A Study of Bipolar Fuzzy Soft Sets and Its Application in Decision-Making Problems

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1. Introduction

Education is considered to be the core need and right of every individual and in every discipline, namely, in engineering and agriculture. The dream of a healthy, peaceful, and prosperous world can be made true on the basis of education. This strengthens an individual personality building and keeps updated with new knowledge, and experiences also help to understand the modern world and advancements. It is therefore considered as a tool for eradicating poverty [1]. A theory about human capital says that education is the source of bringing positive change to human life. It builds his personality by improving his intellectual capabilities and provides him with a path to upgrade his hidden talents and also provides a way of increasing his earning [2]. The students need better education and knowledge when they are getting admission for higher education. It is seen that mostly students select the university (such as engineering or medical) according to the opinion and experiences of their fathers and senior students but it may go wrong as with the passage of time, universities change their policies and there is a need to see the certain that a student look for while getting admission. There is a need of a decision-making tool to help student for the correct selection. Decision making of multiple parameters is dynamic and difficult. The difficulty of the decision-making process with multicriteria is due to multiple parameters. Selecting the best university (such as engineering or medical) is the multicriteria decision-making method which has many problems.

The analytical hierarchy process (AHP) is a recent addition to various approaches used to determine the relative importance of a set of activities or criteria. The novel aspect and major distinction of this approach are that it structures any complex, multiperson, multicriteria, and multiperiod problem hierarchically.
L. A. Zadeh [3] gave the idea of fuzzy set theory to deal with the vagueness and uncertainties occurring while decision making. It is human nature to have dual thinking. Zhang in [4] gave the idea of bipolarity to overcome the double-sided thinking nature of human while decision making. It provides more flexibility while modeling human thinking as compared with fuzzy.

The idea of soft sets described by Molodtsov [5] is an extension of fuzzy sets which is used to describe uncertainties. Alkhazaleh and Salleh gave the idea of soft expert sets where they gave the idea of including the opinion of experts for decision making. Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is a famous approach to deal with the multicriteria decision-making method that was first proposed by Hwang and Yoon [6]. It is a multicriteria decision-making method to find a best choice among set of alternatives.

The idea of the classical TOPSIS method is discussed by Chen Hwang and Lofti in which the rating and weights of the criteria are given by crisp values [7]. As in real life situations, we deal with vagueness and ambiguity. So, the drawback of the classical TOPSIS is that it does not deal with vague situations for decision making. Therefore, Chen [8] extended the classical TOPSIS to fuzzy TOPSIS to cover the vagueness and uncertainty occurring in real-life situations. To cover the uncertainty and give more precise decision, Dey et al. [9] extended TOPSIS for solving decision-making problems under bipolar neutrosophic environment. Apart from this, some tremendous progress has been extensively done by different researchers in different disciplines (see [10–13]).

Recently, another new technique was introduced by Xiao [14] for improving the performance of fuzzy complex event processing-based decision-making systems. He addressed the issues of intrinsic uncertainty in dynamic input events and overcame the difficulties of operator distribution problem. He proposed the CAFltr system for operator scheduling on fuzzy complex event processing systems based on the technique for order preferences by similarity to an ideal solution. The efficiency of the proposed technique has been checked through an application on the stream base system.

Lots of work have been done on decision-making processes and are readily available (see [15, 16]). Decision maker always gets confused on what technique to be applied to carry out the decision process. Choosing a wrong technique can lead to imprecise and ambiguous results. Therefore, it is very important to choose a right technique at right time. Though all the techniques that are presented so far are correct and accepted across the world, it is always hard to choose a best technique. The best method implies to be the method giving the fastest results or sometimes researchers look for cheapest methods to be the best method. A technique with lots of complicated calculations could be time consuming and needs expertise to solve one.

The gaps in the previous methods were because of their inability to cover human judgmental thinking that is on both positive and negative sides. This drawback has been overcome by the bipolar fuzzy concept see [17, 18] which further innovates the new concept. On the other hand, by including soft sets, the order of preferences see, [19, 20] is created; thus, a more precise result can be obtained. Thus, the optimal decision is obtained by using bipolar fuzzy TOPSIS approach.

Briefly speaking, the objective of this research is based on overcoming the hurdles and problems by using bipolar fuzzy which covers human dual behavior combined with soft sets which give order of preferences thus giving a bipolar fuzzy soft set decision approach.

The significance of this study is that it provides a correct decision-making method when there is ambiguity and dual human responses. It gives us basis of dealing with human nature by implementing bipolar fuzzy numbers. It is the best way to deal with two-sided human behavior as bipolar has two sides like agree and disagree. Bipolar fuzzy set is defined in the interval (−1, +1), where −1 indicates the negative opinion whereas +1 indicates the positive opinion, so it provides a wide space of dealing with uncertainties [21–23].

In this research to cover the vagueness and imprecision, a trapezoidal bipolar fuzzy number is assigned to each linguistic value. The role of bipolar is very important as bipolarity provides precision and flexibility to the system when compared with crisp and fuzzy models. The human decision making is vague and is double sided so it allows us to make decision making in bipolar fuzzy environment, and using soft sets gave us the order of preference thus making the data precise.

2. Preliminaries

The role of bipolarity is very essential in many research fields. It provides more flexibility as compared with fuzzy and Boolean logic. Zhang [4] defined the concept of bipolarity as a wide variety of human decision making is based on double-sided or bipolar judgmental thinking on a positive side and a negative side, for instance, cooperation and competition, common interests and conflict interests, friendship and hostility, likelihood and unlikelihood, and effect and side effect.

The concept of soft expert sets is defined by Salleh (2011) as follows.

Definition 1. A pair \((F, A)\) is called a soft expert set over \(U\), where \(F\) is a mapping given by \(F: A \rightarrow P(U)\) where \(P(U)\) denotes the power set of \(U\).

3. Proposed Bipolar Fuzzy Multicriteria Decision-Making Technique and Soft Sets Algorithm

3.1. Bipolar Fuzzy Analytical Hierarchical Process. Analytical hierarchy process (AHP) is used to estimate criterion weights through pairwise comparison tables. Through this comparison, the decision makers are allowed to assign weight to the parameters, and thus alternatives are compared without any difficulty. We convert crisp value into trapezoidal bipolar fuzzy numbers. By using these numbers, we construct a pairwise decision matrix defined as follows:
which are the trapezoidal bipolar fuzzy numbers. Here, $M$ and $N$ represent the positive and negative responses of the bipolar fuzzy number.

Then, aggregated bipolar fuzzy weights are assigned with respect to the positive and negative responses of each criterion. These are represented as follows:

\[
\mu_i = \left[ (w_{1j}, w_{2j}, w_{3j}, w_{4j}), (w'_{1j}, w'_{2j}, w'_{3j}, w'_{4j}) \right],
\]

where $j = 1, 2, \ldots, m$ and $\mu_i$ is calculated by taking the average of the weights with respect to positive and negative responses.

The defuzzification process to obtain crisp values of weights using the ranking function is as follows:

\[
\varepsilon_{ij} = \left( \frac{w_{1j} + w_{2j} + w_{3j} + w_{4j}}{4} + \frac{-w_{1j} - w_{2j} - w_{3j} - w_{4j}}{2} \right) - \left( \frac{w'_{1j} + w'_{2j} + w'_{3j} + w'_{4j}}{4} + \frac{-w'_{1j} - w'_{2j} - w'_{3j} - w'_{4j}}{2} \right),
\]

To get normalized weights, we use the following formula:

\[
\sum_{i=1}^{n} W_i = \left[ \frac{\varepsilon_1}{\sum_{k=1}^{m} \varepsilon_k}, \frac{\varepsilon_2}{\sum_{k=1}^{m} \varepsilon_k}, \ldots, \frac{\varepsilon_k}{\sum_{k=1}^{m} \varepsilon_k} \right].
\]

The sum of the normalization must be equal to 1.

\[
\sum_{i=1}^{n} W_i = 1.
\]
3.2 Bipolar Fuzzy TOPSIS Method. The technique for order preference by similarity to ideal solution gives a framework to handle this type of problem. This model is based on the modified bipolar fuzzy TOPSIS method. The main steps include the construction of a decision matrix, weighted standardized decision matrix, determination of bipolar fuzzy positive ideal solution and negative ideal solution, calculation of the Euclidean distance, and computation of the score of alternatives and rank the preference order. The multi-criteria decision-making problem is defined by set of alternatives and parameters. Let the alternatives be $E_1, E_2, \ldots, E_n$, and $H_1, H_2, \ldots, H_m$ are the parameters, respectively. The crisp information based on different criteria is then converted into bipolar fuzzy numbers, and then it is represented in the bipolar decision matrix. The bipolar decision matrix is calculated as

$$M = \begin{pmatrix} m_{11,n_{11}} & m_{12,n_{12}} & \cdots & m_{1n,n_{1n}} \\ m_{21,n_{21}} & m_{22,n_{22}} & \cdots & m_{2n,n_{2n}} \\ \vdots & \vdots & \ddots & \vdots \\ m_{m1,n_{m1}} & m_{m2,n_{m2}} & \cdots & m_{mn,n_{mn}} \end{pmatrix}_{m \times n},$$

in which possible alternatives are $E_{11}, E_{12}, \ldots, E_{nm}$ and evaluation criteria are presented as $H_1, H_2, \ldots, H_m$. The weights are then assigned by the decision makers, and the weight of attributes is represented by

$$W = (w_1, w_2, \ldots, w_n)^T.$$ (8)

These weights are assigned by the faculty on the basis of set rules defined by the institute and $\sum_{i=1}^{n} W_i = 1$. The standardized matrix of criterion weights is given as follows:

$$X = [x_{ij}]_{m \times n}, \quad i = 1, 2, \ldots, n; \quad j = 1, 2, \ldots, m,$$ (9)

where $x_{ij}$ are the weights of $m$ criteria assigned by $n$ decision makers. The standardized matrix is obtained by assigning linguistic scales to the criteria by the decision makers which is then transformed into trapezoidal bipolar fuzzy numbers. The weighted standardized decision matrix is represented as

$$X = w_i \otimes M,$$ (10)

where $w_i$ represents the weights matrix and $M$ represents the bipolar fuzzy decision matrix based on bipolar fuzzy weighted positive value and bipolar fuzzy weighted negative value, respectively. The bipolar fuzzy weighted positive value and negative value is calculated as

$$s_{ij} = w_i \otimes m_{ij}, u_{ij} = w_i \otimes n_{ij}.$$ (11)

Here, $s_{ij}$ and $u_{ij}$ show the bipolar weighted positive value and negative value. The bipolar fuzzy positive and negative ideal solutions are calculated as follows:
Mathematical Problems in Engineering

Goal: Selection of best University

Figure 3: Hierarchical structure model of universities flow chart.

Table 2: Linguistic terms for weight of parameters.

<table>
<thead>
<tr>
<th>Linguistic variables</th>
<th>Bipolar fuzzy numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very low</td>
<td>[(0.0, 0.0, 0.1, 0.2), (0.7, 0.8, 0.9, 1)]</td>
</tr>
<tr>
<td>Low</td>
<td>[(0.1, 0.2, 0.3, 0.4), (0.6, 0.7, 0.8, 0.9)]</td>
</tr>
<tr>
<td>Neutral</td>
<td>[(0.4, 0.5, 0.5, 0.6), (0.5, 0.5, 0.6, 0.7)]</td>
</tr>
<tr>
<td>High</td>
<td>[(0.6, 0.7, 0.7, 0.8), (0.1, 0.2, 0.3, 0.3)]</td>
</tr>
<tr>
<td>Very high</td>
<td>[(0.8, 0.9, 1.0, 1.0), (0.0, 0.1, 0.2, 0.2)]</td>
</tr>
</tbody>
</table>

Table 3: Linguistic terms for weight of parameters.

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<thead>
<tr>
<th>Linguistic variables</th>
<th>Bipolar fuzzy numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very bad</td>
<td>[0.0, 0.0, 0.1, 0.2], (0.7, 0.8, 0.9, 1)]</td>
</tr>
<tr>
<td>Bad</td>
<td>[0.1, 0.2, 0.3, 0.4], (0.6, 0.7, 0.8, 0.9)]</td>
</tr>
<tr>
<td>Neither good nor bad</td>
<td>[0.4, 0.5, 0.5, 0.6], (0.5, 0.5, 0.6, 0.7)]</td>
</tr>
<tr>
<td>Good</td>
<td>[0.6, 0.7, 0.7, 0.8], (0.1, 0.2, 0.3, 0.3)]</td>
</tr>
<tr>
<td>Very good</td>
<td>[0.8, 0.9, 1.0, 1.0], (0.0, 0.1, 0.2, 0.2)]</td>
</tr>
</tbody>
</table>

\[ t^* = \{ x_1^*, x_2^*, \ldots, x_n^* \} = \{(\max x_{ij} \mid j \in B), (\min x_{ij} \mid j \in C) \mid i = 1, 2, \ldots, m \}, \]
\[ t^- = \{ x_1^-, x_2^-, \ldots, x_n^- \} = \{(\min x_{ij} \mid j \in B), (\max x_{ij} \mid j \in C) \mid i = 1, 2, \ldots, m \}. \]

The distance of each alternative from bipolar fuzzy positive ideal solution and negative ideal solution is calculated as follows:

\[ \Delta^+ = \frac{1}{2} \sum_{i=1}^{m} s_{ij} - \max x_i^2 + (u_{ij} - \min x_{ij})^2, \]

(13)

\[ \Delta^- = \frac{1}{2} \sum_{i=1}^{m} s_{ij} - \min x_i^2 + (u_{ij} - \max x_{ij})^2. \]

The distance from positive ideal solution and \( \Delta^- \) is the distance from negative ideal solution.

The flow chart of the proposed methodology defined in Section 3.2 is presented in Figure 1.

3.3. Proposed Algorithm in Multicriteria Decision-Making Using Soft Expert Sets. The concept of soft expert sets is defined by Alkhazaleh and Salleh [24], and it has many uses in economics, decision making, social sciences, mathematics, and so on. In this study, we have proposed a bipolar fuzzy multicriteria decision-making method. We proposed an algorithm in which we compress the parameters to obtain a precise result by developing a preference level of the criteria. The following steps are involved in soft set technique:

1. Input a bipolar fuzzy soft expert set \((F, U)\).
(2) Find the agree-bipolar fuzzy soft expert set and disagree-bipolar fuzzy soft expert set by using
\[ \mu^+ (u_j) - \nu^- (u_j), \] (15)
where \( \mu^+ (u_j) \) is the positive response and \( \nu^- (u_j) \) is the negative response about each \( u_j \in U \).

(3) Find the highest value for agree-BF soft expert set and disagree-BF soft expert set by using
\[ \max (e_i) = \text{Highest value}, \]
\[ e_i = \text{Alternatives} (e_1, e_2, e_3). \] (16)

(4) Find scores for agree-bipolar fuzzy soft expert set by using
\[ A_j = \sum_i e_{ij}. \] (17)

(5) Find scores for disagree-bipolar fuzzy soft expert set by using
\[ D_j = \sum_i e_{ij}. \] (18)

(6) By using agree-bipolar fuzzy soft expert set and disagree-bipolar fuzzy soft expert set, find the final scores using
\[ s_j = A_j - D_j, \] (19)
where \( s_j \) represent the final scores, \( A_j \) is agree-bipolar fuzzy soft expert set, and \( D_j \) is disagree-bipolar fuzzy soft expert set.
Nowadays, it has become very important for the students to take right decision in choosing a university while getting admission. Students need to focus on every aspect of the university that they want to get facilitated with while studying there. For the evaluation of the best university, the following parameters need to be compared and analysed. Table 1 shows the parameters to choose the best university. The source of Table 1 is the decision makers referred as the faculty members who work in the respective universities as mentioned above. Table 1 presents the different criteria gathered from the decision makers.

The hierarchical structure model of selected universities is represented by the flow chart in Figure 3.

We provide weightage among different alternatives on the basis of the selected criteria; the following five sets of linguistic scales are used for the denied alternatives: very bad (VB), bad (B), neither good nor bad (NGBN), good (G), very good (VG), no (N), partially no (PN), neutral (Ne), partially yes (PY), yes (Y), very low (VL), low (L), neutral (Ne), high (H), and very high (VH), respectively. The bipolar fuzzy numbers are presented in Table 2.

The linguistic terms are assigned weights with respect to bipolar fuzzy numbers as shown in Tables 3 and 4.

The data are collected through questionnaires from the decision makers, and weights are assigned in linguistic terms for each parameter. The bipolar fuzzy numbers are used to compute the weights. It is to be noted that the sum of weights must always be equivalent to 1.

### 4. Numerical Computation

We have used the proposed method in education sector to deal with an issue of selecting a best university while getting admission. We have used a soft set technique to make the result more precise and obtain better results. The present research work depends on primary data collection. Data were collected from the decision makers who work in education sector. Different parameters have been developed to study the performance of alternatives. The selection of university depends on eleven parameters such as HEC ranking, location, environment, fee structure, faculty, infrastructure, library, research quality, merit, curricular activities, and career guidance. These linguistic terms are then changed into trapezoidal bipolar fuzzy numbers chosen by the decision makers. The three alternatives of the study were basically three universities, namely, National University of Science and Technology (NUST), Quaid-e-Azam University (QAU), and International Islamic University (IIU).

### 4.1. Parameters of Study

Nowadays, it has become very difficult for the students to take right decision in choosing a university while getting admission. Students need to focus on every aspect of the university that they want to get facilitated with while studying there. For the evaluation of the best university, the following parameters need to be compared and analysed. Table 1 shows the parameters to choose the best university. The source of Table 1 is the decision makers referred as the faculty members who work in the respective universities as mentioned above. Table 1 presents the different criteria gathered from the decision makers.

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### 5. Results and Discussion

#### 5.1. BF-AHP Results along with Weight of Parameters

The weights gathered through AHP as shown in Table 5 totally depend on the judgment of the decision makers. AHP has an advantage that it is easy to utilize and provide with pairwise comparison. The weights are normalized for multicriteria decision making.

It is clear from the beginning that the sum of the weights must be equivalent to 1. So, there is a need to normalize the weights to obtain sum of 1 which are shown in Table 5.

The bar chart shown in Figure 4 depicts all the eleven criteria with their weights, respectively. The criterion weights can be further used for decision making as these weights are suitable for further process.

#### 5.2. BF-TOPSIS Results and Scoring of Alternatives

TOPSIS is the frequently used among many decision-making techniques due to its simplicity of steps and precision. Its ultimate goal is to identify an alternative with distance closest to the positive ideal solution and farthest to negative ideal solution. In this study, we have highlighted one of the socialistic issue and applied TOPSIS on that for decision making. We have gathered information from three different universities and want to decide that which university is the best for getting admission in all the three alternatives. These alternatives are studied on basis of fifteen
### Table 8: Weighted normalized decision matrix.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>E1</th>
<th>Alternatives</th>
<th>E2</th>
<th>E3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1$</td>
<td>([0.07, 0.08, 0.09, 0.09, 0.00, 0.00, 0.01, 0.01])</td>
<td>([0.07, 0.08, 0.09, 0.09, 0.00, 0.00, 0.01, 0.01])</td>
<td>([0.05, 0.06, 0.06, 0.00, 0.00, 0.01, 0.02, 0.02])</td>
<td></td>
</tr>
<tr>
<td>$H_2$</td>
<td>([0.02, 0.03, 0.03, 0.04, 0.02, 0.02, 0.02, 0.02])</td>
<td>([0.02, 0.03, 0.03, 0.04, 0.02, 0.02, 0.02, 0.02])</td>
<td>([0.02, 0.02, 0.02, 0.02, 0.02, 0.02, 0.02, 0.02])</td>
<td></td>
</tr>
<tr>
<td>$H_3$</td>
<td>([0.02, 0.02, 0.02, 0.02, 0.00, 0.00, 0.00, 0.00])</td>
<td>([0.02, 0.02, 0.02, 0.02, 0.00, 0.00, 0.00, 0.00])</td>
<td>([0.00, 0.00, 0.00, 0.01, 0.01, 0.01, 0.01, 0.01])</td>
<td></td>
</tr>
<tr>
<td>$H_4$</td>
<td>([0.02, 0.03, 0.03, 0.03, 0.00, 0.00, 0.00, 0.00])</td>
<td>([0.11, 0.12, 0.13, 0.13, 0.01, 0.01, 0.01, 0.01])</td>
<td>([0.00, 0.00, 0.00, 0.00, 0.01, 0.01, 0.01, 0.01])</td>
<td></td>
</tr>
<tr>
<td>$H_5$</td>
<td>([0.07, 0.10, 0.12, 0.12, 0.01, 0.01, 0.02, 0.02])</td>
<td>([0.07, 0.08, 0.08, 0.09, 0.01, 0.02, 0.03, 0.03])</td>
<td>([0.08, 0.09, 0.09, 0.10, 0.01, 0.02, 0.03, 0.03])</td>
<td></td>
</tr>
<tr>
<td>$H_6$</td>
<td>([0.08, 0.09, 0.10, 0.11, 0.01, 0.01, 0.02, 0.02])</td>
<td>([0.05, 0.06, 0.07, 0.08, 0.02, 0.07, 0.04, 0.04])</td>
<td>([0.00, 0.09, 0.11, 0.11, 0.01, 0.02, 0.02, 0.02])</td>
<td></td>
</tr>
<tr>
<td>$H_7$</td>
<td>([0.07, 0.08, 0.09, 0.09, 0.00, 0.01, 0.02, 0.02])</td>
<td>([0.07, 0.08, 0.09, 0.09, 0.00, 0.01, 0.02, 0.02])</td>
<td>([0.05, 0.06, 0.06, 0.07, 0.02, 0.02, 0.03, 0.04])</td>
<td></td>
</tr>
<tr>
<td>$H_8$</td>
<td>([0.08, 0.09, 0.10, 0.10, 0.01, 0.01, 0.02, 0.02])</td>
<td>([0.05, 0.06, 0.07, 0.08, 0.02, 0.07, 0.04, 0.04])</td>
<td>([0.05, 0.06, 0.06, 0.07, 0.02, 0.02, 0.03, 0.04])</td>
<td></td>
</tr>
<tr>
<td>$H_9$</td>
<td>([0.09, 0.10, 0.11, 0.12, 0.00, 0.02, 0.02, 0.02])</td>
<td>([0.11, 0.12, 0.13, 0.13, 0.01, 0.02, 0.02, 0.02])</td>
<td>([0.07, 0.08, 0.08, 0.10, 0.03, 0.04, 0.05, 0.06])</td>
<td></td>
</tr>
<tr>
<td>$H_{10}$</td>
<td>([0.04, 0.04, 0.04, 0.05, 0.00, 0.01, 0.02, 0.02])</td>
<td>([0.03, 0.04, 0.04, 0.04, 0.02, 0.02, 0.03, 0.03])</td>
<td>([0.02, 0.03, 0.04, 0.04, 0.02, 0.03, 0.03, 0.03])</td>
<td></td>
</tr>
</tbody>
</table>
Table 9: Bipolar fuzzy positive and negative ideal solutions.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>PIS</th>
<th>NIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_1$</td>
<td>[0.07, 0.08, 0.09, 0.09], (0.0, 0.00, 0.01, 0.01)</td>
<td>[0.05, 0.06, 0.06, 0.07], (0.00, 0.01, 0.02, 0.02)</td>
</tr>
<tr>
<td>$H_2$</td>
<td>[0.02, 0.03, 0.03, 0.04], (0.01, 0.03, 0.02, 0.02)</td>
<td>[0.02, 0.02, 0.03, 0.03], (0.01, 0.02, 0.02, 0.02)</td>
</tr>
<tr>
<td>$H_3$</td>
<td>[0.02, 0.02, 0.02, 0.02], (0.00, 0.00, 0.00, 0.00)</td>
<td>[0.01, 0.01, 0.01, 0.01], (0.00, 0.00, 0.01, 0.01)</td>
</tr>
<tr>
<td>$H_4$</td>
<td>[0.02, 0.03, 0.03, 0.03], (0.00, 0.00, 0.00, 0.00)</td>
<td>[0.02, 0.02, 0.02, 0.02], (0.01, 0.01, 0.01, 0.02)</td>
</tr>
<tr>
<td>$H_5$</td>
<td>[0.11, 0.12, 0.13, 0.13], (0.00, 0.01, 0.02, 0.02)</td>
<td>[0.09, 0.10, 0.11, 0.12], (0.00, 0.02, 0.03, 0.03)</td>
</tr>
<tr>
<td>$H_6$</td>
<td>[0.09, 0.10, 0.12, 0.12], (0.00, 0.02, 0.02, 0.02)</td>
<td>[0.07, 0.08, 0.08, 0.09], (0.01, 0.02, 0.03, 0.03)</td>
</tr>
<tr>
<td>$H_7$</td>
<td>[0.08, 0.09, 0.11, 0.11], (0.00, 0.01, 0.02, 0.02)</td>
<td>[0.05, 0.06, 0.07, 0.08], (0.02, 0.07, 0.04, 0.04)</td>
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<td>$H_8$</td>
<td>[0.07, 0.08, 0.09, 0.09], (0.00, 0.01, 0.01, 0.01)</td>
<td>[0.05, 0.06, 0.06, 0.07], (0.02, 0.02, 0.03, 0.04)</td>
</tr>
<tr>
<td>$H_9$</td>
<td>[0.08, 0.09, 0.10, 0.10], (0.00, 0.01, 0.02, 0.02)</td>
<td>[0.05, 0.06, 0.06, 0.07], (0.02, 0.03, 0.04, 0.04)</td>
</tr>
<tr>
<td>$H_{10}$</td>
<td>[0.11, 0.12, 0.13, 0.13], (0.00, 0.01, 0.02, 0.02)</td>
<td>[0.07, 0.08, 0.08, 0.10], (0.03, 0.04, 0.05, 0.06)</td>
</tr>
<tr>
<td>$H_{11}$</td>
<td>[0.04, 0.04, 0.04, 0.05], (0.00, 0.01, 0.02, 0.02)</td>
<td>[0.02, 0.03, 0.04, 0.04], (0.02, 0.02, 0.03, 0.03)</td>
</tr>
</tbody>
</table>

Table 10: Ranking and scores of alternatives.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Scores</th>
<th>Ranking</th>
</tr>
</thead>
<tbody>
<tr>
<td>E1</td>
<td>0.775931</td>
<td>1</td>
</tr>
<tr>
<td>E2</td>
<td>0.567748</td>
<td>2</td>
</tr>
<tr>
<td>E3</td>
<td>0.2832</td>
<td>3</td>
</tr>
</tbody>
</table>

Figure 5: Scores of alternatives.

Table 11: Agree-bipolar fuzzy soft expert set.

<table>
<thead>
<tr>
<th>E</th>
<th>$e_1$</th>
<th>$e_2$</th>
<th>$e_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(h_1, p, 1)$</td>
<td>3.7</td>
<td>3.7</td>
<td>2.8</td>
</tr>
<tr>
<td>$(h_2, p, 1)$</td>
<td>3.7</td>
<td>3.7</td>
<td>2.8</td>
</tr>
<tr>
<td>$(h_3, p, 1)$</td>
<td>3.7</td>
<td>3.7</td>
<td>2.8</td>
</tr>
<tr>
<td>$(h_4, q, 1)$</td>
<td>3.7</td>
<td>3.7</td>
<td>2.8</td>
</tr>
<tr>
<td>$(h_5, q, 1)$</td>
<td>3.7</td>
<td>2.8</td>
<td>2.8</td>
</tr>
<tr>
<td>$(h_6, q, 1)$</td>
<td>3.7</td>
<td>3.7</td>
<td>2.8</td>
</tr>
<tr>
<td>$(h_7, r, 1)$</td>
<td>2.8</td>
<td>3.7</td>
<td>2.8</td>
</tr>
<tr>
<td>$(h_8, r, 1)$</td>
<td>3.7</td>
<td>3.7</td>
<td>2.8</td>
</tr>
<tr>
<td>$(h_9, r, 1)$</td>
<td>3.7</td>
<td>3.7</td>
<td>2.8</td>
</tr>
<tr>
<td>$(h_{10}, r, 1)$</td>
<td>2.8</td>
<td>2.8</td>
<td>3.7</td>
</tr>
</tbody>
</table>

aggregated decision matrix of alternatives is represented in Table 7.

Next, we need to find the weighted normalized decision matrix which is obtained by multiplying the aggregated matrix with the weighted matrix. The weights were first obtained using analytical hierarchical process. The obtained weighted normalized decision matrix is shown in Table 8.

The maximum and minimum values are highlighted in Table 8. The next step is to find bipolar fuzzy positive and negative ideal solutions. The calculated bipolar fuzzy positive and negative ideal solutions are shown in Table 9.

The final scores of the alternatives are computed. Results are displayed in Table 10 which clearly shows that alternative 1 which is National University of Science and Technology has got the highest score 0.7774 and is ranked first, second is Quaid-e-Azam University with score 0.602, and third is International Islamic University getting the score of 0.3241. The ranking of the alternatives is shown in Table 10 by which we have concluded that the best university is NUST.
In Figure 5, the highest score is alternative 1, i.e., \( E_1 \) (NUST university) and is the best university and alternative 2 which is \( E_2 \) (Quaid-e-Azam University) has got the second highest score and is ranked number 2, and \( E_3 \) (International Islamic University) has obtained lowest scores and is ranked number 3.

### 6. Proposed Bipolar Fuzzy Soft Expert Sets Algorithm

The next step is to compute soft sets of the criteria. We have used the soft expert set technique. On the basis of students priority, few of parameters are selected as soft sets and have questioned the decision makers for the agree and disagree of the selected soft sets.

Input a weighted bipolar fuzzy soft expert set \((F, U)\), where alternatives are \( E = (e_1, e_2, e_3) \) which from the universe and the chosen parameter are \( H = (h_1, h_2, h_3) \), where \( h_i (i = 1, 2, 3, 4) \) are “HEC Ranking,” “Environment,” “Fee structure,” and “Merit,” and \( Z = (p, q, r) \) be set of decision makers.

\[
(F, U) = \left\{ (h_1, p, 1), \begin{bmatrix} e_1 & e_2 & e_3 \\ (0.8, 0.9, 1.0, 1.0) & (0.8, 0.9, 1.0, 1.0) & (0.6, 0.7, 0.7, 0.8) \end{bmatrix} \right\},
\]

\[
(h_1, p, 1), \begin{bmatrix} e_1 & e_2 & e_3 \\ (0.8, 0.9, 1.0, 1.0) & (0.8, 0.9, 1.0, 1.0) & (0.6, 0.7, 0.7, 0.8) \end{bmatrix},
\]

\[
(h_1, q, 1), \begin{bmatrix} e_1 & e_2 & e_3 \\ (0.8, 0.9, 1.0, 1.0) & (0.8, 0.9, 1.0, 1.0) & (0.6, 0.7, 0.7, 0.8) \end{bmatrix},
\]

\[
(h_1, r, 1), \begin{bmatrix} e_1 & e_2 & e_3 \\ (0.8, 0.9, 1.0, 1.0) & (0.8, 0.9, 1.0, 1.0) & (0.6, 0.7, 0.7, 0.8) \end{bmatrix},
\]

\[
(h_2, p, 1), \begin{bmatrix} e_1 & e_2 & e_3 \\ (0.8, 0.9, 1.0, 1.0) & (0.8, 0.9, 1.0, 1.0) & (0.6, 0.7, 0.7, 0.8) \end{bmatrix},
\]

\[
(h_2, q, 1), \begin{bmatrix} e_1 & e_2 & e_3 \\ (0.8, 0.9, 1.0, 1.0) & (0.6, 0.7, 0.7, 0.8) & (0.6, 0.7, 0.7, 0.8) \end{bmatrix},
\]

\[
(h_2, r, 1), \begin{bmatrix} e_1 & e_2 & e_3 \\ (0.8, 0.9, 1.0, 1.0) & (0.6, 0.7, 0.7, 0.8) & (0.6, 0.7, 0.7, 0.8) \end{bmatrix},
\]

\[
(h_3, p, 1), \begin{bmatrix} e_1 & e_2 & e_3 \\ (0.8, 0.9, 1.0, 1.0) & (0.8, 0.9, 1.0, 1.0) & (0.8, 0.9, 1.0, 1.0) \end{bmatrix},
\]

\[
(h_3, q, 1), \begin{bmatrix} e_1 & e_2 & e_3 \\ (0.8, 0.9, 1.0, 1.0) & (0.8, 0.9, 1.0, 1.0) & (0.8, 0.9, 1.0, 1.0) \end{bmatrix},
\]

\[
(h_3, r, 1), \begin{bmatrix} e_1 & e_2 & e_3 \\ (0.8, 0.9, 1.0, 1.0) & (0.8, 0.9, 1.0, 1.0) & (0.8, 0.9, 1.0, 1.0) \end{bmatrix},
\]

\[
(h_4, p, 1), \begin{bmatrix} e_1 & e_2 & e_3 \\ (0.8, 0.9, 1.0, 1.0) & (0.8, 0.9, 1.0, 1.0) & (0.4, 0.5, 0.5, 0.6) \end{bmatrix},
\]

\[
(h_4, q, 1), \begin{bmatrix} e_1 & e_2 & e_3 \\ (0.6, 0.7, 0.7, 0.8) & (0.8, 0.9, 1.0, 1.0) & (0.4, 0.5, 0.5, 0.6) \end{bmatrix},
\]

\[
(h_4, r, 1), \begin{bmatrix} e_1 & e_2 & e_3 \\ (0.6, 0.7, 0.7, 0.8) & (0.6, 0.7, 0.7, 0.8) & (0.8, 0.9, 1.0, 1.0) \end{bmatrix},
\]

\[
(20)
\]
Values of agree-BF soft expert set and disagree-BF soft expert set are found which are shown in Tables 11 and 12. The bipolar fuzzy TOPSIS method. The comparison of the bipolar fuzzy method of order preference by similarity to ideal solution (BF-TOPSIS). The results which are based on the positive and negative aspects are covered by the bipolar fuzzy TOPSIS method. The comparison of the existing and modified methods is as follows.

\[ \begin{align*}
(h_1, p, 0), & \frac{e_1}{(0.0, 0.1, 0.2, 0.2)}, \frac{e_2}{(0.0, 0.1, 0.2, 0.2)}, \frac{e_3}{(0.1, 0.2, 0.3, 0.3)} \bigg) \\
(h_1, q, 0), & \frac{e_1}{(0.0, 0.1, 0.2, 0.2)}, \frac{e_2}{(0.0, 0.1, 0.2, 0.2)}, \frac{e_3}{(0.1, 0.2, 0.3, 0.3)} \\
(h_1, r, 0), & \frac{e_1}{(0.0, 0.1, 0.2, 0.2)}, \frac{e_2}{(0.0, 0.1, 0.2, 0.2)}, \frac{e_3}{(0.1, 0.2, 0.3, 0.3)} \\
(h_2, p, 0), & \frac{e_1}{(0.0, 0.1, 0.2, 0.2)}, \frac{e_2}{(0.0, 0.1, 0.2, 0.2)}, \frac{e_3}{(0.1, 0.2, 0.3, 0.3)} \\
(h_2, q, 0), & \frac{e_1}{(0.0, 0.1, 0.2, 0.2)}, \frac{e_2}{(0.0, 0.1, 0.2, 0.2)}, \frac{e_3}{(0.1, 0.2, 0.3, 0.3)} \\
(h_2, r, 0), & \frac{e_1}{(0.0, 0.1, 0.2, 0.2)}, \frac{e_2}{(0.0, 0.1, 0.2, 0.2)}, \frac{e_3}{(0.5, 0.5, 0.6, 0.7)} \\
(h_3, p, 0), & \frac{e_1}{(0.0, 0.1, 0.2, 0.2)}, \frac{e_2}{(0.0, 0.1, 0.2, 0.2)}, \frac{e_3}{(0.0, 0.1, 0.2, 0.2)} \\
(h_3, q, 0), & \frac{e_1}{(0.0, 0.1, 0.2, 0.2)}, \frac{e_2}{(0.0, 0.1, 0.2, 0.2)}, \frac{e_3}{(0.0, 0.1, 0.2, 0.2)} \\
(h_3, r, 0), & \frac{e_1}{(0.0, 0.1, 0.2, 0.2)}, \frac{e_2}{(0.0, 0.1, 0.2, 0.2)}, \frac{e_3}{(0.0, 0.1, 0.2, 0.2)} \\
(h_4, p, 0), & \frac{e_1}{(0.0, 0.1, 0.2, 0.2)}, \frac{e_2}{(0.0, 0.1, 0.2, 0.2)}, \frac{e_3}{(0.5, 0.5, 0.6, 0.7)} \\
(h_4, q, 0), & \frac{e_1}{(0.1, 0.2, 0.3, 0.3)}, \frac{e_2}{(0.0, 0.1, 0.2, 0.2)}, \frac{e_3}{(0.5, 0.5, 0.6, 0.7)} \\
(h_4, r, 0), & \frac{e_1}{(0.1, 0.2, 0.3, 0.3)}, \frac{e_2}{(0.0, 0.1, 0.2, 0.2)} \\
\end{align*} \]

\[21\]

7. Validation and Robustness

We check the accuracy of the proposed bipolar fuzzy TOPSIS modeling technique by validation and robustness. From the scores obtained in Table 13, it can be seen that the highest score is 34.6 for alternative 1, that is, NUST, so the optimal decision is to choose alternative 1 as the best university. Also, it is clear that by applying soft expert set technique, optimal result remains same as obtained by the BF-TOPSIS method. This method is then applied on the same alternatives for university selection. At the end, we get the same optimal solution. We conclude from Table 13 that \( E_1 \) has obtained the highest scores and is the best university among the other two universities.

8. Conclusion

Decision making in education-based problems is so common nowadays. Students also pay for counselors and advisors for the right advice and right decision to take. Bipolar fuzzy and soft sets are combined together in this study to solve the problems that occur while taking multicriteria decisions. Fuzzy covers the vagueness, and bipolar covers human dual behavior. Soft sets on the other side give more precise results by giving order of preference. Hence, we have demonstrated a modified bipolar fuzzy soft set decision-making method. We have used the TOPSIS method for decision making as the computational steps of TOPSIS are simple and results are easy to evaluate, which is named as the bipolar fuzzy method of order preference by similarity measure to ideal solution (BF-TOPSIS). The results which are based on the positive and negative aspects are covered by the bipolar fuzzy TOPSIS method. The comparison of the existing and modified methods is as follows.

8.1. Existing Method. The existing method deals directly with the alternatives which could be very time consuming and can cause errors. The existing method does not give precise results. The existing decision-making methods in [15, 16] based on fuzzy TOPSIS and fuzzy soft sets simply consider the best optimal solution by distance measure and preferences. The method fails when the decision maker wants to assess the variables based on judgmental thinking.

8.2. Proposed Method. The bipolar fuzzy AHP minimizes this issue by allowing pairwise comparison and by giving
hierarchy. Using soft sets a preference level is made, and thus more compressed and precise results are obtained. Then, bipolar fuzzy TOPSIS is introduced to assess the linguistic variables based on dual thinking. Our proposed method can efficiently achieve the optimum solution with preferences and positive and negative aspects.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

References