Research Article

The Analysis of a Cracked Material under Combined Unsymmetric Thermal Flux and Symmetric Linear Mechanical Loading

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An improved partially permeable crack model is put forward to deal with the problem of a single crack embedded in an orthotropic or isotropic material under combined unsymmetric thermal flux and symmetric linear mechanical loading. With the application of the Fourier transform technique (FTT), the thermoelastic field is given in a closed form. Numerical results show combined unsymmetric linear thermal flux, symmetric linear mechanical loading, and dimensionless thermal conductivity, and the coefficient has influences on fracture parameters. For the improved partially permeable crack, the mode II stress intensity factor and the energy release rate might be zero or positive under combined unsymmetric thermal flux and symmetric linear mechanical loading. Therefore, closure of the crack tip region need not be considered under combined unsymmetric thermal flux and symmetric linear mechanical loading when making use of fracture parameters as a criterion.

1. Introduction

An elastic solid's expansion and contraction from changes in temperature is inhibited for the external constraints and the mutual constraints between the internal parts of the solid, resulting in thermal stress [1, 2]. So, it is essential to address the thermoelastic field of cracked solid in practical engineering [3–5]. A great many of studies have been published to consider the fracture behaviors of various cracks which are embedded in an orthotropic solid under thermal loading [6–14]. By making use of the J-integral, Wilson and Yu obtained the stress intensity factor [11]. Chen and Zhang addressed the thermoelasticity problem of cracked orthotropic solid [12]. Rizk investigated the problem of the cracked orthotropic semifinite material [15]. Wu et al. applied the Fourier transform technique to compute the II stress intensity factors of two collinear cracks under anti-symmetrical linear thermal flux [16]. Afterwards, Wu et al. gave the solution of a single crack in the closed form under

quadratic thermomechanical loading by using the Fourier transform technique [17].

In the abovementioned studies, the partially permeable crack model becomes more widely available [18–21]. According to the mathematical intuition, an improved partially permeable crack model in Figure 1(a) can be raised as

$$Q_{1c} = -h_c \Delta T + \varepsilon Q_1, \quad (1)$$

where Q_{1c} and Q_1 denote the heat flux per thickness to the crack surface and initial heat flux; h_c stands for the thermal conductivity on the crack region. The coefficient ε is regarded as a constant and in line with the complex and real situation, i.e., the nonoptimal cases of complicated crack surface. In general, ε proves to be negative or positive relying on the thermoelastic field.

An improved partially permeable crack model is presented to consider the problem of a single crack embedded in an orthotropic or isotropic material under combined unsymmetric linear thermal flux and symmetric linear

FIGURE 1: (a) A single crack embedded in an orthotropic or isotropic material under unsymmetric linear thermal flux. (b) A single crack embedded in an orthotropic or isotropic material under symmetric linear mechanical loading.

mechanical loading. With application of the Fourier transform technique, the thermoelastic partial differential equations (PDE) are converted to singular integral equations. The thermoelastic field is given in the closed form by solving singular integral equations. The obtained results reveal physical quantities that have influences on fracture parameters for cracked solid under combined unsymmetric linear thermal flux and symmetric linear mechanical loading.

2. Problem Statement

A single crack embedded in an orthotropic or isotropic material under combined unsymmetric linear thermal flux and symmetric linear mechanical loading is shown in Figures 1(a) and 1(b).

The constitute equations are given based on the state of plane stress [22].

$$\sigma_{xx} = c_{11} \frac{\partial u}{\partial x} + c_{12} \frac{\partial v}{\partial y} - \beta_1 T, \quad (2)$$

$$\sigma_{yy} = c_{12} \frac{\partial u}{\partial x} + c_{22} \frac{\partial v}{\partial y} - \beta_2 T, \quad (3)$$

$$\sigma_{xy} = c_{66} \left[\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right], \quad (4)$$

where

$$\begin{aligned} c_{11} &= \frac{E_{xx}}{1 - \nu_{xy}\nu_{yx}}, \\ c_{22} &= \frac{E_{yy}}{1 - \nu_{xy}\nu_{yx}}, \\ c_{12} &= \frac{E_{xx}\nu_{yx}}{1 - \nu_{xy}\nu_{yx}} = \frac{E_{yy}\nu_{xy}}{1 - \nu_{xy}\nu_{yx}}, \\ \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} &= \begin{bmatrix} c_{11} & c_{12} \\ c_{12} & c_{22} \end{bmatrix} \begin{bmatrix} \alpha_{xx} \\ \alpha_{yy} \end{bmatrix}, \end{aligned} \quad (5)$$

where σ_{yy} and σ_{xy} are the components of stress, E_{xx} and E_{yy} represent Young's moduli, T is the temperature, ν_{xx} and ν_{yy} denote Poisson's ratios, u and v indicate the components of elastic displacement, $c_{66} = G_{xy}$ represents the shear modulus, and α_{xx} and α_{yy} are the coefficients of linear expansion. Using the following equation,

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xy}}{\partial y} = 0, \quad (6)$$

$$\frac{\partial \sigma_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} = 0,$$

one obtains

$$c_{11} \frac{\partial^2 u}{\partial x^2} + c_{66} \frac{\partial^2 u}{\partial y^2} + (c_{12} + c_{66}) \frac{\partial^2 v}{\partial x \partial y} = \beta_1 \frac{\partial T}{\partial x}, \quad (7)$$

$$c_{66} \frac{\partial^2 v}{\partial x^2} + c_{22} \frac{\partial^2 v}{\partial y^2} + (c_{12} + c_{66}) \frac{\partial^2 u}{\partial x \partial y} = \beta_2 \frac{\partial T}{\partial y}. \quad (8)$$

Based on the Fourier heat conduction, one has

$$Q_x = -\lambda_x \frac{\partial T}{\partial x}, \quad (9)$$

$$Q_y = -\lambda_y \frac{\partial T}{\partial y},$$

where Q_x and Q_y are the components of the heat flux; λ_x and λ_y are the coefficient of heat conduction. Furthermore, based on equilibrium equation, one has

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = 0. \quad (10)$$

Using the thermal equilibrium equation, one has

$$\lambda^2 \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0, \quad (11)$$

where

$$\lambda = \sqrt{\frac{\lambda_x}{\lambda_y}}. \quad (12)$$

In this study, the crack-face boundary conditions are used as follows.

$$Q_y^I(x, 0) - Q_y^{II}(x, 0) = \frac{Q_1 x}{2a^2}, \quad -a < x < a, \quad (13a)$$

$$\sigma_{yy}^{II}(x, 0) = \sigma_{yy}^I(x, 0) = -\frac{\sigma_1}{a}|x|, \quad -a < x < a. \quad (13b)$$

The superscripts *I* and *II* denote the physical quantities of $y > 0$ and $y < 0$ region, respectively. Q_1 and σ_1 are the prescribed constants. By using the improved partially permeable crack model, the crack-face boundary conditions can be given as

$$\sigma_{xy}^{II}(x, 0) = \sigma_{xy}^I(x, 0) = 0, \quad 0 < x < a, \quad (14a)$$

$$\begin{aligned} Q_y^I(x, 0) - Q_y^{II}(x, 0) &= -\frac{(Q_1 - Q_{1c})x}{2a^2}, \quad 0 < x < a, \\ \sigma_{yy}^{II}(x, 0) &= \sigma_{yy}^I(x, 0) = -\frac{\sigma_1}{a}|x|, \quad 0 < x < a. \end{aligned} \quad (14b)$$

Based on the antisymmetry of thermal flux and symmetry of mechanical loading, only half of the field (i.e., $x > 0$) is considered under combined unsymmetric thermal flux and symmetric linear mechanical loading.

$$\sigma_{xy}^{II}(x, 0) = \sigma_{xy}^I(x, 0) = 0, \quad 0 < x < a, \quad (15)$$

$$Q_y^I(x, 0) - Q_y^{II}(x, 0) = -\frac{(Q_1 - Q_{1c})x}{2a^2}, \quad 0 < x < a, \quad (16)$$

$$\sigma_{yy}^{II}(x, 0) = \sigma_{yy}^I(x, 0) = -\frac{\sigma_1}{a}|x|, \quad 0 < x < a, \quad (17)$$

where

$$Q_{1c} = -h_c(T^I(x, 0) - T^{II}(x, 0)) + \varepsilon Q_1. \quad (18)$$

Moreover, factors such as stresses, elastic displacements, temperature, and thermal flux across the crack-free region of the horizontal x -axis comply with the following conditions.

$$\begin{aligned} \sigma_{xy}^I(x, 0) &= \sigma_{xy}^{II}(x, 0), \\ \sigma_{yy}^I(x, 0) &= \sigma_{yy}^{II}(x, 0), \quad x > a \text{ or } x < -a, \end{aligned} \quad (19)$$

$$\begin{aligned} u^I(x, 0) &= -u^{II}(x, 0), \\ v^I(x, 0) &= -v^{II}(x, 0), \quad x > a \text{ or } x < -a, \end{aligned} \quad (20)$$

$$\begin{aligned} T^I(x, 0) &= T^{II}(x, 0), \\ Q_y^I(x, 0) &= Q_y^{II}(x, 0), \quad x > a \text{ or } x < -a. \end{aligned} \quad (21)$$

3. Solution Procedure

3.1. Temperature Field. Due to equation (11) is irrelevant to the elastic strain, the temperature field can be solved out first. Considering the antisymmetry of thermal flux, the solution of equation (11) is written with application of FTT.

$$T^{I,II}(x, y) = \int_0^{+\infty} g^\pm(\xi) e^{-\xi \delta^\pm \lambda y} \sin(\xi x) d\xi, \quad (22)$$

where $g^\pm(\xi)$ is unknown and will be solved. $\delta^+ = 1$ and $\delta^- = -1$ denote $y > 0$ and $y < 0$, respectively. From (9), one gets

$$\begin{aligned} Q_x^{I,II}(x, y) &= -\lambda_x \int_0^\infty \xi g^\pm(\xi) e^{-\xi \delta^\pm \lambda y} \cos(\xi x) d\xi, \\ Q_y^{I,II}(x, y) &= \lambda_y \lambda \int_0^\infty \delta^\pm \xi g^\pm(\xi) e^{-\xi \delta^\pm \lambda y} \sin(\xi x) d\xi. \end{aligned} \quad (23)$$

Making use of the second relation in equation (21), one gets

$$g^+(\xi) = -g^-(\xi). \quad (24)$$

By the aid of the first relation in equations (21) and (17), one attains

$$\int_0^{+\infty} g^+(\xi) \sin(\xi x) d\xi = 0, \quad x > a, \quad (25)$$

$$\int_0^{+\infty} g^+(\xi) \sin(\xi x) d\xi = -\frac{(Q_1 - Q_{1c})x}{4a^2 \lambda \lambda_y}, \quad 0 < x < a. \quad (26)$$

For the solution of equations (25) and (26), the auxiliary function $\gamma(x)$ is defined as

$$\gamma(x) = \frac{\partial [T^I(x, 0) - T^{II}(x, 0)]}{\partial x}. \quad (27)$$

Using inverse Fourier transform, one has

$$g^+(\xi) \xi = \frac{1}{\pi} \int_0^a \gamma(s) \cos(\xi s) ds. \quad (28)$$

Substituting equations (28) into (26), one gets

$$\frac{2}{\pi} \int_0^a \gamma(s) ds \int_0^{+\infty} \sin(\xi x) \cos(\xi s) d\xi = -\frac{(Q_1 - Q_{1c})x}{2a^2 \lambda \lambda_y}. \quad (29)$$

From the known result [23],

$$\int_0^{+\infty} \cos(\xi s) \sin(\xi x) d\xi = \frac{1}{2} \left(\frac{1}{x-s} + \frac{1}{x+s} \right). \quad (30)$$

Equation (29) can be expressed as follows:

$$\frac{1}{\pi} \int_{-a}^a \gamma(s) \frac{1}{s-x} ds = \frac{(Q_1 - Q_{1c})x}{2a^2 \lambda \lambda_y}. \quad (31)$$

Equation (31) is a singular integral equation including the Cauchy kernel [24], and the solution can be depicted as

$$\gamma(x) = \frac{1}{\pi\sqrt{a^2 - x^2}} \int_{-a}^a \frac{\sqrt{a^2 - s^2}}{x - s} \frac{(Q_1 - Q_{1c})s}{2a^2\lambda\lambda_y} ds + \frac{C}{\sqrt{a^2 - x^2}} \quad (32)$$

Considering the following condition,

$$\int_{-a}^a \gamma(s) ds = 0. \quad (33)$$

After some computations, one has $C = 0$. Equation (32) can be given as

$$\gamma(s) = \frac{(Q_1 - Q_{1c})}{4a^2\lambda\lambda_y\sqrt{a^2 - x^2}} (2x^2 - a^2). \quad (34)$$

In addition, with application of equations (27) and (34), the temperature change on the crack is given as

$$T^I(x, 0) - T^{II}(x, 0) = -\frac{Q_1 - Q_{1c}}{4a^2\lambda\lambda_y} x\sqrt{a^2 - x^2}, \quad (35)$$

which is consistent with that in [16] for a single crack of length $2a$.

3.2. Elastic Field. In order to solve equations (7) and (8), $u_1^{I,II}(x, y)$ and $v_1^{I,II}(x, y)$ are written as [25]

$$\begin{aligned} u_1^{I,II}(x, y) &= u_1^{I,II}(x, y) + u_2^{I,II}(x, y), \\ v_1^{I,II}(x, y) &= v_1^{I,II}(x, y) + v_2^{I,II}(x, y), \end{aligned} \quad (36)$$

where $u_1^{I,II}(x, y)$ and $v_1^{I,II}(x, y)$ correspond to the general solutions under $T^{I,II}(x, y) = 0$. $u_2^{I,II}(x, y)$ and $v_2^{I,II}(x, y)$ are the peculiar solutions under thermal flux. With application of Fourier transform technique, $u_1^{I,II}(x, y)$ and $v_1^{I,II}(x, y)$ can be written as

$$u_1^{I,II}(x, y) = \sum_{j=1}^2 \int_0^{+\infty} g_j^{\pm}(\xi) e^{-\xi\delta^{\pm}\rho_j y} \cos(\xi x) d\xi, \quad (37)$$

$$v_1^{I,II}(x, y) = \sum_{j=1}^2 \int_0^{+\infty} \eta_j \delta^{\pm} g_j^{\pm}(\xi) e^{-\xi\delta^{\pm}\rho_j y} \sin(\xi x) d\xi, \quad (38)$$

and $g_j^{\pm}(\xi)$ will be solved. ρ_j , ($j = 1, 2$) ($\text{Re}\rho_j > 0$) are chosen as the roots of the following character equation.

$$c_{22}c_{66}\rho^4 + (c_{12}^2 + 2c_{12}c_{66} - c_{12}c_{22})\rho^2 + c_{11}c_{66} = 0, \quad (39)$$

where

$$\eta_j = \frac{c_{11} - c_{66}\rho_j^2}{(c_{12} + c_{66})\rho_j}. \quad (40)$$

Furthermore, $u_2^{I,II}(x, y)$ and $v_2^{I,II}(x, y)$ are chosen as

$$u_2^{I,II}(x, y) = \sum_{j=1}^2 \int_0^{+\infty} g^{*\pm}(\xi) e^{-\delta^{\pm}\xi\lambda y} \cos(\xi x) d\xi, \quad (41)$$

$$v_2^{I,II}(x, y) = \sum_{j=1}^2 \int_0^{+\infty} \delta^{\pm} L^{*\pm}(\xi) e^{-\delta^{\pm}\xi\lambda y} \sin(\xi x) d\xi. \quad (42)$$

Substituting equations (41) and (42) into (7) and (8), one has

$$\begin{bmatrix} g^{*\pm}(\xi) \\ L^{*\pm}(\xi) \end{bmatrix} = \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} \frac{g^{\pm}(\xi)}{\xi}, \quad (43)$$

where

$$\begin{bmatrix} K_1 \\ K_2 \end{bmatrix} = \begin{bmatrix} c_{11} - c_{66}\lambda^2 & -(c_{12} + c_{66})\lambda \\ (c_{12} + c_{66})\lambda & c_{66} - c_{22}\lambda^2 \end{bmatrix}^{-1} \begin{bmatrix} \beta_1 \\ \beta_2\lambda \end{bmatrix}. \quad (44)$$

Based on equations (2)–(4), (37), (38), (41), and (42), the stresses are rewritten as

$$\begin{aligned} \sigma_{xx}^{I,II}(x, 0) &= -\sum_{j=1}^2 \int_0^{+\infty} (c_{11} - c_{12}\gamma_j\eta_j) \xi g_j^{\pm}(\xi) \sin(\xi x) d\xi \\ &\quad - (c_{11}M_1 - c_{12}\lambda M_2 - \beta_1) \int_0^{+\infty} g^{\pm}(\xi) \sin(\xi x) d\xi, \end{aligned} \quad (45a)$$

$$\begin{aligned} \sigma_{yy}^{I,II}(x, 0) &= -\sum_{j=1}^2 \int_0^{+\infty} (c_{12} - c_{22}\gamma_j\eta_j) \xi g_j^{\pm}(\xi) \sin(\xi x) d\xi \\ &\quad - (c_{12}M_1 - c_{22}\lambda M_2 - \beta_2) \int_0^{+\infty} g^{\pm}(\xi) \sin(\xi x) d\xi, \end{aligned} \quad (45b)$$

$$\begin{aligned} \sigma_{xy}^{I,II}(x, 0) &= -c_{66} \left[\sum_{j=1}^2 \int_0^{+\infty} \delta^{\pm} (\gamma_j + \eta_j) \xi g_j^{\pm}(\xi) \cos(\xi x) d\xi \right. \\ &\quad \left. + \int_0^{+\infty} \delta^{\pm} (M_1\lambda + M_2) g^{\pm}(\xi) \cos(\xi x) d\xi \right]. \end{aligned} \quad (45c)$$

To obtain the explicit solution of the problem, two parts are divided. One part is what is induced by mechanical loading ($-\sigma_1|x|/a$). The other part is what is resulted from thermal flux ($-Q_1x/2a^2$). The solution procedure is neglected for simplicity under mechanical loading ($-\sigma_1|x|/a$). The elastic displacement on the crack is obtained with application of the Fourier transform technique (FTT) [17].

$$v^I(x, 0) = \frac{\sigma_1(\eta_1\rho_2 - \eta_2\rho_1)x\sqrt{a^2 - x^2}}{2a\omega_1(\rho_2 + \eta_2)} + O(1). \quad (46)$$

Furthermore, one can obtain the stresses field as follows:

$$\sigma_{yy}^{I,II}(x, 0) = \frac{2x}{\pi a\sqrt{x^2 - a^2}} \sigma_1 + O(1). \quad (47)$$

Then, the elastic field under linear thermal flux will be given in the closed form. With application of thermal flux, it fits the condition as follows.

$$\sigma_{yy}^I(x, 0) = \sigma_{yy}^{II}(x, 0) = 0, \quad 0 < x < a. \quad (48)$$

With application of equations (48) and (45b), one arrives at

$$\int_0^{+\infty} \left[\sum_{j=1}^2 g_j^+(\xi) \cos(\xi x) + \frac{M_1 g^+(\xi)}{\xi} \cos(\xi x) \right] d\xi = 0, \quad x > a, \quad (50)$$

$$\int_0^{+\infty} \sum_{j=1}^2 (\rho_j + \eta_j) \xi g_j^+(\xi) \cos(\xi x) d\xi + \int_0^{+\infty} (M_1 \lambda + M_2) g^+(\xi) \cos(\xi x) d\xi = 0, \quad 0 < x < a. \quad (51)$$

To give the solvation of equations (50) and (51), the auxiliary function $\chi(x)$ can be defined as

$$\chi(x) = \frac{\partial u^I(x, 0)}{\partial x}. \quad (52)$$

Applying equation (52) and inverse Fourier transform, one has

$$\sum_{j=1}^2 (c_{12} - c_{22} \rho_j \eta_j) g_j^+(\xi) = (c_{22} \lambda M_2 + \beta_2 - c_{12} M_1) \frac{g^+(\xi)}{\xi}. \quad (49)$$

With application of equation (15) and the first relation in equation (20), the dual integral equations are given:

$$\sum_{j=1}^2 g_j^+(\xi) \xi + M_1 g^+(\xi) = -\frac{2}{\pi} \int_0^a \chi(s) \sin(\xi s) ds. \quad (53)$$

Utilizing equations (49) and (53), one obtains

$$g_1^+(\xi) \xi = \frac{c_{22} \lambda M_2 + \beta_2 - c_{22} \rho_2 \eta_2 M_1}{c_{22} (\rho_2 \eta_2 - \rho_1 \eta_1)} g^+(\xi) - \frac{c_{22} \rho_2 \eta_2 - c_{12}}{c_{22} (\rho_2 \eta_2 - \rho_1 \eta_1)} \frac{2}{\pi} \int_0^a \chi(s) \sin(\xi s) ds, \quad (54)$$

$$g_2^+(\xi) \xi = \frac{c_{22} \rho_1 \eta_1 M_1 - c_{22} \lambda M_2 - \beta_2}{c_{22} (\rho_2 \eta_2 - \rho_1 \eta_1)} g^+(\xi) - \frac{c_{12} - c_{22} \rho_1 \eta_1}{c_{22} (\rho_2 \eta_2 - \rho_1 \eta_1)} \frac{2}{\pi} \int_0^a \chi(s) \sin(\xi s) ds. \quad (55)$$

With application of equations (53)–(55), one obtains

$$2 \int_0^a \chi(s) ds \int_0^{+\infty} \sin(\xi s) \cos(\xi x) d\xi = \pi \omega_2 \int_0^{+\infty} g^+(\xi) \cos(\xi x) d\xi, \quad 0 < x < a, \quad (56)$$

where

$$\omega_2 = \frac{H_1}{H_2}, \quad (57)$$

with

$$\begin{aligned} H_1 &= (\rho_1 + \eta_1) (c_{22} \rho_2 \eta_2 M_1 - c_{22} \lambda M_2 - \beta_2) + (\rho_2 + \eta_2) \\ &\quad (c_{22} \lambda M_2 + \beta_2 - c_{22} \rho_1 \eta_1 M_1) + c_{22} (M_1 \lambda + M_2) (\rho_1 \eta_1 - \rho_2 \eta_2), \\ H_2 &= (\rho_1 + \eta_1) (c_{22} \rho_2 \eta_2 - c_{12}) + (\rho_2 + \eta_2) (c_{12} - c_{22} \rho_1 \eta_1). \end{aligned} \quad (58)$$

With the knowledge of equations (30), and (56) can be rewritten as

$$\frac{1}{\pi} \int_{-a}^a \frac{\chi(s)}{x-s} ds = \omega_2 \int_0^{+\infty} g^+(\xi) \sin(\xi x) d\xi. \quad (59)$$

With application of equation (22) and the inverse Fourier transform, one has

$$\begin{aligned} \int_0^{+\infty} g^+(\xi) \cos(\xi x) d\xi &= \frac{1}{\pi} \int_{-a}^a \frac{T^+(s) - T^-(s)}{x-s} ds \\ &= -\frac{(Q_1 - Q_{1c})}{16a^2 \lambda \lambda_y} (2x^2 - a^2). \end{aligned} \quad (60)$$

Based on the singular integral theory with the Cauchy kernel, equation (59) is obtained as

$$\chi(x) = \frac{\omega_2(Q_1 - Q_{1c})}{16a^2\pi\lambda_y\sqrt{a^2 - x^2}} \int_{-a}^a \frac{\sqrt{a^2 - s^2}}{x - s} (2s^2 - a^2) ds + \frac{E}{\sqrt{a^2 - x^2}}. \quad (61)$$

After some calculations, $E = 0$. Equation (61) can be given as

$$\chi(x) = \frac{\omega_2(Q_1 - Q_{1c})}{8a^2\lambda_y\sqrt{a^2 - x^2}} (x^3 - a^2x). \quad (62)$$

With application of equation (52), one obtains

$$u^I(x, 0) = \int_{-a}^x \chi(s) ds = \frac{\omega_2(Q_1 - Q_{1c})}{24a^2\lambda_y\sqrt{a^2 - x^2}} (a^2 - x^2) \sqrt{a^2 - x^2}. \quad (63)$$

From equations (45c), (54), (55), and (63), one can obtain the shearing stresses as follows.

$$\sigma_{xy}^{I,II}(x, 0) = \frac{c_{66}H_1(Q_1 - Q_{1c})}{16a^2\lambda_y c_{22}(\rho_2\eta_2 - \rho_1\eta_1)} \left(\frac{a^2x - x^3}{\sqrt{x^2 - a^2}} \right). \quad (64)$$

3.3. Crack Tip Field. Using equations (18) and (35), one has

$$Q_{1c} = \frac{E_c Q_1 x \sqrt{a^2 - x^2} + 4\varepsilon Q_1 a^2 \lambda}{E_c x \sqrt{a^2 - x^2} + 4a^2 \lambda}. \quad (65)$$

In particular, when $\varepsilon = 0$, one has

$$Q_{1c} = \frac{E_c Q_1 x \sqrt{a^2 - x^2}}{E_c x \sqrt{a^2 - x^2} + 4a^2 \lambda}. \quad (66)$$

The quantity $E_c = h_c/\lambda_y$ stands for the dimensionless thermal conductivity. In equation (65), when $E_c \rightarrow \infty$ or

$E_c \rightarrow 0$, $Q_{1c} \rightarrow Q_1$ or $Q_{1c} \rightarrow \varepsilon Q_1$. The observations reveal εQ_1 can be considered to be created by the thermal flux or other large loading such as mechanical loading.

Furthermore, the stress intensity factor is of much importance for analysis of the behavior of cracked solid. The mode I stress intensity factor and the mode II stress intensity factor are given as

$$K_I = \lim_{x \rightarrow a^+} \sqrt{2\pi(x-a)} \sigma_{yy}^I(x, 0), \quad (67)$$

$$K_{II} = \lim_{x \rightarrow a^+} \sqrt{2\pi(x-a)} \sigma_{xy}^I(x, 0). \quad (68)$$

Next, the corresponding intensity factors to characterize opening crack displacement are defined as

$$K_v^{COD} = \lim_{x \rightarrow a^+} \sqrt{\frac{\pi}{2(a-x)}} v^I(x, 0), \quad (69)$$

$$K_u^{COD} = \lim_{x \rightarrow a^+} \sqrt{\frac{\pi}{2(a-x)}} u^I(x, 0). \quad (70)$$

With application of equations (47), (64), (46), (63), (67), (68), (69), and (70), one has

$$K_I = \frac{2\sigma_1 \sqrt{\pi a}}{\pi},$$

$$K_{II} = 0, \quad (71)$$

$$K_v^{COD} = \frac{\sigma_1(\eta_1\rho_2 - \eta_2\rho_1)\sqrt{\pi a}}{2\pi\omega_1(\rho_2 + \eta_2)},$$

$$K_u^{COD} = 0.$$

The energy release rate (G) has an important effect on the analysis of crack growth, and the energy release rate is defined as [25]

$$G = \lim_{\delta \rightarrow 0} \frac{1}{\delta} \int_0^\delta \sigma_{xy}^I(x+a, 0) u^I(a-x, 0) + \sigma_{yy}^I(x+a, 0) v^I(a-x, 0) dx. \quad (72)$$

4. Numerical Results

This section presents a research on the influence of combined unsymmetric linear thermal flux, symmetric linear mechanical loading, the dimensionless thermal resistance and the coefficient on the heat flux to the crack surface, the mode II stress intensity factor (K_{II}), and the energy release rate (G). For convenience, the isotropic material constants are chosen as shown in [26] (Table 1).

Figure 2 shows Q_1/Q_{1c} versus E_c with $x/a = 0.25, 0.5, 0.75, 0.99$ for $\varepsilon = 0.01$. The cases of $E_c = 20, E_c = 40, E_c = 50$, and $E_c = 60$ indicate the heat conduction is 19 times, 39 times, 49 times, and 59 times, respectively, stronger than thermal conductivity of the orthotropic material. $E_c = 0$ or

$E_c \rightarrow \infty$ corresponds to the thermally fully impermeable or permeable case.

Figure 3 presents Q_1/Q_{1c} versus ε with $x/a = 0.25, 0.5, 0.75, 0.99$ for $E_c = 1$. It is seen from Figure 3 when ε increases, Q_1/Q_{1c} increases. The case of $E_c = 1$ correlates with the situation that the crack region is identical to that of the isotropic material.

Figure 4 presents K_{II} versus Q_1 . In the case of purely unsymmetric thermal flux, K_{II} remains zero.

Figure 5 shows G , as computed using equation (72), for the improved partially permeable crack with a variety of crack lengths. In the case of purely unsymmetric thermal flux or symmetric tension stress, the value of G may turn zero or positive. It means that symmetric linear mechanical

TABLE 1: Constants of steel.

E_{xx} (GPa)	E_{yy} (GPa)	G_{xy} (GPa)	ν_{xy}	ν_{yx}	α_{xx} ($10^{-5}/^{\circ}\text{C}$)	α_{yy} ($10^{-5}/^{\circ}\text{C}$)	λ_x (w/m $^{\circ}\text{C}$)	λ_y (w/m $^{\circ}\text{C}$)
205	205	79.2	0.3	0.303	1.14	1.14	48.6	48.6

FIGURE 2: Q_1/Q_{1c} versus E_c with $x/a = 0.25, 0.5, 0.75, 0.99$ for $\varepsilon = 0.01$. FIGURE 3: Q_1/Q_{1c} versus ε with $x/a = 0.25, 0.5, 0.75, 0.99$ for $E_c = 1$.

loading rather than unsymmetric thermal flux has an effect on G .

It is revealed that dimensionless thermal conductivity, the coefficient, and the size of crack have great effects on the heat flux per thickness to the crack surface, the mode II stress intensity factor, and the energy release rate. In the case of purely unsymmetric thermal flux, the mode II stress intensity factor and the energy release rate is zero. The mode II stress intensity factor and the energy release rate may be zero or positive under combined unsymmetric thermal flux and symmetric linear mechanical loading. The mode II stress

FIGURE 4: K_{II} versus Q_1 . FIGURE 5: G versus tension stress with $2a = 0.25, 0.5, 1, 2$ mm for $Q_1 = 10, 20, 30, 40 \text{ J}/(\text{m}^2 \times \text{s})$.

intensity factor and the energy release rate may be zero or positive under combined unsymmetric thermal flux and symmetric linear mechanical loading. Contrary to the situations under symmetric thermal flux and symmetric mechanical loading [17], such consideration need not be taken when using the energy release rate as a criterion.

5. Conclusions

An improved partially permeable crack model is put forward to deal with the problem of a crack embedded in an orthotropic or isotropic solid under combined unsymmetric

thermal flux and symmetric linear mechanical loading. Using the Fourier transform technique, the jumps of temperature, elastic displacements on the crack, and so on are obtained in the closed form. Numerical results show combined unsymmetric linear thermal flux, symmetric linear mechanical loading, dimensionless thermal conductivity, and the coefficient have great influences on fracture parameters. For the extended partially permeable crack, the mode II stress intensity factor and the energy release rate may be zero or positive under combined unsymmetric thermal flux and symmetric linear mechanical loading. Therefore, closure of the crack tip region need not be considered under combined unsymmetric thermal flux and symmetric linear mechanical loading when making use of fracture parameters as a criterion.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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