

## Research Article

# Catching Critical Transition in Engineered Systems

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A variety of engineered systems can encounter critical transitions where the system suddenly shifts from one stable state to another at a critical threshold. The critical transition has aroused vital concerns for its potentially disastrous impacts. We validate an often taken-for-granted hypothesis that the failure of engineered systems can be attributed to the respective critical transitions and show how early warning signals are closely associated with critical transitions. We demonstrate that it is feasible to use early warning signals to predict system failures. Our findings open a new path to forecast failures of engineered systems with a generic method and provide supporting evidence for the universal existence of critical transition in dynamical systems at multiple scales.

## 1. Introduction

Critical transition, i.e., the dramatic shift of states in complex dynamical systems, is increasingly recognized as a vital concern for its potentially disastrous impacts in many social-ecological systems [1, 2]. Despite the difficulty in predicting such transitions, recent years witnessed significant progresses in identifying early warning signals [3–9] by analyzing the observed time series [10–12]. For example, critical slowing down (CSD) [10] as well as other indicators like the changing skewness [13] have been found to be present in a wide spectrum of complex systems ranging from ecosystems [14–16], neurons in mammalian cortices [17], and climates [18] to financial markets [19] when approaching the tipping point. A detailed review of past research dedicated to CSD indicators and associated tipping points and promising research directions for future research has been conducted by Nazarimehr et al. [20]. Besides, some researchers have explored the approaches of predicting the tipping points [21] and discussed the new scientific understanding of social problems through a complex systems perspective, such as crowd disasters, crime, terrorism, wars, and epidemics [22]. It is thus intriguing to explore the viability of exploiting the early warning signals to detect the tendency toward failure in engineered systems, which also exhibits intricate stochastic

behavior under complex working conditions. However, it is still an open problem to establish the universality of the above early warning indicators in both natural and engineered systems. Here we demonstrate the feasibility of using early warning signals to predict system failures in a variety of engineered systems from various domains. We show how early warning signals, i.e., CSD and deviating skewness, which can be extracted by performing a Fluctuation Mining (FM) procedure to capture the inherent stochastic behaviors, are closely associated with critical transitions at the points of systems approaching failures. Our results suggest that early warning signals are prevalent in dynamic systems. We anticipate that finding early warning signals through critical transition analysis will open a new path to predict the failures of engineered systems by offering a generic method.

Complex dynamical systems can undergo critical transitions when arriving at a specific threshold [23, 24]. The existence of such tipping points is the result of the inherent nonlinearity and stochasticity in the dynamical systems [25]. Due to the global concern on the sustainability of the complex social-ecological systems, intensive research efforts have been dedicated to identifying early warning signals for critical transitions [26]. It is increasingly recognized that a wide range of systems exhibit CSD and deviating skewness, which are caused by the fact that it is harder for a system

approaching the critical transition to return to the equilibrium upon perturbations. Various studies suggest that these early warning signals can be recognized in the intrinsic stochastic fluctuations of many social-ecological systems and these signals provide an essential solution to predict potential natural disasters [2].

Despite their seemingly vast difference from social-ecological systems, engineered systems such as jet engines, locomotives, and bearings are also dynamical systems demonstrating nonlinear behaviors and stochastic fluctuations under the complex interplay of internal and environmental factors [27, 28]. The failure of engineered systems may incur disastrous impacts since the operation of modern society hinges on their smooth functioning. One example is the tragic high-speed train accident [29] happening in Eschede (Germany) in 1998, which was caused by the fatigue-induced crack of a wheel. Intuitively, the process from the suddenly accelerated worsening of cracking to the final derailment bears significant similarity to the critical transition in natural systems. Here, we explore the universality of early warning signals in engineered systems.

Discovering potential failures in engineered systems is extremely challenging. Although these systems are designed to offer well-defined behaviors, the complex interactions within their components and with the working environment tend to be too complex for exhaustive analytical modeling [30]. The problem can be more involved if a positive loop of feedback exists in the system. Even if the system dynamics approaching failure can be derived through physics-of-failure or reliability analysis [31], the working conditions and the environment cannot be fully reproduced for a complete treatment of various idiosyncrasies in the system. The globalization of the manufacturing process also hinders a comprehensive understanding of the domain knowledge. Precise prediction of failures in engineered systems in a timely fashion is still a worldwide challenge. It is thus appealing to devise effective generic early warning indicators that require less or even no domain knowledge.

## 2. Methods and Results

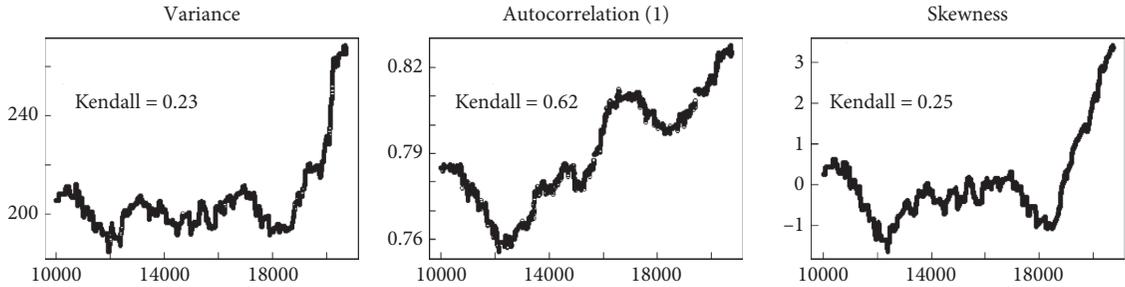
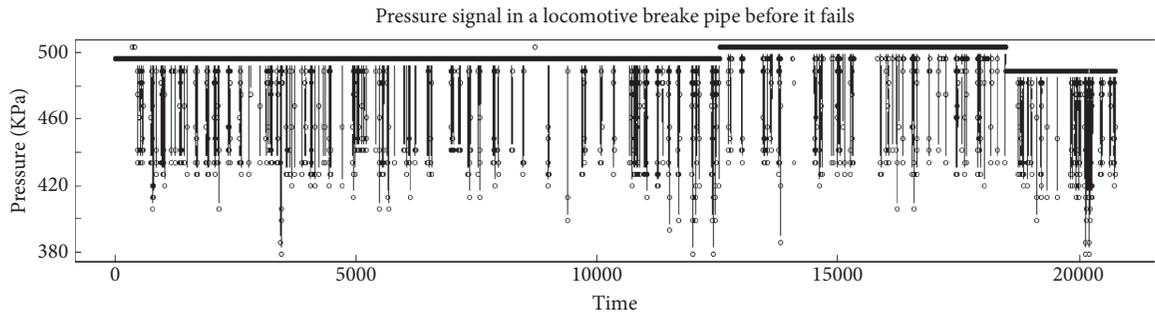
Hypothesizing that the critical transition is strongly correlated with the failure of engineered systems, we examine four datasets from different engineered systems, that is, an airbrake system of diesel locomotive (a pneumatic system), a turbofan engine (a mechatronic system), an IGBT (a power electronic system), and a rolling bearing (a mechanic system), to investigate whether early warning signals can be identified. The airbrake dataset is gathered from a commercially operating locomotive, while the remaining three datasets are public prognostics benchmarks available by NASA Prognostics Center of Excellence (PCoE) [32]. In the NASA PCoE Datasets, the turbofan engine data are generated in an industry-strength simulation environment [33], whereas the IGBT and bearing datasets are produced through aging experiments [34, 35]. All the datasets are univariate or multivariate time series collected by sensors during the process from normal operating to running into failure. With respect to data availability, the airbrake dataset

is gathered from a commercially operating locomotive, while the remaining three are public prognostics benchmarks made available by NASA Prognostics Center of Excellence (PCoE) at the website: <https://ti.arc.nasa.gov/tech/dash/groups/pcoe/prognostic-data-repository/>.

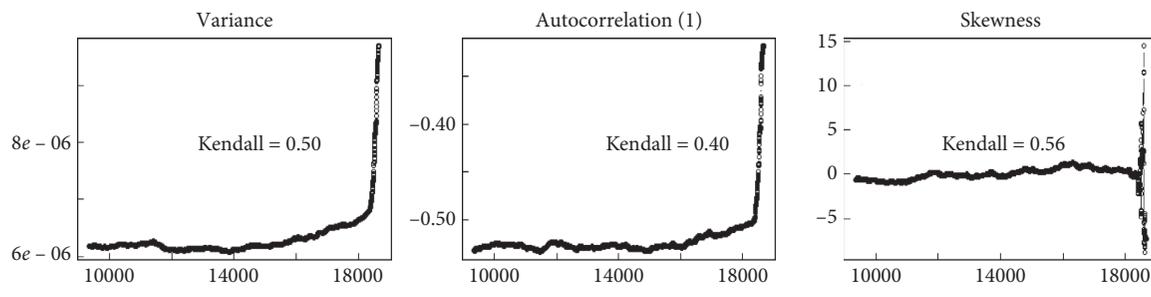
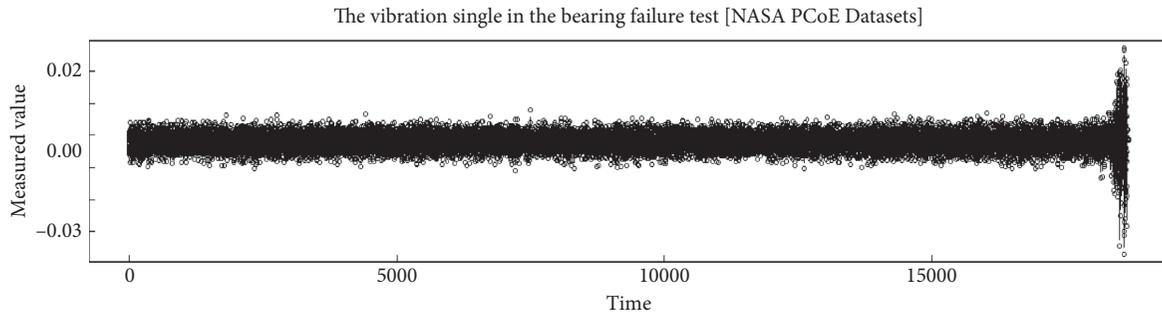
With proper preprocessing, we derive the variance, autocorrelation, and skewness characteristics of the time series. Although they are from considerably different domains, all the four systems unanimously exhibit the typical symptoms of CSD and deviating skewness (Figure 1 and Supplementary Figure S1). Such an observation seems to suggest that early warning indicators can serve as the precursor of system failure.

The dynamic behaviors of an engineered system are generally reflected by one or more observable signals, which can be discretely sampled into time series. The values in a time series are determined by both an operational trend (i.e., main trend) and stochastic fluctuations. In the case of the complex dynamic social-ecological system, a detrending [11] procedure is often necessary to extract the stochastic components for analyzing early warning signals [1]. As rather obvious variations can be observed in the curves of variance, autocorrelation, and skewness (Figure 1), we devise a Fluctuation Mining (FM) framework to ascertain stochastic fluctuations that is not directly associated with the designed operation of engineered systems (Figures 2(a)–2(c)). The framework consists of a data cleansing step that removes the failure-insensitive components in the observed signals and a detrending step that eliminates the dominant trend, i.e., the overall tendency of change, in the time series. For instance, the application of the FM on the IGBT dataset successfully reveals a much more obvious propensity of increasing variance and autocorrelation as well as deviating skewness (Figures 2(f)–2(h) and Supplementary Figure S2). In addition, although early warning signals can be consistently identified in all the four engineering systems from different domains, we still need to verify that the observations with a 50% window size are not a coincidence. Thus, the sensitive test is implemented that analyzed the indicators with different window sizes (Figure S3).

The findings so far have validated the existence of early warning signals in engineered systems with forthcoming failures. The correspondence between the critical transition and the system failure, however, is still an open problem. We endeavored Student's  $t$ -test [36], the Gaussian kernel-based probability density test [36], the Akaike Information Criterion- (AIC-) based ARIMA model analysis [37], and the phase space analysis [38] on the four engineered systems before and after they reach their failures (Figure 3 and Supplementary Figure S4). The analysis results consistently suggest that the systems undergo an abrupt change of state. For example, the bearing system remains stable under normal operation and diverges to an unstable state upon failure (Figures 3(b)–3(e)), the turbofan engine witnesses a complete change of the underlying dynamical function (Figure 3(g)), and in the case of the IGBT system, the normal functioning stage and failure stage reveal significant varying limit cycles in the phase portrait (Figures 3(i) and 3(k)). The above analysis shows strong evidence on the equivalence



(a)



(b)

FIGURE 1: Continued.

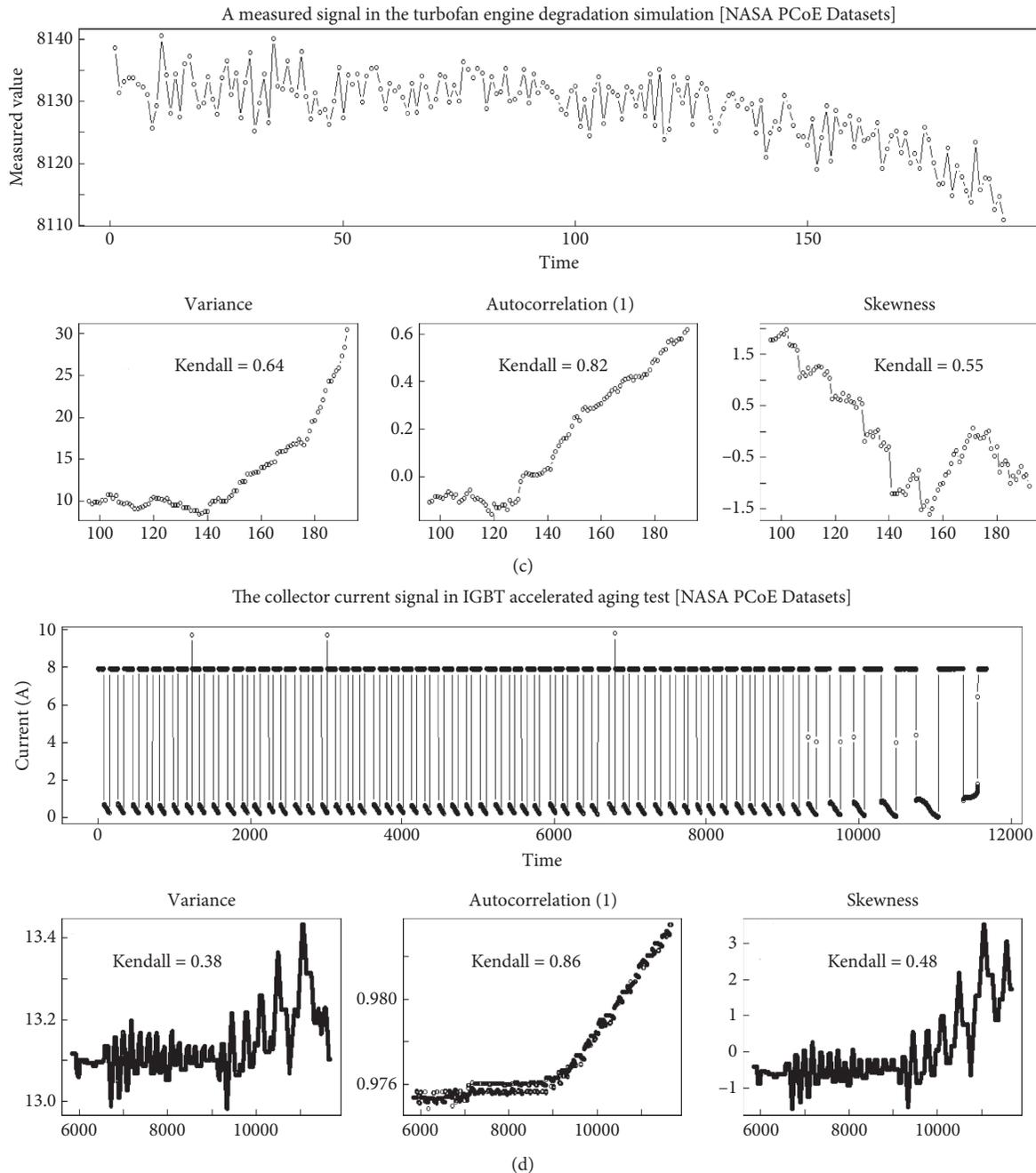


FIGURE 1: The presence of early warning signals in terms of the CSD (i.e., increasing variance and autocorrelation) and deviating skewness in four engineered systems: the airbrake system of a locomotive, the bearing, the turbofan engine, and the IGBT. a(1), the air pressure signal in the brake pipe of a locomotive in the period of two days before the failure of unrecoverable pressure loss. The signal was collected by the Train Control/Management System (TCMS) of the locomotive every second. It was preprocessed by eliminating the data corresponding to the parking states. b(1), the vibration signal from a bearing test platform collected by NASA PCoE while the bearing is operating until breaking down. c(1), an unspecified signal in the turbofan engine degradation simulation collected by NASA PCoE while the engine runs to failure. d(1), the collector current signal of a power IGBT during accelerated aging tests collected by NASA PCoE. a(2–4), b(2–4), c(2–4), and d(2–4), the early warning signal indicators. a(2), b(2), c(2), and d(2), the variance of the respective signal. a(3), b(3), c(3), and d(3), the lag 1 autocorrelation of the respective signal. a(4), b(4), c(4), and d(4), the skewness of the respective signal. The sliding window to compute variance, autocorrelation, and skewness is chosen as 50% of the span of time series for all the four systems. There are strong evidences of CSD and deviating skewness in all the four systems when they approach system failures. An increasing trend in both variance and lag 1 autocorrelation (i.e., CSD) can be clearly recognized. Clear deviations are observed in the skewness indicator. For instance, the airbrake and the IGBT systems show a strong tendency of increase before the system failure. The skewness in the bearing system significantly oscillates when approaching failure, while the skewness in the case of the turbofan has a consistent tendency of decreasing from the original value at around 1.5. Despite the clear trend, strong fluctuations in the variance, autocorrelation, and skewness are observed in all the cases, suggesting systematic disturbances existing in the original time series. For example, the IGBT data were collected from the accelerated aging

test by periodically switching on and off the device. It is the transient current after switching off that reflects the inherent resilience [32]. The times series of the collector current mainly be two components: the switch-on current signal (which is failure-insensitive) at around 8 A and the switch-off current signal (which is failure-sensitive) exponentially converging to zero  $d(1)$ ). We show how to derive a more evident tendency in the early warning signals by focusing on failure-sensitive components in Figure 2. (a) Pressure signal in a locomotive brake pipe before it fails. (b) The vibration signal in the bearing failure test (NASA PCoE Datasets) locomotive brake pipe before it fails. (c) A measured signal in the turbofan engine degradation simulation (NASA PCoE Datasets). (d) The collector current signal in IGBT accelerated aging test (NASA PCoE Datasets).

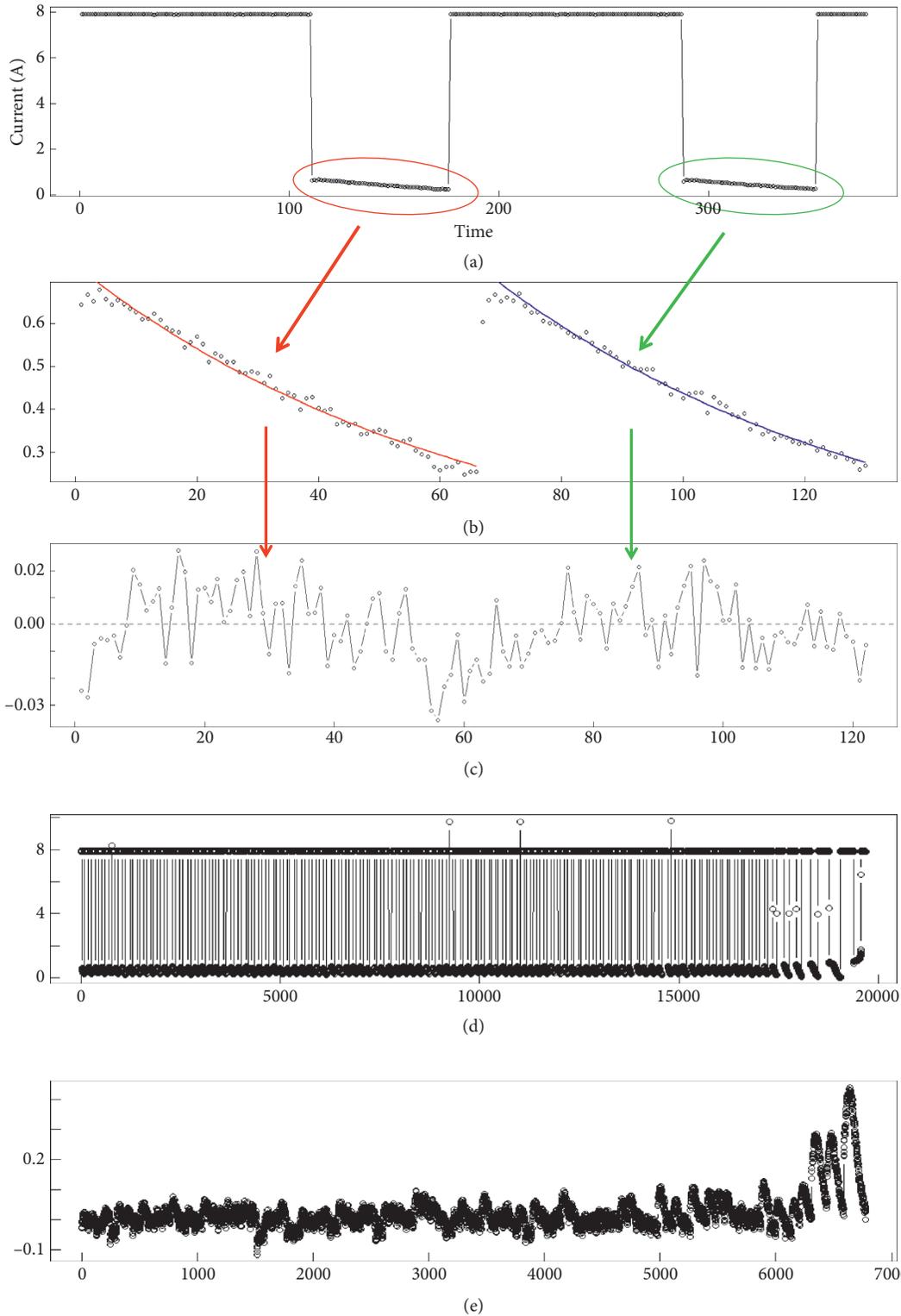


FIGURE 2: Continued.

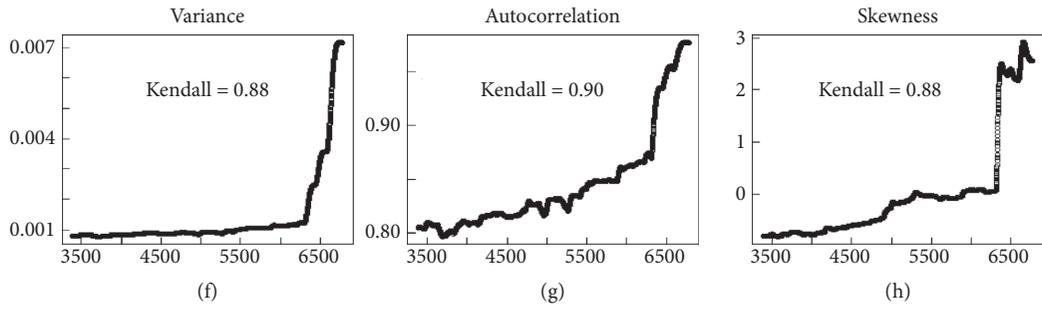


FIGURE 2: The Fluctuation Mining (FM) process and the identification of intrinsic stochastic fluctuations. (a–c) The processing flow of FM on the collector current of the IGBT system. (a) A snippet of the time series of collector current collected from the IGBT system. (b) The failure-sensitive component, i.e., the switch-off current that exponentially converges to 0, in the collector current. (c) The fluctuations derived after performing FM on the previous snippet of the time series. (d) The original time series of collector current collected from the IGBT accelerated aging test. (e) The fluctuations derived after FM for (d). (f–h) The variance, autocorrelation, and skewness indicators after FM. Analysis of the signals after the FM process shows strong evidence of CSD and deviating skewness.

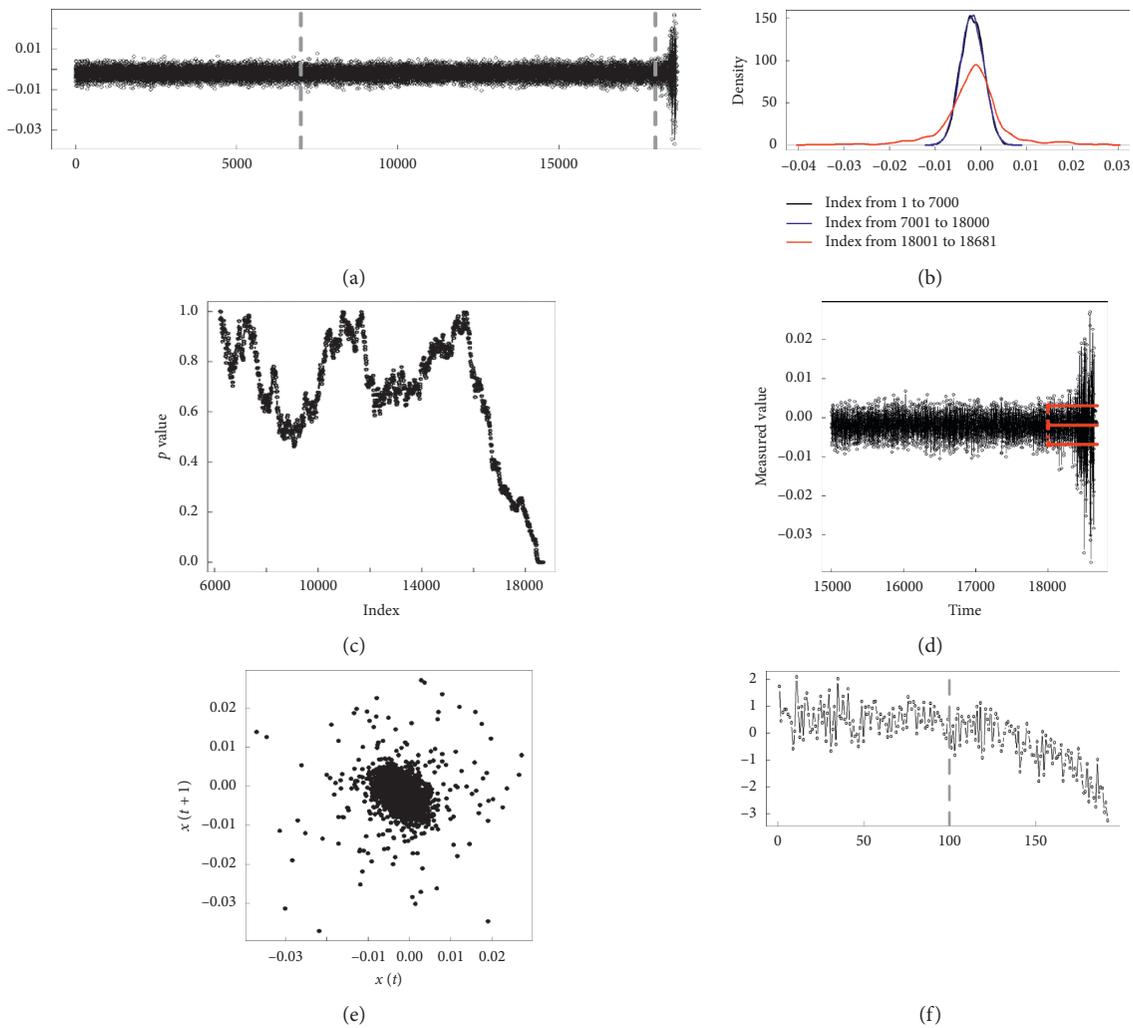


FIGURE 3: Continued.

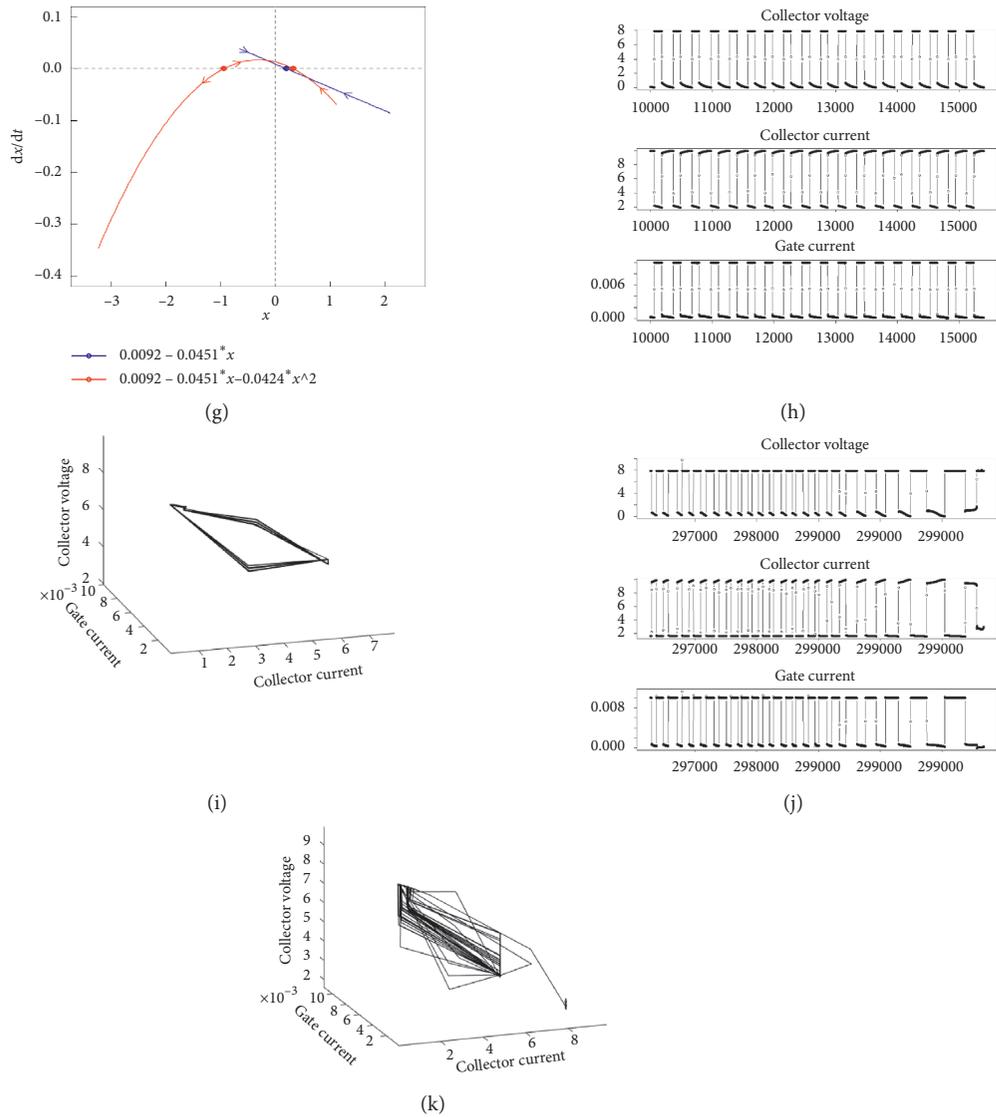


FIGURE 3: Critical transition analysis of the engineered systems when running into failures with various methods. (a) The original time series of the vibration signal collected from the bearing system approaching failure, with the dashed line segmenting the signals into three sections for succeeding analysis. (b–e) Critical transition analysis for (a). (b) The Gaussian kernel-based probability density estimates for the three sections of the vibration signal. The density of the third section significantly differs from the first two. (c) The results of Student’s  $t$ -test of the vibration amplitude signal with the true value of the mean are the mean of a normally functioning section. The tested  $p$  value shows an evident decrease to below 0.01, which significantly denies the null hypothesis, indicating that the system evolves into a different state when getting into system failure. (d) Forecasting with a 95% confidence interval (enclosed in the red bars) derived from the AIC-based ARIMA model for the third section of the vibration signal. The failure in encompassing the vibration signal in the section suggests a phase transient. (e) Logistic map for phase space analysis of the vibration signal. The first two sections are rather concentrated while the third section diverges to unstable. (b–e) A critical transition in the bearing system when it is arriving a system failure. (f) A time series of the turbofan engine system during the degradation simulation process with the dashed line separating the signals into two sections at a departure point. (g) The phase portrait of the two sections of the time series signal of the turbofan engine system. A governing dynamical equation,  $dx/dt = 0.0092 - 0.0451x$ , is learned from the left section and a system equation,  $dx/dt = 0.0131 - 0.0261x - 0.0424x^2$ , for the right section [36]. It shows that the left section has only one stable equilibrium point, while the right section has two, of which the one at the negative axis potentially drives the system to unstable (i.e., the corresponding system failure). h(1–3) and j(1–3), two snippets of the three selected signals of the IGBT system at the normal functioning stage and the close-to-failure stage. (i, k) The phase portraits of the signals in h(1–3) and j(1–3). The phase portrait of the normal functioning stage has a clear limit cycle serving as the basin of attractor, while the portrait of the close-to-failure stage reveals divergent behaviors indicating the occurrence of critical transition. More analysis of critical transition can be found in the supplemental material. The above analysis of the engineered systems suggests that the systems undergo certain critical transitions when running into system failures.

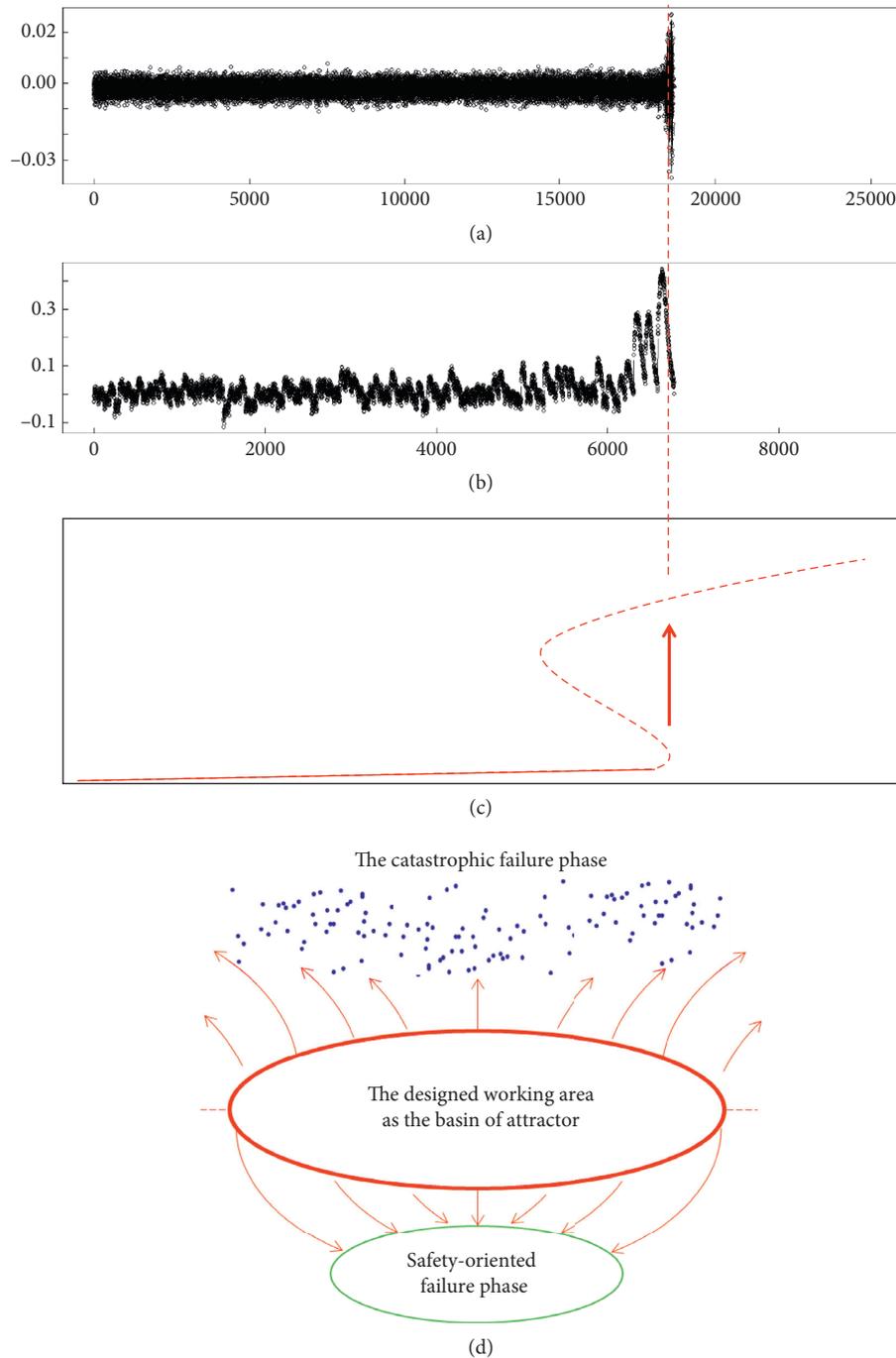


FIGURE 4: The potential catastrophic bifurcation and critical transition in engineered systems. (a) The abrupt state change in the vibration signal of the bearing system while running into failure. (b) The abrupt state change in the fluctuation of the IGBT collector current extracted with FM. (c) The hypothesized catastrophic bifurcation at the system failure. The catastrophic bifurcation can be mapped to the sudden regime shifts in (a) and (b). Considering the nonlinear phenomenon like hysteresis, the above hypothesis is likely to hold. (d) The proposed three phases, a nominally designed working phase that is equivalent to the basin of attractions, a predefined safety-oriented phase when the system initializes the self-protection after certain failures happened, and an unstable failure phase exhibiting out-of-control performance, in engineered systems with dynamical behaviors. The catastrophic bifurcation, i.e., critical transition, occurs when the system departs from the basin of attraction and loses its resilience as the system fails.

between the critical transition and the system failure in the engineered systems. Such correspondence further demonstrates the applicability of the early warning signals in predicting incoming system failures.

Our findings are the first to recognize the critical transition as a common property in engineered systems, and early warning signals can be effectively used in predicting failures in such systems. The discoveries have implications in

several aspects. First, we offer evidence that the phenomenon of early warning signals is prevalent in both natural and engineered systems. It is probably not surprising that there are no absolute boundaries between the two as the complex dynamics are fundamentally universal. Second, we validate an often taken-for-granted hypothesis that the failure of engineered systems can be attributed to the respective critical transitions. Third, it is feasible to catch disastrous bifurcations with early warning indicators, which can be identified by investigating the fluctuations in an engineered system. Such a discovery paves the way toward a generic methodology to predict and prevent potential disasters caused by the failure of engineered systems.

### 3. Discussion

Critical transition, as shown to have a strong correlation with system failure, is likely to be the aftermath of a catastrophic bifurcation [39]. Taking the bearing system and the IGBT system as examples, the sudden shift in the fluctuation signals exhibits the features of bifurcations (Figures 4(a) and 4(b)). In fact, nonlinear phenomena like hysteresis, which are closely related to bifurcations, can be found in many engineered systems, e.g., hysteretic actuators in control systems, positive feedback-based circuits in electronics, airstream in aerodynamics, elastic deformation of materials, and magnetic and electrical hysteresis in ferromagnetic materials [40]. Hence, we anticipate that the catastrophic bifurcation, which can be triggered by a small perturbation of a latent parameter, to be a common phenomenon in engineered systems approaching failures (Figure 4(c)). We propose that the engineered systems with dynamical behaviors can be abstracted to have three phases: a nominal design phase that is equivalent to the basin of attraction, a predefined safety-oriented phase when the system initializes the self-protection after certain faults happened, and an unstable failure phase exhibiting out-of-control performance (Figure 4(d)). The critical transition occurs when the system departs from the basin of attraction and loses its resilience [41] as the system fails.

### Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

### Conflicts of Interest

The authors declare that they have no conflicts of interest.

### Authors' Contributions

J.H. and Y.D. proposed the idea and wrote the manuscript. T.M. and F.H. performed the analysis.

### Acknowledgments

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### Supplementary Materials

This supplementary file mainly includes some necessary information related to data source descriptions, as well as additional analysis on early warning signals in the systems, additional validation of the FM process, sensitivity analysis results, and additional verification of critical transition in the engineered systems. (*Supplementary Materials*)

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