

Research Article

Game-Based Multiobjective Optimization of Suspension System for In-Wheel Motor Drive Electric Vehicle

Tang Feng ^{1,2} and Lin Shu ³

¹Department of Automobile and Mechanical Engineering, Anhui Communications Vocational & Technical College, Hefei 230051, China

²School of Electrical Engineering and Automation, Hefei University of Technology, Hefei 230009, China

³School of Automotive and Transportation Engineering, Hefei University of Technology, Hefei 230009, China

Correspondence should be addressed to Tang Feng; tfok@163.com

Received 17 January 2021; Revised 26 March 2021; Accepted 30 March 2021; Published 27 April 2021

Academic Editor: Chongfeng Wei

Copyright © 2021 Tang Feng and Lin Shu. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Since the driving motor is embedded in the wheel, the unsprung mass and the wheel rotational inertia of the in-wheel motor drive electric vehicle both increase, which not only affect the vehicle smoothness but also worsen the motor's working condition due to its own vertical vibration. The evaluation index of in-wheel motor's vertical vibration is introduced on the basis of vehicle smoothness analysis. The parameters' optimization of vibration absorber and suspension are carried out, respectively, and the optimization results show the contradictory relationship between smoothness objective and the motor's vertical vibration acceleration objective. Regarding the contradictory indices of the smoothness and the motor's vertical acceleration as the objective function, a multiobjective optimization scheme is designed. Then, the orthogonal experimentation and fuzzy clustering method are applied in the multiobjective optimized design based on the game decision analysis, and the Nash equilibrium and cooperative competition game theory are used to optimize the parameters of suspension and vibration absorber. The optimized results verify the game relation between the optimization variables, the game optimization obtains better optimized results than traditional linear weight sum method, and the in-wheel motor functional stability and the vehicle smoothness can be both achieved. Compared with the traditional complex iterative process and the manmade preassurance weight allocation, the game optimization has the advantages of less iterations, faster convergence, and less influence by human experience.

1. Introduction

New energy automobiles have become the forward-looking field in the car industry. The driven technology of the new energy automobiles and the traditional internal combustion engine cars shares a great deal of differences, among which the in-wheel motor drive electric vehicles embrace a promising prospect as a new car driven method. Compared with motor integrated power driven, the in-wheel motor drive has a big advantage, which is more flexible and has less complicated mechanic driven system. Nevertheless, it also has some disadvantages. The electric wheels in the in-wheel drive vehicles combine the electric machines, wheel hubs, retarding mechanisms, and brakes

together, greatly increasing the unsprung mass and rotational inertia, worsening the vibration caused by the in-wheel motor drive, and causing the vibration between the neighbour parts and the rotors, which brings bad influence on the vehicles' smoothness and the safety of the tires touching the ground. How to reduce and even get rid of this kind of bad effect has become one of the key problems needed to be solved in the development of in-wheel drive electric vehicles.

Focusing on the above problems, currently, the scholars have already conducted initial research on the related questions to reduce the influence of the in-wheel drive motor on vehicle vibration. References [1–5] pointed out that, because of the introduction of the in-wheel motors, the

unsprung mass of the vehicle is increased clearly, promoting the dynamic load of the tires and the vibration acceleration of the car greatly to affect the ground-touching features of the tires and the smoothness of the vehicle. In order to suppress the in-wheel vibration and improve vehicle ride comfort, the in-wheel motor in [6] was considered as a dynamic vibration absorber, which was isolated from the unsprung mass by using a spring and a damper. A Rojas et al. [7] conducted a deep and thorough study on the active and passive suspensions of the in-wheel motor drive cars and the in-wheel motors including the damping suspension devices, and the study results revealed that the half and active suspension system could reduce the unsprung mass vertical vibration acceleration. But it did little improvement in the wheel ground-touching load, while the design of the motor suspension could effectively improve the wheel ground-touching load and the sprung mass vertical vibration acceleration. Guo-bao Ning et al. [8] from Tongji University analysed the transformation of the vehicle vertical vibration caused by the in-wheel motor system and put forward the idea of transferring the motor mass from the unsprung load with the method of mass transferring as the mass of the vibration absorbing unit to lower the wheel motor load. Scholars like Ma Ying [9] used the minimum root-mean-square value of the vehicle smoothness index as the optimized target and conducted optimization on the in-wheel motor suspension motors. The result showed that the optimized suspension structure of the in-wheel motor could not only eliminate the negative effect exerted on the vertical operation of the in-wheel motors but also improve the wheel ground-touching operations and improve the smoothness and safety of the electric cars.

However, with the development of the in-wheel motor technology, the orientation of the in-wheel motor drive cars is not limited to the travel tool in the city. Its optimized analysis is not limited to the traditional smoothness optimization. As the core part of the in-wheel motor electric cars, the operation, and condition of the in-wheel motor directly exert influences on the safety and cruising ability of the car. The research on in-wheel motor has gained more and more attention. Wei Tong et al. [10] used 1/4 vehicle motor force models to introduce the appraisal index of the vehicle smoothness and the motor vehicle vibration acceleration, based on the relative root-mean-square value and the transmission characteristics, among which the motor vehicle vibration is closely related to the features of the vehicle. It provides available ranges for matching the vehicle performances. In [11], it was proposed that the magnet gap deformation will lead to unbalanced magnetic force, which was a critical vibration source to the vehicle vertical dynamics. For this problem, an in-wheel-motor-driven electric vehicle without a speed reducer was considered as the research subject, and the results demonstrated that the magnetic force had an influence on vehicle vertical dynamics, which provided some theoretical basis for the design and optimization of the in-wheel motor drive electric vehicle.

Besides, the related optimized design analysis and research on in-wheel motors have progressed greatly [12, 13],

among which, the game theory is applied more and more often in recent years in engineering optimized design as a competitive nature-driven theory and method. Suheyła Özyıldırım [14] put forward the optimized method based on the Nash equilibrium and noncompetition game theory in a relatively systematic way, realizing the optimization on the nonrenewable resource models, with calculating speedup by 90% compared with the traditional genetic algorithm. Periaux et al. [15] applied multiple-aimed Nash equilibrium game method in the numerous engineering examples of the airfoil aerodynamic promotion motor optimization, multiple-wing optimization design, and multiple-standard reverse power optimization. Zhong Chen et al. [16] adopted the competition and cooperation game models to conduct multiple-targeted optimized design on the Baihe Dam's shape, reaching a better design plan. Hong-bo Zhang et al. [17] applied Nash equilibrium on the collision analysis of automobiles, confronted with complicated automobile analysis problems, and the game optimization method presented good robustness, with faster convergence during the problem-solving process, meeting the multiple-targeted demands. It could be seen that the game analysis had solved many practical and effective engineering problems.

Many related solutions have been put forward to overcome the negative impact brought by the torque ripple of the in-wheel motors on electric automobiles. While the suspension vibration and the in-wheel motor vibration are both affected by each other. Only observing the vibration of the suspension to get good ride comfort may lead the in-wheel motor over vibration. Due to the limited space in the wheel for the motor, the over vibration of the vehicle motor will shorten its using lifetime; meanwhile, this also affects the other parts working in the wheel or the vehicle. In this paper, we will discuss different suspension types with the in-wheel motor. During different suspension types, suspension parameters selection will get different vibration performance. Thus, the parameter optimization method and strategy are very important to improve the suspension performance. In this paper, it is focused on the two optimized schemes on the in-wheel motor drive vehicle, which are suspension parameters optimization and vibration absorber optimization, respectively. Based on vehicle's smoothness analysis, the evaluation index of vertical vibration of the in-wheel motor is introduced. The parameters of suspension and vibration absorber are optimized, respectively, and the two optimization schemes are compared and analysed. Then, regarding the contradictory smoothness index and the in-wheel vertical acceleration as the two objective functions, the suspension parameters' multiobjective optimization scheme is carried out, which could better consider the smoothness of the whole vehicle and the stability of the motor vibration. Finally, the orthogonal experimentation and fuzzy clustering method are applied in the multiobjective optimization design based on the game decision analysis, realizing the transfer of the multiobjective optimization to game-related problems and the division of the game player side. The Nash equilibrium and the cooperation and competition equilibrium are adopted, respectively, to solve the best solution of the related parameters with the profit function as

optimization target. Compared with the traditional optimized result, the optimization of the game theory is further analysed, and the effect of the multiobjective optimization with the help of the game theory is validated.

2. Suspension Model of In-Wheel Motor Drive Vehicle

The torque ripple of the in-wheel motor exerts influences on various functional parameters of the suspension system, especially on the unsprung mass wheel, deteriorating the ground-catching ability of the wheels and harm to the smooth travel of the vehicles. At the same time, the vertical vibration of the in-wheel motor will also deteriorate the motor's working condition. In [18], the regular solutions were pointed out: (1) to conduct torque ripple control over the motor or to optimize the motor types; (2) to optimize or to reallocate the automobile suspension; (3) to design an in-wheel motor vibration absorbing system in particular. This paper mainly focuses on the influence exerted by the electric wheels on the vehicles' vertical motor forces, instead of the function of the motor itself. Therefore, the study primarily is on the latter two solutions in this paper. The suspension parameter optimization model and the vibration absorbing optimization model are established, respectively.

2.1. Suspension Parameter Optimization Model. The simplified 1/4 vertical vibration model of the vehicle with the in-wheel motor is established as shown in Figure 1.

Because the tire damping is relatively low, the tire damping is neglected. The motion equation of the 1/4 vehicle vertical vibration model with in-wheel motor is built as follows:

$$\begin{aligned} (m_1 + m_3)\ddot{x}_1 + k_2(x_1 - x_2) + c_2(\dot{x}_1 - \dot{x}_2) + k_1(x_1 - x_0) &= 0, \\ m_2\ddot{x}_2 + k_2(x_2 - x_1) + c_2(\dot{x}_2 - \dot{x}_1) &= 0. \end{aligned} \quad (1)$$

The root-mean-square value of the car body's acceleration is $\sigma_{\ddot{x}_2} = \sqrt{4\pi^2 G_q(n_0)n_0^2 u \int_0^\infty |H(j\omega)|_{\ddot{x}_2 \sim \dot{x}_0}^2 df}$, the root-mean-square value of the suspension dynamic deflection is $\sigma_{f_d} = \sqrt{4\pi^2 G_q(n_0)n_0^2 u \int_0^\infty |H(j\omega)|_{f_d \sim \dot{x}_0}^2 df}$, and the root-mean-square value of the wheels' relative loading is $\sigma_{F_d/G} = \sqrt{4\pi^2 G_q(n_0)n_0^2 u \int_0^\infty |H(j\omega)|_{F_d/G \sim \dot{x}_0}^2 df}$.

In the above formula, $G_q(n_0)$ is the average value of the road spectrum, n_0 is the reference space frequency, and u is the vehicle speed. $|H(j\omega)|_{\ddot{x}_2 \sim \dot{x}_0}$ is the frequency characteristics of the acceleration of car body \ddot{x}_2 to the random road input speed \dot{x}_0 , $|H(j\omega)|_{f_d \sim \dot{x}_0}$ is the amplitude-frequency characteristics of suspension dynamic deflection to the random road input speed \dot{x}_0 , $|H(j\omega)|_{F_d/G \sim \dot{x}_0}$ is the amplitude-frequency characteristics of wheels' relative loading to random road input speed \dot{x}_0 , x_0 is the input of the uneven degree of the road, x_j ($j = 1, 2, 3$) is the related mass position movement, m_1 is the unsprung mass (mass of the wheel), m_2 is the sprung mass (mass of the 1/4 car body), m_3 is the in-wheel motor mass, c_1 is the wheel damping coefficient, c_2 is

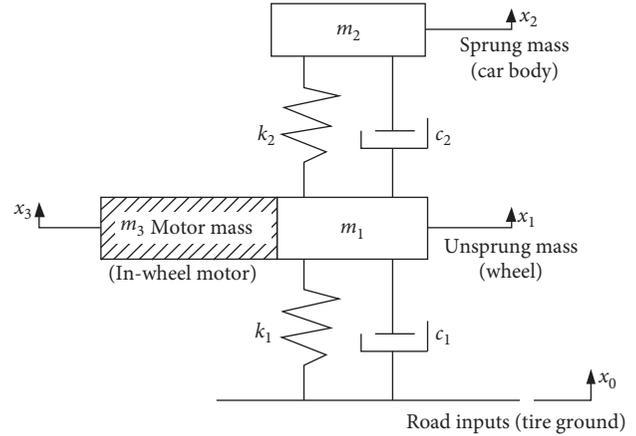


FIGURE 1: Vertical vibration model of 1/4 in-wheel motor drive electric vehicle.

the vehicle suspension damping coefficient, k_1 is the tire stiffness, and k_2 is the vehicle suspension stiffness. The system parameters of the target vehicle are shown in Table 1.

2.2. Vibration Absorbing Optimization Model. The introduction of the in-wheel motor brings about the severe vertical negative effect on the vehicle. On the premise of not changing the suspension structure and parameters of the initial design in Section 2.1, the special in-wheel motor vibration absorbing system is installed. The 1/4 in-wheel motor vehicle vertical vibration model with vibration absorbing device [10, 19] is set up and shown in Figure 2.

The wheel, vehicle body, and in-wheel motor's vertical displacement are x_1 , x_2 , and x_3 , respectively. With the origin of coordinates staying in the respective equilibrium points, the road displacement input is x_0 and its dynamic differential equation is

$$M \begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{pmatrix} + C \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{pmatrix} + K \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} c_1 \\ 0 \\ 0 \end{pmatrix} x_0 + \begin{pmatrix} k_1 \\ 0 \\ 0 \end{pmatrix} x_0. \quad (2)$$

In the above formula, M is the quality matrix, C is the damping matrix, and K is the stiffness matrix.

$$\begin{aligned} M &= \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \\ C &= \begin{pmatrix} c_1 + c_2 + c_3 & -c_2 & -c_3 \\ -c_2 & c_2 & 0 \\ -c_3 & 0 & c_3 \end{pmatrix}, \\ K &= \begin{pmatrix} k_1 + k_2 + k_3 & -c_2 & -k_3 \\ -k_2 & k_2 & 0 \\ -k_3 & 0 & k_3 \end{pmatrix}, \end{aligned} \quad (3)$$

TABLE 1: System parameters of the target vehicle.

Parameter	m_1 (kg)	m_2 (kg)	m_3 (kg)	k_1 (N/m)	k_2 (N/m)	c_1 (N/(m/s))	c_2 (N/(m/s))
Value	45.4	317.5	50	192000	15000	50	1500

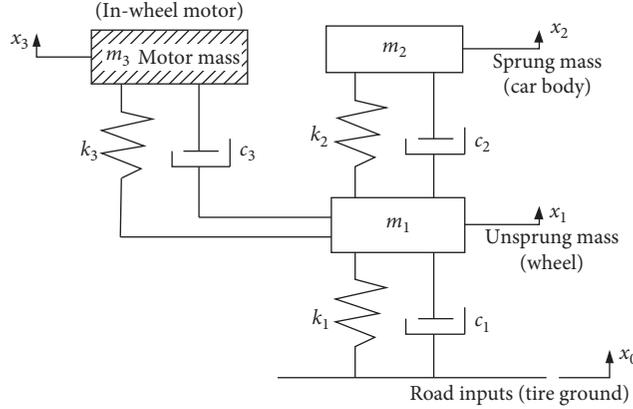


FIGURE 2: Vertical vibration model of the 1/4 in-wheel motor vehicle contained with vibration absorber.

where c_3 is the damping coefficient of vibration absorbing system between the motor and the wheel and k_3 is the spring stiffness between the motor and the wheel.

The in-wheel motor's vertical displacement and the in-wheel motor's vertical displacement relative to the wheel, that is, the root-mean-square value of the in-wheel motor dynamic deflection, are

$$\sigma_{x_3 \sim \dot{x}_0} = \sqrt{4\pi^2 G_q(n_0) n_0^2 u \int_0^\infty |H(j\omega)|_{x_3 \sim \dot{x}_0}^2 df}, \quad (4)$$

$$\sigma_{x_3 - x_1 \sim \dot{x}_0} = \sqrt{4\pi^2 G_q(n_0) n_0^2 u \int_0^\infty |H(j\omega)|_{x_3 - x_1 \sim \dot{x}_0}^2 df}, \quad (5)$$

where $|H(j\omega)_{x_3 \sim \dot{x}_0}|$ and $|H(j\omega)_{x_3 - x_1 \sim \dot{x}_0}|$ represent the frequency response of in-wheel motors' motion distance of x_3 and the relative motion distance of the in-wheel motor $x_3 - x_1$ to the vertical speed input \dot{x}_0 , respectively.

3. Parameter Optimization of Suspension and Vibration Absorber

3.1. Suspension Parameter Optimization. Suspension could be used to buffer the impact on the in-wheel motor drive electric vehicle caused by the rough road and reduce the vibration to ensure the smooth travel of the vehicle. The suspension parameter optimization aims at fulfilling the realistic demand of the vehicle wheels to reach an optimal suspension parameter value. The in-wheel motor is the drive core of in-wheel motor drive electric vehicle. Its vertical vibration analysis is one of the crucial subjects in the research. The covibration frequency belt of the car-carried in-wheel motor has the characteristics of large changing range, big covibration peak, slow decreasing speed, and so on [20].

When matching the in-wheel vehicle parameters, the motor vibration's effect has to be noted particularly.

According to the standard GB/T 4970-1996, the car's smoothness relies on the random input of driving trial method. The suspension parameter optimization scheme is set as follows: selecting the road average spectrum $G_q(n_0)$ as $0.0000256 m^3$ (level C) and the random road speed 80 km/h as working encouragement. The evaluation index of vertical vibration of the in-wheel motor $\sigma_{\ddot{x}_3}$ is introduced on the basis of vehicles' smoothness optimization index $J_{\text{vao}} = \sigma_{\ddot{x}_2} + \sigma_{f_d} + \sigma_{F_d/G}$, and the optimization is carried out for the model in Section 2.1 to get the optimal suspension damping and stiffness parameter. $\sigma_{\ddot{x}_2}$ is the root-mean-square value of the car body's acceleration, σ_{f_d} is the root-mean-square value of the suspension dynamic deflection, and $\sigma_{F_d/G}$ is the root-mean-square value of the wheels' relative loading.

m_3 and m_2 are solid connections, so the vibration of m_3 is basically the same as m_2 . Therefore, the vibration of m_3 is about the relative dynamic load of the wheel.

The variation range of the variables in the optimization process is $(1/2)k_2 \leq k_2 \leq (3/2)k_2$ and $(1/2)c_2 \leq c_2 \leq (3/2)c_2$. The suspension dynamic deflection f_d should be properly allocated with its limit distance $[f_d]$; otherwise, the suspension would constantly bump into the limited area. Thus, the restriction $0 \leq f_d \leq 0.1$ is added.

3.2. Vibration Absorber Optimization. The introduction of the in-wheel motors brings about the severe vertical negative effect on the vehicle. Adoption of the method in 3.1 could have some improvement, but the challenges are great in terms of the parameter adjustment and matching involved in the vehicle stability in the method [21]. To solve this problem, the in-wheel motor in [6] was considered as a dynamic vibration absorber, which was isolated from the wheel shaft and hub by using an in-wheel spring and a damper, whereby the rigid connections between the motor

and the hub were replaced by flexible connections. Based on the model in Section 2.2, the vibration absorber optimization scheme is designed in this section, and the vibration absorber parameters k_3 and c_3 are optimized by using the same working condition, evaluation index, and objective function as the suspension parameter optimization scheme.

The vertical vibration acceleration can be calculated by equation (4). Substituting the restriction of the suspension limit distance $0 \leq f_d \leq 0.1$ into formula (5), we obtain $\sigma_{x_3-x_1 \sim x_0} \leq 0.00287m$, and other restrictions keep the same with the suspension optimization scheme. Mark the smoothness optimization index as J_{vao} , and the optimization comparison results are shown in Table 2.

3.3. Comparison and Analysis of Optimization Results.

The comparison results of two optimization schemes show that the two schemes both improve the smoothness of electric vehicle. The vibration absorber optimization scheme is superior to the suspension parameter optimization scheme in terms of vehicle smoothness. In the suspension optimization, the in-wheel motor acceleration increased by 1.2% with a little deterioration, while in the vibration absorbing optimization, the in-wheel motor acceleration increased by 19.7% with bigger deterioration.

From the above analysis, it could be seen that the vibration absorbing scheme enjoys an obvious improvement in the vehicle's smoothness compared with the initial design. However, the condition of the in-wheel motor deteriorates greatly. In suspension parameter optimization, the smoothness improvement is not that obviously compared with the initial design and the corresponding working condition of the in-wheel motor in this scheme deteriorates only a little.

For this circumstance, reference [22] mentioned that this is caused by the newly installed vibration absorber. After installing the vibration absorber, the coupling relationship of the system increases, and the outer force leads to the covibration of the main system and the vibration absorbing system. Under the circumstance of the same input energy of the road, the vibration energy is separated by the main system and the vibration absorbing system. By adjusting the vibration absorber damping and spring, the vibration of the system comes near to the road vibration frequency and thus resonance happens as a result. The majority of the energy is absorbed and under the circumstance of the same input energy of the road, it reaches the goal of decreasing the energy of the main system. But compared with the time when the dynamic vibration absorber is not installed, m_3 is in the free vibration end and the system mass is reduced greatly compared with the former solid connection. Though part of the vibration is absorbed by damping and spring, generally speaking, the ability to resist the road input vibration is reduced. If the road input energy is large, the independent takeover of the vibration should be taken by m_3 but this would lead to worse working conditions.

Therefore, it is required to design a new multiobjective optimization of suspension parameter, by adjusting the parameters k_2 , c_2 , k_3 , and c_3 to enable both the suspension system's smoothness and in-wheel motor vertical vibration acceleration in a better working condition.

TABLE 2: Comparison of the main evaluation indexes of three optimization schemes.

Index	Smoothness evaluation index, J_{vao}	Root-mean-square value of the in-wheel motor acceleration $\sigma_{\ddot{x}_3}$
Suspension parameter optimization	1.7577	0.3978
Vibration absorber optimization	1.1717	0.4765
Multiple-objective optimization	1.2009	0.4102

4. Multiple-Objective Optimal Design

Based on the analysis in Section 3.3, choosing the vehicle main suspension damping coefficient and stiffness, the damping coefficient and stiffness of the vibration absorbing system between motor and wheel as the optimization variables of the multiple-objective optimization, that is, the optimization variable, is $x = \{k_2, k_3, c_2, c_3\}$ in the optimization model. Considering the in-wheel motor drive electric vehicle's demand for smoothness and the working condition and lifespan of in-wheel motor, a multiple-objective optimal design is needed. Mark the multiple-objective optimization's smoothness evaluation index J_{vao} as the first optimal objective and the root-mean-square value of the in-wheel motor acceleration $\sigma_{\ddot{x}_3}$ as the second objective. To fully coordinate the conflicts and confrontation between the first and second optimization objective, the model in Section 2.2 is used as the optimization model, including the variables of k_2 , c_2 , k_3 , and c_3 . To better compare the optimization results, the initial design parameter $\{k_3, c_3\} = \{45000, 4500\}$ is set.

Firstly, the classic linear weight sum method is adopted to solve this multiple-objective optimization problem. It is shown as follows:

$$\begin{cases} \min z(x) = \sum_{i=1}^k q_i f_i(x), \\ s.t. x \in X. \end{cases} \quad (6)$$

In the above formula, q_i is the weight and $\sum_{i=1}^k q_i = 1$. By using different weight parameters to solve the above optimal problem, the solution could be reached as follows.

4.1. Objective Function. The smoothness evaluation index J_{vao} and the root-mean-square values of in-wheel motor acceleration $\sigma_{\ddot{x}_3}$ are used that as the objective function in the multiple-objective optimization, which are expressed as $f_1(x)$ and $f_2(x)$, respectively. And we have the following formula:

$$z(x) = q_1 f_1(x) + q_2 f_2(x) \longrightarrow \min. \quad (7)$$

4.2. Restrictions. The changing ranges of the optimization variables in the multiple-objective optimization process are as follows:

$$\begin{aligned}
\frac{1}{2}k_2 &\leq k_2 \leq \frac{3}{2}k_2, \\
\frac{1}{2}k_3 &\leq k_3 \leq \frac{3}{2}k_3, \\
\frac{1}{2}c_2 &\leq c_2 \leq \frac{3}{2}c_2, \\
\frac{1}{2}c_3 &\leq c_3 \leq \frac{3}{2}c_3.
\end{aligned} \tag{8}$$

4.3. Optimization Result and Its Analysis. Because there is not enough former theoretical support for solutions, it takes nine groups of different weight parameters for optimization, regarding the average value [23] as the optimal result shown in Table 2. As a representative, the nine groups of different weight parameters are selected for the multiple-objective optimization, shown in Table 3.

Compared with the suspension parameter optimization, the multiple-objective optimization smoothness evaluation index J_{vao} is reduced by 30%, and compared with the vibration absorbing optimization, it increased by 2%. Compared with the suspension parameter optimization, the in-wheel motor acceleration increased by 3%, and compared with the vibration absorbing optimization, it is reduced by 20%. Compared with the initial design, multiple-objective optimization smoothness evaluation index J_{vao} is reduced by 31%, and the in-wheel motor acceleration only increased by 4%. Compared with the suspension parameter optimization, the vibration absorbing optimization, and multiple-objective optimization, the solution of the multiobjective optimization scheme is more balanced, which is more comprehensive and reasonable than the original design.

5. Game Optimization

5.1. Basic Principles of Game Theory. Game means an action competitive or with protesting nature. From the perspective of methodology, the game theory is a vector optimization consisting of objective function vector and strategic vector. From the perspective of countermeasure, the game mainly studies the decision when the decision-makers act with each other and the equilibrium formed by the decision under some restrictions [24].

From the above sections, it could be seen that, in the suspension parameter optimization, the improvement of the smoothness is smaller compared with the initial design. The in-wheel motor working condition is only deteriorated a little bit, while in the vibration absorbing optimization, the smoothness is obviously improved compared with the initial design and the corresponding in-wheel motor's working condition deteriorates greatly.

It could be seen that the smoothness evaluation index is in a relative resistance relation with the index, that is, the acceleration of in-wheel motors, meeting the requirement of the game equilibrium optimization. The optimization is actually to form an individual advantage and a decision-making process of taking profits separately from each side. By game decisions'

TABLE 3: Multiobjective optimization weighting parameters.

q_1	0.1	0.2	0.3	0.4	0.5	0.6	0.4	0.7	0.8	0.9
q_2	0.9	0.8	0.7	0.6	0.5	0.4	0.6	0.3	0.2	0.1

mutual compromise, the final optimization results eliminate the gaining contradictions between the two, thus forming an equilibrium scheme taking both the vehicle's smoothness and the motor's vibration smoothness into account.

5.2. Description of the Game Equilibrium Optimization. Game equilibrium optimization is represented by Nash noncompetitive game and cooperation-competitive game, the main differences of which lie in whether the game individuals abide by the unified agreement. The cooperative competition game focuses more on the whole profit and team cooperation, rather than the cooperative competition game, which only considers its self-interest to realize the maximization of its profits [25]. In the specific formula, the cooperative competition game replaces the profit function of the Nash equilibrium game by the profit weighting function.

$$F_i = q_{ii} \frac{f_i(s_i^*, \bar{s}_i^{(0)})}{f_i(s_i^{(0)}, \bar{s}_i^{(0)})} + \sum_{j=1(j \neq i)}^m q_{ij} \frac{f_i(s_i^*, \bar{s}_i^{(0)})}{f_i(s_i^{(0)}, \bar{s}_i^{(0)})} \quad (i, j = 1, 2, \dots, m). \tag{9}$$

The Nash equilibrium game roughly consists of the following four steps [26]:

- (1) By calculating the sensitivity of the variables in the game, it divides the optimization variables X_1, X_2, \dots, X_m into the strategic combination $S = \{S_1, S_2, \dots, S_m\}$ in the game side, and then game contest is conducted.
- (2) In the strategic combination S , the initial and feasible strategy combination $s^{(0)} = \{s_1^{(0)}, s_2^{(0)}, \dots, s_m^{(0)}\}$ is randomly generated.
- (3) Mark $\bar{s}_1^{(0)}, \bar{s}_2^{(0)}, \dots, \bar{s}_m^{(0)}$ as the related complements of $s_1^{(0)}, s_2^{(0)}, \dots, s_m^{(0)}$ in $s^{(0)}$. The optimization targets F_1, F_2, \dots, F_m are the profit functions of the game f_1, f_2, \dots, f_m , respectively; that is, $F_1(s) = f_1(s), \dots, F_m(s) = f_m(s)$. Fixing $\bar{s}_1^{(0)}, \bar{s}_2^{(0)}, \dots, \bar{s}_m^{(0)}$, the proper objective optimization game method (Nash equilibrium or Cooperative competition game) is selected to carry out the single objective optimization for strategic combination S_1, S_2, \dots, S_m . That is, the best decision s_i^* is solved for any i^{th} game side ($i = 1, 2, \dots, m$), making the game gaining the profit $F_i(s_i^*, \bar{s}_i^{(0)}) \rightarrow \min$ and meeting the existing restrictions.
- (4) Let $s^{(1)} = s_1^* \cup s_2^* \cup \dots \cup s_m^*$, and calculate the distance (norm) between the two (former and latter) strategic combinations to see if it meets the convergence criterion $\|s^{(1)} - s^{(0)}\| \leq \varepsilon = 10^{-6}$. If it is satisfied, the game process is over; if not, use $s^{(1)}$ replacing $s^{(0)}$ and then transfer to step (3) to go over the process. The framework of Nash equilibrium game is shown in Figure 3.

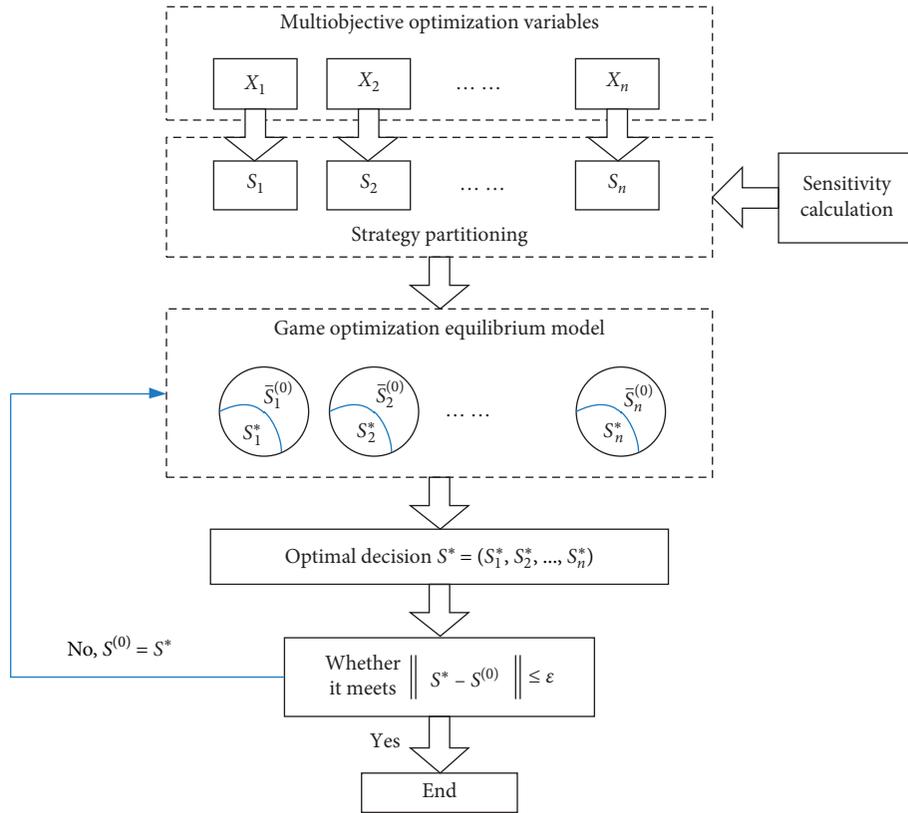


FIGURE 3: Framework of Nash equilibrium game.

5.3. *Game Optimization Calculation.* Choosing the same multiobjective optimization model, optimization target, optimization function, and restrictions with those in Section 4 to set up game optimization model, mark the game optimization’s smoothness evaluation index as J_{vao} for the game optimization calculation [27].

Firstly, conduct single objective optimization to the objective function J_{vao} , $\sigma_{\dot{x}_3}$, respectively, and get Table 4.

The optimization target in this paper is the nonlinear optimization target, very sensitive to searching space. The calculation efficiency of using general strategy division is relatively low. Therefore, it uses the orthogonal test in the experimental division to calculate the sensitivity of the various optimization variables to the objective functions.

It is known that this optimization contains four variables and two objective functions. According to the orthogonal test rule [28], the orthogonal experiment table $L_8(2^7)$ is set up. The first five lists of the table are selected to arrange the orthogonal experiment. The calculation formula of the sensitivity is as follows:

$$|S_{ij}| \approx |R_j| / |\max\{x_{j1}, x_{j2}, \dots\} - \min\{x_{j1}, x_{j2}, \dots\}|, \quad (10)$$

where i is the number of factor level, j is the number of factor column, R_j is the range of factor x_j ’s average value \bar{K}_{ij} , $\bar{K}_{ij} = T_{ij}/a_j$, a_j is the number of horizontal repetitions of the scheduled factors in the j th column, and T_{ij} is the total value of the i level of the factor arranged in the j th column whose magnitude is the sum of the a_j test results at the j th level.

The orthogonal experiment table of the various objective functions is set up, getting the sensitivities of various optimization variables to each objective function shown in Tables 5–8 in detail.

We get the test variables x_1, x_2, x_3 , and x_4 , that is, optimization variables of k_2, c_2, k_3 , and c_3 , and the sensitivities to the two objective functions are separately shown as

$$\begin{aligned} S(J_{\min}) &= \{4.525 \times 10^{-5} \quad 1.6675 \times 10^{-4} \quad 3.25 \times 10^{-6} \quad -2.5 \times 10^{-7}\}, \\ S(\sigma_{\dot{x}_3}) &= \{-1.05 \times 10^{-5} \quad 0.0000205 \quad 0.0001165 \quad -9 \times 10^{-6}\}. \end{aligned} \quad (11)$$

The sensitivity of each variable corresponds to one by one and influence factors $\Delta_1, \Delta_2, \Delta_3, \Delta_4$ are then extracted which are $\Delta_1 = \{4.525 \times 10^{-5} \quad -0.0000105\}$, $\Delta_2 = \{1.6675 \times 10^{-4} \quad 0.0000205\}$, $\Delta_3 = \{3.25 \times 10^{-6} \quad 0.0001165\}$, $\Delta_4 = \{-2.5 \times 10^{-7} \quad -9 \times 10^{-6}\}$.

Based on the fuzzy clustering method [29], the fuzzy similar equivalent matrix is obtained as follows:

$$I = \begin{bmatrix} 1.0000 & 0.9958 & 0.8136 & 1.0000 \\ 0.9958 & 1.0000 & 0.9017 & 0.9969 \\ 0.8136 & 0.9017 & 1.0000 & 0.8532 \\ 1.0000 & 0.9969 & 0.8532 & 1.0000 \end{bmatrix}. \quad (12)$$

Choose the confidence level 0.92 to divide the matrix. From the fuzzy clustering method, the strategic set of game

TABLE 4: Single objective optimization results.

Objective variables	J_{vao}	$\sigma_{\dot{x}_3}$
Suspension stiffness, k_2 (N/m)	11000	13813
Suspension damping, c_2 (N/(m/s))	750	1493
Suspension stiffness, k_3 (N/m)	55336	25000
Suspension damping, c_3 (N/(m/s))	1292	4500
Root-mean-square value of the body's acceleration	0.9038	1.2995
Root-mean-square value of suspension dynamic deflection	0.0124	0.0081
Root-mean-square value of wheels' relative loading	0.2321	0.2633
Root-mean-square value of the in-wheel motor acceleration	0.5087	0.383
Objective function, J_{vao}	1.1484	1.5709

TABLE 5: Level table of J_{vao} optimization factors.

Factors	x_1	x_2	x_3	x_4
Level 1	10950	700	55286.58	1242.84
Level 2	11050	800	55386.58	1342.84

TABLE 6: Orthogonal experimental results of the J_{vao} optimization.

Factors	x_1	x_2	x_3	x_4	J_{gop}
Experiment 1	10950	700	55286.58	1242.836	1.1389
Experiment 2	10950	700	55286.58	1342.836	1.139
Experiment 3	10950	800	55386.58	1242.836	1.156
Experiment 4	10950	800	55386.58	1342.836	1.1559
Experiment 5	11050	700	55386.58	1242.836	1.1438
Experiment 6	11050	700	55386.58	1342.836	1.1438
Experiment 7	11050	800	55286.58	1242.836	1.1602
Experiment 8	11050	800	55286.58	1342.836	1.1601
K_{j1} (J_{gop})	1.14745	1.141375	1.14955	1.149725	—
K_{j2} (J_{gop})	1.151975	1.15805	1.149875	1.1497	—
R_j (J_{gop})	0.004525	0.016675	0.000325	-2.50E-05	—
S_j (J_{gop})	4.53E-05	0.00016675	3.25E-06	-2.50E-07	—

TABLE 7: Level table of $\sigma_{\dot{x}_3}$ optimization factors.

Factors	x_1	x_2	x_3	x_4
Level 1	13763.02	1443.72	24950	4450
Level 2	13863.02	1543.72	25050	4550

TABLE 8: Orthogonal experimental results of the $\sigma_{\dot{x}_3}$ optimization.

Factors	x_1	x_2	x_3	x_4	J_{gop}
Experiment 1	13763.02	1443.72	24950	4450	0.3834
Experiment 2	13763.02	1443.72	24950	4550	0.3832
Experiment 3	13763.02	1543.72	25050	4450	0.394
Experiment 4	13763.02	1543.72	25050	4550	0.3938
Experiment 5	13863.02	1443.72	25050	4450	0.399
Experiment 6	13863.02	1443.72	25050	4550	0.395
Experiment 7	13863.02	1543.72	24950	4450	0.3839
Experiment 8	13863.02	1543.72	24950	4550	0.3847
K_{j1} ($\sigma_{\dot{x}_3}$)	0.3886	0.3886	0.3838	0.390075	—
K_{j2} ($\sigma_{\dot{x}_3}$)	0.3891	0.39065	0.39545	0.389175	—
R_j ($\sigma_{\dot{x}_3}$)	-0.00105	0.00205	0.01165	-0.0009	—
S_j ($\sigma_{\dot{x}_3}$)	-0.0000105	0.0000205	0.0001165	-9E-06	—

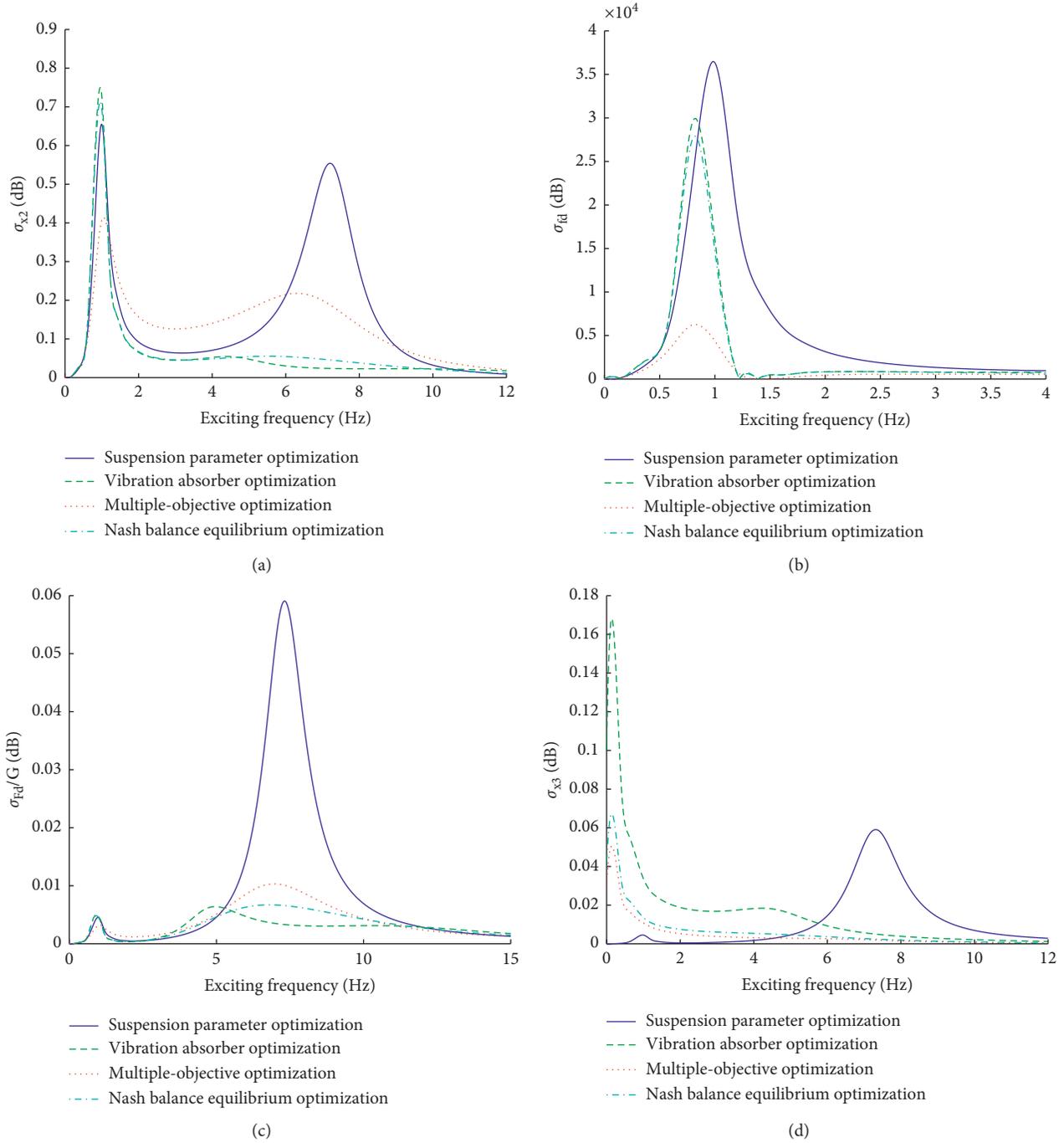


FIGURE 4: Optimization results of Nash equilibrium optimization. (a) Spectrometer of root-mean-square value of vehicle body acceleration. (b) Spectrometer of root-mean-square value of the suspension dynamic deflection. (c) Spectrometer of root-mean-square value of the wheel relative loading. (d) Spectrometer of root-mean-square value of motor acceleration.

side f_1 is $S_1 = \{x_1, x_2, x_4\}$, that is, $\{k_2, c_2, c_3\}$, and the strategic set of game side f_2 is $S_1 = \{x_3\}$, that is, $\{k_3\}$. The main parameters in the initial design are used as the initial and feasible strategy combination $s^{(0)} = \{s_1^{(0)}, s_2^{(0)}\} = \{15000 \ 1500 \ 45000 \ 4500\}$, which is the starting point of the iteration, where $s_1^{(0)} = \{x_1^{(0)}, x_2^{(0)}, x_4^{(0)}\} = \{k_2$

$(0), c_2^{(0)}, c_3^{(0)}\} = \{15000, 1500, 4500\}$, $s_2^{(0)} = \{x_3^{(0)}\} = \{45000\}$. Thus, the complements of $s_1^{(0)}$ and $s_1^{(0)}$ in $s^{(0)}$ are $\bar{s}_1^{(0)} = s_2^{(0)} = \{45000\}$ and $\bar{s}_2^{(0)} = s_1^{(0)} = \{15000, 1500, 4500\}$, respectively.

According to step (3), construct the profit function $F_1(s) = f_1(s), \dots, F_m(s) = f_m(s)$. The profit function $F_i(s_i^*, s_i^{(0)}) \rightarrow \min$ is used as the optimal game objective to

TABLE 9: Results comparison by various optimization schemes.

Evaluation index	Suspension parameter optimization	Vibration absorber optimization	Multiobjective optimization	Nash equilibrium	Cooperative competition
Suspension stiffness, k_2 (N/m)	11437	15000	11002	11600	11061
Suspension damping, c_2 (N/(m/s))	1236	1500	750	1414.2	778
Vibration absorber stiffness, k_3 (N/m)	—	56631	45773	53232	45403
Vibration absorber damping, c_3 (N/(m/s))	—	1325	2857	1398	2182
Root-mean-square value of the body's acceleration $\sigma_{\ddot{x}_2}$	1.3418	0.9264	0.9283	0.9274	0.9303
Root-mean-square value of suspension dynamic deflection σ_{fd}	0.0181	0.0125	0.0126	0.0129	0.0122
Root-mean-square value of wheels' relative loading $\sigma_{F_d/G}$	0.3978	0.2328	0.2599	0.233	0.2456
Root-mean-square value of the in-wheel motor acceleration $\sigma_{\ddot{x}_3}$	0.3978	0.4765	0.4102	0.4221	0.4028
Smoothness J_{vao}	1.7577	1.1717	1.2009	1.1734	1.1881
Iterations	—	—	—	7	11

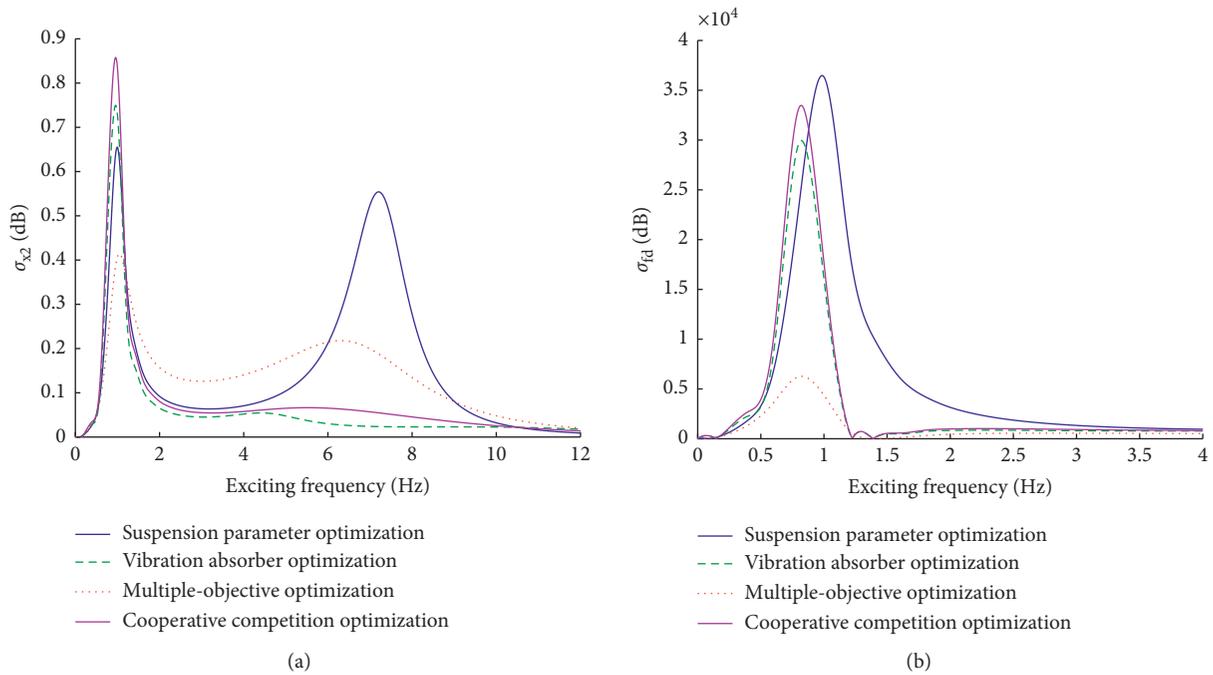


FIGURE 5: Continued.

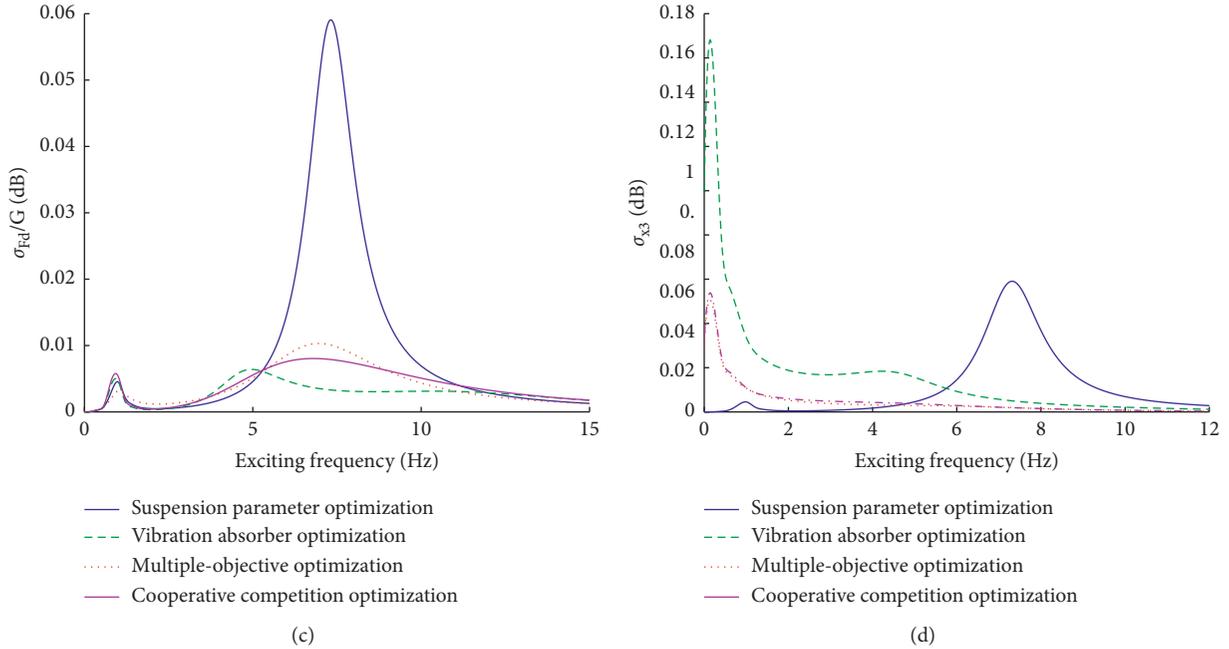


FIGURE 5: Results of cooperative competition game optimization. (a) Spectrometer of root-mean-square value of vehicle body acceleration. (b) Spectrometer of root-mean-square value of the suspension dynamic deflection, (c) Spectrometer of root-mean-square value of the wheel relative loading, and (d) Spectrometer of root-mean-square value of motor acceleration.

conduct Nash equilibrium optimization. The results are shown in Table 9 and Figure 4.

The cooperative competition game optimization is conducted using formula (9) as the profit function of the game optimization and the results are shown in Table 9 and Figure 5.

5.4. Optimization Result Comparison and Analysis Based on the Game. According to the above results' comparison, the evaluation index presents more reasonable equilibrium relative to the suspension parameter and vibration absorber optimization, which are more close to multiple-objective optimization, that is, the traditional weighting optimization scheme. But compared with the traditional and complex iterative process and the manmade preassurance weight allocation, the game optimization has the advantages of less iterations, faster convergence, and less influenced by human factor, which is more efficient. Meanwhile, vertically comparing the optimization parameters in the different optimizations, it is easy to find that although the game optimization results are similar, the optimization parameters k_2 , c_2 , k_3 , and c_3 are not close to each other.

The basic optimal parameters k_2 , c_2 , k_3 , and c_3 in Nash equilibrium optimization are basically those values between the suspension parameter optimization and the vibration absorber optimization or close to that of either group of parameters. In Nash equilibrium optimization, the specific comparison results k_2 ($11437 < 11600 < 15000$), c_2 ($1236 < 1414 < 1500$), k_3 , and c_3 are close to those in vibration

absorber optimization, while the optimization parameters k_2 , c_2 , k_3 , and c_3 in the cooperative competition optimization are more close to those in multiobjective optimization, which is due to the different profits focused by the game sides. In Nash equilibrium strategy, the game sides do not have a favour towards the objectives and they only focus on their own profits and the competition relations among the various objectives.

Nash equilibrium optimization, suspension parameters, and vibration absorber optimization have the same optimal parameters k_2 , c_2 , k_3 , and c_3 and the same profits (optimization objective), but they adopt different optimization strategies (suspension optimization and vibration absorber optimization), leading to the profits of each side conflict to each other (smoothness evaluation index and in-wheel motor acceleration have formed a certain contrast). Therefore, the Nash equilibrium could be totally seen as the competition and game between two game strategies of suspension parameters optimization and vibration absorber optimization, and the parameters allocation stays between the two values of the two optimizations. In cooperative competition equilibrium game strategy, the game sides focus on the whole profit of all the game sides and the objectives remain as cooperative and competitive relationship. The parameters are coordinated better. Similar to the idea of the traditional weight allocation, the parameters' allocation is close to the multiobjective optimization naturally.

To view in a comprehensive way, Nash equilibrium game strategy and cooperative competition game strategy both

achieve the optimization design effect of the in-wheel motor, fulfilling the complex design demand of the in-wheel motors and revealing the effectiveness and efficiency of game optimization.

6. Conclusions

The two common in-wheel motor smoothness optimization models, the suspension parameter optimization model and vibration absorber optimization model, are set up. Based on vehicle's smoothness analysis, the evaluation index of in-wheel motors' vertical vibration is introduced. The suspension stiffness and damping parameters are optimized for the suspension parameter optimization model, and the stiffness and damping parameters of the vibration absorber are optimized for the vibration absorber optimization model. The comparison and analysis of two optimization results show that, compared with the suspension parameter optimization scheme, the vibration absorber optimization scheme can better improve vehicle smoothness, but the corresponding vertical vibration acceleration deterioration of in-wheel motor worsens more.

Comparison analysis between the suspension optimization scheme and the vibration absorber optimization scheme is conducted. On this basis, a multiple-objective optimization scheme is designed. The linear weighting method is used to weigh the smoothness index and the in-wheel motors' vibration acceleration index, and the parameters of suspension stiffness k_2 , suspension damping c_2 , vibration absorber stiffness k_3 , and the vibration absorber damping c_3 are optimized by multiple-objective optimization scheme. The results show that the two optimization objectives are adjusted in a great balance to enable the vehicle in-wheel motor to work in the stable condition while maintaining the good smoothness of the vehicle.

The game optimization is applied to deal with the conflict of the two optimization objectives. Taking advantage of the orthogonal experiment combined with fuzzy clustering method, the multiple-objective optimization is converted to game equilibrium optimization. Based on the sensitivity of the optimization variables to optimization objectives, the different game strategic combinations are divided to conduct game confrontation. Using the main suspension and vibration absorber stiffness and damping coefficients as the optimization variables and the minimum profit function as objective function, the Nash equilibrium and cooperative competition game optimizations are adopted to optimize the parameters of suspension and vibration absorber, and the results are more ideal than that by linear weighting multiobjective optimization.

Combined with the game theory, the internal game relation between different optimization results is analysed. The game optimization has the advantages of the greater convergence and high efficiency. The different optimal equilibriums of the suspension vibration and in-wheel motor vibration have been achieved by the two different game strategies. Compared with the traditional

linear weighting method, this method does not require the weighting of different objectives, which has greater value applied in engineering. Although the optimization design can significantly promote the in-wheel motor drive vehicle vertical vibration performance, the parameters in the system cannot be dynamically regulated according to the vehicle driving conditions. To design the controllers to achieve the suspension and absorber parameters' real-time regulation while driving can further reduce the vehicle vertical vibration. Meanwhile, the motor vibration has the coupling with the suspension system vertical vibration, so paying attention to the coordinated controller design of the motor and suspension considering their characteristics, so as to further reduce the in-wheel motor drive vehicle vertical vibration and improve the system comprehensive performance, is especially worth studying.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This research was supported by the Major Education and Teaching Reform Research Project of Anhui Provincial Department of Education (2019zdjg11); the Natural Science Research Projects in Anhui Universities (KJ2019A1070); the Transportation Vocational Education Research Project (2018YJ203) and Open Fund of State Key Laboratory of Mechanical Behavior and System Safety of Traffic Engineering Structures (KF2020-27).

References

- [1] X. X. Shao, F. Naghdy, and H. P. Du, "Reliable fuzzy H-infinity control for active suspension of in-wheel motor driven electric vehicles with dynamic damping," *Mechanical Systems and Signal Processing*, vol. 87, pp. 21–25, 2017.
- [2] Z. Li, L. Zheng, W. Gao, and Z. Zhan, "Electromechanical coupling mechanism and control strategy for in-wheel-motor-driven electric vehicles," *Institute of Electrical and Electronics Engineers Transactions on Industrial Electronics*, vol. 66, no. 6, pp. 4524–4533, 2019.
- [3] B. Xu, C. Xiang, Y. Qin, P. Ding, and M. Dong, "Semi-active vibration control for in-wheel switched reluctance motor driven electric vehicle with dynamic vibration absorbing structures: concept and validation," *Institute of Electrical and Electronics Engineers Access*, vol. 6, no. 2018, pp. 60274–60285, 2018.
- [4] S. Nie, Y. Zhuang, F. Chen, Y. Wang, and S. Liu, "A method to eliminate unsprung adverse effect of in-wheel motor-driven vehicles," *Journal of Low Frequency Noise, Vibration and Active Control*, vol. 37, no. 4, pp. 955–976, 2018.
- [5] Z. Wang, C. Yong, Z. Li et al., "Simulation analysis and optimization of ride quality of in-wheel motor electric

- vehicle,” *Advances in Mechanical Engineering*, vol. 10, no. 5, pp. 1–10, 2018.
- [6] M. Liu, F. Gu, J. Huang, C. Wang, and M. Cao, “Integration design and optimization control of a dynamic vibration absorber for electric wheels with in-wheel motor,” *Energies*, vol. 10, no. 12, pp. 2069–2091, 2017.
- [7] A. E. Rojas Rojas, H. Niederkofler, and J. Willberger, “Comfort and safety enhancement of passenger vehicles with in-wheel motors,” *SAE Technical Papers*, vol. 10, pp. 4–6, 2010.
- [8] G. B. Ning and G. Wan, “The present research situation of the influences on vehicle vertical performances induced by direct wheel drives system,” *Automobile Technology*, vol. 3, pp. 21–25, 2007.
- [9] Y. Ma, Z. X. Deng, and D. Xie, “Analysis and optimization of in-wheel motor suspension configuration,” *Journal of Central South University (Science and Technology)*, vol. 9, no. 45, pp. 3008–3014, 2013.
- [10] W. Tong and Z. C. Hou, “Analyses on the vertical characteristics and motor vibration of an electric vehicle with motor-in-wheel drive,” *Automobile Engineering*, vol. 4, no. 36, pp. 398–403, 2013.
- [11] D. Tan and C. Lu, “The influence of the Magnetic force generated by the in-wheel motor on the vertical and lateral coupling dynamics of electric vehicles,” *Institute of Electrical and Electronics Engineers Transactions on Vehicular Technology*, vol. 65, no. 6, pp. 4655–4668, 2016.
- [12] D. Tan, *Dynamics and Structure Optimization of the In-Wheel Motor System with Rubber Bushing*, South China University of Technology, Guangzhou, China, 2013.
- [13] Z. Li and L. Zheng, “Integrated design of active suspension parameters for solving negative vibration effects of switched reluctance-in-wheel motor electrical vehicles based on multi-objective particle swarm optimization,” *Journal of Vibration and Control*, vol. 25, no. 3, pp. 639–654, 2019.
- [14] S. Özyıldırım and N. Alemdar, “Learning the optimum as a Nash equilibrium,” *Journal of Economic Dynamics & Control*, vol. 24, pp. 483–499, 2000.
- [15] J. Periaux, H. Q. Chen, B. Mantel, M. Sefrioui, and H. T. Sui, “Combining game theory and genetic algorithms with application to DDM-nozzle optimization problems,” *Finite Elements in Analysis and Design*, vol. 37, no. 5, pp. 417–429, 2001.
- [16] Z. Li, N. G. Xie, and Y. M. Zhang, “Cooperative game method for structural optimization with multiple objectives,” *Engineering Mechanics*, vol. 4, no. 26, pp. 32–37, 2009.
- [17] H. B. Zhang, L. Gu, and Y. Z. Xu, “Vehicle crashworthiness multi-objective optimization based on game theory,” *Automotive Engineering*, vol. 7, no. 30, pp. 553–556, 2008.
- [18] T. F. Ma, N. W. Xue, Z. X. Li et al., “The analysis of the influence factors of vehicle handling and stability and study and evaluation on handling and stability,” *Machinery Design & Manufacture*, vol. 4, pp. 122–123, 2005.
- [19] R. He and R. J. Zhang, “Research and development of in-wheel motor drive technology,” *Journal of Chongqing University of Technology (Natural Science)*, vol. 7, no. 29, pp. 10–18, 2015.
- [20] Y. M. Sun, *Study on Suspension Vertical Mechanical-Electrical Coupled Vibration of In-Wheel Motor Electrical Vehicle*, Anhui Polytechnic University, Wuhu, China, 2015.
- [21] W. Tong, *Research on the Vertical Performance and Motor Vibration of Electrical Vehicle Driven by In-Wheel Motors*, Tsinghua University, Beijing, China, 2013.
- [22] Y. E. Zhao, J. W. Zhang, and X. Han, “Design and study on the dynamic-damper mechanism for an in-wheel motor individual drive electric vehicle,” *Mechanical Science and Technology for Aerospace Engineering*, vol. 3, no. 27, pp. 395–398, 2008.
- [23] H. W. Tang, *Practical Methods of Optimization*, Dalian University of Technology Press, Dalian, China, 2004.
- [24] R. G. Fan, *Game Theory*, Wuhan University Press, Wuhan, China, 2011.
- [25] XG. Na, *Game and Decision-Making*, Foreign Language Teaching and Research Press, Beijing, China, 2011.
- [26] J. Tan and L. Wang, “A game-theoretic framework for vehicle-to-grid frequency regulation considering smart charging mechanism,” *Institute of Electrical and Electronics Engineers Transactions on Smart Grid*, vol. 8, no. 5, pp. 2358–2369, 2017.
- [27] Z. R. Shen, *The Research and Application of Multi-Objective Optimization Design Method Based on Game Theory*, Zhejiang University, Hangzhou, China, 2013.
- [28] L. Q. Ren, *Optimum Design and Analysis of Experiments*, Higher Education Press, Beijing, China, 2001.