Research Article

A Comparative Study between GLP and GBWM

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When decision-makers' judgments are uncertain, they often express their opinions using grey linguistic variables. Once used, the data often retains its grey nature throughout all subsequent decision-making iterations. Multicriteria decision-making (MCDM) is a tool used when making complicated decisions and in circumstances where several criteria require evaluation to choose the most desirable option. Grey data serves as the basis for several MCDM methods. This paper compares two MCDM methods, Grey-Linear-Programming (GLP) and Grey-Best-Worst-Method (GBWM), in terms of the weights of decision criteria and their rankings. Moreover, Grey-The Technique for Order of Preference by Similarity to Ideal Solution (GTOPSIS) was used to rank the weights of the two methods. Study findings demonstrated that GBWM requires more mathematical calculations than GLP, based on linear programming's classic simplex method. On the other hand, when GTOPSIS follows GLP, the alternative rank does not change compared to when GTOPSIS followed GBWM. For the applications used in this comparison, GLP procedure is considered simpler than GBWM procedure.

1. Introduction

Recently, decision-makers have introduced many new techniques and methods into their use of MCDM for complex decisions. Among these techniques, the new Best-Worst-Method (BWM) has captured many scholars' attention due to its ability to determine criteria weights using only two vectors. The authors of [1, 2] proposed a BWM as a structured technique to analyze complex decisions using mathematics and psychology. The method provides decision-makers (DMs) the choice of the best and worst criteria.

However, if decision-makers' judgments are uncertain, the values represented cannot be deemed accurate [3]. One possible method to overcome the uncertainty is to collect decision-makers' opinions based on grey linguistic variables [4]. Consequently, the data retains its grey nature throughout the decision-making process. Examples of methods and techniques to solve the uncertainty of decision-makers include Grey-Linear-Programming (GLP), Grey-Analytical-Hierarchy-Process (GAHP), Grey-The Technique for Order of Preference by Similarity to Ideal Solution (GTOPSIS), and Grey-Best-Worst-Methods (GBWM) [5–7].

This paper compares the effect of using GLP and GBWM criteria weights on ranking the decision alternatives produced by the GTOPSIS technique. The study uses data from five case studies in the literature for comparison. The question under investigation is as follows: which method is superior, GLP, or GBWM, without impacting the final decision alternatives?

This paper is organized as follows: Section 2 explores GLP and GBWM methods and their uses in the literature. Section 3 describes the GLP procedure. Section 4 illustrates the use of GLP and GBWM techniques in calculating the criteria weights using five different case studies from the literature. Moreover, alternatives ranking comparisons using integrated GLP-GTOPSIS and GBWM-GTOPSIS are presented in Section 4. The last section contains the conclusions of the study.

2. Literature Review

Over the past several decades, MCDM has experienced extraordinary growth to solve complex decisions in numerous areas. Consequently, its role in various fields of

application has increased dramatically, especially with the development of new methods and the improvement of old methods.

One of the developed methods is GLP, derived from linear programming (LP), a technique used to guide quantitative decisions. The model rests on the use of a linear function that is minimized or maximized by various constraints. In other terms, analysts use the method to represent complex relationships through a linear function and calculate the optimum value [8]. Even though a given problem may be complex, LP simplifies the problem through a linear function. The method is useful in guiding decisions in engineering and the social and physical sciences. LP's popularity rests on its use of optimization theory concepts such as decomposition, duality, generalizations, and the importance of convexity [9].

Paper [5] developed GLP as a new technique to solve problems through the whitenization of grey numbers (GNs) using the standard theory of ordinary LP methodology. GLP is a grey systems analysis model for decision-making in the presence of uncertainty. As an advancement of the conventional LP technique, a Grey LP problem differs from a typical problem. Its objective function's coefficients and technological constants and the coefficients of its constraints are grey rather than real numbers.

GBWM, on the other hand, is a new technique added to MCDM. Created for multiple-criteria, multiple-expert decision-making problems, GBWM can use grey linguistic variables as input data for covering uncertainty. The study in [2] proposed ordinary BWM, which requires fewer comparisons on criteria than the Analytical Hierarchy Process (AHP) method [10]. BWM can also deduce more consistent weights due to fewer comparisons than AHP [2]. Moreover, BWM can be considered as a robust approach for multi-criteria decision-making and defining criteria weights used for evaluating alternatives based on a set of decision criteria [6]. In BWM, the decision-maker identifies two criteria (the best-to-others and the others-to-worst) once the decision criteria are identified: the best-to-others criterion plays the most significant role in the decision-making; the others-to-worst criterion holds the least significant role [6]. Subsequently, the decision-maker specifies the best criterion over all other criteria and their preferences of the worst criterion over the other criteria using a predefined scale of numbers (1 to 9). Therefore, the higher number in the best-to-others vector indicates the importance of the best criterion over all other criteria. Likewise, the higher number in the other-to-worst vector indicates the importance of the other criteria over the worst criterion.

The study in [6] proposed GBWM, to use grey linguistic variables as input data to address the uncertainty problems associated unknown and incomplete data. The GBWM deploys a linear model capable of presenting global-optimum weights for MCDM problems. GBWM can define criteria weights with grey numbers and render a crisp solution using experts' needs. It systematically compares pairs of decision criteria.

The authors of [7] developed GTOPSIS as an effective technique for measuring and ranking value chain performance and ranking using quantitative criteria and determining the ranking order of different firms' performance. Moreover, researchers have established the GTOPSIS method as valuable for enabling MCDM under vagueness and uncertainty [7]. Thus, GTOPSIS is well-suited to group decision-making problems in an uncertain environment. Researchers stress that this method will be of particular value to managers in resolving decision-making problems in their companies in the future [7].

3. The Proposed GLP Methodology

In this section, the GLP procedure will be illustrated while GBWM and GTOPSIS were thoroughly explained by [6, 7], respectively.

GNs are an effective tool to handle grey system data because a GN is an indeterminate number whose probable range is known but lacks a defined position within its boundaries [5]. Therefore, a GN, say A , can be expressed mathematically as $A \in [a, b] = \{x \in R: a \leq x \leq b\}$ where R is the set of the real numbers. If $a = b$, then A is called a white number, and if $A \in (-\infty, +\infty)$, then A is called a black number.

Based on the definition of GN, the GLP technique is expressed in the following steps.

Step 1. Let $w(A)$ be the white number with the highest probability to be the representative value of the GN $A [a, b]$. The technique of determining the value of $w(A)$ is called whitenization of A . One usually defines $w(A) = (1 - t)a + tb$, with t in $[0, 1]$. This is known as the equal weight whitenization. When the distribution of A is unknown, one takes $t = 1/2$, which gives that $w(A) = (a + b)/2$.

The authors of [5, 11] presented general facts on the whitenization of GNs.

Step 2. Use the general form of GLP as follows:

$$\begin{aligned} &\text{maximize or minimize the linear expression, } F = A_1x_1 + A_2x_2 + \cdots + A_nx_n, \\ &\text{subject to constraints of the form: } A_{i1}x_1 + A_{i2}x_2 + \cdots + A_{in}x_n \leq (\geq) B_i, \end{aligned} \quad (1)$$

where $i = 1, 2, \dots, m, j = 1, 2, \dots, n$,

$$x_j > 0, \quad (2)$$

and A_j, A_{ij}, B_i are GNs.

Step 3. Convert the values of the decision variables in the optimal solution to GNs with the desired Rank of Greyness (RoG). The GN $A[a, b]$ has RoG equal to x , with x in R , if and only if $b - a = x$. Then, $\text{RoG}(A) = x$. Therefore, for a white number A we have $\text{RoG}(A) = 0$. Based on [5], the smaller the value of the chosen RoG, the more creditable the vague expression of the problem's optimal solution. Note that if the values $w(A) = y$ and $\text{RoG}(A) = x$ are known, then the GN $A \in [a, b]$ can be easily determined by solving the system of equations

$$\begin{aligned} w(A) &= y, \\ b - a &= x. \end{aligned} \quad (3)$$

For example, if $w(A) = 4$ and $\text{RoG}(A) = 2$, then $a + b = 8$ and $b - a = 2$, which gives that $a = 3$ and $b = 5$.

4. Numerical Examples

This section compares GLP and GBWM techniques using five different case studies from the literature. The first three cases derived data from [6], while data for the fourth and fifth cases were extracted from [12, 13]. Example 1 presents the weights ranking comparison between GLP and GBWM. Alternatives ranking comparisons between GLP-GTOPSIS and GBWM-GTOPSIS are presented in examples 2 and 3, respectively.

4.1. Example 1. This example uses secondary data derived from a previous study [6]. The collected data for an MCDM problem related to an automobile purchase includes the following criteria: price, quality, comfort, safety, and style. Three experts comprised the group for decision-making. Table 1 presents the evaluation linguistic variables scale for this problem. Tables 2 and 3 present the degree of performance best to others and others to worst using GLP. Table 4 presents a comparative ranking between GBWM and GLP. Figure 1 shows a comparison consistency ratio length between GLP and GBWM methods.

Using the GLP method to rank criteria gave identical ranking results derived from GBWM. Moreover, the consistency ratio length in Figure 1 shows that the GLP is more precise than GBWM.

4.2. Example 2. This example consists of secondary data from a previous study [12]. The MCDM problem seeks the selection of a socially sustainable supplier for an Iranian manufacturing company. This problem involved ten experts; the study in [12] supplied complete data for one expert. Table 5 presents the evaluation linguistic variables scale for this problem. The degree of performance of best-to-others and others-to-worst using GLP is shown in Tables 6 and 7.

TABLE 1: Linguistic variables of decision-makers.

Grey BWM			Grey LP
Equally important	EI	[1, 1]	1
Weakly important	WI	[2/3, 3/2]	1.083
Fairly important	FI	[3/2, 5/2]	2
Very important	VI	[5/2, 7/2]	3
Absolutely important	AI	[7/2, 9/2]	4

TABLE 2: The degree of preferences of the best criterion to the other criteria.

Best-to-others		Price	Quality	Comfort	Safety	Style
Expert 1	Best criterion: price	EI	WI	VI	VI	AI
		1	1.083	3	3	4
Expert 2	Best criterion: price	EI	FI	AI	AI	VI
		1	2	3	4	3
Expert 3	Best criterion: quality	WI	EI	AI	VI	AI
		1.083	1	4	3	4

Table 8 provides a comparative ranking between GBWM and GLP. The decision matrix for the GTOPSIS method and final alternative ranking results are shown in Tables 9 and 10.

Table 10 provides a comparison of the two methods for ranking the alternatives. The final ranking results were identical in the table, regardless of the order of criteria weights in Table 8.

Seven criteria were included in this study as follows: Work Health and Safety (WHS), Training Education and Community Influence (TECI), Contractual Stakeholders Influence (CSI), Occupational Health and Safety Management System (OHSMS), the Interests and Rights of Employees (IRE), the Rights of Stakeholders (RS), Information Disclosure (ID), and Employment Practices (EP).

4.3. Example 3. The data in this example are secondary data from a previous study [13]. The MCDM problem is to select a suitable medical waste disposal location for Madrid General Hospital. The study in [13] thoroughly explained the evaluation scale and the best-to-others and others-to-worst vectors. Table 11 presents the criteria's optimal weights showing that the dimension of level one criteria ranking result using the GLP method resulted in the same ranking result as the GBWM method. The sub-criteria ranking using the GLP method resulted in only 36% of the same sub-criteria ranking as the GBWM method.

The criteria weights are derived from GLP method used in GTOPSIS for dual experts. Decision matrix for GTOPSIS is presented in Table 12. The weighted normalized weight and final evaluation and ranking of alternatives are presented in Tables 13 and 14, respectively.

The alternative ranking result using the GLP-GTOPSIS method is identical in the first two positions compared to GBWM-GTOPSIS. There was a slight difference in the middle positions, which may not be of much significance since the best alternative is always a preferred option.

TABLE 3: The degree of preferences of each criterion to the worst criterion.

Others-to-worst						
Expert 1			Expert 2		Expert 3	
	Worst criterion: style		Worst criterion: safety		Worst criterion: comfort	
Price	AI	4	AI	4	VI	3
Quality	VI	3	AI	4	AI	4
Comfort	FI	2	WI	1.083	EI	1
Safety	FI	2	EI	1	FI	2
Style	EI	1	FI	2	WI	1.083

TABLE 4: A comparative ranking.

Expert 1						
Criteria	GWBM paper (2020)			This paper (GLP)		
	Grey weights		Rank	Grey weights		Rank
Price	0.264	0.400	1	0.307	0.407	1
Quality	0.244	0.373	2	0.244	0.344	2
Comfort	0.104	0.160	3	0.081	0.181	3
Safety	0.104	0.160	3	0.081	0.181	3
Style	0.081	0.106	4	0.035	0.135	4
Consistency ratio = 0–0.0519			Consistency ratio = 0.0167–0.0384			
Expert 2						
Criteria	GWBM paper (2020)			This paper (GLP)		
	Grey weights		Rank	Grey weights		Rank
Price	0.333	0.457	1	0.350	0.450	1
Quality	0.177	0.318	2	0.190	0.290	2
Comfort	0.098	0.136	4	0.070	0.170	4
Safety	0.063	0.096	5	0.030	0.130	5
Style	0.126	0.191	3	0.110	0.210	3
Consistency ratio = 0.0106–0.0561			Consistency ratio = 0.0450–0.0565			
Expert 3						
Criteria	GWBM paper (2020)			This paper (GLP)		
	Grey weights		Rank	Grey weights		Rank
Price	0.251	0.310	2	0.254	0.354	2
Quality	0.272	0.466	1	0.319	0.419	1
Comfort	0.083	0.103	5	0.038	0.138	5
Safety	0.186	0.107	3	0.086	0.186	3
Style	0.133	0.083	4	0.052	0.152	4
Consistency ratio = 0–0.0535			Consistency ratio = 0.0176–0.0391

FIGURE 1: Consistency ratio length of GBWM vs. GLP.

TABLE 5: Linguistic variables of decision-makers.

Grey BWM			Grey LP
Equally importance	EqI	[1, 1]	1
Weak importance	WI	[1, 5/2]	1.75
Little importance	LI	[5/2, 7/2]	3
Moderate importance	MI	[7/2, 9/2]	4
Moderate plus importance	MpI	[9/2, 11/2]	5
Strong importance	SI	[11/2, 13/2]	6
Strong plus importance	SpI	[13/2, 15/2]	7
Very strong importance	VsI	[15/2, 17/2]	8
Extreme importance	ExI	[17/2, 20/2]	9.25

TABLE 6: The degree of preferences of the best criterion to the other criteria.

Best-to-others		WHS	TECI	CSI	OHSMS	IRE	RS	ID	EP
Expert 1	Best criterion: EP	LI 3	MI 4	LI 3	WI 1.75	SI 6	MpI 5	MI 4	EqI 1

TABLE 7: The degree of preferences of each criterion to the worst criterion.

Others-to-worst				Expert 1	
				Worst criterion: IRE	
WHS			LI		3
TECI			WI		1.75
CSI			LI		3
OHSMS			MI		4
IRE			EqI		1
RS			LI		3
ID			MI		4
EP			SI		6

TABLE 8: A comparative ranking result.

Expert 1						
Criteria	GWBM paper (2020)			This paper (GLP)		
	Grey weights		Rank	Grey weights		Rank
WHS	0.480	0.660	4	0.066	0.166	4
TECI	0.240	0.540	6	0.037	0.137	6
CSI	0.480	0.660	3	0.066	0.166	3
OHSMS	0.670	0.750	2	0.149	0.249	2
IRE	0.130	0.140	8	0.000	0.087	7
RS	0.200	0.340	7	0.019	0.119	8
ID	0.270	0.490	5	0.037	0.137	5
EP	0.960	1.000	1	0.236	0.336	1

TABLE 9: Decision matrix for GTOPSIS method.

Criteria	Supplier 1		Supplier 2		Supplier 3		Supplier 4		Supplier 5	
WHS	4	6	6	8	0	2	4	6	2	4
TECI	0	2	8	10	6	8	2	4	4	6
CSI	4	6	6	8	4	6	6	8	0	2
OHSMS	2	4	4	6	8	10	0	2	2	4
IRE	8	10	8	10	2	4	2	4	8	10
RS	8	10	6	8	8	10	8	10	6	8
ID	0	2	4	6	4	6	2	4	4	6
EP	2	4	2	4	2	4	0	2	2	4

TABLE 10: A comparative of the final alternatives results from GBWM-GTOPSIS vs. GLP-GTOPSIS.

Alternatives	Expert 1							
	GBWM paper (2019)				This paper (GLP)			
	S^+	S^-	P_i	Rank	S^+	S^-	P_i	Rank
Supplier 1	0.501	0.666	0.484	2	0.218	0.259	0.543	2
Supplier 2	0.493	0.682	0.580	1	0.198	0.286	0.590	1
Supplier 3	0.607	0.554	0.476	3	0.251	0.226	0.473	3
Supplier 4	0.666	0.515	0.435	5	0.281	0.202	0.418	5
Supplier 5	0.647	0.504	0.438	4	0.268	0.199	0.426	4

TABLE 11: Comparative criteria weight ranking result.

Criteria	GBWM technique				GLP technique			
	Weight of level #1	Weight of level #2	Overall weight	Rank	Weight of level #1	Weight of level #2	Overall weight	Rank
D1	[0.260–0.261]			2	[0.261–0.361]			2
C1		[0.619–0.619]	[0.161–0.161]	3		[0.437–0.537]	[0.114–0.194]	1
C2		[0.274–0.476]	[0.071–0.124]	4		[0.284–0.384]	[0.074–0.139]	5
C3		[0.105–0.171]	[0.027–0.044]	9		[0.127–0.227]	[0.033–0.082]	9
D2	[0.618–0.624]			1	[0.449–0.549]			1
C4		[0.101–0.112]	[0.062–0.069]	6		[0.158–0.258]	[0.071–0.142]	4
C5		[0.128–0.167]	[0.079–0.104]	5		[0.106–0.206]	[0.047–0.113]	7
C6		[0.067–0.071]	[0.041–0.044]	8		[0.086–0.186]	[0.039–0.102]	8
C7		[0.303–0.363]	[0.187–0.226]	1		[0.183–0.283]	[0.082–0.155]	3
C8		[0.279–0.285]	[0.172–0.178]	2		[0.215–0.315]	[0.096–0.173]	2
D3	[0.096–0.114]			3	[0.138–0.238]			3
C9		[0.174–0.290]	[0.016–0.033]	11		[0.226–0.326]	[0.031–0.078]	11
C10		[0.242–0.242]	[0.023–0.027]	10		[0.395–0.495]	[0.054–0.118]	6
C11		[0.582–0.582]	[0.056–0.066]	7		[0.227–0.327]	[0.031–0.078]	10

TABLE 12: Decision matrix for GTOPSIS method.

Criteria	Weight	A1	A2	A3	A4	A5
Decision-maker #1						
C1	0.114–0.194	5.500–6.000	3.500–4.000	2.000–2.000	1.500–2.000	4.000–5.000
C2	0.074–0.139	3.000–3.500	3.000–3.000	2.500–4.000	2.000–4.000	3.000–3.000
C3	0.033–0.082	4.000–5.000	3.000–5.000	1.000–4.000	4.000–5.000	2.500–3.000
C4	0.071–0.142	5.000–5.000	3.500–5.000	3.000–3.500	2.000–3.000	1.000–2.500
C5	0.047–0.113	4.000–5.000	1.000–2.000	4.500–5.000	3.500–5.000	2.500–3.000
C6	0.039–0.102	2.500–4.000	6.500–7.000	5.000–5.000	6.000–7.000	3.000–4.000
C7	0.082–0.155	5.000–5.000	3.000–3.000	1.500–2.000	2.500–3.000	3.000–4.000
C8	0.096–0.173	1.000–2.000	7.500–8.000	4.500–5.000	1.000–2.000	2.000–2.000
C9	0.031–0.077	4.000–4.000	2.000–2.500	5.000–7.000	1.000–2.500	5.000–5.000
C10	0.054–0.118	1.000–1.500	5.000–5.000	4.000–5.000	4.000–6.000	2.000–2.000
C11	0.031–0.078	6.000–6.500	3.000–3.000	1.500–2.000	4.500–5.000	1.000–2.000
Decision-maker #2						
C1	0.114–0.194	5.000–5.500	3.000–3.500	2.000–2.000	1.000–1.500	3.000–4.000
C2	0.074–0.139	3.500–4.000	3.000–3.000	1.000–2.500	4.000–6.000	3.000–3.000
C3	0.033–0.082	3.000–4.000	5.000–5.000	4.000–7.000	4.000–5.000	2.000–2.500
C4	0.071–0.142	5.000–5.000	2.000–3.500	3.500–4.000	3.000–4.000	2.500–4.000
C5	0.047–0.113	3.000–4.000	2.000–3.000	4.000–4.500	2.000–3.500	2.000–2.500
C6	0.039–0.102	1.000–2.500	6.000–6.500	5.000–5.000	5.000–6.000	4.000–5.000
C7	0.082–0.155	5.000–5.000	3.000–3.000	1.000–1.500	2.000–2.500	2.000–3.000
C8	0.096–0.173	2.000–3.000	7.000–7.500	4.000–4.500	2.000–3.000	2.000–2.000
C9	0.031–0.077	4.000–4.000	2.500–3.000	3.000–5.000	2.500–4.000	5.000–5.000
C10	0.054–0.118	1.500–2.000	5.000–5.000	3.000–4.000	6.000–8.000	2.000–2.000
C11	0.031–0.078	6.500–7.000	3.000–3.000	1.000–1.500	4.000–4.500	2.000–3.000

TABLE 13: The weighted normalized decision matrix.

Criteria	A1	A2	A3	A4	A5	Z^+	Z^-
Decision-maker #1							
C1	0.105–0.194	0.066–0.129	0.038–0.064	0.028–0.064	0.076–0.162	1	0.250
C2	0.055–0.121	0.055–0.104	0.046–0.139	0.037–0.139	0.055–0.104	1	0.500
C3	0.022–0.068	0.027–0.068	0.005–0.054	0.027–0.082	0.013–0.041	1	0.166
C4	0.071–0.142	0.050–0.142	0.042–0.099	0.028–0.085	0.014–0.071	1	0.200
C5	0.038–0.113	0.009–0.045	0.042–0.113	0.033–0.113	0.023–0.067	1	0.200
C6	0.013–0.058	0.036–0.102	0.027–0.073	0.033–0.102	0.016–0.058	1	0.357
C7	0.082–0.155	0.049–0.093	0.024–0.062	0.041–0.093	0.049–0.124	1	0.300
C8	0.012–0.433	0.090–0.173	0.054–0.108	0.012–0.043	0.024–0.043	1	0.125
C9	0.017–0.044	0.008–0.027	0.022–0.077	0.004–0.027	0.022–0.055	1	0.142
C10	0.009–0.029	0.045–0.098	0.036–0.098	0.036–0.118	0.018–0.039	1	0.166
C11	0.029–0.078	0.014–0.036	0.007–0.024	0.021–0.060	0.004–0.024	1	0.153
Decision-maker #2							
C1	0.104–0.194	0.062–0.123	0.042–0.070	0.020–0.053	0.062–0.141	1	0.181
C2	0.043–0.092	0.037–0.069	0.012–0.057	0.049–0.139	0.037–0.069	1	0.166
C3	0.014–0.047	0.023–0.058	0.019–0.082	0.019–0.058	0.009–0.029	1	0.285
C4	0.071–0.142	0.028–0.099	0.050–0.113	0.042–0.113	0.035–0.113	1	0.400
C5	0.031–0.100	0.021–0.075	0.042–0.113	0.042–0.113	0.021–0.062	1	0.444
C6	0.006–0.039	0.036–0.102	0.030–0.078	0.030–0.078	0.024–0.078	1	0.153
C7	0.082–0.155	0.049–0.093	0.016–0.046	0.032–0.077	0.032–0.093	1	0.200
C8	0.025–0.069	0.090–0.173	0.051–0.104	0.025–0.069	0.025–0.046	1	0.266
C9	0.025–0.062	0.015–0.046	0.018–0.077	0.015–0.062	0.031–0.077	1	0.500
C10	0.010–0.029	0.034–0.073	0.020–0.059	0.041–0.118	0.013–0.029	1	0.187
C11	0.029–0.078	0.013–0.033	0.004–0.016	0.018–0.050	0.009–0.033	1	0.142

TABLE 14: Final evaluation and ranking of alternatives using the GLP weights.

Alternative	S^+	S^-	P_i	Rank (GLP)	Rank (GBWM)
A1	0.349	0.443	0.5644	2	2
A2	0.323	0.371	0.5649	1	1
A3	0.448	0.262	0.4192	4	3
A4	0.416	0.331	0.4383	3	4
A5	0.440	0.262	0.3818	5	5

TABLE 15: Summary comparison between the two methods.

Application	GLP	GBWM	GLP-GTOPSIS	GBWM-GTOPSIS
[6]	Simplicity Requires less time Precise	Requires more time		
[12]			Straightforward calculations	Complex calculations
[13]			Easier calculations	Complicated calculations Sensitive

5. Discussion and Conclusion

There are several useful MCDM methods used for problems within an environment of uncertainty and vagueness. Uncertainty problems drive the use of grey numbers. This paper compared two MCDM methods, GLP and GBWM, in terms of the weights of decision criteria and their rankings in five different cases. In the first three cases, the weights ranking comparison was identical regardless of the differences in the values.

In the fourth case, the comparison used GLP, followed by GTOPSIS. The alternative ranking in GLP followed by GTOPSIS remained unchanged regardless of the differences in values when GTOPSIS followed GBWM.

In the last case, the comparison used GLP, followed by GTOPSIS. The rankings of the first two alternatives were identical. However, there was a slight change in the alternatives ranking in the middle positions, which is not of great importance, given that the first two ranking positions were identical.

Our findings showed that GBWM requires more mathematical calculations compared to GLP based on the classical simplex method of linear programming. Table 15 summarizes the differences between the two methods. The methods used in the five cases were having integral, derivative, and complex calculations. However, the GLP method is much simpler than the methods used in all five cases. GLP used the simplex method and yielded the same final ranking/weight result.

As shown in Table 15, using GLP is simpler than GBWM and has more straightforward calculations requiring less time. On the other hand, when GTOPSIS followed GLP, calculating the alternative ranking took less time than GBWM-GTOPSIS. The advantages of using GLP are clear, simpler, and computationally more efficient than GBWM for the applications used in this comparison. Moreover, using GLP gave identical results in the first, second, and last positions in all cases included in this paper. Only the middle positions in the last case observed slight changes which do not affect the choice of the best alternative. Therefore, GLP is superior to the three different methodologies used in the five cases that used complex integrative computations.

Data Availability

The data were extracted from three different papers, and a copy of each paper is attached in the supplemental files. The three papers are as follows: 1- Mahmoudi, A., Mi, X., Liao, H., Feylizadeh, M. R.; Turskis, Z. (2020). Grey Best-Worst Method for Multiple Experts Multiple Criteria Decision Making under Uncertainty. *Informatica* (Netherlands), 31 (2), 331–357. <https://doi.org/10.15388/20-INFOR409> 2- Bai, C., Kusi-Sarpong, S., Badri Ahmadi, H.; Sarkis, J. (2019). Social sustainable supplier evaluation and selection: a group decision-support approach. *International Journal of Production Research*, 57 (22), 7046–7067. <https://doi.org/10.1080/00207543.2019.1574042> 3- Yazdani, M., Tavana, M., Pamučar, D.; Chatterjee, P. (2020). A rough based multi-criteria evaluation method for healthcare waste disposal location decisions. *Computers and Industrial Engineering*, 143 (March), 106394. <https://doi.org/10.1016/j.cie.2020.106394>.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Supplementary Materials

1. The data in example 1 were derived from [6]. The data are shown in Table 1 to Table 4. Table 1: linguistic variables of decision-makers (based on [6]). Table 2: the degree of preferences of the best criterion to the other criteria (based on [6]). Table 3: the degree of preference of each criterion to the worst criterion (based on [6]). Table 4: the weight of each criterion based on the opinions of decision-makers (based on [6]). 2. The data in example 2 were derived from [12]. The data are shown in Table 5 to Table 9. Table 5: the best and worst attributes determined by experts 1–10 (based on [12]).

Table 6: the linguistic responses and grey number of the best-to-others evaluation matrix for Expert 1 (based on [12]). Table 7: the linguistic responses and grey number of the others-to-worst evaluation matrix for Expert 1 (based on [12]). Table 8: the linguistic responses and grey number of the others-to-worst evaluation matrix for Expert 1 (based on [12]). Table 9: decision matrix for GTOPSIS method (based on [12]). 3. The data in example 2 were derived from [13]. The data are shown in Table 10 to Table 13. Table 10: the best-to-others (BO) and the others-to-worst (OW) vectors obtained by the experts' evaluations (based on [13]). Table 11: the optimal values (weights) of the criteria (based on [13]). Table 12: decision matrix for GTOPSIS method (based on [13]). Table 13: the rank of the alternatives (based on [13]). (*Supplementary Materials*)

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