/\rho}}\right] \quad (7)$$

where $f(RN(\xi_i)) = \begin{cases} f(\underline{\xi}_i) = \frac{\underline{\xi}_i}{\sum_{i=1}^{n}\underline{\xi}_i}; \\ f(\bar{\xi}_i) = \frac{\bar{\xi}_i}{\sum_{i=1}^{n}\bar{\xi}_i}. \end{cases}$ *represents a grey function.*

Proof.. We need to prove that Eq. (7) is valid. According to the operational laws of IRNs, we obtain:

$$RN\left(\xi_j^{q}\right) = \left[\frac{\underline{\xi}_j}{1+\left\{q\left(\frac{1-\underline{\xi}_i}{\underline{\xi}_i}\right)^{\rho}\right\}^{1/\rho}}, \frac{\bar{\xi}_j}{1+\left\{q\left(\frac{1-\bar{\xi}_j}{\bar{\xi}_j}\right)^{\rho}\right\}^{1/\rho}}\right], \prod_{j=1}^{n} RN\left(\xi_j^{q}\right) = \left[\frac{\sum_{j=1}^{n}\underline{\xi}_j}{1+\left\{q\sum_{\substack{j=1\\ j\neq i}}^{n}\left(\frac{1-f(\underline{\xi}_j)}{f(\underline{\xi}_j)}\right)^{\rho}\right\}^{1/\rho}}, \frac{\sum_{j=1}^{n}\bar{\xi}_j}{1+\left\{q\sum_{\substack{j=1\\ j\neq i}}^{n}\left(\frac{1-f(\bar{\xi}_j)}{f(\bar{\xi}_j)}\right)^{\rho}\right\}^{1/\rho}}\right]$$

and: $$\left(\prod_{j=1}^{n} RN\left(\xi_j^{q}\right)\right)^{\frac{1}{n-1}} = \left[\frac{\sum_{i=1}^{n}\underline{\xi}_j}{1+\left\{\frac{q}{n-1}\sum_{\substack{j=1\\ j\neq i}}^{n}\left(\frac{1-f(\underline{\xi}_j)}{f(\underline{\xi}_j)}\right)^{\rho}\right\}^{1/\rho}}, \frac{\sum_{i=1}^{n}\bar{\xi}_j}{1+\left\{\frac{q}{n-1}\sum_{\substack{j=1\\ j\neq i}}^{n}\left(\frac{1-f(\bar{\xi}_j)}{f(\bar{\xi}_j)}\right)^{\rho}\right\}^{1/\rho}}\right]$$

Furthermore, we obtain:

$$RN(\xi_i^{p})\left(\prod_{j=1}^{n} RN\left(\xi_j^{q}\right)\right)^{\frac{1}{n-1}} = \left[\frac{\sum_{i=1}^{n}\underline{\xi}_i}{1+\left\{p\left(\frac{1-f(\underline{\xi}_i)}{f(\underline{\xi}_i)}\right)^{\rho}+\frac{q}{n-1}\sum_{\substack{j=1\\ j\neq i}}^{n}\left(\frac{1-f(\underline{\xi}_j)}{f(\underline{\xi}_j)}\right)^{\rho}\right\}^{1/\rho}}, \frac{\sum_{i=1}^{n}\bar{\xi}_i}{1+\left\{p\left(\frac{1-f(\bar{\xi}_i)}{f(\bar{\xi}_i)}\right)^{\rho}+\frac{q}{n-1}\sum_{\substack{j=1\\ j\neq i}}^{n}\left(\frac{1-f(\bar{\xi}_j)}{f(\bar{\xi}_j)}\right)^{\rho}\right\}^{1/\rho}}\right]$$

and

$$\sum_{\substack{i=1\\i\neq j}}^{n} RN(\xi_i^{\,p})\left(\prod_{j=1}^{n} RN\left(\xi_j^{q}\right)\right)^{\frac{1}{n-1}} = \left[\sum_{i=1}^{n}\underline{\xi}_j - \frac{\sum_{i=1}^{n}\underline{\xi}_i}{1+\left\{\frac{1}{\sum_{\substack{i=1\\i\neq j}}^{n}\left(p\left(\frac{1-\underline{\xi}_i}{\underline{\xi}_i}\right)^{\rho}+\frac{q}{n-1}\sum_{\substack{j=1\\j\neq i}}^{n}\left(\frac{1-\underline{\xi}_j}{\underline{\xi}_j}\right)^{\rho}\right)}\right\}^{1/\rho}}, \sum_{i=1}^{n}\bar{\xi}_j - \frac{\sum_{i=1}^{n}\bar{\xi}_i}{1+\left\{\frac{1}{\sum_{\substack{i=1\\i\neq j}}^{n}\left(p\left(\frac{1-\bar{\xi}_i}{\bar{\xi}_i}\right)^{\rho}+\frac{q}{n-1}\sum_{\substack{j=1\\j\neq i}}^{n}\left(\frac{1-\bar{\xi}_j}{\bar{\xi}_j}\right)^{\rho}\right)}\right\}^{1/\rho}}\right],$$

Thereafter:

$$\frac{1}{n}\sum_{i,j=1}^{n} RN(\xi_i^{\,p})\left(\prod_{j=1}^{n} RN\left(\xi_j^{q}\right)\right)^{\frac{1}{n-1}} = \left[\sum_{j=1}^{n}\underline{\xi}_j - \frac{\sum_{j=1}^{n}\underline{\xi}_i}{1+\left\{\frac{n}{\sum_{\substack{i=1\\i\neq j}}^{n}\left(p\left(\frac{1-\underline{\xi}_i}{\underline{\xi}_i}\right)^{\rho}+\frac{q}{n-1}\sum_{\substack{j=1\\j\neq i}}^{n}\left(\frac{1-\underline{\xi}_j}{\underline{\xi}_j}\right)^{\rho}\right)}\right\}^{1/\rho}}, \sum_{j=1}^{n}\bar{\xi}_j - \frac{\sum_{j=1}^{n}\bar{\xi}_i}{1+\left\{\frac{n}{\sum_{\substack{i=1\\i\neq j}}^{n}\left(p\left(\frac{1-\bar{\xi}_i}{\bar{\xi}_i}\right)^{\rho}+\frac{q}{n-1}\sum_{\substack{j=1\\j\neq i}}^{n}\left(\frac{1-\bar{\xi}_j}{\bar{\xi}_j}\right)^{\rho}\right)}\right\}^{1/\rho}}\right]$$

Therefore:

$$DGBM^{p,q,\rho}(RN(\xi_1), RN(\xi_2), \ldots, RN(\xi_n)) = \left(\frac{1}{n}\sum_{i,j=1}^{n} RN(\xi_i^{\,p})\left(\prod_{j=1}^{n} RN\left(\xi_j^{q}\right)\right)^{\frac{1}{n-1}}\right)^{\frac{1}{p+q}}$$

$$= \left[\frac{\sum_{i=1}^{n}\underline{\xi}_i}{1+\left\{\frac{1}{p+q}\frac{n}{\frac{1}{\sum_{\substack{i=1\\i\neq j}}^{n}\left(p\left(\frac{1-f(\underline{\xi}_i)}{f(\underline{\xi}_i)}\right)^{\rho}+\frac{q}{n-1}\sum_{\substack{j=1\\j\neq i}}^{n}\left(\frac{1-f(\underline{\xi}_j)}{f(\underline{\xi}_j)}\right)^{\rho}\right)}}\right\}^{1/\rho}}, \frac{\sum_{i=1}^{n}\bar{\xi}_i}{1+\left\{\frac{1}{p+q}\frac{n}{\frac{1}{\sum_{\substack{i=1\\i\neq j}}^{n}\left(p\left(\frac{1-f(\bar{\xi}_i)}{f(\bar{\xi}_i)}\right)^{\rho}+\frac{q}{n-1}\sum_{\substack{j=1\\j\neq i}}^{n}\left(\frac{1-f(\bar{\xi}_j)}{f(\bar{\xi}_j)}\right)^{\rho}\right)}}\right\}^{1/\rho}}\right] \square$$

Appendix B

Theorem 2 (*(Idempotency):*). *Assuming that $RN(\xi_i) = [\underline{\xi}_i, \bar{\xi}_i]$; $(j = 1, 2, \ldots, n)$ is a collection of IRNs in R, if $RN(\xi_i) = RN(\xi)$, then:* $DGBM^{p,q,\rho}\{RN(\xi_1), RN(\xi_2), \ldots, RN(\xi_n)\} = DGBM^{p,q,\rho}\{RN(\xi), RN(\xi), \ldots, RN(\xi)\}$.

Proof:. Since $RN(\xi_i) = RN(\xi)$, i.e. $\underline{\xi}_i = \underline{\xi}$, $\bar{\xi}_i = \bar{\xi}$, then:

$$DGBM^{p=1,q=1,\rho=1}\{RN(\xi_1), RN(\xi_2), ..,RN(\xi_n)\} = \{RN(\xi), RN(\xi), ..,RN(\xi)\}$$

$$\left[\frac{\sum_{i=1}^{n}\underline{\xi}_i}{\left(1+\left\{\frac{1}{p+q}\frac{n}{\frac{1}{\left(\sum_{\substack{i=1\\ i\neq j}}^{n}\left(p\left(\frac{1-f(\underline{\xi}_i)}{f(\underline{\xi}_i)}\right)^{\rho}+\frac{q}{n-1}\sum_{\substack{j=1\\ j\neq i}}^{n}\left(\frac{1-f(\underline{\xi}_j)}{f(\underline{\xi}_j)}\right)^{\rho}\right)\right)}}\right\}\right)^{1/\rho}}, \frac{\sum_{i=1}^{n}\bar{\xi}_i}{\left(1+\left\{\frac{1}{p+q}\frac{n}{\frac{1}{\left(\sum_{\substack{i=1\\ i\neq j}}^{n}\left(p\left(\frac{1-f(\bar{\xi}_i)}{f(\bar{\xi}_i)}\right)^{\rho}+\frac{q}{n-1}\sum_{\substack{j=1\\ j\neq i}}^{n}\left(\frac{1-f(\bar{\xi}_j)}{f(\bar{\xi}_j)}\right)^{\rho}\right)\right)}}\right\}\right)^{1/\rho}}\right] = \left[\frac{2\underline{\xi}}{\left(1+\left\{\frac{1}{1+1}\frac{2}{\sum_{\substack{i,j=1\\ i\neq j}}^{2}\frac{1}{\left((1+1)\left(\frac{1-\underline{\xi}}{\underline{\xi}}\right)^{\rho}\right)}}\right\}\right)^{1/\rho}}, \frac{2\bar{\xi}}{\left(1+\left\{\frac{1}{1+1}\frac{2}{\sum_{\substack{i,j=1\\ i\neq j}}^{2}\frac{1}{\left((1+1)\left(\frac{1-\bar{\xi}}{\bar{\xi}}\right)^{\rho}\right)}}\right\}\right)^{1/\rho}}\right]$$

$$= \left[\frac{\underline{\xi}}{\left(1+\left\{\frac{1}{\frac{1}{\left(\left(\frac{1-\underline{\xi}}{\underline{\xi}}\right)^{\rho}\right)}}\right\}\right)^{1/\rho}}, \frac{\bar{\xi}}{\left(1+\left\{\frac{1}{\frac{1}{\left(\left(\frac{1-\bar{\xi}}{\bar{\xi}}\right)^{\rho}\right)}}\right\}\right)^{1/\rho}}\right] = \left[\frac{\underline{\xi}}{1+\left\{\left(\frac{1-\underline{\xi}}{\underline{\xi}}\right)^{\rho}\right\}^{1/\rho}}, \frac{\bar{\xi}}{1+\left\{\left(\frac{1-\bar{\xi}}{\bar{\xi}}\right)^{\rho}\right\}^{1/\rho}}\right] = [\underline{\xi}, \bar{\xi}] = RN(\xi)$$

The proof of Theorem 2 is completed. □

Theorem 3 (*(Boundedness):*). *Assuming that* $RN(\xi_i) = [\underline{\xi}_i, \bar{\xi}_i]$; $(j = 1, 2, ...,n)$ *is a collection of IRNs in R, and letting* $RN(\xi^-) = [\min \underline{\xi}_i, \min \bar{\xi}_i]$ *and* $RN(\xi^+) = [\max \underline{\xi}_i, \max \bar{\xi}_i]$,*then*

$RN(\xi^-) \leqslant DGBM^{p,q,\rho}(RN(\xi_1), RN(\xi_2), ...,RN(\xi_n)) \leqslant RN(\xi^+)$.

Proof:. Let $RN(\xi^-) = \min(RN(\xi_1), RN(\xi_2), ...,RN(\xi_n)) = [\min \underline{\xi}_i, \min \bar{\xi}_i]$and $RN(\xi^+) = \max(RN(\xi_1), RN(\xi_2), ...,RN(\xi_n)) = [\max \underline{\xi}_i, \max \bar{\xi}_i]$. □

Then, it can be stated that $\underline{\xi}^- = \min_i(\underline{\xi}_i)$, $\bar{\xi}^- = \min_i(\bar{\xi}_i)$, $\underline{\xi}^+ = \max_i(\underline{\xi}_i)$ and $\bar{\xi}^+ = \max_i(\bar{\xi}_i)$. Based on that, the following inequalities can be formulated:

$RN(\xi^-) \leqslant RN(\xi_i) \leqslant RN(\xi^+)$;
$\min_i(\underline{\xi}_i) \leqslant \underline{\xi}_i \leqslant \max_i(\underline{\xi}_i)$;
$\min_i(\bar{\xi}_i) \leqslant \bar{\xi}_i \leqslant \max_i(\bar{\xi}_i)$.

According to the inequalities shown above, it can be concluded that $RN(\xi^-) \leqslant DGBM^{p,q,\rho}(RN(\xi_1), RN(\xi_2), ..,RN(\xi_n)) \leqslant RN(\xi^+)$ holds.

Appendix C

Theorem 5. *Assuming that* $RN(\xi_j) = \left[\underline{\xi}_j, \bar{\xi}_j\right]$;$(j = 1, 2, ...,n)$ *is a collection of GNs in R, then the DNGBM operator is defined as follows:*

$$DNGBM^{p,q,\rho}(RN(\xi_1), RN(\xi_2), ...,RN(\xi_n)) = \frac{1}{p+q}\prod_{i,j=1}^{n}(pRN(\xi_i) + qRN(\xi_j))^{\frac{w_iw_j}{1-w_i}}$$

$$= \left[\sum_{i=1}^{n}\underline{\xi}_i - \frac{\sum_{i=1}^{n}\underline{\xi}_i}{\left(1+\left\{\frac{1}{(p+q)w_iw_j}\frac{1-w_i}{\left(\sum_{\substack{i,j=1\\ i\neq j}}^{n}\frac{1}{p\left(\frac{f(\underline{\xi}_i)}{1-f(\underline{\xi}_i)}\right)^{\rho}+q\left(\frac{f(\underline{\xi}_j)}{1-f(\underline{\xi}_j)}\right)^{\rho}}\right)}\right\}\right)^{1/\rho}}, \sum_{i=1}^{n}\bar{\xi}_i - \frac{\sum_{i=1}^{n}\bar{\xi}_i}{\left(1+\left\{\frac{1}{(p+q)w_iw_j}\frac{1-w_i}{\left(\sum_{\substack{i,j=1\\ i\neq j}}^{n}\frac{1}{p\left(\frac{f(\bar{\xi}_i)}{1-f(\bar{\xi}_i)}\right)^{\rho}+q\left(\frac{f(\bar{\xi}_j)}{1-f(\bar{\xi}_j)}\right)^{\rho}}\right)}\right\}\right)^{1/\rho}}\right] \quad (8)$$

where $w_j \in [0, 1]$ *represents the weight coefficient of* $RN(\xi_i)$, $i = 1, 2, ..., n$, $\sum_{j=1}^{n} w_j = 1$ *and* $f(RN(\xi_i)) = \begin{cases} f(\underline{\xi}_i) = \frac{\underline{\xi}_i}{\sum_{i=1}^{n} \underline{\xi}_i}; \\ f(\bar{\xi}_i) = \frac{\bar{\xi}_i}{\sum_{i=1}^{n} \bar{\xi}_i}. \end{cases}$ *represents a grey function.*

Proof.. We need to prove that Eq. (8) is valid. According to the operational laws of IRNs, we obtain:

$$pRN(\xi_i) = \left[\underline{\xi}_i - \frac{\underline{\xi}_i}{1 + \left\{ p\left(\frac{\underline{\xi}_i}{1 - \underline{\xi}_i}\right)^{\rho} \right\}^{1/\rho}}, \bar{\xi}_i - \frac{\bar{\xi}_i}{1 + \left\{ p\left(\frac{\bar{\xi}_i}{1 - \bar{\xi}_i}\right)^{\rho} \right\}^{1/\rho}} \right]$$

and

$$qRN(\xi_j) = \left[\underline{\xi}_j - \frac{\underline{\xi}_j}{1 + \left\{ q\left(\frac{\underline{\xi}_j}{1 - \underline{\xi}_j}\right)^{\rho} \right\}^{1/\rho}}, \bar{\xi}_j - \frac{\bar{\xi}_j}{1 + \left\{ q\left(\frac{\bar{\xi}_j}{1 - \bar{\xi}_j}\right)^{\rho} \right\}^{1/\rho}} \right]$$

Furthermore, we obtain:

$$pRN(\xi_i) + qRN(\xi_j) = \left[\underline{\xi}_i + \underline{\xi}_j - \frac{\underline{\xi}_i + \underline{\xi}_j}{1 + \left\{ p\left(\frac{\underline{\xi}_i}{1 - \underline{\xi}_i}\right)^{\rho} + q\left(\frac{\underline{\xi}_j}{1 - \underline{\xi}_j}\right)^{\rho} \right\}^{1/\rho}}, \bar{\xi}_i + \bar{\xi}_j - \frac{\bar{\xi}_i + \bar{\xi}_j}{1 + \left\{ p\left(\frac{\bar{\xi}_i}{1 - \bar{\xi}_i}\right)^{\rho} + q\left(\frac{\bar{\xi}_j}{1 - \bar{\xi}_j}\right)^{\rho} \right\}^{1/\rho}} \right]$$

and

$$(pRN(\xi_i) + qRN(\xi_j))^{\frac{w_i w_j}{1 - w_i}} = \left[\frac{\underline{\xi}_i + \underline{\xi}_j}{1 + \left\{ \frac{1}{1 - w_i} \frac{w_i w_j}{p\left(\frac{\underline{\xi}_i}{1 - \underline{\xi}_i}\right)^{\rho} + q\left(\frac{\underline{\xi}_j}{1 - \underline{\xi}_j}\right)^{\rho}} \right\}^{1/\rho}}, \frac{\bar{\xi}_i + \bar{\xi}_j}{1 + \left\{ \frac{1}{1 - w_i} \frac{w_i w_j}{p\left(\frac{\bar{\xi}_i}{1 - \bar{\xi}_i}\right)^{\rho} + q\left(\frac{\bar{\xi}_j}{1 - \bar{\xi}_j}\right)^{\rho}} \right\}^{1/\rho}} \right]$$

Thereafter,

$$\prod_{\substack{i,j=1 \\ i \neq j}}^{n} (pRN(\xi_i) + qRN(\xi_j))^{\frac{w_i w_j}{1 - w_i}} = \left[\frac{\sum_{i=1}^{n} \underline{\xi}_i}{1 + \left\{ \frac{w_i w_j}{1 - w_i} \sum_{\substack{i,j=1 \\ i \neq j}}^{n} \left(\frac{1}{p\left(\frac{f(\underline{\xi}_i)}{1 - f(\underline{\xi}_i)}\right)^{\rho} + q\left(\frac{f(\underline{\xi}_j)}{1 - f(\underline{\xi}_j)}\right)^{\rho}} \right) \right\}^{1/\rho}}, \frac{\sum_{i=1}^{n} \bar{\xi}_i}{1 + \left\{ \frac{w_i w_j}{1 - w_i} \sum_{\substack{i,j=1 \\ i \neq j}}^{n} \left(\frac{1}{p\left(\frac{f(\bar{\xi}_i)}{1 - f(\bar{\xi}_i)}\right)^{\rho} + q\left(\frac{f(\bar{\xi}_j)}{1 - f(\bar{\xi}_j)}\right)^{\rho}} \right) \right\}^{1/\rho}} \right]$$

Therefore,

$$DNGBM_w^{p,q,\rho}(RN(\xi_1), RN(\xi_2), ..., RN(\xi_n)) = \frac{1}{p+q}\prod_{i,j=1}^{n}(pRN(\xi_i) + qRN(\xi_j))^{\frac{w_i w_j}{1-w_i}}$$

$$= \left[\sum_{i=1}^{n}\underline{\xi}_i - \frac{\sum_{i=1}^{n}\underline{\xi}_i}{\left\{1 + \left\{\frac{1}{(p+q)w_i w_j}\frac{1-w_i}{\sum_{i,j=1, i\neq j}^{n}\left(\frac{1}{p\left(\frac{f(\underline{\xi}_i)}{1-f(\underline{\xi}_i)}\right)^{\rho} + q\left(\frac{f(\underline{\xi}_j)}{1-f(\underline{\xi}_j)}\right)^{\rho}}\right)}\right\}\right\}^{1/\rho}}, \sum_{i=1}^{n}\bar{\xi}_i - \frac{\sum_{i=1}^{n}\bar{\xi}_i}{\left\{1 + \left\{\frac{1}{(p+q)w_i w_j}\frac{1-w_i}{\sum_{i,j=1, i\neq j}^{n}\left(\frac{1}{p\left(\frac{f(\bar{\xi}_i)}{1-f(\bar{\xi}_i)}\right)^{\rho} + q\left(\frac{f(\bar{\xi}_j)}{1-f(\bar{\xi}_j)}\right)^{\rho}}\right)}\right\}\right\}^{1/\rho}}\right] \square$$

Appendix D

Theorem 6.. *Assume that*$w_j = (1/n, 1/n, ..., 1/n)$, $(j = 1, 2, ..., n)$. *Then*:

$DNGBM_w^{p,q,\rho}(RN(\xi_1), RN(\xi_2), ..., RN(\xi_n)) = DBM^{p,q,\rho}(RN(\xi_1), RN(\xi_2), ..., RN(\xi_n))$, where: $IGNDBM^{p,q,\rho}$represents the IRN Dombi-Bonferroni mean operator.

Proof.. Since $w_j = (1/n, 1/n, ..., 1/n)$, according to Eq. (8) the following equation can be obtained:

$DNGBM_w^{p,q,\rho}(RN(\xi_1), RN(\xi_2), ..., RN(\xi_n))$=

$$= \left[\sum_{i=1}^{n}\underline{\xi}_i - \frac{\sum_{i=1}^{n}\underline{\xi}_i}{\left\{1 + \left\{\frac{1}{(p+q)n^2}\frac{\frac{n-1}{n}}{\sum_{i,j=1, i\neq j}^{n}\left(\frac{1}{p\left(\frac{f(\underline{\xi}_i)}{1-f(\underline{\xi}_i)}\right)^{\rho} + q\left(\frac{f(\underline{\xi}_j)}{1-f(\underline{\xi}_j)}\right)^{\rho}}\right)}\right\}\right\}^{1/\rho}}, \sum_{i=1}^{n}\bar{\xi}_i - \frac{\sum_{i=1}^{n}\bar{\xi}_i}{\left\{1 + \left\{\frac{1}{(p+q)n^2}\frac{\frac{n-1}{n}}{\sum_{i,j=1, i\neq j}^{n}\left(\frac{1}{p\left(\frac{f(\bar{\xi}_i)}{1-f(\bar{\xi}_i)}\right)^{\rho} + q\left(\frac{f(\bar{\xi}_j)}{1-f(\bar{\xi}_j)}\right)^{\rho}}\right)}\right\}\right\}^{1/\rho}}\right]$$

$$= \left[\frac{\sum_{i=1}^{n}\underline{\xi}_i}{\left\{1 + \left\{\frac{1}{(p+q)}\frac{n(n-1)}{\sum_{i,j=1, i\neq j}^{n}\left(\frac{1}{p\left(\frac{f(\underline{\xi}_i)}{1-f(\underline{\xi}_i)}\right)^{\rho} + q\left(\frac{f(\underline{\xi}_j)}{1-f(\underline{\xi}_j)}\right)^{\rho}}\right)}\right\}\right\}^{1/\rho}}, \frac{\sum_{i=1}^{n}\bar{\xi}_i}{\left\{1 + \left\{\frac{1}{(p+q)}\frac{n(n-1)}{\sum_{i,j=1, i\neq j}^{n}\left(\frac{1}{p\left(\frac{f(\bar{\xi}_i)}{1-f(\bar{\xi}_i)}\right)^{\rho} + q\left(\frac{f(\bar{\xi}_j)}{1-f(\bar{\xi}_j)}\right)^{\rho}}\right)}\right\}\right\}^{1/\rho}}\right]$$

$= DBM^{p,q,\rho}(RN(\xi_1), RN(\xi_2), ..., RN(\xi_n))$ $\square$

Appendix E

(a) Assuming that$RN(\xi_j) = \left[\underline{\xi}_j, \bar{\xi}_j\right]$; $(j = 1, 2, \ldots, n)$ is a collection of IRNs in R and if $p, \rho \geq 0$, $q = 0$, then the IRN Geometric Bonferroni Mean operator can be transformed to Dombi arithmetic mean (DM) operator as follows:

$$D^{p,0,\rho}(RN(\xi_1), RN(\xi_2), \ldots, RN(\xi_n)) = \left(\frac{1}{n}\sum_{i=1}^{n} RN(\xi_i^{p})\right)^{\frac{1}{p}} =$$

$$= \left[\frac{\sum_{i=1}^{n}\underline{\xi}_i}{1+\left\{\frac{1}{p}\frac{1}{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{p\left(\frac{1-f(\underline{\xi}_i)}{f(\underline{\xi}_i)}\right)^{\rho}}}\right\}^{1/\rho}}, \frac{\sum_{i=1}^{n}\bar{\xi}_i}{1+\left\{\frac{1}{p}\frac{1}{\frac{1}{n}\sum_{i=1}^{n}\frac{1}{p\left(\frac{1-f(\bar{\xi}_i)}{f(\bar{\xi}_i)}\right)^{\rho}}}\right\}^{1/\rho}}\right]$$

Proof:. If $q = 0$, the following results are obtained.

$$DGBM^{p,0,\rho}\{RN(\xi_1), RN(\xi_2), \ldots, RN(\xi_n)\} = \left(\frac{1}{n}\sum_{i,j=1}^{n} RN(\xi_i^{p})\left(\prod_{j=1}^{n} RN\left(\xi_j^{0}\right)\right)^{\frac{1}{n-1}}\right)^{\frac{1}{p+0}}$$

$$= \left[\frac{\sum_{i=1}^{n}\underline{\xi}_i}{1+\left\{\frac{1}{p+0}\frac{n}{\frac{1}{\sum_{\substack{i=1\\ i\neq j}}^{n}\left(p\left(\frac{1-f(\underline{\xi}_i)}{f(\underline{\xi}_i)}\right)^{\rho}+\frac{0}{n-1}\sum_{\substack{j=1\\ j\neq i}}^{n}\left(\frac{1-f(\underline{\xi}_j)}{f(\underline{\xi}_j)}\right)^{\rho}\right)}}\right\}^{1/\rho}}, \frac{\sum_{i=1}^{n}\bar{\xi}_i}{1+\left\{\frac{1}{p+0}\frac{n}{\frac{1}{\sum_{\substack{i=1\\ i\neq j}}^{n}\left(p\left(\frac{1-f(\bar{\xi}_i)}{f(\bar{\xi}_i)}\right)^{\rho}+\frac{0}{n-1}\sum_{\substack{j=1\\ j\neq i}}^{n}\left(\frac{1-f(\bar{\xi}_j)}{f(\bar{\xi}_j)}\right)^{\rho}\right)}}\right\}^{1/\rho}}\right]$$

Therefore, we obtain

$$D^{p,0,\rho} = \left(\frac{1}{n}\sum_{i,j=1}^{n} RN(\xi_i^{p})\right)^{\frac{1}{p}}$$

$$= \left[\frac{\sum_{i=1}^{n}\underline{\xi}_i}{1+\left\{\frac{1}{p}\frac{n}{\frac{1}{\sum_{\substack{i=1\\ i\neq j}}^{n}\left(p\left(\frac{1-f(\underline{\xi}_i)}{f(\underline{\xi}_i)}\right)^{\rho}\right)}}\right\}^{1/\rho}}, \frac{\sum_{i=1}^{n}\bar{\xi}_i}{1+\left\{\frac{1}{p}\frac{n}{\frac{1}{\sum_{\substack{i=1\\ i\neq j}}^{n}\left(p\left(\frac{1-f(\bar{\xi}_i)}{f(\bar{\xi}_i)}\right)^{\rho}\right)}}\right\}^{1/\rho}}\right] \quad \square$$

(b) Assuming that$RN(\xi_j) = \left[\underline{\xi}_j, \bar{\xi}_j\right]$; $(j = 1, 2, \ldots, n)$ is a collection of IRNs in R and if $p, \rho \geq 0$, $q = 0$, then the IRN DNGBM operator can be transformed to IRN Dombi generalized weighted geometric operator as follows:

$$DG^{p,0,\rho}(RN(\xi_1), RN(\xi_2), ..., RN(\xi_n)) = \frac{1}{p}\prod_{i=1}^{n}(RN(\xi_i))^{w_i}$$

$$= \left[\sum_{i=1}^{n}\underline{\xi}_i - \frac{\sum_{i=1}^{n}\underline{\xi}_i}{1+\left\{\frac{1}{p}\frac{1}{w_i\sum_{i=1}^{n}\left(\frac{1-f(\underline{\xi}_i)}{f(\underline{\xi}_i)}\right)^{\rho}}\right\}^{1/\rho}}, \sum_{i=1}^{n}\bar{\xi}_i - \frac{\sum_{i=1}^{n}\bar{\xi}_i}{1+\left\{\frac{1}{p}\frac{1}{w_i\sum_{i=1}^{n}\left(\frac{1-f(\bar{\xi}_i)}{f(\bar{\xi}_i)}\right)^{\rho}}\right\}^{1/\rho}}\right]$$

Proof:. If $q = 0$, the following results are obtained.

$$DG^{p,0,\rho}(RN(\xi_1), RN(\xi_2), ..., RN(\xi_n)) = \frac{1}{p+0}\prod_{i,j=1}^{n}(pRN(\xi_i) + 0 \cdot RN(\xi_j))^{\frac{w_iw_j}{1-w_i}}$$

$$= \left[\sum_{i=1}^{n}\underline{\xi}_i - \frac{\sum_{i=1}^{n}\underline{\xi}_i}{1+\left\{\frac{1}{(p+0)w_iw_j}\frac{1-w_i}{\sum_{\substack{i,j=1\\i\neq j}}^{n}\left(\frac{1}{p\left(\frac{f(\underline{\xi}_i)}{1-f(\underline{\xi}_i)}\right)^{\rho}+0\left(\frac{f(\underline{\xi}_j)}{1-f(\underline{\xi}_j)}\right)^{\rho}}\right)}\right\}^{1/\rho}}, \sum_{i=1}^{n}\bar{\xi}_i - \frac{\sum_{i=1}^{n}\bar{\xi}_i}{1+\left\{\frac{1}{(p+0)w_iw_j}\frac{1-w_i}{\sum_{\substack{i,j=1\\i\neq j}}^{n}\left(\frac{1}{p\left(\frac{f(\bar{\xi}_i)}{1-f(\bar{\xi}_i)}\right)^{\rho}+0\left(\frac{f(\bar{\xi}_j)}{1-f(\bar{\xi}_j)}\right)^{\rho}}\right)}\right\}^{1/\rho}}\right]$$

Therefore, we obtain

$$DG^{p,0,\rho}(RN(\xi_1), RN(\xi_2), ..., RN(\xi_n)) = \left[\sum_{i=1}^{n}\underline{\xi}_i - \frac{\sum_{i=1}^{n}\underline{\xi}_i}{1+\left\{\frac{1}{p}\frac{1}{w_i\sum_{i=1}^{n}\left(\frac{1-f(\underline{\xi}_i)}{f(\underline{\xi}_i)}\right)^{\rho}}\right\}^{1/\rho}}, \sum_{i=1}^{n}\bar{\xi}_i - \frac{\sum_{i=1}^{n}\bar{\xi}_i}{1+\left\{\frac{1}{p}\frac{1}{w_i\sum_{i=1}^{n}\left(\frac{1-f(\bar{\xi}_i)}{f(\bar{\xi}_i)}\right)^{\rho}}\right\}^{1/\rho}}\right] \square$$

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